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The Limits of Custodial Symmetry [a]

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We introduce a toy model implementing the proposal of using a custodial symmetry to protect the $Zb_L\bar{b}_L$ coupling from large corrections. This "doublet-extended standard model" adds a weak doublet of fermions (including a heavy partner of the top quark) to the particle content of the standard model in order to implement an $O(4)\times U(1)_X \sim SU(2)_L\times SU(2)_R\times P_{LR}\times U(1)_X$ symmetry that protects the $Zb_L\bar{b}_L$ coupling. This symmetry is softly broken to the gauged $SU(2)_L\times U(1)_Y$ electroweak symmetry by a Dirac mass M for the new doublet; adjusting the value of M allows us to explore the range of possibilities between the $O(4)$-symmetric ($M\to 0$) and standard-model-like ($M\to\infty$) limits.

1 Introduction

Agashe [2] et al. have shown that the constraints on beyond the standard model physics related to the $Zb_L\bar{b}_L$ coupling can, in principle, be loosened if the global $SU(2)_L\times SU(2)_R$ symmetry of the electroweak symmetry breaking sector is actually a subgroup of a larger global symmetry of both the symmetry breaking and top quark mass generating sectors of the theory. In particular, they propose that these interactions preserve an $O(4)\sim SU(2)_L\times SU(2)_R\times P_{LR}$ symmetry, where P_{LR} is a parity interchanging $L\leftrightarrow R$. The $O(4)$ symmetry is then spontaneously broken to $O(3)\sim SU(2)_V\times P_{LR}$, breaking the elecroweak interactions but protecting g_{Lb} from radiative corrections, so long as the left-handed bottom quark is a P_{LR} eigenstate.

In this talk we report on the construction of the simplest $O(4)$-symmetric extension of the SM.[1], the doublet-extended standard model or DESM.

1.1 The Model

We extend the global $SU(2)_L\times SU(2)_R$ symmetry of the Higgs sector of the SM to an $O(4)\times U(1)_X\sim SU(2)_L\times SU(2)_R\times P_{LR}\times U(1)_X$ for both the symmetry breaking and top quark mass generating sectors of the theory. As usual, only the electroweak subgroup, $SU(2)_L\times U(1)_Y$, of this global symmetry is gauged; our model does not include additional electroweak gauge bosons. The global $O(4)$ spontaneously breaks to $O(3)\sim SU(2)_V\times P_{LR}$ which will protect g_{Lb} from radiative corrections,[2] provided that the left-handed bottom quark is a parity eigenstate: $P_{LR}b_L=\pm b_L$. The additional global $U(1)_X$ group is included to ensure that the light t and b eigenstates, the ordinary top and bottom quarks, obtain the correct hypercharges.

We therefore introduce a new doublet of fermions $\Psi\equiv(\Omega,T')$. The left-handed component,

[a] Speaker at conference: R. Sekhar Chivukula. This report is a shortened version of previously published work.[1]

Table 1: Charges of the fermions under the various symmetry groups in the model. Note that, as discussed in the text, other T_R^3 and Q_X assignments for the Ω_R and T'_R states are possible.

	t'_L	b_L	Ω_L	T'_L	t'_R	b_R	Ω_R	T'_R
T_L^3	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0	$\frac{1}{2}$	$-\frac{1}{2}$
T_R^3	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	-1	0	0
Q	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{5}{3}$	$\frac{2}{3}$
Y	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{7}{6}$	$\frac{7}{6}$	$\frac{2}{3}$	$-\frac{1}{3}$	$\frac{7}{6}$	$\frac{7}{6}$
Q_X	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{7}{6}$	$\frac{7}{6}$

Ψ_L joins with the top-bottom doublet $q_L \equiv (t'_L, b_L)$ to form an $O(4) \times U(1)_X$ multiplet

$$\mathcal{Q}_L = \begin{pmatrix} t'_L & \Omega_L \\ b_L & T'_L \end{pmatrix} \equiv \begin{pmatrix} q_L & \Psi_L \end{pmatrix} , \tag{1}$$

which transforms as a $(2, 2^*)_{2/3}$ under $SU(2)_L \times SU(2)_R \times U(1)_X$. The parity operation P_{LR}, which exchanges the $SU(2)_L$ and $SU(2)_R$ transformation properties of the fields, acts on $\mathcal{Q}_L$ as:

$$P_{LR}\mathcal{Q}_L = -\left[(i\sigma_2)\, \mathcal{Q}_L \,(i\sigma_2)\right]^T = \begin{pmatrix} T'_L & -\Omega_L \\ -b_L & t'_L \end{pmatrix} \tag{2}$$

exchanging the diagonal components, while reversing the signs of the off-diagonal components. The t' and T' states mix to form mass eigenstates corresponding to the top quark (t) and a heavy partner (T).

We assign the minimal right-handed fermions charges that accord with the symmetry-breaking pattern we envision: the top and bottom quarks will receive mass via Yukawa terms that respect the full $O(4) \times U(1)_X$ symmetry, while the exotic states will have a dimension-three mass term that explicitly breaks the large symmetry to $SU(2)_L \times U(1)$. The charges of all the fermions are listed in Table 1.

Now, let us describe the symmetry-breaking pattern and fermion mass terms explicitly. Spontaneous electroweak symmetry breaking proceeds through a Higgs multiplet that transforms as a $(2, 2^*)_0$ under $SU(2)_L \times SU(2)_R \times U(1)_X$:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} v + h + i\phi^0 & i\sqrt{2}\,\phi^+ \\ i\sqrt{2}\,\phi^- & v + h - i\phi^0 \end{pmatrix} . \tag{3}$$

Again, the parity operator P_{LR} exchanges the diagonal fields and reverses the signs of the off-diagonal elements. When the Higgs acquires a vacuum expectation value, the longitudinal W and Z bosons acquire mass and a single Higgs boson remains in the low-energy spectrum. The Higgs multiplet has an $O(4) \times U(1)_X$ symmetric Yukawa interaction with the top quark:

$$\mathcal{L}_{\text{Yukawa}} = -\lambda_t \mathrm{Tr}\left(\overline{\mathcal{Q}}_L \cdot \Phi\right) t'_R \;+ \text{h.c.}\,. \tag{4}$$

that contributes to generating a top quark mass.[b]

Next we break the full $O(4) \times U(1)_X$ symmetry to its electroweak subgroup. We do so first by gauging $SU(2)_L \times U(1)_Y$. In addition, we wish to preserve the $O(4)$ symmetry of the top quark mass generating sector in all dimension-4 terms, but break it softly by introducing a dimension-3 Dirac mass term for Ψ,

$$\mathcal{L}_{\text{mass}} = -M\;\bar{\Psi}_L \cdot \Psi_R + h.c. \tag{5}$$

[b]Here we neglect m_b and any other Yukawa interactions.

that explicitly breaks the global symmetry to $SU(2)_L \times U(1)_Y$. Note that we therefore expect that any flavor-dependent radiative corrections to the $Zb_L\bar{b}_L$ coupling will vanish in the limit $M \to 0$, as the protective parity symmetry is restored; alternatively, as $M \to \infty$, the larger symmetry is pushed off to such high energies that the resulting theory looks more and more like the SM.

1.2 *Mass Matrices and Eigenstates*

When the Higgs multiplet acquires a vacuum expectation value and breaks the electroweak symmetry, masses are generated for the top quark, its heavy partner T and the exotic fermion Ω through the mass matrix:

$$\mathcal{L}_{\text{mass}} = -\begin{pmatrix} t'_L & T'_L \end{pmatrix} \begin{pmatrix} m & 0 \\ m & M \end{pmatrix} \begin{pmatrix} t'_R \\ T'_R \end{pmatrix} - M\bar{\Omega}_L\Omega_R + \text{h.c} \,, \tag{6}$$

where

$$m = \frac{\lambda_t v}{\sqrt{2}} \,. \tag{7}$$

Diagonalizing the top quark mass matrix yields mass eigenstates t (corresponding to the SM top quark) and T (a heavy partner quark), with corresponding eigenvalues

$$m_t^2 = \frac{1}{2}\left[1 - \sqrt{1 + \frac{4m^4}{M^4}}\right] M^2 + m^2 \,, \qquad m_T^2 = \frac{1}{2}\left[1 + \sqrt{1 + \frac{4m^4}{M^4}}\right] M^2 + m^2 \,. \tag{8}$$

The mass eigenstates are related to the original gauge eigenstates through the rotations whose mixing angles are given by

$$\sin\theta_R = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{1 - 2m^2/M^2}{\sqrt{1 + 4m^4/M^4}}} \,, \quad \sin\theta_L = \frac{1}{\sqrt{2}}\sqrt{1 - \frac{1}{\sqrt{1 + 4m^4/M^4}}} \,. \tag{9}$$

From these equations the decoupling limit $M \to \infty$ is evident: m_t approaches its SM value as in Eq. (7), the $t - T$ mixing goes to zero, and T becomes degenerate with Ω. Conversely, in the limit $M \to 0$, the full $O(4) \times U(1)_X$ symmetry is restored and only the combination $T'_L + t'_L$ couples to t_R with mass m. For phenomenological discussion, it will be convenient to fix m_t at its experimental value and express the other masses in terms of m_t and the ratio $\mu \equiv M/m$.

2 δg_{Lb}, αS, and αT

We now display the value of the $Z\bar{b}_L b$ coupling, g_{Lb}, in our model[1] (as a function of μ for fixed m_t), and compare with the values given by experiment and the SM, as illustrated in Fig. (1). The (solid blue) curve shows how g_{Lb} varies with μ in our model; we required g_{Lb} to match the SM value with $m_t = 172$ GeV and $v = 246$ GeV as $\mu \to \infty$. We see that g_{Lb} in our model is slightly more negative than (i.e. slightly farther from the experimental value than) the SM value for $\mu > 1$, agrees with the SM value for $\mu = 1$, and comes within $\pm 1\sigma$ of the experimental value only for $\mu < 1$. Given the shortcomings of the small-μ limit, this is disappointing.

Furthermore, in Figure 2 we show the DESM predictions[1] for the oblique parameters[5,6,7] $[\alpha S^{th}(\mu), \alpha T^{th}(\mu)]$ using $m_h = 117$ GeV, and illustrating the successive mass-ratio values $\mu =$ 3, 4, ..., 20, ∞; the point $\mu = \infty$ corresponds to the SM limit of the DESM and therefore lies at the origin of the αS - αT plane. From this figure, we observe directly that the 95%CL lower limit on μ for $m_h = 115$ GeV is about 20, while for any larger value of m_h the DESM with $\mu \leq 20$ is excluded at 95%CL. In other words, the fact that a heavier m_h tends to worsen the fit of

Figure 1: The solid (blue) curve shows the DESM model's prediction for g_{Lb} The thick horizontal line corresponds to $g_{Lb}^{ex} = -0.4182$, while the two horizontal upper and lower solid lines bordering the shaded band correspond to the $\pm 1\sigma$ deviations [4]. The SM prediction is given by the dashed horizontal line. The leading-log contribution is shown by the dotted curve.

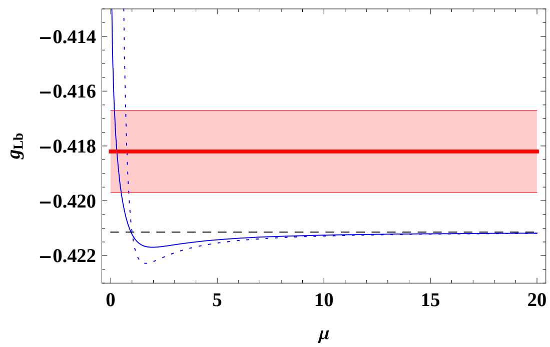
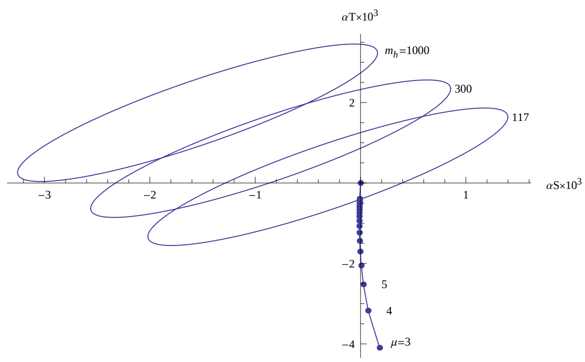


Figure 2: The dots represent the theoretical predictions of the DESM (with m_h set to the reference value 115 GeV), showing how the values of αS and αT change as μ successively takes on the values 3, 4, 5, ..., 20, ∞. The three ellipses enclose the 95%CL regions of the αS - αT plane for the fit to the experimental data performed in [3]; they correspond to Higgs boson mass values of $m_h = 115\,\text{GeV}$, $300\,\text{GeV}$, and $1\,\text{TeV}$. Comparing the theoretical curve with the ellipses shows that the minimum allowed value of μ is 20, for $m_h = 117$ GeV.



even the SM ($\mu \to \infty$) to the electroweak data is exacerbated by the new physics contributions within the DESM. The bound $\mu \geq 20$ corresponding to a DESM with a 115 GeV Higgs boson also implies, at 95%CL, that $m_T \geq \mu\, m_t \cong 3.4$ TeV, so that the heavy partners of the top quark would likely be too heavy for detection at LHC.

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