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**Theis' Contribution and Parallel Developments
in the Earth Sciences**

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The theoretical material is presented, however, primarily as a working hypothesis, with the hope that those who may be interested will assist in the perfection of the methods of investigation of these and correlated problems.

Willard Gardner and John A. Widtsoe, 1921

Introduction

That Charles V. Theis has a unique position in the history of groundwater hydrology is well attested by this conference, which seeks to pay tribute to his memory. Theis made his noteworthy contribution on unsteady groundwater flow during a period when exciting progress was being made in many disciplines closely related to groundwater hydrology. It is therefore appropriate that we use this opportunity provided by the Theis commemoration to reflect upon the evolution of ideas in hydrogeology during a score of years extending from the early 1920's to the early 1940's. Such a reflection is worthwhile, not only to get a historical sense for the development of those ideas, but also to place the contribution of Theis in perspective with other distinguished contributions of that period.

Contribution of Theis

Perhaps the single most significant contribution of Theis is his 1935 paper. This contribution has two sides to it: the practical and the philosophical. The *practical significance* of the Theis paper is that it provided the groundwater hydrologist with a powerful tool for interpreting data from *transient field tests* and for making predictions about the evolution of aquifers. The *philosophical significance* of the Theis contribution is that it emphasized the importance of time and provided a practical way of

incorporating time in the analysis of groundwater systems. Theis justified the mathematics by invoking an analogy with heat flow and by identifying the similarity between specific heat and the storage coefficient.

The groundwork for the acceptance of Theis' suggestion had already been laid by Oscar E. Meinzer (1928), who, by drawing upon field lore and upon the emerging ideas of Karl Terzaghi, argued that elastic compression of aquifers must account for a bulk of the water mined from aquifers by pumping. The phenomenological formalism of the storage term suggested by Theis was elegantly provided by Charles E. Jacob (1940), who showed from first principles that the storage term includes pore compaction as well as water expansion. Despite the physical and mathematical soundness of the principles, the success of the Theis methodology perhaps owes a great deal to the contribution of a colleague of his in the U.S. Geological Survey, Leland K. Wenzel (1942), who, in assembling field methods for determining aquifer permeability, introduced the technique of *type-curve matching* using a log-log plot of time versus drawdown. To this day, this is the only method which allows interpretation of the response over the *entire time domain* and makes possible estimates of aquifer permeability *as well as* aquifer storativity.

Related Developments in the Earth Sciences

In pursuing his research Theis was motivated by the fact that the existing theory of groundwater hydrology depended entirely on the concept of steady state. Indeed Theis shows surprise when he states, "This analogy[†] has been recognized at least since the work of Slichter, but apparently no attempt has been made to introduce the function of time into the mathematics of ground-water hydrology." It is appropriate to take Theis' reaction of surprise as an inspiration to examine parallel developments in the fields of Soil Mechanics, Petroleum Engineering and Soil Physics.

Karl Terzaghi

In his paper on compressibility of aquifers Meinzer drew upon many evidences to establish the importance of porosity decrease in accounting for storage depletion. Among the evidences presented by Meinzer is the laboratory experiment of Terzaghi on compacted sand from a 1925 publication of Terzaghi in the English language. However, Terzaghi had earlier formulated his experimental observations on the deformation of saturated clays into a cogent theory drawing upon the analogy of transient heat conduction (Terzaghi, 1923) and solved the one-dimensional transient flow problem. Terzaghi followed this with a formal statement of consolidation theory in the prestigious First International Congress on Applied Mechanics at Delft, Holland (Terzaghi, 1924). These two papers are considered by some to have laid the foundation for

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(†) hydraulic pressure being analogous to temperature, pressure gradient to thermal gradient, permeability to thermal conductivity, and specific yield to specific heat

the new discipline of soil mechanics. It is conceivable that Theis was not aware of these two papers because they were both in the European literature. Note that Terzaghi and Theis solved the same governing equation but for different geometries and different boundary conditions, as necessitated by the needs of their professional practice.

William Hurst, Morris Muskat

Just as Meinzer and other field hydrogeologists dating back to the late 19th Century noticed the correlation between groundwater storage depletion and fluid pressure decline in major aquifer systems (e.g., the Dakota Sandstone in the Western U.S.; the Central Australian artesian basin), petroleum production engineers had for decades been aware of the correlation between declining oil production and declining fluid pressures in oil reservoirs. The latter correlation was far more dramatic because oil reservoirs are, in general, closed systems, whereas groundwater systems are invariably open, being subject to recharge and discharge. The economical importance of this correlation led to some outstanding research efforts among petroleum reservoir engineers during the 1930's.

Two prominent researchers of this era were William Hurst of the Humble Oil and Refining Company and Morris Muskat of the Gulf Research and Development Corporation. They independently addressed many problems related to transient flow of fluids to wells and pioneered many new developments in petroleum reservoir engineering. In a relatively obscure publication, T. V. Moore, R. J. Schilthuis and William Hurst (1933) presented what is perhaps the first exposition of transient well-test analysis. They considered unsteady flow to a well of finite diameter in a circular reservoir with prescribed conditions on the external and internal boundaries. At the well bore they considered both a prescribed pressure condition and a prescribed flux-boundary condition. The detailed mathematical analysis relating to this method was published about a year later by William Hurst (1934). In the same year Morris Muskat (1934) comprehensively addressed very similar transient problems related to the flow of compressible fluids to wells. Both Hurst and Muskat took advantage of many already existing solutions in the heat-conduction literature compiled by H. S. Carslaw (1921). In addition, Muskat also introduced some new solutions, "not even mentioned in the standard (English) texts on heat conduction."

In defining the storage parameter, both Hurst and Muskat considered only fluid expansion (fluid compressibility). Consequently, both used *fluid density* as the dependent variable. Although Theis on the one hand and Hurst and Muskat on the other were interested broadly in the radial flow of fluids to wells, they differed noticeably in the details of the problems addressed. Hurst and Muskat were dealing with closed reservoirs penetrated by deep, finite-diameter wells with considerable dead storage. Thus they chose to study "production-well" testing as the priority problem. On the

other hand, Theis, a groundwater hydrologist, was interested in open groundwater systems with large, areally extensive (infinite for practical purposes) aquifers. Hence Theis pioneered "interference" testing, in which a well could be conveniently approximated as a line source. This interference-testing concept would begin to interest petroleum engineers only about a decade or so later.

L. A. Richards

Following the rigorous extension of Darcy's law to unsaturated systems by Edgar Buckingham (1907), soil physicists were studying the transient migration of moisture in soils with considerable interest. Willard Gardner and John A. Widtsoe (1921) were keenly aware of the analogy between transient moisture movement in unsaturated soils and the unsteady heat-conduction process. They attempted to simulate experimentally observed transient moisture profiles in soil columns using soil moisture diffusivity as the controlling parameter. Unfortunately, they only had limited success in their endeavor because they assumed that moisture diffusivity is independent of moisture content, whereas moisture diffusivity in unsaturated media is in fact a strong function of moisture content.

The formal exposition of transient unsaturated flow as a nonlinear parabolic partial differential equation was done by Lorenzo A. Richards (1931). He decomposed soil moisture diffusivity into hydraulic conductivity and soil moisture capacity and recognized that both are functions of the dependent variable pressure head. Because the rate of desaturation with reference to fluid pressure in unsaturated materials may often far exceed in magnitude the other two components of storativity, namely, the compressibility of the matrix and the compressibility of the liquid, Richards restricted attention solely to rate of desaturation as the mechanism governing the storage parameter. Note that Theis, Terzaghi, Hurst and Muskat considered linear differential equations in which both permeability and storativity are treated as constants in time. These linear differential equations are amenable to solutions by analytical methods. Hence these researchers were able to obtain solutions for practical problems of interest. However, the functional dependence of both permeability and storativity on the dependent variable render the unsaturated flow equation to be nonlinear and extremely difficult to solve by analytical methods. Therefore, Richards did not attempt to solve the equation for any practical problem of interest.

Concluding Remarks

From the foregoing it is apparent that during the period 1920-1940 several outstanding researchers, motivated by diverse physical perceptions, helped establish time as an important dimension in understanding hydrogeologic systems. Here we use the phrase "hydrogeologic systems" in its broadest scope, encompassing systems of interest in groundwater hydrology, geotechnical engineering, petroleum engineering and soil physics. The diversity in the perspectives of these researchers is reflected in the fact that for the dependent variable in the governing equation, Theis used *drawdown*, Hurst and Muskat used *fluid density* and the soil physicists used *fluid saturation*.

This brief article is an attempt to reflect upon the parallel development of ideas among some distinguished earth scientists of the early 20th Century. It seems appropriate to present these thoughts on an occasion dedicated to honoring Theis for his valuable contributions. A detailed analysis of these developments and their interrelationships requires far more attention and thought than what has gone into the present work. The publications cited here merely include what are believed to be the earliest and most well known. The thoughts presented above suggest that perhaps an in-depth analysis of these developments would be a worthwhile future endeavor.

A Postscript

The philosophical importance of Theis' contribution lies in the recognition of the importance of the time dimension and the related recognition that time cannot be formally considered without the storage term. In a broad sense, the storage term provides an interesting basis for unifying the diverse perspectives of Gardner, Terzaghi, Richards, Hurst, Muskat, Theis and others mentioned to above.

Meinzer in his 1928 paper clearly recognized the importance of fluid expansion as well as pore compression in groundwater storage. In 1940 Jacob combined these two processes in formally deriving the storage term. Additionally, investigations by soil physicists as well as by groundwater hydrologists studying unconfined aquifers has well established the fact that desaturation is a third independent process governing groundwater storage. It stands to reason, then, that these diverse perspectives could be synthesized into a single framework if the three independent processes governing storage, namely, matrix deformation, fluid expansion and pore desaturation, could be combined to form a single generalized coefficient to represent the storage term suggested by Theis.

This synthesis, nonetheless, is not trivial. Indeed, to achieve the synthesis we need to await the introduction of yet another important concept in the soil mechanics literature in 1955 by Alan W. Bishop of England. This development relates to the extension of Terzaghi's effective stress concept to materials, not fully saturated with water. Terzaghi, who considered fully saturated materials, suggested that pore fluid

pressure change has a one to one correspondence with effective stress change. That is, a unit decline in pore pressure is equivalent to a unit increase in effective stress. Studying the engineering properties of compacted clays, Bishop (1955) pointed out that in unsaturated soils the correspondence between change in pore pressure and change in effective stress is not one to one. Bishop proposed a transfer function, χ ($0 < \chi < 1$), which is a strong function of saturation, to account for the fact that large changes in moisture suction may not induce any change in effective stress. Although in his later work he recognized that the transfer function could be a function of other factors besides water saturation, Bishop's introduction of this concept of a nonlinear transfer function set the stage for formally integrating the three processes. Thanks to these developments, we are now in a position to carry out unified analysis of transient flow of water in deformable saturated-unsaturated media (e.g., Narasimhan, 1975).

Acknowledgements

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