Title
ARTIFICIAL SINGULARITY IN THE MULTICHANNEL ND-1 EQUATIONS OF THE NEW STRIP APPROXIMATION

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Berkeley, California

Errata ARTIFICIAL SINGULARITY IN THE MULTICHANNEL ND^{-1} EQUATIONS OF THE NEW STRIP APPROXIMATION

Shu-Yuan Chu

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The Wiener-Hopf operator $O_\ell(s,s')$ introduced in Sec. II, when both s and s' tend to the strip boundary σ simultaneously, has the behavior:

$$O_\ell(s,s') \underset{s,s'\to\sigma}{\propto} \frac{\left(\frac{\sigma - s}{\sigma - s'}\right)^a - \left(\frac{\sigma - s'}{\sigma - s}\right)^a}{s' - s}$$

and correspondingly is not square integrable. Thus the simple prescription we gave is not valid unless all λ_{ij} with $i \neq j$ vanish. The reader should disregard the previous argument, starting with Eq. (III.2), and consider instead the following. Substracting out the singular part of Eq. (III.1) down to the highest threshold s_M, we have

$$N_i(s) = B_i(s) + \int_{s_\mu}^{s_M} U_{i\mu}(s,s')N_\mu(s')ds' + \int_{s_M}^{\sigma} K_{i\mu}(s,s')N_\mu(s')ds'$$

$$- \lambda_{i\mu} \int_{s_M}^{\sigma} k(s,s')N_\mu(s')ds' \quad \text{for} \quad i = 1, \cdots, n \quad . \qquad \text{(III.2')}$$

Then we define functions N_i^0 through the following equations:

$$N_i(s) = N_i^0(s) - \lambda_{i\mu} \int_{s_M}^{\sigma} k(s, s')N_\mu(s')ds' \quad \text{for} \quad i = 1, \ldots, n \tag{III.3'}$$

Now consider the λ_{ij} as elements of an $n \times n$ matrix Λ. Let S be the orthogonal matrix which diagonalizes Λ (since Λ is real and symmetric, S always exists); i.e.,

$$S\Lambda S^{-1} = \Lambda^D \qquad S^{-1} = S^T$$

where Λ^D is a diagonal matrix:

$$\Lambda_{ij}^D = e_i \delta_{ij} \quad \text{for} \quad i, j = 1, \ldots, n \quad .$$

From (III.3') we then have

$$S_{i\mu}N_\mu = S_{i\mu}N^0 - S_{i\mu}\lambda_{\mu\nu}S^{-1}_{\nu\rho} S_{\rho\tau}kN_\tau$$

or

$$\bar{N}_i = \bar{N}_i^0 - e_i \int_{s_M}^{\sigma} k\bar{N}_i \quad \text{for} \quad i = i, \ldots, n \tag{III.4'}$$

where

$$\bar{N}_i = S_{i\mu}N_\mu$$

$$N_i^0 = S_{i\mu}N_\mu^0$$

so

$$N_i = S^{-1}_{i\mu}\bar{N}_\mu$$

$$N^0_i = S^{-1}_{i\mu}\bar{N}^0_\mu \quad \text{for} \quad i = 1, \cdots, n \quad . \qquad \text{(III.5')}$$

Equations (III.4') can be inverted by the Wiener-Hopf method to give:

$$\bar{N}_i \underset{s>s_M}{=} O_i\bar{N}^0_i \quad \text{for} \quad i = 1, \cdots, n \quad . \qquad \text{(III.6')}$$

Thus we have:

$$N_i(s) = B_i(s) + \int_{s_\mu}^{s_M} U_{i\mu}(s,s')N_\mu(s')ds' + S^{-1}_{\mu\nu}\int_{s_M}^{\sigma} (U_{i\mu}O_\nu)(s,s')\bar{N}^0_\nu(s')ds'$$

$$\text{for} \quad s < s_M \quad i = 1, \cdots, n \qquad \text{(III.7'a)}$$

where

$$(U_{i\mu}O_\nu)(s,s') = \int_{s_M}^{\sigma} ds''\, U_{i\mu}(s,s'')O_\nu(s'',s')$$

$$N^0_i(s) = B_i(s) + \int_{s_\mu}^{s_M} U_{i\mu}(s,s')N_\mu(s')ds' + \int_{s_M}^{\sigma} K_{i\mu}(s,s')N_\mu(s')ds'$$

or

$$\bar{N}^0_i(s) = \bar{B}_i(s) + S_{i\mu}\int_{s_\mu}^{s_M} U_{\mu\nu}(s,s')N_\nu(s')ds' + S_{i\mu}S^{-1}_{\nu\rho}\int_{s_M}^{\sigma} (K_{\mu\nu}O_\rho)(s,s')\bar{N}^0_\rho(s')ds'$$

$$\text{for} \quad s_M < s < \sigma \quad i = 1, \cdots, n \qquad \text{(III.7'b)}$$

where $\bar{B}_i(s) = S_{i\mu}B_\mu(s)$

$$(K_{\mu\nu}O_\rho)(s,s') = \int_{s_M}^{\sigma} ds'' K_{\mu\nu}(s,s'')O_\rho(s'',s') \quad .$$

Equations (III.7') are a system of coupled integral equations with $N_i(s)$ for $s_1 < s < s_M$ and $\bar{N}_i^0(s)$ for $s_M < s < \sigma$ as unknown functions. The functions $U_{ij}(s,s')$ are square integrable for $s_1 < s < s_M$, $s_1 < s' < \sigma$, and the functions $K_{ij}(s,s')$ are square integrable for $s_M < s,s' < \sigma$. The singular function $O_\nu(s'',s')$ no longer appears above but only folded with these non-singular functions, so the difficulty explained at the beginning of this errata will not arise. The functions $(U_{i\mu}O_\nu)(s,s')$ can easily be shown to be square integrable for $s_1 < s < s_M$, $s_1 < s' < \sigma$ and likewise the functions $(K_{\mu\nu}O_\rho)(s,s')$ for $s_M < s,s' < \sigma$. Thus we have achieved a system of Fredholm equations.

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ARTIFICIAL SINGULARITY IN THE MULTI-CHANNEL ND^{-1} EQUATIONS OF THE NEW STRIP APPROXIMATION*

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ABSTRACT

It is shown that the Wiener-Hopf technique introduced by Chew in the single-channel case to remove the artificial singularity near the strip boundary can be applied straightforwardly to the multichannel case. The singular ND^{-1} equations are reduced to a system of Fredholm equations which can be solved directly by numerical methods. No new Wiener-Hopf operators arise.

I. INTRODUCTION

It has been advocated that an understanding of the two-body problem, provided metastable particles and enough channels are included, will suffice for many purposes in studying strong interactions.[1] The N/D equations in the new strip approximation furnish a good basis for actual calculations with a single channel. The artificial singularity near the strip boundary can be removed by the Wiener-Hopf technique described in reference 2. It is interesting to see whether the same technique can be applied to the multichannel case.

In Section II we review briefly the result obtained in reference 2. In Section III the matrix generalization of the N/D equations[3] to the multichannel case is made. The resulting ND^{-1} equations are reduced to a system of Fredholm equations. Only the Wiener-Hopf operator already occurring in the single-channel case turns out to be needed.

II. REVIEW OF THE 1-CHANNEL CASE

The integral equation in question is (I-1) of reference 2:

$$N_\ell(s) = B_\ell^p(s) + \frac{1}{\pi}\int_{s_0}^{s_1} \frac{B_\ell^p(s') - B_\ell^p(s)}{s' - s}\, \rho_\ell(s')N_\ell(s')ds' \quad . \tag{I.1}$$

B_ℓ^p has a logarithmic branch point near s_1:

$$B_\ell^p(s) \xrightarrow[s\to s_1]{} -\frac{1}{\pi}\, \mathrm{Im}\, B_\ell^p(s_1)\, \ln\,(s_1 - s) \quad .$$

Writing (I.1) symbolically, we have:

$$N_\ell = B_\ell^p + UN_\ell \quad ,$$

where U represents the integral operator, i.e.,

$$(Uf)(s) = \frac{1}{\pi} \int_{s_0}^{s_1} \frac{B_\ell^p(s') - B_\ell^p(s)}{s' - s} \rho_\ell(s')f(s')ds' \quad .$$

Separating out the singular part,

$$N_\ell = B_\ell^p + KN_\ell - \lambda_\ell kN_\ell \quad , \tag{I.2}$$

where

$$U = K - \lambda_\ell k \quad ,$$

with

$$\lambda_\ell = \rho_\ell(s_1) \text{ Im } B_\ell^p(s_1) \quad ,$$

and

$$(kf)(s) = \frac{1}{\pi^2} \int_{s_0}^{s_1} ds' \frac{\ln (s_1 - s') - \ln (s_1 - s)}{s' - s} f(s') \quad ,$$

we can write (I.2) as two equations;

$$N_\ell = N_\ell^0 - \lambda_\ell kN_\ell \quad , \tag{I.3}$$

$$N_\ell^0 = B_\ell^p + KN_\ell \quad . \tag{I.4}$$

Equation (I.3) can be solved to give

$$N_\ell = O_\ell N_\ell^0 \quad ,$$

where O_ℓ depends only on λ_ℓ, and has the following behavior near s_1:

$$O_\ell(s,s') \underset{\substack{s \to s_1 \\ s' \text{ fixed}}}{\propto} (s_1 - s)^{-a_\ell}$$

$$\underset{\substack{s' \to s_1 \\ s \text{ fixed}}}{\propto} (s_1 - s')^{-a_\ell} , \quad \text{with } 0 < a_\ell < \frac{1}{2} .$$

(Vigdor Teplitz has been able to evaluate the operator O_ℓ without difficulty.[4] Thus we have a Fredholm equation for N_ℓ^0 :

$$N_\ell^0 = B_\ell^p + KO_\ell N_\ell^0 , \tag{I.5}$$

since the operator KO_ℓ can be shown to be square integrable. Or, if we multiply by O_ℓ on both sides of (I.5), we have a Fredholm equation for N_ℓ,

$$N_\ell = O_\ell B_\ell^p + O_\ell K N_\ell ,$$

which is the form pointed out by Kreps.

III. GENERALIZATION TO THE N-CHANNEL CASE

We may employ the matrix formulation of the N/D method[3] straightforwardly. Suppressing the ℓ index, we have

$$N_{ij}(s) = B_{ij}^p(s) + \frac{1}{\pi} \sum_{\mu=1}^{n} \int_{s_\mu}^{\sigma} \frac{B_{i\mu}^p(s') - B_{i\mu}^p(s)}{s' - s} \rho_\mu(s') N_{\mu j}(s') ds' .$$

$$\text{for } i,j = 1, \cdots, n , \tag{II.1}$$

where i,j are the channel indices, σ is the strip width (common to all channels), s_μ the thresholds of the μth channel, and the ρ_μ's are the appropriate phase-space factors.

Writing (II.1) symbolically, we have

$$N_{ij} = B^p_{ij} + \sum_{\mu=1}^{n} U_{i\mu} N_{\mu j} \quad .$$

The first point to notice is that only those N_{ij}'s of the same column index j are coupled together. In the following we shall thus consider a particular column and suppress the index j. Separating out the singular part as in the single-channel case, we have

$$N_i = B^p_i + \sum_{\mu=1}^{n} (K_{i\mu} N_\mu - \lambda_{i\mu} k_\mu N_\mu) \quad \text{for} \quad i = 1, \cdots, n, \tag{II.2}$$

where

$$(k_\mu f)(s) = \frac{1}{\pi^2} \int_{s_\mu}^{\sigma} \frac{\ln(\sigma - s') - \ln(\sigma - s)}{s' - s} f(s') ds'$$

and

$$\lambda_{i\mu} = \rho_\mu(\sigma) \operatorname{Im} B^p_{i\mu}(\sigma) \quad .$$

Next we define functions N^0_i through the equations

$$N_i = N^0_i - \lambda_{ii} k_i N_i \quad \text{for} \quad i = 1, \cdots, n, \tag{II.3}$$

where repeated Latin indices are not summed over.

Comparing with Eq. (I.3), we see that $N_i = O_i N^0_i$, where O_i is the same operator as discussed in Section I, depending only on λ_{ii} and s_i. From (II.2) we have the equations for N^0_i:

$$N_i^0 = B_i^p + \sum_{\mu=1}^{n} K_{i\mu} O_\mu N_\mu^0 - \sum_{\mu\neq i} \lambda_{i\mu} K_\mu N_\mu \quad \text{for} \quad i = 1, \cdots, n ,$$

while, from (II.3),

$$k_i N_i = \frac{N_i^0 - N_i}{\lambda_{ii}} = \frac{1}{\lambda_{ii}} (1 - O_i) N_i^0 \quad \text{for} \quad i = 1, \cdots, n .$$

Thus we achieve coupled equations for N_i^0:

$$N_i^0 = B_i^p + \sum_{\mu=1}^{n} K_{i\mu} O_\mu N_\mu^0 - \sum_{\mu\neq i} \frac{\lambda i\mu}{\lambda\mu\mu} (1 - O_\mu) N_\mu^0 \quad \text{for} \quad i = 1, \cdots, n . \tag{II.4}$$

Note that, since O_i are different for different i, the Kreps' form can't be restored, and apart from the second summation on the right-hand side, the equations are very similar to the single-channel case. Since all kernels are square integrable, we have a system of coupled Fredholm equations which can be solved by the usual technique of matrix inversion. The required number of mesh points is increased only in proportion to the number of channels.

The application of this method to the $\pi\pi$, $K\bar{K}$ (or $\pi\omega$) channel is under investigation.

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FOOTNOTES AND REFERENCES

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