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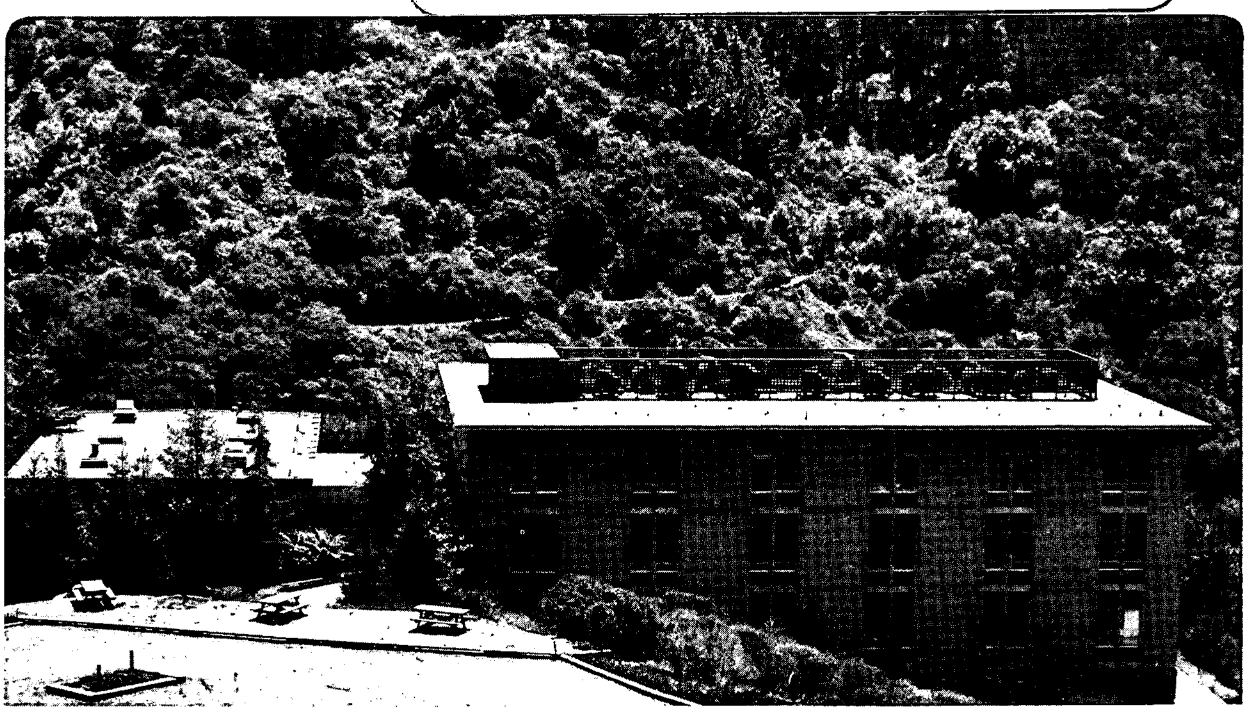

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SUPERCONDUCTIVITY: PHENOMENOLOGY*

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SUPERCONDUCTIVITY: PHENOMENOLOGY

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ABSTRACT

I.- THE SUPERCONDUCTING STATE.

(a) The superconducting transition temperature; (b) Zero resistivity; (c) The Meissner effect; (d) The isotope effect; (e) Microwave and optical properties; (f) The superconducting energy gap.

II.- THE GINZBURG-LANDAU EQUATIONS.

(a) The coherence length; (b) The penetration depth; (c) Flux quantization; (d) Magnetic-field dependence of the energy gap; (e) Quantum interference phenomena; (f) The Josephson effect.

August 2, 1988

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I. THE SUPERCONDUCTING STATE.

IA. Background Information.

SUPERCONDUCTIVITY is a phase, a state of matter (in the sense that ice and steam are phases of water and diamond and graphite are phases of pure carbon) observed *only* in some solids, mostly metals [1-3].

The SUPERCONDUCTING STATE has several characteristic properties:

1.- When it exists for a given substance, it exists only at temperatures below a so-called TRANSITION TEMPERATURE, T_c, and in general down to the absolute zero of the temperature scale ($0\,K = -273.15 \quad C$).

2.- It exhibits *d.c.* ZERO RESISTIVITY, i.e. infinite conductivity for zero-frequency measurements (an effect discovered in mercury by Kamerlingh Onnes in 1911),

$$\rho(\omega = 0; T < T_c) = 0 \ . \tag{1}$$

3.- It exhibits, for weak magnetic fields, perfect DIAMAGNETISM, i.e. its magnetic susceptibility in Gaussian units is given by

$$\chi_M = - (1/4\pi) \ , \tag{2}$$

which means that magnetic flux lines are completely expelled from the superconductor and that there is a force pushing superconductors away from magnetic fields. This effect, known as the MEISSNER EFFECT, was discovered by Meissner and Ochsenfeld in 1933.

4.- There is a minimum energy value -- called an ENERGY GAP [4] -- for exciting *electronically* the system away from its state of lowest energy (the so-called ground state). This energy gap

$$E_G = 2|\Delta| \ , \tag{3}$$

was conjectured theoretically by F. London in 1935, deduced from thermodynamic data in 1946, observed by infrared measurements in 1957 and by electron tunneling in 1960.

Infrared absorption measurements in 1937, down to frequencies of the order of 10^{14} cycles per second showed no differences between the normal and superconducting phases. Microwave measurements by H. London in 1940 gave no appreciable absorption of electromagnetic radiation by superconductors up to a frequency of 10^9 cycles per second. These two experiments together gave upper and lower bounds to the superconducting energy gap.

In 1957 Glover and Tinkham were successful in reaching the far infrared region of the electromagnetic spectrum and observed, for lead, a sudden drop in the absorption as the frequency was decreased.

In 1960 Giaever discovered that the current/voltage characteristics of sandwiches consisting of a superconductor and either another superconductor or a normal metal, separated by a thin oxide insulating layer were nonlinear, and that the non-linearity could be easily interpreted in terms of an energy gap in the spectrum of the superconductor(s).

5.- There is a surprising dependence on the transition temperature, T_c, on the isotopic mass of the atomic nuclei of the superconductor. (It is surprising that a phase which is electric and magnetic in nature, and therefore caused by the *electrons*, depends in any fashion on nuclear parameters, in particular the mass of the *nuclei*.) This is the so-called ISOTOPE EFFECT, was discovered in 1950, and establishes that

$$M^{\alpha} \cdot T_c = constant \quad , \tag{4}$$

where M is the nuclear mass and, for various metals, the exponent α takes the values:

0.485 for Pb, 0.415 for Sn, 0.150 for Ti, 0.065 for Ru, and -0.015 for Ir.

6.- In addition to the effect of high temperatures, superconductivity can be destroyed (with a return to the normal state) by either a large enough electric current $I > I_c$, or a large enough magnetic field $H > H_{c2}$. (It should be mentioned that in some superconductors, the so-called type II superconductors, for intermediate field strengths $H_{c1} < H < H_{c2}$, the magnetic flux lines partially penetrate the superconductor but do not destroy the superconducting state.) The quantities I_c, H_{c1}, and H_{c2}, are called the CRITICAL CURRENT, and the first and second CRITICAL MAGNETIC FIELDS, respectively.

7.- Superconductivity is a MACROSCOPIC QUANTUM PHENOMENON, with amplitudes and phases associated with the energy gap parameter Δ. Therefore interference and diffraction effects can be achieved, in particular the JOSEPHSON EFFECT [3]. These effects can be fruitfully employed in processing, storing, and retrieving information, i.e. in computer technology.

IB. Theory.

The currently, universally accepted theory of superconductivity, known as the BCS THEORY was formulated [5] by Bardeen, Cooper and Schrieffer in 1957. The theory in its most general form states that, if metallic mobile electrons interact ATTRACTIVELY with each other, then they will condense into a ground state with:

(1) an energy gap in the excitation spectrum;

(2) zero resistivity;

(3) the Meissner effect; and

(4) a phase transition to the normal metallic state at a transition temperature T_c.

There is an important issue to resolve. How can two electrons -- which are charged particles with identical negative charges, and therefore experience a strong Coulomb-force repulsion -- attract one another? The answer is: by polarizing the crystal lattice. [An instructive simile is the attraction that two billiard balls experience when placed on a rubber membrane: one billiard ball falls readily into the depression caused by the other ball, hence it is attracted by the other ball.] Since the polarization of the crystal lattice depends on the mass of the nuclei which form it, the strength of the electron-electron attraction, which is caused by the lattice polarization, depends the mass of the nuclei, i.e. there is an ISOTOPE EFFECT.

The BCS theory yields, in general, an integral equation for the energy gap parameter Δ, and another integral equation for the transition temperature T_c. These integral equations depend on the electronic structure of the metal, and on the details of the attractive interaction between the electrons. As an example of their theory, Bardeen, Cooper and Schrieffer introduced a very simple model, the so-called BCS MODEL, for which the integral equations can be analytically solved, and that yields

$$\Delta \;=\; 1.76\,k\;T_c \;=\; 2\hbar\,\omega_D\,\exp[-1/NV] \quad , \tag{5}$$

where k is Boltzmann's constant, ω_D is the vibration (Debye) frequency of the lattice, N is the number of available electronic states per unit energy in the solid (density of states at the Fermi level), and V is the strength of the attractive (lattice mediated) electron-electron interaction. This formula gives an isotope effect because ω_D, a lattice frequency, depends on the nuclear mass [it is proportional to $M^{-1/2}$].

This simple BCS model gives a good idea of how the BCS theory works: the transition temperature can be increased (i) by increasing ω_D, (ii) by increasing N, or (iii) by increasing V. [It should be remarked that the influence of both N and V on T_c is much more dramatic than the simple proportionality of T_c and ω_D.] According to formula (5) there is no maximum transition temperature; T_c can be increased without limit by finding solids with larger and larger N, V, and ω_D.

In fact formula (5) is not accurate: it is only a simple model. A very good and accurate theory, based on the BCS theory, was developed by Eliashberg and McMillan [6] which, starting from precise experimental information about the crystal lattice vibrations, could accurately -- by numerical methods -- calculate the gap parameter Δ and the transition temperature T_c. The main results of this theory are presented in the Appendix. With a precision of a few percent, the equations yield excellent results for the transition temperature T_c and the isotope effect exponent α in several well studied cases, mostly transition metals. Numerical experiments performed with the Eliashberg-McMillan equations produced, for sensible input of lattice vibration spectra, superconducting transition temperatures which never exceeded 40 K. Therefore, although no rigorous limit was established for a MAXIMUM SUPERCONDUCTING TRANSITION TEMPERATURE, the belief among most specialists was that such an upper bound existed, and that it was in the range of 30 K to 40 K.

IC. History of the Highest Superconducting Transition Temperatures.

The Table below shows the history of the experimentally found highest superconducting transition temperatures:

YEAR	T_c [K]	SUBSTANCE	Notes and References.
1911	4.2	Hg	[1]
~ 1913	7.2	Pb	
1933	9.5	Nb	
1941	16.0	NbN	
1953	17.1	V_3Si	
1960	18.05	Nb_3Sn	
1969	20.8	$NbAlGe$	
1973	23.2	Nb_3Ge	[7]
1986	~ 30	$La-Ba-Cu-O$	[8,9]
1988	~ 30	$Ba-K-Bi-O$	highest T_c superconductor without Cu [10]
1986	39	$La-Sr-Cu-O$	[11]
1987	~ 92	$RE-Ba-Cu-O$	RE = various rare earths [12,13].
1988	~ 105	$Bi-Sr-Ca-Cu-O$	[14]
1988	125	$Tl-Ca-Ba-Cu-O$	[15,16]
1987	~ 230	$RE-Ba-Cu-O$	not reproducible, unstable! [17]

As can be seen, from 1911 to 1973 the increase in maximum observed transition temperatures was a more-or-less linear function of about 0.3 K per year. No temperature was found to violate the (wrongly believed) upper bound.

For the sake of comparison it should be remembered that liquid helium boils at 4.5 K, liquid hydrogen at 20.7 K, liquid neon at 27.2 K, and liquid nitrogen (i.e. liquid air) at 77.4 K. These are the most commonly used refrigerants, and any technology based on superconductivity will have its running costs determined, almost exclusively, by the refrigeration costs. The discovery of superconducting Nb_3Ge in 1973 was considered a major breakthrough, since for the first time the liquid-hydrogen barrier was crossed. Needless to say the events of the last two years can be considered, by any standards, fantastic: first the liquid-neon barrier was broken; soon thereafter the liquid-air temperature was surpassed; and -- if the elusive and unstable very high temperatures reported [17] but easily lost, are both confirmed and stabilized -- it seems that the dream of room-temperature superconductivity is now within accessible reach.

II.- THE GINZBURG-LANDAU EQUATIONS.

IIA. General formulation.

In 1950 Ginzburg and Landau[4,18–23] (GL) proposed a phenomenological theory of superconductivity, which was independent of the microscopic aspects of the phenomenon. The theory was quantum-mechanical, in the sense that included coherent, macroscopic quantum effects. It was a pioneering theory which, independently of the mechanisms responsible for superconductivity, is still valid today. It contains such diverse phenomena as magnetic-field penetration depths, coherence lengths, magnetic-field flux quantization, magnetic-field dependence of the energy gap (order parameter), and the Josephson effect. It can be applied to all superconductors, as well as to superfluid ^{3}He, and has become the prototype theory to study a whole class of phenomena related to second-order phase transitions.

The GL theory introduces a complex order parameter ψ which is allowed to vary in space. Originally GL interpreted ψ as an amplitude, and $|\psi|^2$ as the density of the "superconducting" electrons (they envisioned a superconductor as two interpenetrating electron fluids, the non-dissipative, non-resistive "superconducting" electron fluid, and the dissipative, resistive "normal" electron fluid). In 1959, however, Gor'kov[24,25] proved, using his own formulation of the BCS theory, that for temperatures below and close to T_c, equations identical to those of GL could be obtained, and that the GL parameters ψ could be interpreted (except for a trivial constant of proportionality) as the BCS energy-gap parameter Δ.

The starting point of the GL theory is the introduction of a magnetic Helmholtz free energy F_{SH} for the superconductor, derived from plausibility arguments

$$F_{SH} = \int_{superconductor} d^3r \left[F_{N0} + \Delta F(|\psi|^2) + (1/2m) \left| -i\hbar\nabla\psi - (e*/c)\mathbf{A}\psi \right|^2 + (1/8\pi)\, H^2(\mathbf{r}) \right] . \qquad (6)$$

Here F_{N0} is the free-energy density of the normal state in the absence of a magnetic field; ΔF is the difference of free-energy densities between the superconducting and the normal states (also in the absence of a field) and is a function of $|\psi|^2$. The third term is the gauge invariant "superconducting kinetic energy", and the last term is the magnetic field energy in the superconductor. The vector potential is $\mathbf{A}$, $\mathbf{H}$ is the magnetic field, and $e*$ is an effective charge, known now to be the charge of a a "Cooper pair"

$$e* = 2e \ . \qquad (7)$$

All terms in (6) are functions of the position $\mathbf{r}$, and change with the magnitude and direction of the magnetic field.

Because the proper variables of the magnetic Helmholtz free energy are the temperature T and the magnetization $\mathbf{M}$, where

$$\mathbf{M} = (1/4\pi) \int_{all\ space} d^3r \left[\mathbf{H}(\mathbf{r}) - \mathbf{H}_0 \right] , \qquad (8)$$

$$\mathbf{H}_0 = applied\ \ magnetic\ \ field \ ,$$

F_{SH} is not continuous at the critical fields. The function which is continuous at H_{c1} and H_{c2}, and whose proper variables are T and $\mathbf{H}_0$, is the Gibbs free energy G_{SH}, given by

$$G_{SH} = F_{SH} - \mathbf{M}\cdot\mathbf{H}_0 \ . \qquad (9)$$

Substitution of (6) and (8) into (9) yields

$$G_{SH} = G_{NH} + \int_{superconductor} d^3r \left[\Delta F(|\psi|^2) + (1/2m) \left| -i\hbar\nabla\psi - (e*/c)\mathbf{A}\psi \right|^2 \right] +$$

$$(1/8\pi) \int_{all\ space} d^3r \left[\mathbf{H}(\mathbf{r}) - \mathbf{H}_0 \right]^2 , \qquad (10)$$

where

$$G_{NH} = \int_{superconductor} d^3r \left[F_{N0} + H_0^2 / 8\pi \right] .$$

It should be noted that the last term in (10) is to be integrated over *the whole space* (both in the superconductor and outside). Minimization of G with respect to the four functions ψ and A [or equivalently ψ and H] yields the famous GL equations:

$$\nabla^2 \mathbf{A} = \frac{ie^*\hbar}{mc} \left[\psi^* \nabla \psi - \psi \nabla \psi^* \right] + \frac{4\pi e^{*2}}{mc^2} |\psi|^2 \mathbf{A} , \tag{11}$$

$$\nabla \times \mathbf{A} = \mathbf{H}_0 \quad on\ surface , \tag{12}$$

$$\frac{\partial \Delta F}{\partial \psi^*} + \frac{1}{2m} \left[-i\hbar \nabla - \frac{e^*}{c} \mathbf{A} \right]^2 \psi = 0 , \tag{13}$$

$$\left[i\hbar \nabla \psi + \frac{e^*}{c} \mathbf{A} \psi \right]_{perpendicular} = 0 , \tag{14}$$

where the London gauge

$$\nabla \cdot \mathbf{A} = 0$$

has been chosen.

IIB. Penetration Length and Coherence Length.

In singly connected samples with no penetration of the magnetic flux into the bulk superconductor, the phase of ψ can be chosen so that ψ is real throughout the sample. In particular for a one-dimensional, singly connected problem, with quantities varying along the x-axis, and with magnetic field and vector potential given by

$$\mathbf{H} = [\, 0 \,, 0 \,, H(x) \,] ,$$

$$\mathbf{A} = [\, 0 \,, A(x) \,, 0 \,] ,$$

the equations (11)-(14) become

$$(d^2A \,/\, dx^2) = (4\pi e*^2/mc^2)\, \psi^2 A \quad , \tag{15}$$

$$(dA \,/\, dx) = H_0 \quad on\ surface \quad , \tag{16}$$

$$\frac{\partial \Delta F}{\partial \psi} + \frac{e*^2}{mc^2} A^2 \psi = \frac{\hbar^2}{m} \frac{d^2\psi}{dx^2} \quad , \tag{17}$$

$$(d\psi \,/\, dx) = 0 \quad on\ surface \quad . \tag{18}$$

For the free-energy difference ΔF, the original GL derivation used a power-series expansion in $|\psi|^2$, and neglected all terms higher than the second. That expansion is still commonly used, and is known to be valid for superconductors at temperatures close to T_c:

$$\Delta F = \frac{H_{cb}^2}{8\pi} \left[-2 \left[\frac{\psi}{\psi_T} \right]^2 + \left[\frac{\psi}{\psi_T} \right]^4 \right] \quad , \tag{19}$$

where H_{cb} is the thermodynamic bulk critical field, and ψ_T is the equilibrium value of ψ in the bulk, at temperature T, in the absence of a magnetic field.

One of the simplest and most instructive cases to solve is that of the superconducting half-space, with a constant magnetic field H_0 applied parallel to the surface at $x = 0$. Integration of (15)-(19), under the assumption of small changes in ψ near the surface, yields for $x > 0$

$$\frac{\psi(x, H_0)}{\psi_T} = 1 - \frac{\kappa_o}{(2 - \kappa_o^2)\sqrt{8}} \left[\frac{H_0}{H_{cb}} \right]^2 \left[e^{-\frac{\sqrt{2}\kappa_o x}{\lambda_L}} - \tfrac{1}{2}\kappa_o \; e^{-\frac{2x}{\lambda_L}} \right] \quad , \tag{20}$$

and

$$H(x) \approx H_0 \exp\left(-x/\lambda_L\right) \quad , \tag{21}$$

where λ_L, which governs the decay of the magnetic field into the superconductor, is the London penetration depth

$$\lambda_L^2 = \frac{mc^2}{4\pi e*^2\, \psi_T^2} \quad , \tag{22}$$

and κ_o is a dimensionless constant

$$\kappa_o = (\sqrt{2}e* \,/\, \hbar c)\, \lambda_L^2\, H_{cb} \quad . \tag{23}$$

Two remarks are necessary at this point. First, there are two length scales in the problem: (i) the decay length for magnetic fields, λ_T, and (ii) the decay length, $(\lambda_L / \sqrt{2}\kappa_o)$, for the order parameter ψ, given by the first exponent in (20). Second, Gor'kov has shown[24,25] that

$$\kappa_o \approx 0.96\, \lambda_L\, \xi_o^{-1} \ , \tag{24}$$

where ξ_o is Pippard's electromagnetic coherence length[26,27], now known to be related to the energy gap parameter

$$\xi_o \approx \hbar\, v_F / \Delta \tag{25}$$

(v_F is the Fermi velocity of the electrons in the metal). Values of ξ_o are small (< 0.707) for the soft, type I superconductors [0.01 for Al; 0.3 for Pb], whereas it takes large values (> 0.707) for the hard, type II superconductors [~ 8 for V; extremely large for the new, high T_c materials].

A type I superconductor excludes a magnetic field from its bulk completely. If the magnetic field is increased there is a value, H_c for which the superconductivity is suddenly destroyed, the system returns to the normal state, and the magnetic field penetrates the specimen completely. A type II superconductor excludes the field completely up to a value H_{c1}. Above H_{c1} the field is partially excluded, although the specimen remains superconducting and exhibits zero resistivity. At a higher field, H_{c2}, the flux penetrates completely, superconductivity is destroyed and the specimen returns to its normal state.

IIC. Flux quantization.

In many applications (thin specimens, weak magnetic fields, etc.), the order parameter ψ can be considered to have a constant magnitude $n^{1/2}$, although its phase $\theta(\mathbf{r})$ can vary appreciably in space,

$$\psi = n^{1/2}\, e^{i\theta(\mathbf{r})} \ . \tag{26}$$

From standard quantum-mechanical arguments the electrical supercurrent is given in this case by the usual formula

$$\mathbf{j} = \frac{1}{2m}\left[\psi^* \left[-i\hbar\,\nabla - \frac{e^*}{c}\mathbf{A}\right]\psi + \psi\left[i\hbar\,\nabla - \frac{e^*}{c}\mathbf{A}\right]\psi^*\right] = \frac{ne^*}{m}\left[\hbar\,\nabla\theta - \frac{e^*}{c}\mathbf{A}\right] \ . \tag{27}$$

Deep inside any superconductor the electric current is zero and, therefore, from (27) one obtains

$$e^* \mathbf{A} = \hbar c \; \nabla\theta \; . \tag{28}$$

In a multiply connected sample one can find a closed path C which encircles a non-superconducting region where there may be a magnetic field. Line integration of (28) over that path, use of Stokes theorem and knowledge that ψ must be single-valued yields

$$\int_{closed\ C} \mathbf{A} \cdot ds = \int_{area\ C} \nabla\times\mathbf{A} \cdot d\sigma = \int_{area\ C} \mathbf{H} \cdot d\sigma = \Phi$$

$$= \frac{\hbar c}{e^*} \int_{closed\ C} \nabla\theta \cdot ds = \frac{\hbar c}{e^*} \cdot 2\pi\nu \; , \tag{29}$$

where Φ is the magnetic-field flux, and ν is an arbitrary integer. In other words (29) can be written

$$\Phi = \nu \;\; \Phi_0 \; = \; \nu \; \cdot \; 2.0678\times 10^{\,-7} \; gauss \; cm^2 \; , \tag{30}$$

i.e. if a closed path without currents can be established deep inside a multiply connected superconductor, then the magnetic-field flux encircled by that path is quantized in units of Φ_0.

IID. Phase-current relationship; the Josephson effect.

From the GL equations it can be easily seen that the order parameter ψ has an indeterminate *arbitrary*, *constant* phase. In a given superconductor (called 1) its phase θ_1 is completely arbitrary. If, however, there is nearby a second superconductor (called 2), which is *weakly* connected to the first one, although both phases, θ_1 and θ_2, are indeterminate by the *same* additive constant, the *phase difference* between the two,

$$\delta = \theta_2 - \theta_1 \; ,$$

is an observable meaningful quantity. As can be seen from (27) a change if θ over space is responsible for the existence of a supercurrent. Similarly[3,28] a phase difference between two weakly coupled, spatially close superconductors produces a current flow between them given by

$$J = J_0 \; \sin\delta \; , \tag{31}$$

where J_0, a constant, describes the maximum possible current which may flow between the the two specimens. Equation (31) is Josephson's famous *d.c.* equation relating current and phase difference. It is implicit in the GL equations and applies to any system with a macroscopic, quantum-mechanical, complex order parameter. It is the consequence[28] of the standard quantum mechanical uncertainty relation between particle number and wave-function phase.

IIE. Magnetic-field dependence of the energy gap.

Detailed solution of (15)-(19) for thin films[4] clearly exhibit a field dependence of the *amplitude* of the order parameter $|\psi|$ on applied magnetic fields H_0. As the field is increased the value of $|\psi|$ decreases, and there is a value H_f for which it goes (either continuously or discontinuously) to zero and the film becomes normal. It is found that H_f depends on H_{cb}, d and the London penetration depth (22), and that the $|\psi|$ transition to zero at H_f is discontinuous if

$$d / \lambda_L > \sqrt{5} .$$

These results, and many others obtained from the solution of the GL equations for a variety of geometries and situations, have been confirmed by superconducting tunnelling experiments.

IIF. Quantum interference phenomena.

Finally the facts that:

(i) the order parameter ψ is complex; (ii) ψ must be single valued; (iii) the magnetic field **H** couples to it in a gauge invariant form and therefore is directly related to the phase θ of ψ; and (iv) the GL equations are non linear;

result in an enormous wealth of interference and diffraction effects which can be fruitfully used in designing interesting electronic devices[29,30]. It can be said that, in mastering the science of superconductivity, scientists have promoted quantum mechanics to the macroscopic, everyday-use level.

APPENDIX

The integral equations for the normal and pairing self-energies of a superconductor are [6]

$$\xi(\omega) = [1 - Z(\omega)]\,\omega = \int_{\Delta_0}^{\infty} d\omega' \,\mathrm{Re}\left[\frac{\omega'}{(\omega'^2 - \Delta'^2)^{1/2}}\right] \times \int d\omega_q\, \alpha^2(\omega_q)\, F(\omega_q)\, [D_q(\omega' + \omega) - D_q(\omega' - \omega)]$$

$$\phi(\omega) = \int_{\Delta_0}^{\omega_c} d\omega' \,\mathrm{Re}\left[\frac{\Delta'}{(\omega'^2 - \Delta'^2)^{1/2}}\right] \times \left\{\int d\omega_q\, \alpha^2(\omega_q)\, F(\omega_q)\, [D_q(\omega' + \omega) + D_q(\omega' - \omega)] - \mu^*\right\}$$

where $D_q(\omega) = (\omega + \omega_q - i0^+)^{-1}$, $\Delta(\omega) = \phi(\omega)/Z(\omega)$, and $\Delta_0 = \Delta(\Delta_0)$. $F(\omega)$ is the phonon density of states

$$F(\omega) = \sum_\lambda \int \frac{d^3q}{(2\pi)^3}\, \delta(\omega - \omega_{q\lambda})$$

and $\alpha^2(\omega)$ is an effective electron-phonon coupling function for phonons of energy ω:

$$\alpha^2(\omega)\, F(\omega) = \int_S \frac{d^2p}{v_F} \int_{S'} \frac{d^2p'}{(2\pi)^3 v_F'} \sum_\lambda g^2_{pp'\lambda}\, \delta(\omega - \omega_{p-p'\lambda}) \Big/ \int_S \frac{d^2p}{v_F}$$

where $g^2_{pp'\lambda}$ is the dressed electron-phonon matrix element, $\omega_{q\lambda}$ is the phonon energy for polarization λ and wave number q (reduced to the first zone), and v_F is the Fermi velocity. The two surface integrations are performed over the Fermi surface.

In order to solve these equations, the necessary input consists of two numbers, the electron Coulomb interaction pseudopotential μ^*, and the cut-off ω_c, and the electron-phonon coupling function $\alpha^2(\omega) F(\omega)$.

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