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Chew and Low[1] and Goebel[2] have discussed a method to measure the cross sections for scattering off particles such as pions and neutrons which are not available as free targets in the laboratory. The method consists of extrapolating the cross section for a related but measurable reaction as a function of one of the invariant momentum transfers to a pole of the S matrix for this reaction. The success of the extrapolation depends on the location and strength of other less-well-known singularities of the S matrix. In this note we want to point out the existence of two branch points which appear in the Chew-Low prescription for extrapolation to obtain the differential cross section of the unstable particle. These branch points are due to the constraint of fixed scattering angle, and disappear only when the integration over angle to obtain total cross sections is carried out. We will indicate how to modify the extrapolation to avoid this difficulty.

Suppose we want to measure the differential scattering cross section for the process

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$$a_1 + a_2 \rightarrow a_3 + a_4 , \qquad (1)$$

where a_2 is emitted virtually by the process

$$a_6 \rightleftarrows a_2 + a_5 . \qquad (2)$$

Then, according to Chew and Low, we measure the cross section for the reaction

$$a_1 + a_6 \rightarrow a_5 + a_3 + a_4$$

and extrapolate it in the momentum transfer $S_{56} = (q_6 - q_5)^2$ to the value $S_{56} = m_2{}^2$, keeping fixed $S_{34} = (q_3 + q_4)^2$, the square of the total energy of a_3 and a_4, and $\cos \theta_{14}$, the cosine of the angle between a_1 and a_4, both in the rest frame of a_3 and a_4. The variables q_i and m_i refer to the four-momentum and mass of the particle a_i. In the limit $S_{56} \rightarrow m_2{}^2$, the laboratory-system cross section for this reaction takes the form[1]

$$d\sigma = \frac{\Gamma^2}{4\pi} \frac{m_5}{m_6 \vec{q}_1{}^2} \frac{([S_{34} - (m_1 - m_2)^2][S_{34} - (m_1 + m_2)^2])^{1/2}}{(S_{56} - m_2{}^2)^2}$$

$$\times \frac{d\sigma_0}{d\Omega_4} (S_{34}, S_{14}) d\Omega_4 \, dS_{34} \, dS_{56} , \qquad (3)$$

where $\frac{d\sigma_0}{d\Omega_4}$ is the differential cross section for Process (1) and $\sqrt{4\pi}\,\Gamma$ is the amplitude for Process (2) (coupling constant). For fixed S_{34} and $\cos \theta_{14}$ the invariant momentum transfer $S_{14} = (q_1 - q_4)^2$ is a function of S_{56}.

$$S_{14} = m_1^2 + m_4^2 - \frac{1}{2S_{34}} \Big[(S_{34} + m_1^2 - S_{56})(S_{34} + m_4^2 - m_3^2)$$

$$- ([S_{34} - (m_3 - m_4)^2][S_{34} - (m_3 + m_4)^2]$$

$$\times [S_{56} - (\sqrt{S_{34}} - m_1)^2][S_{56} - (\sqrt{S_{34}} + m_1)^2])^{1/2} \cos\theta_{14} \Big] .$$

Evidently S_{14} is analytic in S_{56} except for square root branch points at $S_{56} = (\sqrt{S_{34}} \pm m_1)^2$. Since $\frac{d\sigma_0}{d\Omega_4}$ is analytic in S_{14} it has also branch points at the same location when considered as a function of S_{56}. However, integration over the scattering angle θ_{14} to obtain the total cross section removes these branch points. This can be seen, for instance, by expanding $\frac{d\sigma_0}{d\Omega_4}$ in a Taylor series in $\sqrt{S_{14}}$, noticing that only even powers of $\cos\theta_{14}$ contribute to the integration.

We note that the minimum experimental value of $\sqrt{S_{34}}$ is $(m_3 + m_4)$. On the other hand, in order that the branch points do not lie between the measured values of S_{56}, $(S_{56} < 0)$ and its value at the pole $(S_{56} = m_2^2)$ it is necessary to consider $(m_1 + m_2) < \sqrt{S_{34}}$. For the case $m_3 + m_4 < m_1 + m_2$, these branch points would forbid extrapolation to the unphysical but interesting region $(m_3 + m_4) < \sqrt{S_{34}} < (m_1 + m_2)$. (For example, in the measurement of $K + K \to \pi + \pi$ by extrapolating $K + N \to Y + \pi + \pi$.)

These branch-point singularities can be avoided by using the invariant momentum transfer S_{14} instead of S_{56} as the variable of extrapolation. Inverting Eq. (4), we express S_{56} as a function of S_{14} for fixed S_{34} and $\cos\theta_{14}$, and substitute it in Eq. (3).

Square root branch points in S_{14} appear now explicitly in Eq. (3), and can be treated exactly. Another alternative is to keep S_{14} instead of $\cos\theta_{14}$ fixed. Since the ranges of values of S_{14} in the physical region of Reactions (1) and (3) are not the same, it will be necessary to perform a second extrapolation, this time for $\frac{d\sigma_0}{d\Omega_4}$ as a function of S_{14} (i.e., $\cos\theta_{14}$), in order to obtain the differential scattering cross section in the nonoverlapping region. On the other hand, this method allows us by extrapolation in S_{56} to obtain the differential cross section $\frac{d\sigma_0}{d\Omega_4}$ in an unphysical range of S_{14}.

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2. C. Goebel, Phys. Rev. Letters 1, 337 (1958).