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DIFFERENTIAL SCATTERING CROSS SECTIONS OF UNSTABLE PARTICLES

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September 22, 1960

ON THE EXTRAPOLATION METHOD TO DETERMINE  
DIFFERENTIAL SCATTERING CROSS SECTIONS OF UNSTABLE PARTICLES\*

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September 22, 1960

Chew and Low<sup>1</sup> and Goebel<sup>2</sup> have discussed a method to measure the cross sections for scattering off particles such as pions and neutrons which are not available as free targets in the laboratory. The method consists of extrapolating the cross section for a related but measurable reaction as a function of one of the invariant momentum transfers to a pole of the S matrix for this reaction. The success of the extrapolation depends on the location and strength of other less-well-known singularities of the S matrix. In this note we want to point out the existence of two branch points which appear in the Chew-Low prescription for extrapolation to obtain the differential cross section of the unstable particle. These branch points are due to the constraint of fixed scattering angle, and disappear only when the integration over angle to obtain total cross sections is carried out. We will indicate how to modify the extrapolation to avoid this difficulty.

Suppose we want to measure the differential scattering cross section for the process

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$$a_1 + a_2 \rightarrow a_3 + a_4, \quad (1)$$

where  $a_2$  is emitted virtually by the process

$$a_6 \rightarrow a_2 + a_3. \quad (2)$$

Then, according to Chew and Low, we measure the cross section for the reaction

$$a_1 + a_6 \rightarrow a_3 + a_3 + a_4$$

and extrapolate it in the momentum transfer  $S_{56} = (q_6 - q_3)^2$  to the value  $S_{26} = m_2^2$ , keeping fixed  $S_{34} = (q_3 + q_4)^2$ , the square of the total energy of  $a_3$  and  $a_4$ , and  $\cos \theta_{14}$ , the cosine of the angle between  $a_1$  and  $a_4$ , both in the rest frame of  $a_3$  and  $a_4$ . The variables  $q_1$  and  $m_1$  refer to the four-momentum and mass of the particle  $a_1$ . In the limit  $S_{56} \rightarrow m_2^2$ , the laboratory-system cross section for this reaction takes the form<sup>1</sup>

$$d\sigma = \frac{\Gamma^2}{4\pi} \frac{m_3}{m_6 q_1} \frac{((S_{34} - (m_1 - m_2)^2)(S_{34} - (m_1 + m_2)^2))^{1/2}}{(S_{56} - m_2^2)^2} \times \frac{d\sigma_0}{d\Omega_4} (S_{34}, S_{14}) d\Omega_4 dS_{34} dS_{56}, \quad (3)$$

where  $\frac{d\sigma_0}{d\Omega_4}$  is the differential cross section for Process (1) and  $\sqrt{4\pi} \Gamma$  is the amplitude for Process (2) (coupling constant). For fixed  $S_{34}$  and  $\cos \theta_{14}$  the invariant momentum transfer  $S_{14} = (q_1 - q_4)^2$  is a function of  $S_{56}$ .

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$$\begin{aligned}
S_{14} = & m_1^2 + m_4^2 - \frac{1}{2S_{34}} \left[ (S_{34} + m_1^2 - S_{56})(S_{34} + m_4^2 - m_3^2) \right. \\
& - (S_{34} - (m_3 - m_4)^2)(S_{34} - (m_3 + m_4)^2) \\
& \left. \times [S_{56} - (\sqrt{S_{34}} - m_1)^2][S_{56} - (\sqrt{S_{34}} + m_1)^2] \right]^{1/2} \cos \theta_{14}.
\end{aligned}$$

Evidently  $S_{14}$  is analytic in  $S_{56}$  except for square root branch points at  $S_{56} = (\sqrt{S_{34}} \pm m_1)^2$ . Since  $\frac{d\sigma_0}{d\Omega_4}$  is analytic in  $S_{14}$  it has also branch points at the same location when considered as a function of  $S_{56}$ . However, integration over the scattering angle  $\theta_{14}$  to obtain the total cross section removes these branch points. This can be seen, for instance, by expanding  $\frac{d\sigma_0}{d\Omega_4}$  in a Taylor series in  $\sqrt{S_{14}}$ , noticing that only even powers of  $\cos \theta_{14}$  contribute to the integration.

We note that the minimum experimental value of  $\sqrt{S_{34}}$  is  $(m_3 + m_4)$ . On the other hand, in order that the branch points do not lie between the measured values of  $S_{56}$ , ( $S_{56} < 0$ ) and its value at the pole ( $S_{56} = m_2^2$ ) it is necessary to consider  $(m_1 + m_2) < \sqrt{S_{34}}$ . For the case  $m_3 + m_4 < m_1 + m_2$ , these branch points would forbid extrapolation to the unphysical but interesting region  $(m_3 + m_4) < \sqrt{S_{34}} < (m_1 + m_2)$ . (For example, in the measurement of  $K + K \rightarrow \pi + \pi$  by extrapolating  $K + \bar{K} \rightarrow Y + \pi + \pi$ .)

These branch-point singularities can be avoided by using the invariant momentum transfer  $S_{14}$  instead of  $S_{56}$  as the variable of extrapolation. Inverting Eq. (4), we express  $S_{56}$  as a function of  $S_{14}$  for fixed  $S_{34}$  and  $\cos \theta_{14}$ , and substitute it in Eq. (3).

Square root branch points in  $S_{14}$  appear now explicitly in Eq. (3), and can be treated exactly. Another alternative is to keep  $S_{14}$  instead of  $\cos \theta_{14}$  fixed. Since the ranges of values of  $S_{14}$  in the physical region of Reactions (1) and (3) are not the same, it will be necessary to perform a second extrapolation, this time for  $\frac{ds_0}{ds_4}$  as a function of  $S_{14}$  (i.e.,  $\cos \theta_{14}$ ), in order to obtain the differential scattering cross section in the nonoverlapping region. On the other hand, this method allows us by extrapolation in  $S_{56}$  to obtain the differential cross section  $\frac{ds_0}{ds_4}$  in an unphysical range of  $S_{14}$ .

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#### REFERENCES

1. G. Chew and F. Low, Phys. Rev. 113, 1640 (1959).
2. C. Goebel, Phys. Rev. Letters 1, 557 (1958).