Title
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There is no dearth of experimental evidence to indicate that real space-time is a four-dimensional manifold. At present there is much less certainty in attributing dimensions to the charge space of heavy particles, and it may be worth while to review various hypotheses and the meager experimental evidence in this regard. The Clebsch-Gordon coefficients for I-spin that appear in the π - N (nucleon) scattering resonance imply at least a two-dimensional space for unitary transformations,[1] which are

[1] E.g., B. L. van der Waerden, Gruppentheoretische Methode, Springer, Berlin (1932).

conveniently represented in the usual terms of rotations in a three-dimensional space.

Some formal convenience attaches to viewing the charge displacement number $a = q - I_z$ of heavy particles as the third component of an independent vector $\underset{\sim}{A}$ in charge space.[2] If $\underset{\sim}{A}$ is to be independent of $\underset{\sim}{I}$,

[2] D. C. Peaslee, in press.

there must be added an independent pair of coordinates for the unitary transformations corresponding to rotations of $\underset{\sim}{A}$, implying a charge space

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of at least four dimensions.[3] If one generalizes by taking scalar charge

[3] A. Pais, Proc. Nat. Acad. Sci. U.S. 40, 484 (1954); A. Salam and J. C. Polkinghorne, Nuovo cimento 2, 685 (1955).

equations always to be the z-component of vector equations, then the proposal[4] for writing $q = I_z + a + b$ is tantamount to introducing a third independent vector $\underline{B}$ in charge space.[5] This requires another pair

[4] R. G. Sachs and S. B. Treiman, Nuovo cimento 2, 1331 (1955).

[5] The introduction of three independent vectors in charge space has been independently suggested by O. Hara and Y. Fujii (private communication).

of independent coordinates for the associated unitary transformations and raises the minimum dimensionality of charge space to six. There is in principle no end to this process: one can postulate additional charge vectors $\underline{C}$, $\underline{D}$, ... ; for n such independent vectors the minimum dimensionality of charge space is $2n$, and there will be $(n - 1)$ distinct hierarchies of strange particles.

To illustrate simply how each independent spin vector requires the addition of two more dimensions, consider the Dirac equation in $2n$ dimensions. Let

$$\gamma_A \gamma_B + \gamma_B \gamma_A = 2\delta_{AB} \qquad A, B = 1 \ldots 2n \tag{1}$$

be matrices for linearizing a quadratic form in $2n$ dimensions. Adjoin to this a two-dimensional space with associated Pauli spin operators σ_x, σ_y, σ_z and corresponding 2×2 unit matrix 1. Then define

$$\Gamma_A = 1\,\gamma_A\,, \qquad A = 1 \ldots 2n - 1$$

$$\Gamma_n, \; \Gamma_{n+1}, \; \Gamma_{n+2} = \sigma_x \gamma_n\,, \; \sigma_y \gamma_n\,, \; \sigma_z \gamma_n \tag{2}$$

so that

$$\Gamma_A \Gamma_B + \Gamma_B \Gamma_A = 2\,\delta_{AB}\,, \quad A, B = 1 \ldots 2n + 2\,. \tag{3}$$

The Γ_A are the Dirac matrices for $2n + 2$ dimensions; for $n = 1$ this procedure corresponds to the $\vec{\rho}$, $\vec{\sigma}$ formulation of the usual Dirac equation. The customary arguments for the four- and six-dimensional[6] cases

[6] D. C. Peaslee, Phys. Rev. 84, 373 (1951).

can be repeated to show that all irreducible representations of Eq. (3) are equivalent to Eq. (2). Thus each increase of dimensionality by two units allows the introduction of an independent spin vector $\underline{\sigma}$; in each such spin space higher spin values follow by vectorial addition of a sufficient number of basic spins $\frac{1}{2}$.

The same procedure illustrates how spaces of $2n + 1$ dimensions also allow at most n independent spin vectors. For by defining $\gamma_{2n+1} = \gamma_1 \gamma_2 \ldots \gamma_{2n}$, we can write Eq. (1) for $A, B = 1 \ldots 2n + 1$ without introducing a new Pauli spin $\vec{\sigma}$. The number of independent spin vectors accordingly increases only with the even-dimensional spaces.

Experimental evidence on the dimensionality of charge space becomes increasingly tenuous with increasing n. A number of advantages appear for four-dimensional charge space with two independent charge vectors,[2,3] and there are several possibilities for experimental study. The possibility of some five-dimensional charge symmetry[7] is not yet ruled out, but would require

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[7] R. W. King and D. C. Peaslee, in press.

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the Λ and N to have identical spins and parities. The six-dimensional hypothesis[4] would require increased statistics on anomalous V-decays for verification.

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