# Quantifier Interpretation and Syllogistic Reasoning: an Individual Differences Account

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## Abstract

It is frequently assumed that interpretational errors can explain reasoning errors. However, the evidence for this position has heretofore been less than convincing. Newstead (1995) failed to show expected relations between Gricean implicatures (Grice, 1975) and reasoning errors, and different measures of illicit conversion (Begg & Denny, 1969; Chapman & Chapman, 1959) frequently fail to correlate in the expected fashion (Newstead, 1989; 1990). This paper examines the relation between interpretation and reasoning using the more configurational approach to classifying subjects' interpretation patterns, described in Stenning & Cox (1995). There it is shown that subjects' interpretational errors tend to fall into clusters of properties defined in terms of *rashness*, *hesitancy* and the subject/predicate structure of inferences. First we show that interpretations classified by illicit conversion errors, though correlated with fallacious reasoning, are equally correlated with errors which cannot be due to conversion of premisses. Then we explore how the alternative method of subject profiling in terms of hesitancy, rashness and subject/predicate affects syllogistic reasoning performance, through analysis in terms of both general reasoning accuracy and the Figural Effect (Johnson-Laird & Bara, 1984). We show that subjects assessed as rash on the interpretation tasks show consistent characteristic error patterns on the syllogistic reasoning task, and that hesitancy, and possibly rashness, interact with the Figural Effect.

## Introduction

How do subjects' explicitly articulated interpretations of the logical force of statements relate to their behaviour when reasoning with these statements? This question is of some theoretical importance for the psychology of reasoning, and cognition in general. The possibility of a link between interpretation and reasoning errors has been used by authors such as Henle (1962) to rationalise reasoning error. There are extensive literatures on interpretation and on reasoning, and it is of some practical and theoretical importance to have a consistent account of data which might naively be expected to be measuring the same structures and processes. But the question is also of practical importance for formal education. Teaching logic involves bringing implicit knowledge of native language and its pragmatics into the explicit focus of attention. Interpretation tasks can be viewed as demands on the subject to explicitly externalise logical relations. Some reasoning tasks may be closer than others to engaging implicit modes of problem solving.

Newstead (1989; 1990; 1995) has extensively explored the commonest theories of the relation between interpretation and reasoning tasks. He investigates the theories of Illicit Conversion, of the logically unconvertible 'All' and 'Some... not' premiss types, and Gricean implicature (Grice, 1975), which predicts inference patterns based on the Maxim of Quantity. He asks whether these 'fallacious' inference patterns, both of which are evidenced in interpretation tasks, are also responsible for errors in syllogistic reasoning tasks, as has frequently been assumed. His general conclusions from analysis of existing data and new experiments are that the relation, between Illicit Conversion errors (1989; 1990) and Gricean errors (1995) in interpretation tasks, and the errors predicted by these theories in syllogistic reasoning tasks, is at best not substantiated and at worst non-existent. His explanation of the lack of the expected transparent relation is expressed in terms of Mental Models theory (Johnson-Laird, 1983), and is basically a depth of processing explanation. Syllogistic reasoning, a logically more complex task than the 'immediate inference' task used in interpretation studies, engages the subject in deeper processing, and when this happens the Gricean 'delusions' drop away.

Our purpose here is to try to arrive at a more positive account of the relation between interpretation and reasoning. We do this on the basis of a more general framework for understanding interpretation data, and for the separation of different individual differences in 'styles' of interpretation and reasoning. Stenning & Cox (1995) questioned the logical basis of the standard 'immediate inference' task used in Newstead (1989) and Newstead (1990). They showed that asking questions that allowed the subject to distinguish *logical independence* from the situation where either a proposition or its negation followed from a premiss, revealed several strong, generalised but distinct individual patterns of response. They distinguished *hesitant* reasoners who tended to respond that candidate conclusions were independent of premisses, from *rash* reasoners who tended to respond either that the candidate conclusion followed, or that its negation followed. But most strikingly, they showed that it was the relation between the subject/predicate structure in premiss and candidate conclusion that most powerfully affected patterns of response—more powerfully, in fact, than quantifier identity. Thus they could distinguish a few groups of subjects on the basis of whether they were rash or hesitant on questions (where *Q1* and *Q2* are quantifiers and may or may not be the same, and *a* and *b* are the terms) of the form *Q1 ab. Does it follow that Q2 ab?* and the form *Q1 ab. Does it follow that Q2 ba?*. The former questions were called *in-place* and the latter *out-of-place* questions, since in the former, the terms occurred in the putative conclusion in the same order as in the premiss, whereas in the latter, the term order was reversed[1].

There are obvious *descriptive* advantages to looking at whole configurations of interpretation behaviour, and to classifying different kinds of subject by configurations of response. Gricean implicature and illicit conversion are generally diagnosed by looking at a small number of responses which leave out the vast majority of the interpretation data. It is also clear from syllogistic reasoning data that more than one kind of subject should be distinguished by qualitatively different reasoning patterns (Ford, 1994; Polk & Newell, 1988). But the approach also offers *theoretical* advantages. The psychological literature on interpretation and reasoning has overwhelmingly focused on errors of *commission*—patterns of inference invalid by the standard of logic. But a configurational approach to interpretation emphasises that there are also errors of *omission*—failures to make inferences which are logically valid—and that there are direct relations between commission on one question and omission on another.

For example, take the two candidate inference patterns *All A are B, so all B are A* and *Some A are B, so some B are A*. Acceptance of the former pattern is well known as 'illicit conversion' error, but failure to accept the latter, perfectly valid inference is unremarked in the psychological literature. But subjects who are generally hesitant toward out-of-place questions will get the first right and the second wrong, whereas subjects who are generally rash on out-of-place questions will get the first wrong and the second right. In Stenning & Cox's (1995) data, both of these patterns linking commission and omission were common, and were exhibited regardless of the quantifiers, though by different groups of subjects. In fact, what drives this interpretation data is attitudes to subject/predicate structuring (in-place vs. out-of-place) in the questioned inferences.

[1] We stress that this definition of rashness should not be taken to imply the possession of some general personality trait, not least because we can distinguish the in-place and out-of-place varieties.

In this paper we seek to explore the relation between this configurational approach to interpretation data and syllogistic reasoning performance. We present a new set of data collected to relate interpretation data to reasoning data, in order to compare the effects of illicit conversion and rashness/hesitancy on reasoning performance. Illicit conversion errors are necessarily restricted to a subset of problems, so we can compare rates of invalid conclusions in conversion-susceptible and unsusceptible problems, for subjects assessed as converters or non-converters in the interpretation tasks.

The link between interpretation and reasoning accuracy is then explored in terms of the rashness/hesitancy configurations. We aim to demonstrate how groups of subjects identified on the basis of their interpretation configurations display different error patterns in the syllogistic reasoning task. Since rashness is here defined as the tendency to infer *more* than is logically valid, i.e. to gain information, rash reasoners should be more liable to produce invalid conclusions than non-rash reasoners; similarly, since hesitancy is the tendency to infer *less* than is logically valid, or to lose information, hesitant reasoners should be more likely to decide that there is No Valid Conclusion (NVC) than non-hesitant reasoners.

Recent work by Ford (1994) raises the possibility of investigating individual differences in relation not only to error tendencies, but also to response biases such as the Figural Effect (Johnson-Laird & Bara, 1984), a robust association linking subject/predicate structuring in premisses to that evidenced in subjects' conclusions. The typical finding is that problems in the ab/bc figure tend to give rise to ac conclusions, even when the converted form is also valid, whereas ba/cb problems tend to give rise to ca conclusions, and ab/cb and ba/bc show no overall bias. Stenning & Yule (1996) have emphasised the importance of problem semantics in the explanation of such effects, so we are interested in whether the Figural Effect might be sensitive to differences in premiss interpretation.

## Method

### Subjects

120 psychology undergraduates from Edinburgh University participated in the interpretation tasks during a Cognitive Psychology lecture. None had any previous training in logic. Subsequently, 40 of these, assessed as EC-converters (6), II-converters (16), both-converters (7) and non-converters (11) on the basis of their performance in the interpretation tasks, were paid £7.50 to participate in the syllogistic reasoning part of the experiment.

### Materials and Procedure

**Interpretation Tasks** The interpretation tasks were the same as those described in Stenning & Cox (1995).

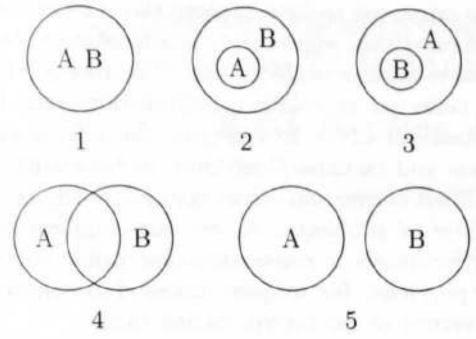


Figure 1: Euler Circle (EC) Task stimuli.

Two questionnaires were used, and their order of presentation was counterbalanced across subjects such that half received the graphical (EC) condition first and half received the sentential (II) condition first.

The Euler Circle (EC) or 'graphical' condition consisted of the five diagrams used by Newstead (1989), and was similar to the EC task presented in that paper (see Figure 1). Diagram 1 was the identity relation (circle 'A' and circle 'B' superimposed); diagram 2 showed a small circle 'A' inside a larger circle 'B'; diagram 3 showed a small circle 'B' inside a larger circle 'A'; diagram 4 showed circles 'A' and 'B' intersecting without either including the other and diagram 5 showed two non-overlapping, disjoint circles 'A' and 'B'. Below the diagrams the 4 premisses were listed in the order ALL, NO, SOME, SOME... NOT. Adjacent to each premiss were the numbers 1 to 5. Subjects were instructed to circle the number(s) of the diagram(s) of which the sentence was true.

The sentential condition was similar to the immediate inference (II) task described by Newstead (1989) with the exception that an additional response option ("Can't tell") was provided. As in Newstead (1989), the questionnaire consisted of four pages. At the top of each page one of the four standard quantified statements was displayed: All A's are B's; No A's are B's; Some A's are B's and Some A's are not B's. These were the premiss statements. Beneath the stimulus statements the four quantified statements were listed (All A's are B's etc.) and the converses of these (All B's are A's etc). These were the response statements. Alongside the eight response statements were response options 'T' (true), 'F' (false) and 'Can't tell'. The order of the four stimulus statement pages was randomised across subjects.

Subjects were instructed to assume the statement at the top of the page was true, and to indicate the truth, falsity or independence of each of the response statements by circling 'T', 'F' or 'can't tell' beside it.

In both EC and II tasks, subjects were instructed to interpret 'some' to mean 'at least one and possibly all'.

Subjects were allowed as much time as they needed to complete the tasks (approximately 20 minutes).

**Syllogistic Reasoning Task** The stimulus materials for the syllogistic reasoning task were booklets containing each of the 64 distinct premiss pairs, in random order, one per page. The terms of the problems were 'A's', 'B's' and 'C's', where the end terms, 'A's' and 'C's', appeared in the first and second premisses respectively, and the middle term 'B's' appeared in both premisses. Subjects were instructed to assume the premisses were true, and to determine whether there was any statement, relating the end terms A and C using one of the quantifiers ALL, NONE, SOME and SOME... NOT, which must then be true. If so, they were instructed to write it in the space on the page below the premisses, and otherwise to write 'No valid conclusion'

Subjects were instructed to assume that some A's, B's and C's existed, and to interpret 'some' to mean 'at least one and possibly all'

Subjects were allowed as much time as they needed to complete the tasks.

## Results

### Illicit conversion and inference

One way to evaluate claims about illicit conversion in syllogistic reasoning is to divide problems into two groups: those which, all else being equal, are susceptible to conversion errors in the sense that extra conclusions follow if illicit conversion is used, and those which are not. We would expect, on the basis of the illicit conversion hypothesis, that subjects assessed as converters on the interpretation tasks, would produce disproportionately high rates of invalid conclusions on conversion-susceptible problems compared to non-converters. All else being equal, there should be no such differences for conversion-unsusceptible problems. So we expect interactions between problem type and conversion group. Overall differences in invalid conclusion rates between problem types are not of interest, since they have different numbers of problems with and without valid conclusions. Table 1 shows the mean percentages of invalid conclusions to problems which are susceptible to illicit conversion errors, and to those which are not, for each of the four conversion-groups of subjects.

To test these predictions, a mixed-model ANOVA was conducted, using percentage invalid conclusions as the dependent variable, and the independent variables problem type (within-subjects conversion-susceptible and unsusceptible problems), EC-conversion and II-conversion (both between subjects - converters and non-converters). Both EC-conversion ($F(1,36) = 9.50$, $p < 0.004$) and II-conversion ($F(1,36) = 6.14$, $p < 0.02$) significantly affected overall percentages of invalid conclusions. Also, there was a significant difference between unsusceptible and susceptible problem types ($F(1,36) = 26.25$, $p < 0.0001$), but this does not concern us here.

| | Conversion group | | | |
|---|---|---|---|---|
| Problem type | Neither | EC | II | Both |
| Unsusceptible | 27.64 | 51.00 | 45.94 | 61.86 |
| Susceptible | 42.73 | 71.33 | 70.94 | 77.14 |
| *N Ss* | 11 | 6 | 16 | 7 |

Table 1: Mean percentages of incorrect conclusions to conversion-susceptible (N=20 problems) and conversion-unsusceptible problems (N=44 problems), for each conversion group.

Notably, there were *no* significant interactions: problem type × EC-conversion ($F(1,36) = 0.03$, *n.s.*); problem type × II-conversion ($F(1,36) = 0.04$, *n.s.*); problem type × II-conversion × EC-conversion ($F(1,36) = 1.25$, *n.s.*).

Therefore, there is no evidence that subjects who illicitly convert premisses in the interpretation task make any more illicit conversion errors in the reasoning task, contrary to hypothesis. However, both EC-converters and II-converters produce more invalid conclusions *overall* in the reasoning task.

## Subject Profiles

Assignment of subjects to profile groups was conducted in the same manner as reported in Stenning & Cox (1995), using the data from the Immediate Inference task. We label a tendency to respond 'can't tell', where T or F is correct, *hesitancy* ($h$) and a tendency to respond either T or F, where 'can't tell' is correct, *rashness* ($r$). These categories can be defined within Q AB questions and Q BA questions, giving a further subdivision into *in-place* ($i$) and *out-of-place* ($o$), where the conclusion sentence preserves or inverts subject-predicate order respectively. Loglinear modelling permitted cutoff points on each dimension to be iteratively adjusted until residuals were minimized. As in Stenning & Cox (1995), there were no subjects who were hesitant on in-place questions, so the three binary dimensions $ro$ (rash out-of-place), $ho$ (hesitant out-of-place) and $ri$ (rash in-place) were sufficient to fit the data. The resulting assignment of subjects to rashness/hesitancy categories is displayed in Figure 2.

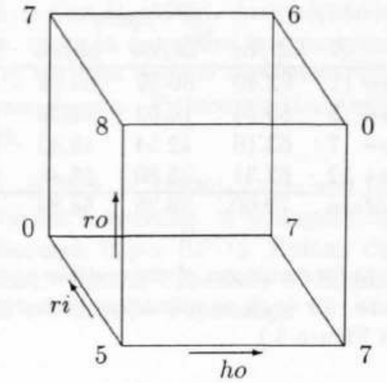


Figure 2: Frequencies of subjects classified as rash or hesitant on out-of-place questions, and rash on in-place questions

## Subject profiles and inference

**Reasoning accuracy** Table 2 shows the effects of the three rashness/hesitancy variables $ro$, $ho$ and $ri$ on percentages of each of four response accuracy categories in the syllogistic reasoning task: correct (valid) conclusions, incorrect (invalid) conclusions, correct NVC responses and incorrect NVC responses. The results of unrelated t-tests for each comparison are also shown. For each of the rashness/hesitancy properties, we expect possession of the property to decrease the number of correct conclusions; the differences are all in the predicted direction, but significant for only the rash properties, $ri$ and $ro$. Rash reasoners should produce more conclusions and fewer NVC responses generally; the predicted differences for incorrect conclusions are both significant, but the NVC differences, while in the predicted direction, are mostly insignificant, except that $ri$ does significantly reduce numbers of correct NVC conclusions. Hesitant reasoners, on the other hand, should produce fewer incorrect conclusions, and more NVC responses overall; however, although the differences are in the right directions, none of these effects reach significance at the $p < 0.05$ level.

| | N | Correct Conc. | Incorrect Conc. | Correct NVC | Incorrect NVC |
|---|---|---|---|---|---|
| $ro-$ | 19 | 25.08% | 40.95% | 26.89% | 3.95% |
| $ro+$ | 21 | 20.46% | 54.84% | 19.05% | 3.72% |
| t (38df) | | 2.069 | -2.112 | 1.638 | 0.166 |
| 1-tail p | | 0.025 | 0.025 | 0.1 | n.s. |
| $ho-$ | 20 | 23.83% | 49.37% | 22.19% | 2.73% |
| $ho+$ | 20 | 21.48% | 47.11% | 23.36% | 4.92% |
| t (38df) | | 1.009 | 0.327 | -0.237 | -1.654 |
| 1-tail p | | n.s. | n.s. | n.s. | 0.1 |
| $ri-$ | 20 | 25.55% | 38.98% | 28.98% | 3.20% |
| $ri+$ | 20 | 19.77% | 57.50% | 16.56% | 4.45% |
| t (38df) | | 2.679 | -2.960 | 2.749 | -0.923 |
| 1-tail p | | 0.01 | 0.005 | 0.005 | n.s. |

Table 2: The effects of rashness/hesitancy on percentages of each of four response accuracy categories.

**The Figural Effect** To a certain extent, the Figural Effect is confounded with the effects of validity, owing to overall differences, between figures, in numbers of valid conclusions with each possible term order. This problem can be overcome by including only *convertible conclusions* in the analysis, defining these as the set of

| | N | ab/bc | ba/cb | ab/cb | ba/bc |
|---|---|---|---|---|---|
| | | *figure* | | | |
| *ri− ho−* | 11 | 82.40 | 30.39 | 50.62 | 56.88 |
| *ri− ho+* | 5 | 85.74 | 16.62 | 64.35 | 58.63 |
| *ri+ ho−* | 7 | 63.16 | 42.54 | 48.43 | 42.82 |
| *ri+ ho+* | 12 | 82.31 | 25.80 | 58.48 | 56.37 |
| *Mean* | | 79.00 | 29.28 | 54.84 | 54.15 |

Table 3: Mean percentages of convertible ac conclusions in each figure, for each combination of levels of *ri* and *ho* (see also Figure 3.)

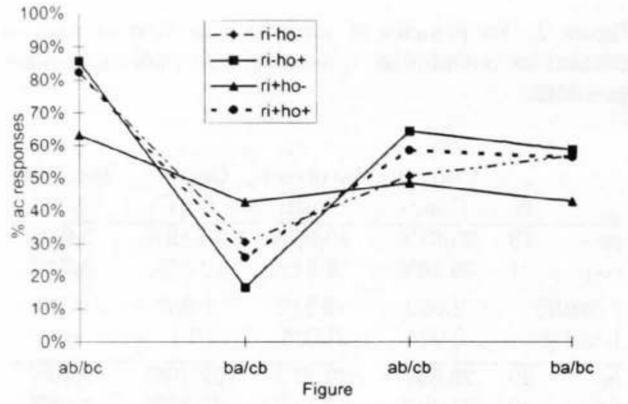


Figure 3: Chart showing effects of *ri* and *ho* on the Figural Effect (data as in Table 3.)

valid conclusions to problems whose strongest conclusions are validly convertible, plus the set of all invalid conclusions. Table 3 shows mean percentages of convertible conclusions with ac term order in each figure, for each combination of *ri* and *ho* (data from 5 subjects were omitted owing to missing values). The same data are plotted on Figure 3, which shows that the most extreme Figural Effect is displayed by $ri-ho+$ subjects, whereas $ri+ho-$ subjects display the least extreme Figural Effect. Subjects who are both ($ri+ho+$) or neither ($ri-ho-$) display intermediate Figural Effects, so the effects of *ri* and *ho* appear to be in opposition to one another.

A three-way mixed-model ANOVA was conducted to assess the effects, on percentage of convertible ac conclusions, of figure (repeated measures with levels ab/bc, ba/cb, ab/cb, ba/bc) and the between-groups variables *ho* and *ri* (with levels *ho+* and *ho−*, and *ri+* and *ri−* respectively.) Since the data departed from sphericity assumptions ($p < 0.004$), the more conservative Greenhouse-Geisser adjusted tail probabilities were used for the ANOVA. First of all, there were no effects, on overall percentage of ac conclusions, of *ri* ($F(1,31) = 0.37$, *n.s.*) or *ho* ($F(1,31) = 0.54$, *n.s.*), nor was there an interaction between them ($F(1,31) = 0.25$, *n.s.*). There *was* a main effect of figure ($F(3,93) = 39.79$, *adjusted* $p < 0.0001$), and there was a significant figure × *ho* interaction ($F(3,93) = 4.03$, *adjusted* $p < 0.02$). The figure × *ri* interaction did not reach significance at the 0.05 level ($F(3,93) = 2.29$, *adjusted* $p > 0.10$), and there was no significant three-way interaction between the variables ($F(3,93) = 0.61$, *n.s.*).

## Conclusions

Traditional approaches to the explanation of reasoning error by appeal to interpretation errors, based on specific 'fallacies', such as the theories of Gricean implicature (Grice, 1975) and Illicit Conversion (Begg & Denny, 1969; Chapman & Chapman, 1959), have had only limited success. Newstead's (1995) observations that Gricean implicatures are not strongly reflected in reasoning data are here generalised to illicit conversion. Tendency to illicitly convert in interpretation tasks is a general predictor of poor reasoning performance, but does not differentially predict illicit conversion errors during reasoning. This general failure of traditional approaches need not be taken to imply that the entire enterprise is misguided however. These theories are simply too specific, when what is required is a more general and *empirical* approach to the question of (mis)interpretation.

When Stenning & Cox's (1995) configurational approach to single-premiss interpretation is applied to reasoning data, more encouraging results emerge. Our preliminary analysis shows measures of syllogistic reasoning accuracy to be generally affected, in the predicted directions, by well-motivated properties based on the single-premiss interpretation data. In particular, 'rashness', as assessed on the basis of single-premiss interpretation, has a recognisable correlate in more complex reasoning performance, which can be simply summarised as a tendency to draw invalid conclusions.

The finding that the individual difference categories interact with the Figural Effect is also an interesting result. Almost all published experiments are analysed without regard for individual differences, and all show extremely robust Figural Effects (Johnson-Laird & Bara, 1984). However, our results identify a group of subjects (actually the ones who are rash both in-place and out-of-place, but not hesitant) who *fail* to show the effect at all. Hesitancy in exchanging subject and predicate, by contrast, results in a much strengthened Figural Effect. On a moment's reflection, these results are not very surprising—subjects who are unwilling to switch subject and predicate in premisses are 'locked in' to Figural conclusions by the superficial structure of syllogistic premiss pairs, whereas those who switch are not so constrained. Although proponents of Mental Models theory might claim that these results can be accommodated in their account of the Figural Effect, this could only be accomplished at the expense of their theory's explanatory power.

Recent semantic analysis of the syllogism by Stenning & Yule (1996) shows that the Figural Effect may usefully be considered as high-order invariant of a wide range of reasoning strategies. The Figural Effect's robustness, we argue, is not due to a unitary reasoning mechanism in the style of Mental Models, but rather to commonalities between numerous disparate implementations of reasoning which tend to share certain abstract characteristics. This type of account leaves room for individual differences in reasoning style, a topic which has been sadly neglected in the reasoning literature, despite the considerable intuitive plausibility of the idea and the increasing availability of empirical evidence for the existence of such cognitive differences.

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