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ABSTRACT

The j-j coupling shell model implies the existence of certain geometrical relations among the spectra of neighboring nuclei. We have considered one such relationship between states of a nucleus that is one proton or neutron away from a closed shell with states of the corresponding closed-shell nucleus. Use of this relationship has enabled us to predict excited-state spins for several nuclei, most of which are in the vicinity of mass number 60. In two cases we predict the existence of states that have not been observed.

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I. INTRODUCTION

It has recently been observed that the j-j coupling shell model implies the existence of certain geometrical relationships among the binding energies and excited states of neighboring nuclides.[1] Such relationships have been useful in the study of the ground- and excited-state properties of nuclei that could be characterized, to a good approximation, by pure (j-j coupled) nucleon configurations. In this paper we discuss a simple theorem relating the excitation spectra of certain nuclide pairs for which the limitation to pure configurations need no longer hold.

We consider a nucleus such that one kind of nucleon, say the protons, have a closed-shell configuration. The configuration of the neutrons is to be completely arbitrary. For such a nucleus we expect to find a set of energy levels corresponding to the different possible total angular momenta to which the neutron configuration can couple. The excitation energy of any such level is given by

$$\Delta E_{J_i} = \left\langle (j)^{2j+1} J_i \,\middle|\, V \,\middle|\, (j)^{2j+1} J_i \right\rangle - \left\langle (j)^{2j+1} J_o \,\middle|\, V \,\middle|\, (j)^{2j+1} J_o \right\rangle . \tag{1}$$

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The state vectors in this expression exhibit explicitly the last proton closed shell (or subshell), which is trivially vector-coupled to the neutron angular momentum. We denote the ground-state neutron angular momentum by J_o, and V denotes the sum of the two-body interaction terms.

Let us now split off one proton from the closed shell (the appropriate fractional parentage coefficient is unity) and recouple the single proton to the neutrons in order to form a new angular momentum J_3. J_3 is then vector-coupled to the remaining $2j$ protons to form the total angular momentum J_i (or J_o). If we rewrite Eq. (1) using the new representation of the state vectors, we discover that[2]

$$(2j+1)\Delta E_{J_i} = (2J_i+1)^{-1}\sum_{J_3}(2J_3+1)\langle j \; J_i, \; J_3 | V | j \; J_i, \; J_3\rangle - (2J_o+1)^{-1}\sum_{J'_3}(2J'_3+1)\langle j \; J_o, \; J'_3 | V | j \; J_o, \; J'_3\rangle \; . \tag{2}$$

The matrix elements that appear on the right-hand side of Eq. (2) are just the first-order energies of a nucleus that differs from the one under consideration by the addition or subtraction of a single proton in state j. It is to be emphasized that the neutron states J_i and J_o are states of the same configuration. It is clear that we are not concerned with the details of the neutron configuration, consequently it is unnecessary to make any assumptions as to the manner in which the particles are coupled.

We note in passing an alternative approach to Eq. (2) in terms of Slater integrals. The interaction between a single proton and the neutron group may be decomposed into a sum of scalar products of irreducible tensor operators[3] of the form

$$V = \sum_K f_K \underset{\sim}{T}^{(K)} \cdot \underset{\sim}{t}^{(K)} .$$

Carrying out the sum indicated by the right-hand side of Eq. (2) leads to the result that every term of V vanishes except for the term with K equal to zero. The proof of the identity is then closely related to the result that only the term containing f_o can survive in the matrix element evaluated for a closed proton shell (E_{J_i}).

The meaning of Eq. (2) should be readily apparent. For each level of the ground-state configuration of the original (closed-shell) nucleus there is a set of levels of an adjacent nucleus (closed shell plus or minus one) corresponding to the several ways in which the proton angular momentum j and the neutron angular momentum J_i can combine to form a resultant J_3. The spacing between the centers of gravity of two such sets is equal to the spacing of the corresponding neutron levels of the original nucleus.

In addition to the conditions already stated we have made use of certain fundamental assumptions pertaining to the model with which we are concerned. These are: (a) that there is j-j coupling for the closed-shell particles (the protons in our example), (b) that the n-p interaction is sufficiently weak for the first-order perturbation theory to be a good approximation, and (c) that the component neutron and proton angular momenta, J_i and j, are "good" quantum numbers in the sense of Condon and Shortley.[4] It would seem to follow that if we can find groups of nuclides for which the sum rule holds with some degree of precision we may conclude that assumptions (a) to (c) give a reasonable description of the properties of such nuclides.

The remainder of the paper is devoted to applications of Eq. (2). In Section II we attempt to predict certain excited states and excited-state spins for nuclides which we expect to satisfy the assumptions (a) to (c). Section III summarizes the results and conclusions

II. APPLICATIONS OF THE SUM RULE

Cr^{53} - Fe^{55} - Co^{56}

We discuss this triad in some detail because it leads to a rather vivid demonstration of both the possibilities and the weaknesses of the method.

The nucleus Fe^{55} is thought to have an $(f_{7/2})^{-2}$ proton configuration and a single $p_{3/2}$ neutron.[5] Excited states have been found at 0.413, 0.932, 1.327, 1.413, 1.84, and 2.17 Mev above ground.[6,7,8] The existence of the 1.84-Mev state is uncertain,[7] and the two members of the 1.33-1.41-Mev doublet have not been seen simultaneously,[7,8] leaving the possibility that these two states are really the same. Of the six listed states we expect four to arise from the coupling of the neutron to the protons when the latter are excited to spin 2. If is found that there are at least twelve different ways of assigning the four spins ($\frac{1}{2}$, 3/2, 5/2, 7/2) so that the center of gravity is at 1.41 Mev, the energy of the 2+ state[9] of Fe^{54}.

In order to resolve this ambiguity one observes that there are purely geometrical relations among the energies of the pairs Fe^{55}-Co^{56} and Fe^{55}-Cr^{53}. We let E_J be the excitation energy of a state of Fe^{55}

of angular momentum J, ϵ_J a state of Cr^{53}, and Δ_J a state of Co^{56} (taking the 4+ state at zero energy). In addition, E'_2 and ϵ'_2 are the excitation energies of the (first excited) 2+ states of Fe^{54} and Cr^{52}, respectively. By uncoupling one of the proton holes from the Fe^{55} wave function we obtain equations between the first-order Fe^{55} and Co^{56} energies. Solving for the Δ_J gives

$$\Delta_2 = \frac{7}{400} \left(15 E_{\frac{1}{2}} + 84 E_{3/2} + 161 E_{5/2} - 260 E'_2\right) ,$$

$$\Delta_3 = \frac{1}{40} \left(35 E_{\frac{1}{2}} - 12 E_{3/2} + 77 E_{5/2} - 100 E'_2\right) , \qquad (3)$$

$$\Delta_5 = \frac{5}{176} \left(-19 E_{\frac{1}{2}} - 36 E_{3/2} + 35 E_{5/2} + 20 E'_2\right) .$$

In an analogous manner one derives three equations relating the states of Cr^{53} to those of Co^{56}. Because the proton configuration of Cr^{53} has an equal number of holes and particles, however, the three equations for the Cr^{53} states can only be of rank two; otherwise it would be possible to prove that a hole-particle interaction is equal to a particle-particle interaction.[10] The linear dependence of the three equations allows us to deduce that in addition to the center-of-gravity theorem the chromium states must satisfy

$$5 \epsilon_{\frac{1}{2}} - 4 \epsilon_{3/2} - 21 \epsilon_{5/2} + 20 \epsilon'_2 = 0 . \qquad (4)$$

In addition it is now possible to relate the chromium and iron states and to find

$$E_{3/2} - E'_2 = \epsilon_{3/2} - \epsilon'_2 \ ,$$

$$20(\epsilon_{\frac{1}{2}} - \epsilon'_2) = 15\ E_{\frac{1}{2}} + 4\ E_{3/2} + 21\ E_{6/2} - 40\ E'_2 \ . \qquad (5)$$

By use of Eqs. (2) - (5) and the measured[9,11,12] spectra of $Cr^{52,53}$ and Co^{56} it seems to be possible to make unique spin assignments for the state of Fe^{55} as given in the first line of the table. We then predict from Eq. (3) that the states of Co^{56} are -0.07(2+), 0(4+), 1.1(5+), and 1.7(3+). Although this result is inconsistent with experiment in that the 4+ state is known to lie lowest,[13] we can account for the inconsistency by noting that Δ_2 is an extremely sensitive function of the Fe^{55} energies. If, in fact, we had chosen the 5/2 level of iron to have an energy of 1.90 instead of 1.84 Mev, a choice not inconsistent with experiment,[6] we would have found the cobalt 2+ state to lie about 100 Kev above the 4+ level. By Way[9] of comparison with experiment, Co^{56} has been tentatively assigned levels at 0.17, 0.96 and 1.75 Mev.

From Eqs. (2), (4), and (5) we now predict the levels of Cr^{53}. These are found to be 0.46(3/2), 1.41(7/2), 1.83(5/2), and 2.34(½) Mev. Experimentally, states are known to lie at 0.57, 1.29 and 2.31 Mev,[11] leaving us in the position of predicting a $5/2^-$ state at about 1.8 Mev in Cr^{53} (line 2 of the table).

It is of some interest to note that we could not find any consistent level schemes that included either the 0.97-Mev state of Cr^{53} or the 0.93-Mev state of Fe^{55}. Thus, our results are in agreement with the conclusions by French and Raz[14] that the former level (and therefore, we would conclude, both levels) arises from the single-particle excitation of a neutron to an $f_{5/2}$ state.[15] The other spin assignments in chromium are not inconsistent with the ℓ_n values calculated by Elwyn and Shull from their d-p stripping measurements.[11]

Some nuclei near Z = 28

Lines 3, 4, and 5 of the table show the results obtained by applying the theorem to three nuclei that lie near the closure of the 20-28 proton shell. In each of these three the supplementary evidence obtained from Coulomb excitation and β-decay experiments appears to vindicate our particular choices of level assignments--always provided, of course, that the model itself is realistic. To illustrate, we remark that the possibility was investigated of assigning the 1.114-Mev state of Cu^{65} as an $f_{5/2}$ single-particle excited state.[15] If this had been done it would have still been possible to satisfy Eq. (2) by including the 1.725-Mev[21] state as part of the quartet. We feel that this assignment is unlikely, however, because of the fact that the transition rate for the 1.114-Mev-to-ground transition is some forty times the single-particle value.[16] If our conclusions are correct, we have no explanation to offer for the absence of a low-lying $f_{5/2}$ proton single-particle excitation.

Zn^{69} and Y^{91}

The next two cases (lines 6 and 7 of table) were considered as a test of the supposition that there is a definite subshell closing at neutron or proton number 38.[5] For the neutron case the supposition appears to be consistent with the results of the center-of-gravity theorem for the pair Zn^{69}-Zn^{68}. This success encouraged us to predict an unobserved 2+ state in Sr^{90}, where the analogous proton configuration is found. If the similarity between neutrons and protons is really close, we should expect the 2+ level of Sr^{90} to have an excitation energy of 0.91 Mev in order that Zn^{69} and Y^{91} would have matching level sequences. There is some evidence for the similarity between neutrons and protons in that both Zn^{69} and Y^{91} appear to have long-lived 9/2 states at 440 and 991 kev, respectively.[9] We suppose this to be an indication of a single-particle g 9/2 excitation.

Au^{199}

We apply the center-of-gravity rule to the pair Au^{199}-Hg^{200} on the assumption that the neutron number 80 marks the closing of a subshell. The spin assignments in line 8 of the table appear to be consistent with the assumption that the ground state[5] of Pt^{199} is 3/2.

The pair Po^{209} - Po^{210}

The β-decay of At^{209} has recently been studied by Stoner.[17] Four cascade γ rays are seen which are attributed to the de-excitation of Po^{209}. These have energies of 780 (M1), 545 (M1 + E2), 195(M1), and 90.8 (E2) kev. The intensity of the 90.8-kev γ ray is quite low, leading one to suspect that only part of the de-excitation proceeds in this way. Using 90.8 kev as the spacing between the two highest levels and juggling the remaining three γ rays to get the best fit to the 2+ level[18] in Po^{210} (1.185 Mev) leads to the decay scheme shown in Fig. 1. The level at 780 kev we suppose to be a single-particle excitation to the $p_{3/2}$ state. The 1.61-Mev level could conceivably be one of the pair arising from the coupling of the spin-$\frac{1}{2}$ particle to the 4+ state of Po^{210}, which lies at 1.431 Mev.[18] If such an interpretation were correct we should also expect to find a 9/2 state with an excitation energy of about 1.3 Mev above ground. There is no evidence that such a state exists.

The case of Si^{29}

According to Mayer we might expect to find evidence for a closed subshell at 14 neutrons or protons.[5] In the light of this expectation it would appear that the pair Si^{28}-Si^{29} satisfies our requirement that one (Si^{28}) be characterized by a closed neutron subshell and that the other have a single neutron outside the closed subshell. On the other hand, one would naively expect the 14 protons that make up the silicon nucleus to similarly arrange themselves in a closed configuration. It would follow that this pair of isotopes is not amenable to analysis by our methods. We think it strange, therefore, that the two silicon isotopes do have excited states that seem to satisfy the center-of-gravity rule (line 9 of table).[19] The conclusion seems to be either that we are dealing with a numerological accident or that the proton and neutron configurations are really different in Si^{28}. If the latter conclusion can be substantiated it would seem to be of interest in connection with the understanding of Coulomb effects in the shell model.

DISCUSSION

We have seen that the j-j coupling shell model, if it is to be taken seriously, carries with itself certain implications concerning relationships among the excitation spectra of neighboring nuclides. It has been our purpose to exploit some of these relationships in an attempt to put the model to test. In so doing we have found ourselves in the position of predicting a sizable number of excited-state spins and, in two cases, the existence of unobserved states. If these predictions should be borne out by experiment then, manifestly, one must regard this as a triumph on the part of the model.

For the most part, consideration has been given to nuclei near mass number 60. This is because we were unable to find a sufficient amount of experimental information for nuclei lying near other closed shells than the ones at 28 and 38 neutrons or protons. It is in some respects unfortunate that this is so, because there is already appreciable evidence that the j-j coupling model gives a good account of itself for nuclei of atomic number up to 28.[1] One would be most interested in examining the situation near, for example, neutron or proton number 50.

There are, perhaps, two reasons why one should not be amazed if the elementary considerations discussed here should give a good account of the situation. In the first place one observes that the level spacings of most of the nuclei we have discussed tend to be sizeable; consequently, there would seem to be an excellent chance that assumption (b) of part I is applicable. Secondly, the nature of our analysis was such that we were concerned only with nuclei that had either closed neutron or closed proton shells. Considerations based upon the collective model[20] would seem to indicate that for such nuclei the shell model might be applicable even at relatively high mass numbers.

We should like to express our appreciation to Professor J. B. French, Professor I. Talmi, and Dr. C. Schwartz for valuable discussions. Dr. Jack M. Hollander, Dr. John O. Newton, and Dr. Donald Strominger have been of great help in their discussions of the experimental situation. And we are most grateful for the computational assistance of Mrs. Ardith Kenney.

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TABLE I

Some Nuclei to which the Center-of-Gravity Rule Has Been Applied

Nucleus	Energy levels (measured) and assigned spins	Comparison nucleus	Energy of 2+ state (predicted) (Mev)	Energy of 2+ state (measured) (Mev)
1. $_{26}Fe^{55}$	0(3/2), 0.413(3/2), 0.932(5/2)†, 1.413(7/2), 1.84(5/2), 2.17(½) (a),(b),(c)	$_{26}Fe^{54}$	1.42	1.41 (d)
2. $_{24}Cr^{53}$	0(3/2), 0.57(3/2), 0.97(5/2)†, 1.29(7/2), ~1.9(5/2)††, 2.31(½) (e),(m)	$_{24}Cr^{52}$	--	1.46 (n)
3. $_{27}Co^{59}$	0(7/2), 1.097(5/2), 1.189(7/2), 1.289(3/2), 1.432(9/2), 1.458(11/2) (e),(f)	$_{28}Ni^{60}$	1.327	1.329 (g)
4. $_{29}Cu^{63}$	0(3/2), 0.669(½), 0.961(5/2), 1.325(7/2), 1.411(3/2) (g),(h)	$_{28}Ni^{62}$	1.167	1.171 (i)
5. $_{29}Cu^{65}$	0(3/2), 0.770(½), 1.114(5/2), 1.482(7/2), 1.623(3/2) (e),(h)	$_{28}Ni^{64}$	1.329	1.34 (e)
6. $_{30}Zn^{69}$	0(½), 0.440(9/2)†, 0.77(5/2), 1.6(3/2) (e)	$_{30}Zn^{68}$	1.1	1.10 (e)
7. $_{39}Y^{91}$	0(½), 0.551(9/2)†, 0.65(5/2), 1.3(3/2) (e)	$_{38}Sr^{90}$	0.9	---††
8. $_{79}Au^{199}$	0(3/2), 0.197(3/2), 0.271(7/2), 0.515(5/2), 0.737(½) (j)	$_{80}Hg^{200}$	0.376	0.375 (k)
9. $_{14}Si^{29}$	0(½), 1.28(3/2), 2.03(5/2) (ℓ)	$_{14}Si^{28}$	1.73	1.78 (ℓ)

TABLE I (Cont.)

Notes

(a) Reference 6

(b) Reference 8

(c) Reference 7

(d) W. W. Buechner and A. Sperduto, Bull. Am. Phys. Soc. Series II, 1, 39 (1956).

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(m) Reference 11

(n) Reference 12

† Assumed to be a single-particle excitation and excluded from the center-of-gravity computation.

†† Not observed. The existence of this level is predicted by the theory.

CAPTIONS

Table I: Some nuclei to which the center-of-gravity theorem has been applied. The second column lists the lower excited states of the nuclei named in the first column. Each such nucleus is characterized by a closed shell plus or minus one particle for either the neutrons or protons. Column three then names the comparison nucleus with the corresponding closed-shell configuration. Column four contains the weighted average of the states listed in column two (omitting any single-particle states) which, by the center-of-gravity theorem, should be equal to the experimental 2+ excitation energy of column five. The spins (in parenthesis) in column two are assigned in a manner calculated to give the best prediction of the 2+ excitation energy and other conditions (see text). Underlined spins indicate measured values.

Figure I: Decay scheme of At^{209} as predicted from consideration of the center of gravity theorem. The first excited state is assigned as a $p_{3/2}$ single particle excitation.[17]

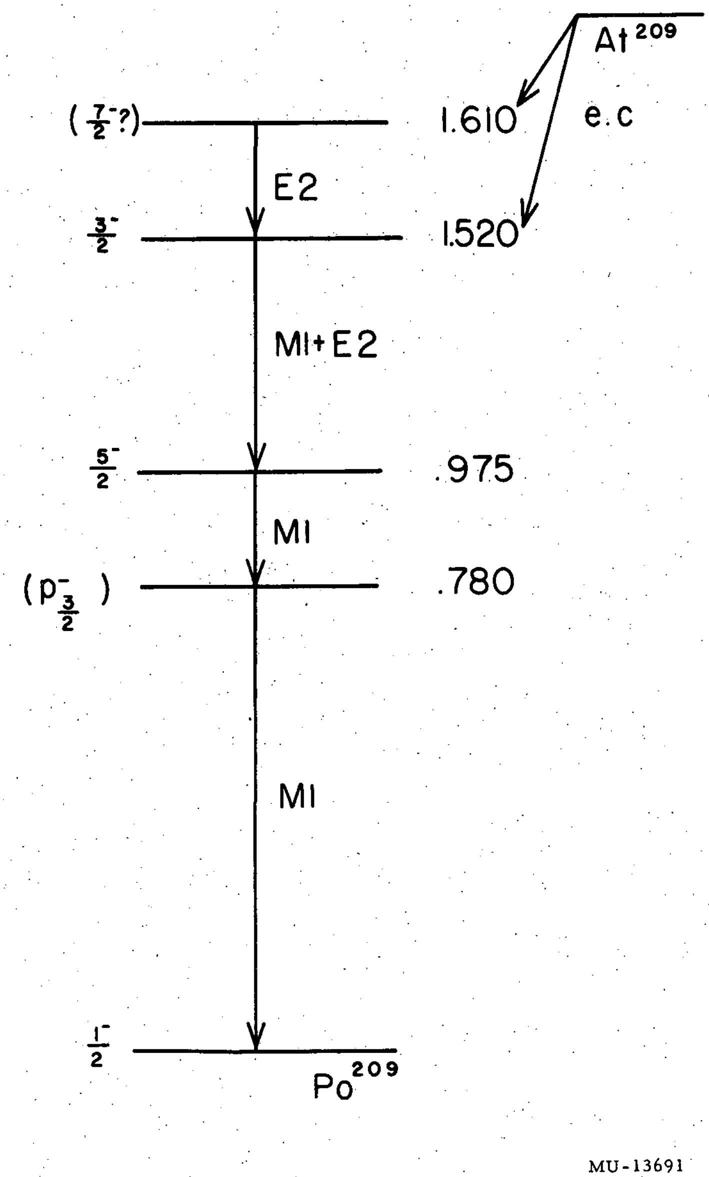
At^{209}
e.c
$(\frac{7}{2}^-?)$
1.610
E2
$\frac{3}{2}^-$
1.520
MI+E2
$\frac{5}{2}^-$
.975
MI
$(p^-_{\frac{3}{2}})$
.780
MI
$\frac{1}{2}^-$
Po^{209}
MU-13691

Fig. 1: