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Contents

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I. IDENTIFICATION

(a) Deck Number: 50-PSPDST "Three-Body Decay".
(b) Programmer: Gordon A. Sutherland, Jr. (March 1961).
(c) Originator: John A. Kadyk.
(d) Machine: IBM 704
(e) Machine Language: Fortran II (704).

II. DETERMINING EQUATIONS IN THE CENTER-OF-MASS (c. m.) SYSTEM

A primary particle of mass m is assumed to decay into three secondary particles of masses m_1, m_2, and m_3, with momenta $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ and total energies E_1, E_2, E_3, respectively.[1] The objective is to generate an arbitrary number of such three-body decay events for which $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ are determined statistically by the phase-space distribution

$$\frac{dN}{dm} = \frac{p_1^2\, dp_1\, d\Omega_1\, p_2^2\, dp_2\, d\Omega_2}{h^6\, dm} V_1 V_2,$$

the angles being measured in some fixed coordinate system

$d\Omega_1$ = solid angle for $\vec{p}_1$
= $\sin\theta_1\, d\theta_1\, d\phi_1$,
$d\Omega_2$ = solid angle for $\vec{p}_2$
= $\sin\theta_2\, d\theta_2\, d\phi_2$,
V_1 = normalization volume for m_1,
and V_2 = normalization volume for m_2.

In the decay plane in the c. m. system, two independent variables must be specified to determine the decay configuration. Since it can be shown that for a fixed momentum of one secondary the locus of the vertex formed by the momentum vectors of the other two secondaries is an ellipsoid, it will be convenient to choose p_1 and ϕ as the two independent variables (see Fig. 1). The integrations over all orientations in space of the decay plane to get the joint distribution of (p_1, ϕ) are automatically accomplished by considering the element of volume between two ellipses, one corresponding to total energy m, and the other to m + dm, for a fixed p_1 and ϕ; such an element dV_p is indicated in the figure.

Since p_1 can be chosen in any direction without restriction, integration over $d\Omega_1$, gives 4π. The distribution function is now

$$\frac{dN}{dm} = \left(\frac{4\pi V_1 V_2}{h^6}\right) p_1^2\, dp_1 \cdot 2\pi r \sin\phi \cdot r \frac{dr d\phi}{dm}, \qquad (1)$$

since $dV_p = 2\pi r \sin\phi \cdot r dr d\phi$.

Evaluation of $r^2(dr/dm)$ in terms of p_1 and ϕ will give the required distribution as follows:

Let A be the semimajor axis and B the semiminor axis; then the semiaxes of the ellipse are given by

[1] The discussion is applied to a decay process, but the analysis for a production process is unchanged if m represents the total energy in the c. m. system.

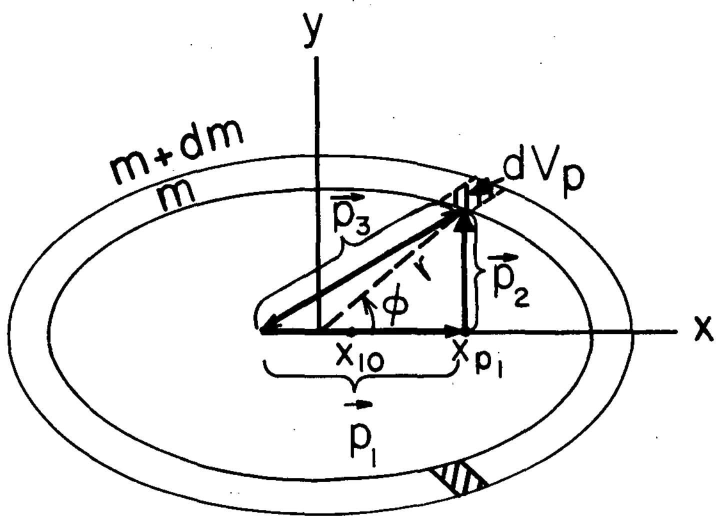

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Fig. 1.

$$A = \frac{m - E_1}{2[(m-E_1)^2 - p_1^2]} \sqrt{[(m-E_1)^2 - p_1^2 - m_2^2 - m_3^2]^2 - 4m_2^2 m_3^2}, \tag{2}$$

$$B = A\sqrt{1 - \left(\frac{p_1}{m-E_1}\right)^2} . \tag{3}$$

The mid-point of the vector p_1 lies at x_{10}:

$$x_{10} = \frac{p_1}{2} \frac{m_2^2 - m_3^2}{(m-E_1)^2 - p_1^2} . \tag{4}$$

It is convenient to abbreviate the following:

$$m - E_1 = a,$$

$$p_1 = b.$$

Since the equation of the ellipse is

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = r^2 \left[\frac{\cos^2\phi}{A^2} + \frac{\sin^2\phi}{B^2}\right] = 1, \tag{5}$$

$r^2(dr/da)$ can be found. The result is

$$r^2 \frac{dr}{da} = A^2\left[\left(\frac{r}{A}\right)^3 \frac{dA}{da} + \frac{A\, ab^2 \sin^2\phi}{(a^2 - b^2)^2} \left(\frac{r}{A}\right)^5\right], \tag{6}$$

where

$$\frac{dA}{da} = \frac{(a^2 - b^2)^3 + 2b^2(a^2 - b^2)(m_2^2 + m_3^2) - (a^2 + b^2)(m_2^2 - m_3^2)^2}{2(a^2 - b^2)^2 \sqrt{(a^2 - b^2 - m_2^2 - m_3^2)^2 - 4m_2^2 m_3^2}},$$

and

$$\frac{r}{A} = \sqrt{\frac{a^2 - b^2}{a^2 - b^2\cos^2\phi}} . \tag{7}$$

The final distribution is

$$\frac{dN}{dm} = \left(\frac{8\pi^2 V_1 V_2}{h^6}\right) p_1^{\,2} A^2 \sin\phi \left\{ \left(\frac{a^2 - b^2}{a^2 - b^2 \cos^2\phi}\right)^{3/2} \frac{dA}{da} \right.$$

$$\left. + \frac{A\, ab^2 \sin^2\phi}{(a^2 - b^2)^2} \left(\frac{a^2 - b^2}{a^2 - b^2 \cos^2\phi}\right)^{5/2} \right\} d\phi\, dp_1. \qquad (8)$$

Integrating this over ϕ, or computing directly the volume between the two ellipses, gives the over-all distribution for p_1 alone:

$$G(p_1)dp_1 = \frac{2\pi^2}{3} \frac{V_1 V_2}{h^6} p_1^{\,2} \sqrt{1 - \frac{2(m_2^{\,2} + m_3^{\,2})}{(m - E_1)^2 - p_1^{\,2}} + \frac{(m_2^{\,2} - m_3^{\,2})^2}{[(m - E_1)^2 - p_1^{\,2}]^2}}$$

$$\left\{ 3(m - E_1)^2 \left[1 - \frac{(m_2^{\,2} - m_3^{\,2})^2}{\{(m - E_1)^2 - p_1^{\,2}\}^2}\right] \right.$$

$$\left. - p_1^{\,2} \left[1 - \frac{2(m_2^{\,2} + m_3^{\,2})}{(m - E_1)^2 - p_1^{\,2}} + \frac{(m_2^{\,2} - m_3^{\,2})^2}{\{(m - E_1)^2 - p_1^{\,2}\}^2}\right] \right\} . \qquad (9)$$

From p_1 and ϕ the configuration in the decay plane is determined:

Endpoint of p_1:

$$x_{p_1} = x_{10} + \frac{p_1}{2} = \frac{p_1}{2}\left[1 + \frac{m_2^{\,2} - m_3^{\,2}}{(m - E_1)^2 - p_1^{\,2}}\right] ; \qquad (10)$$

$$p_2 = \sqrt{(x_{p_1} - x)^2 + y^2} = \sqrt{x_{p_1}^{\,2} - 2r x_{p_1} \cos\phi + r^2}, \qquad (11)$$

where

$$r = A \sqrt{\frac{a^2 - b^2}{a^2 - b^2 \cos^2\phi}} ;$$

$$p_3 = \sqrt{[p_1 - (x_{p_1} - x)]^2 + y^2}$$

$$= \sqrt{\left(\frac{p_1}{2} - x_{10}\right)^2 + 2r\left(\frac{p_1}{2} - x_{10}\right)\cos\phi + r^2}\,; \qquad (12)$$

$$\tan\theta_{12} = \frac{r\sin\phi}{r\cos\phi - x_{p_1}}\,; \qquad (13)$$

$$\tan\theta_{13} = \frac{r\sin\phi}{\left(x_{10} - \frac{p_1}{2}\right) - r\cos\phi}\,; \qquad (14)$$

$$\theta_{23} = 2\pi - (\theta_{12} + \theta_{13})\,. \qquad (15)$$

A. Method

1. Selection of p_1

Of the two variables, p_1 is selected first from the distribution $G(p_1)$. To do this, consider $G(p_1)$ with the coefficient dropped for convenience:

$$G'(p_1) = b^2 \sqrt{1 - \frac{2(m_2^{\,2} + m_3^{\,2})}{a^2 - b^2} + \frac{(m_2^{\,2} - m_3^{\,2})^2}{(a^2 - b^2)^2}}$$

$$\left\{3a^2\left[1 - \frac{(m_2^{\,2} - m_3^{\,2})^2}{(a^2 - b^2)^2}\right] - b^2\left[1 - \frac{2(m_2^{\,2} + m_3^{\,2})}{a^2 - b^2} + \frac{(m_2^{\,2} - m_3^{\,2})^2}{(a^2 - b^2)^2}\right]\right\}, \qquad (16)$$

where $a = m - E_1$, $b = p_1$, and $E_1 = \sqrt{p_1^{\,2} + m_1^{\,2}}$.

The maximum p_1 is computed:

$$p_{1\,max} = \frac{1}{2m}\sqrt{\left[m^2 - (m_1+m_2+m_3)^2\right]\left[m^2 - (m_1-m_2-m_3)^2\right]}\,. \tag{17}$$

The maximum $G'(p_1)$ is supplied in the input:[2] $G_{max}(m, m_1, m_2, m_3)$.

Procedure:

(a) Select 2 random numbers: $0 \leqslant \xi \leqslant 1$,
$0 \leqslant \eta \leqslant 1$;

(b) Select trial p_1: $p_{1t} = \xi\, p_{1\,max}$;

(c) Compute trial $G'(p_{1t})$;

(d) Compare $G'(p_{1t})$ with $\eta\, G_{max}$;

(e) If $G'(p_{1t}) \geqslant \eta\, G_{max}$, then the trial is successful and the program continues.

If $G'(p_{1t}) < \eta\, G_{max}$, then the trial fails, and another trail is attempted with new values of (ξ, η).

2. Selection of ϕ

Dropping the constant coefficient in dN/dm, and inserting the value of p_1 obtained above, we have the distribution in ϕ,

$$f(\phi) = A^2 b^2 \sin\phi \left(\frac{a^2 - b^2}{a^2 - b^2\cos^2\phi}\right)^{3/2} \left\{\frac{dA}{da} + \frac{A\, ab^2 \sin^2\phi}{(a^2 - b^2)(a^2 - b^2\cos^2\phi)}\right\}. \tag{18}$$

This function may be integrated explicitly to give

$$F(\phi) = \int_0^\phi f(\phi)d\phi = A^2 b^2 (a^2 - b^2)^{3/2} \left\{\frac{1}{a^2}\frac{dA}{da}\left(\frac{1}{\sqrt{a^2 - b^2}} - \frac{\cos\phi}{\sqrt{a^2 - b^2\cos^2\phi}}\right)\right.$$

$$\left. + \frac{Ab^2}{3a^3(a^2 - b^2)}\left[\frac{2}{\sqrt{a^2 - b^2}} + \frac{(a^2 + 2b^2)\cos^3\phi - 3a^2\cos\phi}{(a^2 - b^2\cos^2\phi)^{3/2}}\right]\right\}. \tag{18a}$$

[2] Note that an exact value of G_{max} is not necessary, but only a value that is greater than or equal to the exact G_{max}; to the extent that it is greater, the ratio of successes to trials is reduced, but the distribution of p_1 is unaltered.

Since

$$\int_0^{\pi} \frac{dN}{dm} \, d\phi = G(p_1),$$

it is easy to show that

$$F(\pi) = \frac{1}{12} \, G'(p_1) = F_{max}. \qquad (18b)$$

Value ϕ is now selected as follows:

(a) Choose another random number such that $0 \leqslant \zeta \leqslant 1$;

(b) Put $F' = \zeta F_{max} = \zeta \dfrac{G'(p_1)}{12}$

($G'(p_1)$ has just been calculated in selection of p_1);

(c) Solve for ϕ the equation

$G(\phi) = F(\phi) - F' = 0.$

This can probably best be done by an iterative procedure. See Sec. V (Method), for a complete description of solution of $G(\phi) = 0$.

3. Computation of other Momenta and Angles

$$x_{10} = \frac{1}{2} p_1 \frac{m_2^2 - m_3^2}{a^2 - b^2}, \qquad r = A \sqrt{\frac{a^2 - b^2}{a^2 - b^2 \cos^2\phi}},$$

$$x_{p_1} = x_{10} + \frac{1}{2} p_1, \qquad x_{p_2} = \frac{1}{2} p_1 - x_{10},$$

$$p_2 = \sqrt{x_{p_1}^2 - 2rx_{p_1} \cos\phi + r^2},$$

$$p_3 = \sqrt{x_{p_2}^2 + 2rx_{p_2} \cos\phi + r^2},$$

$$\tan\theta_{12} = \frac{r \sin\phi}{r\cos\phi - x_{p_1}},$$

$$\tan\theta_{13} = \frac{r \sin\phi}{(x_{10} - \frac{1}{2} p_1) - r\cos\phi} = -\frac{r \sin\phi}{x_{p_2} + r\cos\phi},$$

and

$$\theta_{23} = 2\pi - (\theta_{12} + \theta_{13}).$$

4. Randomized Orientation of Decay in Space

Perform the orientation as follows:

(a) Pick the direction of $\vec{p}_1$ randomly relative to a fixed coordinate system (X, Y, Z) by specifying the polar coordinates λ, μ according to the joint distribution:

$$p(\lambda, \mu) d\lambda\, d\mu = \frac{1}{4\pi} \sin\lambda\, d\lambda\, d\mu;$$

(b) Establish a primed system of coordinates (x', y', z') rotated relative to the fixed system (see Fig. 2), such that z' lies along $\vec{p}_1$, and the y' axis lies in the Zz' plane, which will then be the decay plane.[3]

The equations relating components of direction cosines along axes in the two systems are then

$$\alpha = \alpha' \sin\mu + \beta' \cos\lambda \cos\mu + \gamma' \sin\lambda \cos\mu,$$

$$\beta = -\alpha' \cos\mu + \beta' \cos\lambda \sin\mu + \gamma' \sin\lambda \sin\mu, \qquad (19)$$

$$\gamma = -\beta' \sin\lambda + \gamma' \cos\lambda.$$

The direction cosines for $\vec{p}_1, \vec{p}_2, \vec{p}_3$ in the primed system are

$$\vec{p}_1:\ \alpha_1' = 0,\ \beta_1' = 0,\ \gamma_1' = 1;$$

$$\vec{p}_2:\ \alpha_2' = 0,\ \beta_2' = \sin\theta_{12},\ \gamma_2' = \cos\theta_{12}; \qquad (20)$$

$$\vec{p}_3:\ \alpha_3' = 0,\ \beta_3' = -\sin\theta_{13},\ \gamma_3' = \cos\theta_{13}.$$

Rotate the decay plane and the x' and y' axes by an angle ν about the z' axis; the rotated system then becomes the (x", y", z") system:

$$\gamma' = \gamma'',\ \alpha' = \alpha'' \cos\nu - \beta'' \sin\nu,\ \beta' = \beta'' \cos\nu + \alpha'' \sin\nu. \qquad (21)$$

The components of $\vec{p}_1$, $\vec{p}_2$, and $\vec{p}_3$ given above are now in the (x", y", z") system. Transforming to the (x', y', z') system gives

$$\alpha_1' = 0,\ \beta_1' = 0,\ \gamma_1' = 1;$$

$$\alpha_2' = -\sin\theta_{12} \sin\nu,\ \beta_2' = \sin\theta_{12} \cos\nu,\ \gamma_2' = \cos\theta_{12}; \qquad (22)$$

$$\alpha_3' = \sin\theta_{13} \sin\nu,\ \beta_3' = -\sin\theta_{13} \cos\nu,\ \gamma_3' = \cos\theta_{13}.$$

[3] If $\cos\lambda = \pm 1$, put $\alpha = \alpha'$, $\beta = \pm\beta'$, and $\gamma = \pm\gamma'$, respectively, using values taken directly from Eqs. (22).

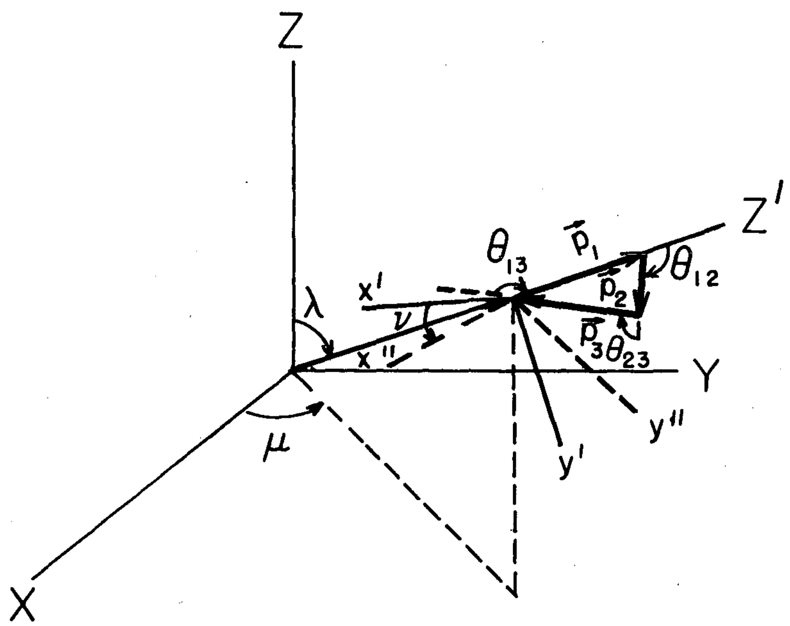

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Fig. 2.

These values are now substituted in Eqs. (19) to obtain the (X, Y, Z) components of $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$, stored in the computer memory for output.

The angles (λ, μ, ν) are chosen with random numbers λ', μ', ν':

$$0 < \lambda' < 1, \qquad 0 < \mu' < 1, \qquad 0 < \nu' < 1;$$

$$\cos \lambda = 2\lambda' - 1, \quad \mu = 2\pi\mu', \qquad \nu = 2\pi\nu' .$$

B. Input and Output

1. Input

m	
m_1	G_{max}
m_2	n = number of trials
m_3	n_s = number of successes

2. Output

p_1	θ_{12}	$p_{1x} = \alpha_1 p_1$	$p_{2x} = \alpha_2 p_2$	$p_{3x} = \alpha_3 p_3$
p_2	θ_{13}	$p_{1y} = \beta_1 p_1$	$p_{2y} = \beta_2 p_2$	$p_{3y} = \beta_3 p_3$
p_3	θ_{23}	$p_{1z} = \gamma_1 p_1$	$p_{2z} = \gamma_2 p_2$	$p_{3z} = \gamma_3 p_3$

n	m	m_3
n_s	m_1	G_{max}
$p_{1\,max}$	m_2	ϕ

III. LORENTZ TRANSFORMATION TO THE LABORATORY SYSTEM

Consider particle m to be moving with momentum $\vec{p}_L$ when it decays into secondaries m_1, m_2, m_3 with previously determined c. m. s. momenta $\vec{p}_1$, $\vec{p}_2$, $\vec{p}_3$ (see Fig. 3). The resulting secondary momenta in the laboratory system (denoted by a subscript "L") are to be determined from the Lorentz transformations.

Input	Output
$p_{1x}, p_{1y}, p_{1z}, m_1, p_1;$	$p_{1Lx}, p_{1Ly}, p_{1Lz}, p_{1L}, E_{1L}, m_1;$
$p_{2x}, p_{2y}, p_{2z}, m_2, p_2;$	$p_{2Lx}, p_{2Ly}, p_{2Lz}, p_{2L}, E_{2L}, m_2;$
$p_{3x}, p_{3y}, p_{3z}, m_3, p_3;$	$p_{3Lx}, p_{3Ly}, p_{3Lz}, p_{3L}, E_{3L}, m_3;$
$\alpha_p, \beta_p, \gamma_p, p_L, m.$	$p_{Lx}, p_{Ly}, p_{Lz}, p_L, E_L, m.$

Derivation of transformations for i = 1, 2, 3:

$$E_i = \sqrt{p_i^{\,2} + m_i^{\,2}} \qquad E_L = \sqrt{p_L^{\,2} + m^2}, \qquad \gamma = \frac{E_L}{m}, \qquad \gamma\beta = \frac{p_L}{m} .$$

Components parallel to $\vec{p}$:

$$\vec{p}_{i\parallel L} = [\gamma(\vec{p}_i \cdot \vec{U}_p) + \gamma\beta E_i]\, \vec{U}_p .$$

Components perpendicular to $\vec{p}$:

$$p_{i\perp L} = \vec{U}_p \times (\vec{p}_i \times \vec{U}_p) = \vec{p}_i - \vec{U}_p(\vec{p}_i \cdot \vec{U}_p),$$

where

$$\vec{U}_p = \frac{\vec{p}_L}{p_L} .$$

Resultant transformed vector:

$$\vec{p}_{iL} = [(\gamma - 1)(\vec{p}_i \cdot \vec{U}_p) + \gamma\beta E_i]\, \vec{U}_p + \vec{p}_i .$$

Thus

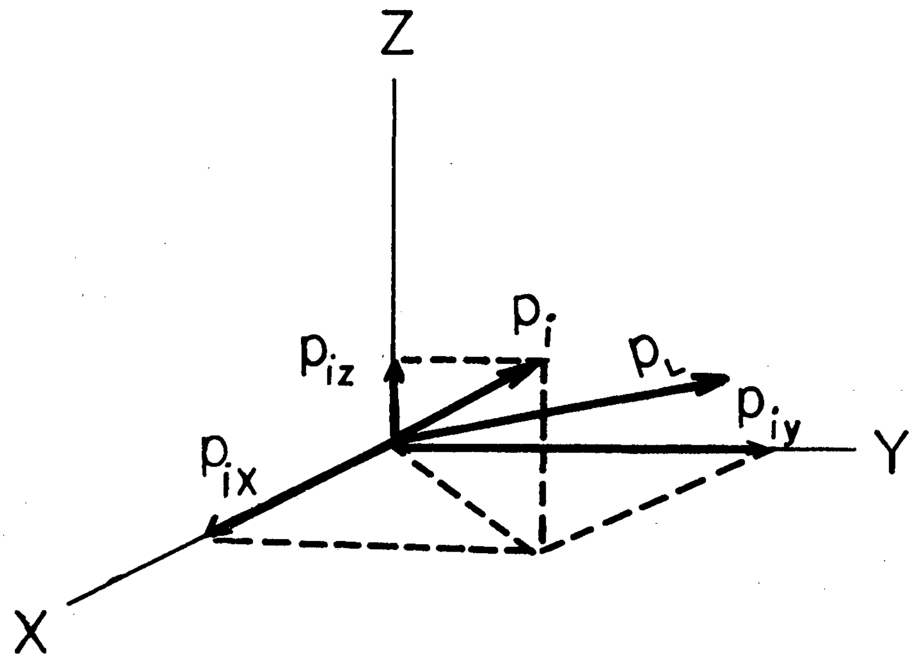

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Fig. 3.

$$\left.\begin{aligned}
p_{iLx} &= [(\gamma - 1)(\vec{p}_i \cdot \vec{U}_p) + \gamma\beta E_i]\,\alpha_p + p_{ix} \\
p_{iLy} &= [(\gamma - 1)(\vec{p}_i \cdot \vec{U}_p) + \gamma\beta E_i]\,\beta_p + p_{iy} \\
p_{iLz} &= [(\gamma - 1)(\vec{p}_i \cdot \vec{U}_p) + \gamma\beta E_i]\,\gamma_p + p_{iz} \\
p_{iL} &= \sqrt{p_{iLx}^2 + p_{iLy}^2 + p_{iLz}^2}
\end{aligned}\right\} \quad \text{for } i = 1, 2, 3,$$

where

$$(\vec{p}_i \cdot \vec{U}_p) = p_{ix}\alpha_p + p_{iy}\beta_p + p_{iz}\gamma_p$$

IV. π^0 DECAY AND THE GAMMA-RAY CONVERSION SUBROUTINE

A. π^0 Decay: $\pi^0 \rightarrow \gamma 1 + \gamma 2$

The π^0 decay subroutine is executed for each of the three secondary masses whose value is 135.00.

The components of the two γ momentum vecotrs, $\vec{p}_{G1}$ and $\vec{p}_{G2}$, normal and parallel to the initial π^0 momentum $\vec{p}_i$ are called, respectively,

$$p_{G1n},\ p_{G1p} \text{ and } p_{G2n},\ p_{G2p}\ .$$

They are found from p_i, m_i as follows (Lorentz transformation):

$$\left.\begin{aligned} p_{Gjp} &= \frac{E_i}{m_i}\ \frac{m_{\pi^0}}{2}\left(\cos\theta^* + \frac{p_i}{E_i}\right) \\ p_{Gjn} &= \pm\ \frac{m_{\pi^0}}{2}\ \sin\theta^*, \end{aligned}\right\}\quad \text{for } j = 1,\ 2$$

with

$$m_{\pi^0} = 135.00.$$

Here

$$\cos\theta^* = 2\eta - 1, \text{ where } \eta \text{ is a random number such that } 0 \leqslant \eta \leqslant 1;$$
$$\sin\theta^* = \sqrt{1 - \cos^2\theta^*}\ .$$

Referring to Fig. 4, where the primed system is constructed with the z' axis along $\vec{p}_i$, and the y' axis in the z' Z plane (cf. Sec. II-A-4 "Randomized Orientation of Decay in Space," and Fig. 2) one finds the following components in the primed system (the z' Z plane is also, at this point, the decay plane):

$$p_{G1z'} = \frac{1}{2}\ (E_i \cos\theta^* + p_i), \qquad p_{G2z'} = \frac{1}{2}\ (-\ E_i \cos\theta^* + p_i);$$

$$p_{G1x'} = 0, \qquad p_{G2x'} = 0;$$

$$p_{G1y'} = \frac{-\ m_{\pi^0}}{2}\ \sin\theta^*, \qquad p_{G2y'} = \frac{m_{\pi^0}}{2}\ \sin\theta^*.$$

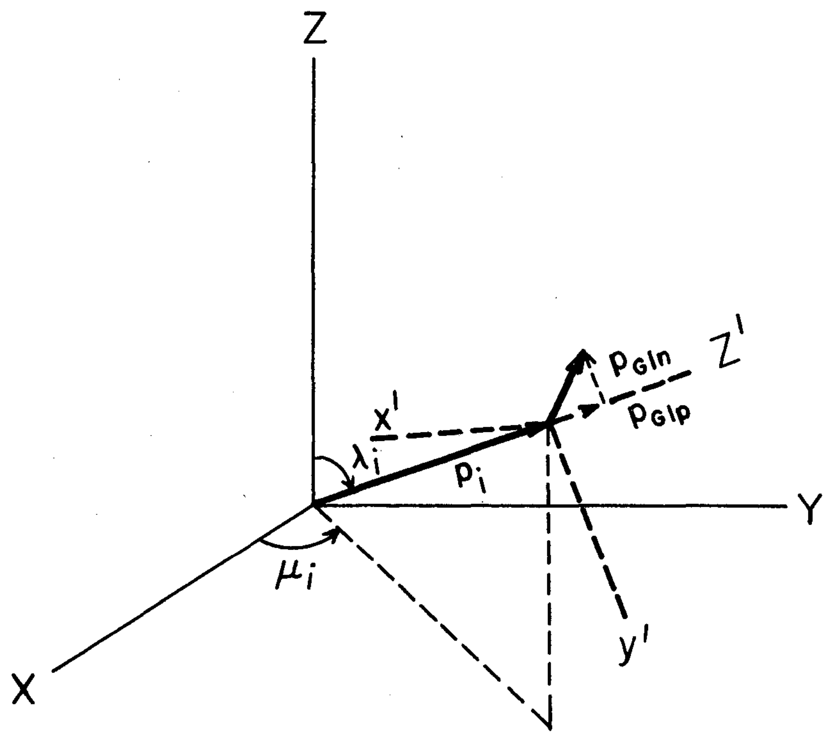

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Fig. 4.

Now, a rotation by angle ν about the z' axis gives the new components in the primed system:

$$p_{G1z'} = \frac{1}{2}(E_i \cos\theta^* + p_i), \qquad p_{G2z'} = \frac{1}{2}(-E_i \cos\theta^* + p_i);$$

$$p_{G1x'} = \frac{m_{\pi^0}}{2} \sin\theta^* \sin\nu, \qquad p_{G2x'} = \frac{-m_{\pi^0}}{2} \sin\theta^* \sin\nu;$$

$$p_{G1y'} = \frac{-m_{\pi^0}}{2} \sin\theta^* \cos\nu, \qquad p_{G2y'} = \frac{m_{\pi^0}}{2} \sin\theta^* \cos\nu.$$

The components in the unprimed system are now found as folows:

$$p_{G1x} = p_{G1x'} \sin\mu_i + p_{G1y'} \cos\lambda_i \cos\mu_i + p_{G1z'} \sin\lambda_i \cos\mu_i;$$

$$p_{G1y} = -p_{G1y'} \cos\mu_i + p_{G1y'} \cos\lambda_i \sin\mu_i + p_{G1z'} \sin\lambda_i \sin\mu_i;$$

$$p_{G1z} = -p_{G1y'} \sin\lambda_i + p_{G1z'} \cos\lambda_i;$$

where

$$\cos\lambda_i = \gamma_i, \quad \sin\lambda_i = \sqrt{1 - \cos^2\lambda_i},$$

$$\cos\mu_i = \frac{\alpha_i}{\sin\lambda_i},$$

and

$$\sin\mu_i = \frac{\beta_i}{\sin\lambda_i},$$

when $(\alpha_i, \beta_i, \gamma_i)$ are direction cosines of π^0.

Similarly, for p_{G2x}, p_{G2y}, p_{G2z}:

Compute:

$$p_{G1} = \sqrt{p_{G1x}^2 + p_{G1y}^2 + p_{G1z}^2},$$

$$p_{G2} = \sqrt{p_{G2x}^2 + p_{G2y}^2 + p_{G2z}^2}.$$

Note:

If $\cos\lambda_i \equiv \pm 1$, then take:

$$p_{G1x} = \frac{m_{\pi^0}}{2} \sin\theta^* \sin\nu = +p_{1x'};$$

$$p_{G1y} = \mp \frac{m_{\pi^0}}{2} \sin\theta^* \cos\nu = \pm p_{1y'};$$

$$p_{G1z} = \pm \frac{1}{2}(E_i \cos\theta^* + p_i) = \pm p_{1z'},$$

and repeat these three equations for p_{G2}.

B. Gamma-Ray Conversion

Input:

From Sec. A above, the momentum and its components are known for each γ ray.

Other input data: Coordinates of the decay point (x_D, y_D, z_D).

Calculations:

Compute direction cosines for $\gamma 1$:

$$\alpha_1 = \frac{p_{G1x}}{p_{G1}}, \qquad \beta_1 = \frac{p_{G1y}}{p_{G1}}, \qquad \gamma_1 = \frac{p_{G1z}}{p_{G1}} .$$

Compute the potential path, FLP, which is the distance measured along the γ-ray direction from the decay point to the fiducial limit of the chamber. This is given by the function POPATH.

$$\text{FLP} = \text{POPATH}\,(z_1, z_2, R, \alpha_1, \beta_1, \gamma_1, x_D, y_D, z_D.$$

where

$$z_1 = 2,$$
$$z_2 = 23,$$
$$R = 13,$$

the fiducial limits of the 12-inch Xenon chamber, in centimeters.

Compute the mean conversion length,[4] in centimeters:

$$\text{FLAMG} = 5.1448 + \frac{135.6738}{p_{G1}} - \frac{1292.6491}{(p_{G1})^2} + \frac{5508.5681}{(p_{G1})^3} .$$

Select a conversion distance according to an exponential distribution:

$$\text{FLGAM} = -(\text{FLAMG})\,(\log_e \eta),$$

where η is a random number, such that $0 < \eta < 1$. Test for γ-ray conversion inside fiducial volume:

(a) if FLGAM-FLP < 0, γ does convert;

(b) if FLGAM-FLP ≥ 0, γ does not convert.

Repeat this set of calculations for $\gamma 2$, keeping the same (x_D, y_D, z_D), but changing to the momentum of $\gamma 2$.

---

[4]This formula was developed by Dr. John Brown, of this Laboratory for use in the xenon bubble chamber. For other materials, an appropriate formula must be substituted.

V. METHOD

Selection of ϕ according to Sec. II-A-2 (Selection of ϕ), Subroutine FOFPHI:

This is a direct integration problem. Define

$$H(x) = H(\cos\phi) = \int_0^{\phi} f d\phi - R(r) \int_0^{\pi} f d\phi,$$

where R(r) is equal to a number produced by the random-number generator, and $f(\phi)$ is as in Eq. (18).

$$H(\cos\phi) = dA/da\ A^2 b(\alpha^2 - 1)^{3/2} \frac{-\cos\phi}{\alpha^2(\alpha^2 - \cos^2\phi)^{1/2}}$$

$$+ \frac{A^3 ab^2(\alpha^2 - 1)^{3/2}}{(a^2 - b^2)} \left(\frac{(2 + \alpha^2)\cos^3\phi - 3\alpha^2\cos\phi}{3\alpha^A(\alpha^2 - \cos^2\phi)^{3/2}} \right)$$

$$+ \text{CAPFL} - 2R(r)\ \text{CAPFL},$$

where $\text{CAPFL} = \dfrac{DA/da\ A^2\ (a^2 - b^2)b^2 a + 2/3\ (A^3\ b^4)}{a^3}$,

$\alpha = \frac{a}{b}$, and $x = \cos\phi$.

H(x) is a monotonically increasing function as x goes from +1 to -1, since $f(\phi) > 0$, and $0 \leqslant \phi \leqslant \pi$. The subroutine then solves the equation

$$x = \frac{x_2 H(x_1) - x_1 H(x_2)}{H(x_1) - H(x_2)}$$

by iteration until $|H(x)| < 0.001\ F'$, where $F' = R(r)\int_0^{\pi} f d\phi = 2R(r) \cdot \text{CAPFL}$; and where $H(x_1) > 0$ and $H(x_2) < 0$; each successive value of x replacing x_1 if $H(x) > 0$, or x_2 if $H(x) < 0$.

The output is PHI = arcos (x). FOFPHT = 0, if x is found within 50 iterations; otherwise FOFPHT = 1.

VI. RESTRICTIONS AND REQUIREMENTS

This program has been prepared to run without a monitor, but may easily be modified to run with a monitor system (except for the main "super" program - see Appendix). The program can be recompiled on the IBM 709 without alteration, to produce a version which can be run on the 709 (except for the main "super" program - see Appendix).

All subroutines are included in the binary deck, including FORTRAN I-0 subroutines and FORTRAN square root, log, arctan, exponential, sine, and cosine subroutines.

This program requires a minimum core size of 17777_8 locations (8192_{10}).

VII. PROGRAM SETUP

A main program must be used to call subroutine PSPDST into play. All other subroutines are called into execution via subroutine PSPDST. The main program must also perform the following tasks:

(a) The values of the masses of the primary particle and the three secondaries must be set prior to executing subroutine PSPDST. The constants are FM, FM1, FM2, and FM3 respectively, and all are located in FORTRAN common. The values for the unit direction cosines of the primary, and the coordinates of the primary and momentum of the primary, must also be set in the main program. These are the constants ALFP, BETP, GAMP, PCOORD(1), PCOORD(2), PCOORD(3), and PL, respectively, all located in FORTRAN common.

The values for G_{max}, the maximum number of trials to be executed, the maximum number of successes allowed, the number of trial failures permitted before the ratio of successful trials to total trials is formed and tested, and the two limiting values that the ratios of successful trials to total trials may assume in cases of failure in the function $G(p_1)$ and the function $F(p_1)$, must all be fixed in the main program. These are the constants GMX, N, NSC, NFAIL, F1, and F2, respectively, forming in the sequence listed the parameters of the subroutine PSPDST calling sequence. These constants are not located in FORTRAN common.

(b) The constants NZZ, LBTRNT, and PIDCAT (See Sec. VIII-B for FORTRAN memory locations), must be fixed as follows:

(1) NZZ = X, where X is the integer value of the tape unit desired for writing the BCD information from all subroutines involved in this program, i.e., if NZZ = 6, then tape 6 is selected by the program.

(2) If LBTRNT = 1, then subroutine LBTRAN is executed. This subroutine performs the calculations for the Lorentz transformations from the c.m.s. to the laboratory system. If LBTRNT $\neq$ 1, then subroutine LBTRAN is not executed.

(3) If PIDCAT = 0, then subroutine PIDAY is not executed. This is the subroutine which makes the π^0-decay and gamma-conversion calculations. If PIDCAT = 1, then subroutine PIDCAY is executed, using results from the original c. m. s. (i. e., directly from subroutine PSPDST). If PIDCAT = 2, then subroutine PIDCAY is executed, using results from the Lorentz transformations to the laboratory system (i. e., from subroutine LBTRAN). All data used by the subroutines LBTRAN and PIDCAY are located in FORTRAN common storage. If, at the termination of subroutine PIDCAY, the constant GMCNVT = -1, then neither gamma ray converted. If GMCNVT = 0, then the first gamma ray did not convert and the second did. If GMCNVT = 1, then the first gamma ray converted and the second did not. If GMCNVT = 2, then both gamma rays converted.

(4) Provision has been established in the present main program to punch out the base value of the number in the function RAN (the random number generator) at the termination of any given execution of PSPDST, and to use this number as the starting base value in RANF for the next PSPDST run. The card containing this number must be the physical first card after the binary deck transition card. If this card is blank, then the base value provided in the function RAN will be used. At the end of the run, this card must be collected from the on-line punch.

(5) Subroutine PDTPUN, the last subroutine called by PSPDST in any given trial, was used to punch data for a special limited experiment, by Dr. John Kadyk. A dummy subroutine has been included for general usage. However, the presence of this subroutine allows the user to develop his own data-producing subroutine. The constant PDTPNT, located in FORTRAN common memory, was used by our special subroutine as a test word, and is available to the user.

See Appendix for further details concerning the main program written by this programmer.

A. Operators' Instructions

An on-line card reader is needed to read the binary program deck. An on-line card punch is expected. A BCD tape is required, the unit number to be determined by the Programmer. The normal stop is HPR 0, 0(0420 0000 0000). At the end of the run, the BCD tape must be saved if DATA is called for. There will always be one card (octal) punched.

VIII. CODING INFORMATION

A. Alphabetical Listing of Subroutines and Functions

Item	Subroutine	No. octal Locations	Reference
1	ABGCMP	266	See Eq. (19), and also Item 22 of this table.
2	ARCOS	52	The ARCCOSINE subroutine.
3	CAPSMA	204	Calculates various constants in Eqs. (2), (3), (4), and (6).
4	CRDPP	131	See Eq. (22), and also Item 22.
5	FOFPHI	577	See Sec. II-A-2 (Selection of ϕ), and also Sec. V (Method).
6	GAMCNV	410	See Sec. IV-B (Gamma-Ray Conversion). Uses subroutine POPATH.
7	GENPR	33	For two random numbers, pg. 11, PROCEDURE
8	GENTRP	115	For three random, as on pg. 15.
9	GOFP	177	See Sec. II-A-1 (Selection of p_1).
10	LBTRAN	544	See Sec. III (Lorentz Transformation to the Laboratory System).
11	LODRAN	See Item (19)	See Item 20.
12	MOMANG	315	See Sec. II-A-3 (Computation of other Momenta and Angles).
13	PDTPUN	22	This is a "dummy" return subroutine.
14	PIDCAY	673	See Sec. IV-A (π^0 Decay: $\pi^0 \rightarrow \gamma 1 + \gamma 2$). Also uses subroutine GAMCNV.
15	PIPCMP	133	See Eq. (20), and also Item 22.
16	PMAX	76	See Eq. (17).
17	POPATH	317	Calculates potential path from a point to the edge of fiducial volumen.
18	PSPDST	1005	Main body of three-body decay experiment. It also calls LBTRAN and PIDCAY.
19	RAN	34 (G5 LC RANF)	Generates random numbers, such that $0 < \gamma < 1$.
20	RANLOD	51	Loads new base element into subroutine RAN. Uses LODRAN. See Sec. VII-b-4.

Item	Subroutine	No. octal Locations	References
21	RANSTR	47	Punches current RAN base element for subsequent uses. Uses STORAN. See Sec. VII-b-4.
22	RNORNT	105	Controls calculations described in Sec. II-A-4 (Randomized Orientation of Decay in Space). Uses subroutine GENTRP, CRDPP, ABGCMP, and PIPCMP.
23	STORAN	See Item 19	
24	SUPER	131	See Appendix (Subroutine ZML).
25	ZML	104	See Appendix (Subroutine ZML).

B. List of FORTRAN Library Subroutines Used by the Program

The number of octal locations used is included in parentheses. Total memory for all subroutines: 12202, octal.

1	ATAN	(124)
2	COS	(154)
3	EXP	(360)
4	EXP(3	(see item 3)
5	LOG	(see item 3)
6	SIN	(see item 2)
7	SQRT	(25)
8	(CSH)	(211)
9	(FIL)	(See item 10)
10	(IOH) I	(1612)
11	(IOH) 0	(see item 10)
12	(LEV)	(143)
13	(RTN)	(See item 12)
14	(SCH)	(132)
15	(STH)	(14)

C. Common Allocation

FORTRAN common storage uses locations 77263 to 77462, octal. The constants NZZ (BCD tape number), LBTRNT (laboratory transformation constant), PIDCAT (PI decay constant), GMCNVT (gamma-conversion test), and PDTPNT (data test) are located in 77462, 77461, 77404, 77374, and 77403, respectively.

APPENDIX: SUBROUTINE ZML

The main "super" program included in this deck calls on the subroutine ZML for reading certain constants into known memory locations (i. e., FORTRAN common storage locations).

ZML is a 704, hand-coded, two-word loader that performs as follows:

(a) Each time a transfer to ZML is executed, ZML reads the first two words of a card in the card reader, i. e., it reads rows 9L and 9R and stores these words in IN1 and IN2, respectively.

(b) It then tests IN1 and IN2 in the following manner. If IN1 is nonzero and IN2 is zero, then the contents of IN1 are planted in the store instruction DOWN, i. e., DOWN STO X, where X is the contents of IN1, and the first two words of the next card in sequence are read into IN1 and IN2. If IN1 is zero and IN2 is nonzero, then the contents of IN2 are stored in the location indicated by the instruction DOWN. The value of DOWN is increased by one, i. e., DOWN STO (X + 1), and the first two words of the next card in sequence are read into IN1 and IN2. If both IN1 and IN2 are nonzero, then a transfer to IN2 is executed, thus implying that the word just read into IN2 is, itself, a transfer command. In all cases, the first two words of each card are expected to be binary.

In the main program, ZML is used to fix the values of the constants PIDCAT and PDTPNT, thus removing the necessity to recompile the main program each time changes in the subroutines PIDCAY and PDTPUN are desired. The last card read by ZML contains a nonzero first word and a TRA 1, 4 (002000400 001) in the second word, thus returning control to the main program.

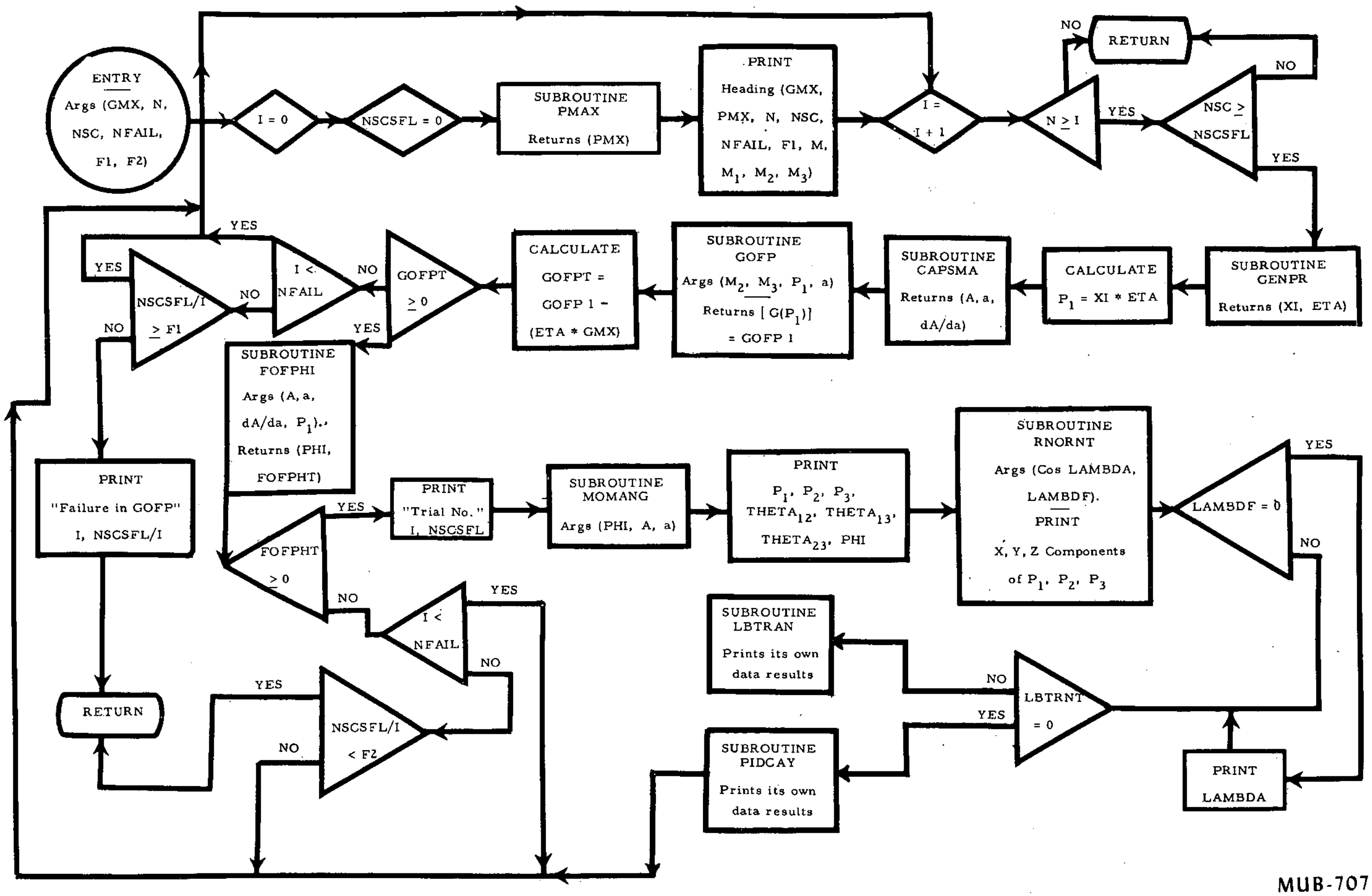

Fig. 5. Flow diagram for subroutine PSPDST.

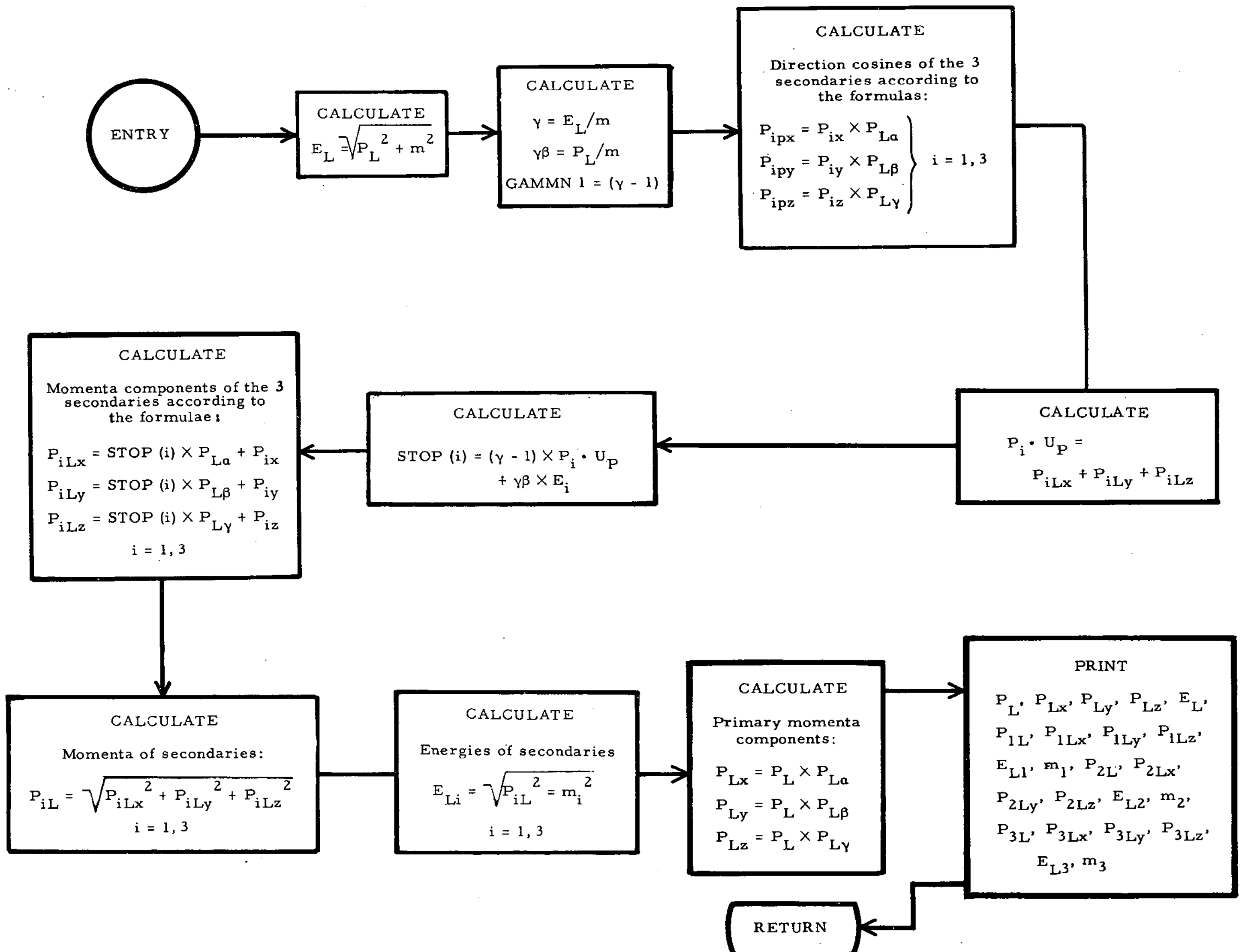

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Fig. 6. Flow diagram for subroutine LBTRAN.

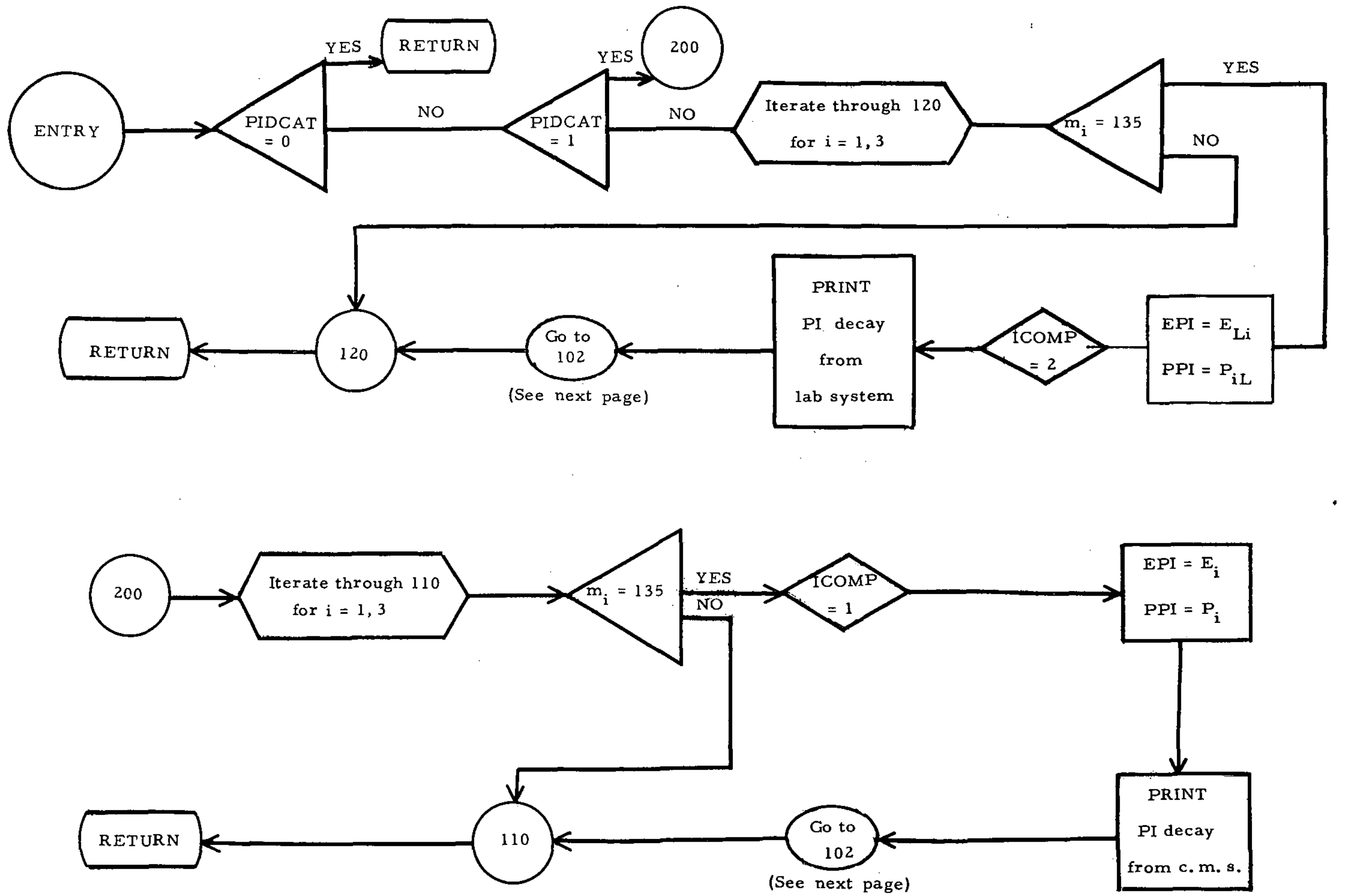

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Fig. 7a. Flow diagram for subroutine PIDCAY: first part.

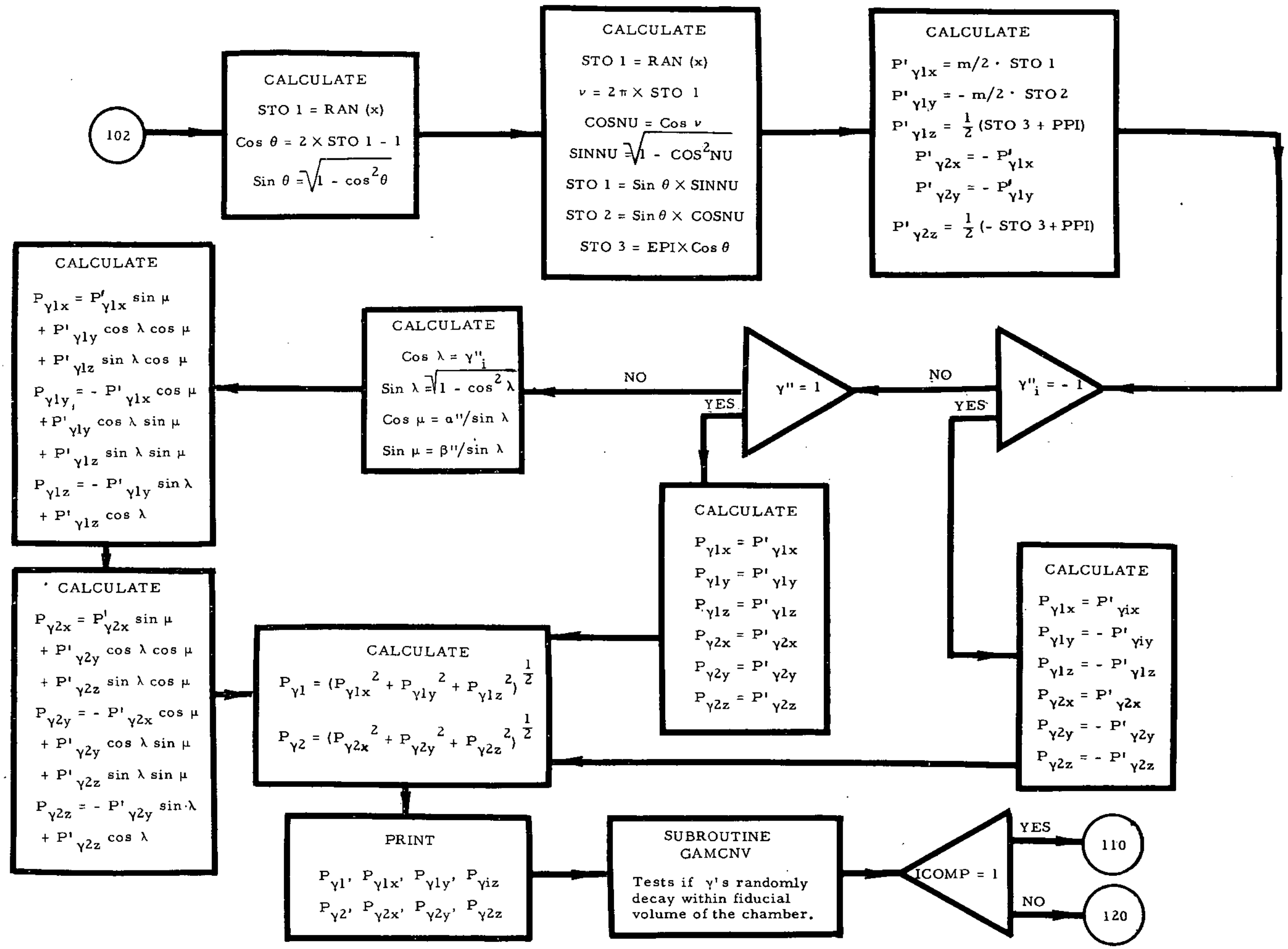

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Fig. 7b. Flow diagram for subroutine PIDCAY: conclusion.

```
C     THIS IS THE MAIN(SUPER) PROGRAM CALLING SUBROUTINE PSPDST.
C
      COMMON NZZ,LBTRNT,ALFPP,BETPP,GAMPP,P1P,P2P,P3P,FM,FM1,FM2,FM3,P1,
     1P2,P3,THTA12,THTA13,THTA23,E,P1LP,P2LP,P3LP,ALFP,BETP,GAMP,PL,
     2PIDCAT,PDTPNT,P1L,P2L,P3L,EL1,EL2,EL3,GMCNVT,PCOORD,PG1X,PG1Y,PG1Z
     3,PG2X,PG2Y,PG2Z,PIGAM1,PIGAM2,FLGAM1,FLGAM2,IPTTYP,IPTREM,XCOORD,
     4YCOORD,ZCOORD,GDIST,P1U,P1UP,P2U,P2UP,P3U,P3UP,G1UX,G1UY,G1UZ,G2UX
     5,G2UY,G2UZ
      DIMENSION ALFPP(3),BETPP(3),GAMPP(3),P1P(3),P2P(3),P3P(3),P1LP(3),
     1P2LP(3),P3LP(3),E(3),PCOORD(3),IPTTYP(8),IPTREM(8),XCOORD(8),YCOOR
     2D(8),ZCOORD(8),GDIST(8),P1U(3),P1UP(3),P2U(3),P2UP(3),P3U(3),P3UP
     3(3)
      CALL RANLOD
      NZZ=6
      LBTRNT=1
      FM=494.000
      FM1=105.660
      FM2=135.000
      FM3=0.
      GMX=44.4E08
      N=100
      NSC=100
      NFAIL=10
      F1=0.10
      F2=0.10
      ALFP=0.
      BETP=1.
      GAMP=0.
      PCOORD(1)=0.0000
      PCOORD(2)=0.0000
      PCOORD(3)=12.500
      PL=494.000
      CALL ZML
      CALL PSPDST(GMX,N,NSC,NFAIL,F1,F2)
      END FILE NZZ
      REWIND NZZ
      CALL RANSTR
      STOP
      END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
IPTTYP	32494	77356	IPTREM	32486	77346	GMCNVT	32508	77374	GDIST	32454	77306	GAMP	32518	77406
GAMPP	32554	77452	G2UZ	32423	77247	G2UY	32424	77250	G2UX	32425	77251	G1UZ	32426	77252
G1UY	32427	77253	G1UX	32428	77254	FM	32542	77436	FM3	32539	77433	FM2	32540	77434
FM1	32541	77435	FLGAM2	32495	77357	FLGAM1	32496	77360	E	32532	77424	EL3	32509	77375
EL2	32510	77376	EL1	32511	77377	BETP	32519	77407	BETPP	32557	77455	ALFP	32520	77410
ALFPP	32560	77460	LBTRNT	32561	77461	NZZ	32562	77462	P1LP	32529	77421	P1L	32514	77402
P1P	32551	77447	P1	32538	77432	P1UP	32443	77273	P1U	32446	77276	P2LP	32526	77416
P2L	32513	77401	P2P	32548	77444	P2	32537	77431	P2UP	32437	77265	P2U	32440	77270
P3LP	32523	77413	P3L	32512	77400	P3P	32545	77441	P3	32536	77430	P3UP	32431	77257
P3U	32434	77262	PCOORD	32507	77373	PDTPNT	32515	77403	PG1X	32504	77370	PG1Y	32503	77367
PG1Z	32502	77366	PG2X	32501	77365	PG2Y	32500	77364	PG2Z	32499	77363	PIDCAT	32516	77404
PIGAM1	32498	77362	PIGAM2	32497	77361	PL	32517	77405	THTA12	32535	77427	THTA13	32534	77426
THTA23	32533	77425	XCOORD	32478	77336	YCOORD	32470	77326	ZCOORD	32462	77316			

STORAGE NOT USED BY PROGKAM

DEC	OCT	DEC	OCT
88	00130	32422	77246

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
PSPDST	1	00001	RANLOD	3	00003	RANSTR	0	00000	ZML	2	00002			

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GMX	87	00127	F2	86	00126	F1	85	00125	NFAIL	84	00124	N	83	00123
NSC	82	00122												

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
2)	64	00100	3)	69	00105	6)	77	00115						

SUBROUTINES NOT PUNCHED FROM LIBRARY

RANLOD ZML PSPDST RANSTR

```
      SUBROUTINE ABGCMP(ALFPP,BETPP,GAMPP,COSLAM,FMU,LAMBDF,ALF,BET,GAM)
      DIMENSION ALFPP(3),BETPP(3),GAMPP(3),ALF(3),BET(3),GAM(3)
      IF (LAMBDF) 200,100,200
  100 IF(COSLAM) 115,200,105
  105 DO 110 I=1,3
      ALF(I)=ALFPP(I)
      BET(I)=BETPP(I)
      GAM(I)=GAMPP(I)
  110 CONTINUE
      GO TO 1000
  115 DO 120 I=1,3
      ALF(I)=ALFPP(I)
      BET(I)=-BETPP(I)
      GAM(I)=-GAMPP(I)
  120 CONTINUE
      GO TO 1000
  200 STO1=SINF(FMU)
      STO2=COSF(FMU)
      STO3=SQRTF(1.-COSLAM*COSLAM)
      STO4=COSLAM
      STO5=STO2*STO4
      STO6=STO1*STO3
      STO7=STO1*STO4
      STO8=STO2*STO3
      DO 210 I=1,3
      ALF(I)=ALFPP(I)*STO1+BETPP(I)*STO5+GAMPP(I)*STO8
      BET(I)=-ALFPP(I)*STO2+BETPP(I)*STO7+GAMPP(I)*STO6
      GAM(I)=-BETPP(I)*STO3+GAMPP(I)*STO4
  210 CONTINUE
 1000 DO 1010 J=1,3
      ALFPP(J)=ALF(J)
      BETPP(J)=BET(J)
      GAMPP(J)=GAM(J)
 1010 CONTINUE
      RETURN
      END (0,1,0,0,0)
```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
100	4	00100	105	5	00104	110	9	00113	115	11	00116	120	15	00125
200	17	00130	210	29	00222	1000	30	00224	1010	34	00233			

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
181	00265	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
COS	1	00001	SIN	2	00002	SQRT	0	00000						

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
STO1	180	00264	STO2	179	00263	STO3	178	00262	STO4	177	00261	STO5	176	00260
STO6	175	00257	STO7	174	00256	STO8	173	00255						

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
9)	169	00251	1)	170	00252	2)	161	00241	3)	163	00243	6)	164	00244

SUBROUTINES NOT PUNCHED FROM LIBRARY

SIN COS SQRT

UCRL-9614

```
      FUNCTION ARCOS(COSX,SINX)
      ARCOS=ATANF(SINX/COSX)
      IF (ARCOS) 1,2,2
    1 ARCOS=ARCOS+3.1415926536
    2 ARCOS=ARCOS+0.
      RETURN
      END (0,1,0,0,0)
```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
1	4	00024	2	5	00027									

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
41	00051	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ATAN	0	00000								

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
ARCOS	40	00050								

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT	DEC	OCT	DEC	OCT
1)	38	00046	3)	31	00037	6)	33	00041				

SUBROUTINES NOT PUNCHED FROM LIBRARY

ATAN

```
SUBROUTINE CAPSMA(CAPA,SMA,DCPDSM)
COMMON XX,FM,FM1,FM2,FM3,P1
DIMENSION XX(20)
ST06=P1*P1
ST08=FM2*FM2
ST09=FM3*FM3
SMA=FM-SQRTF(ST06+FM1*FM1)
ST07=SMA*SMA
ST01=ST07-ST06
ST02=ST08+ST09
ST03=ST01-ST02
ST04=ST08*ST09*4.
ST05=SQRTF(ST03*ST03-ST04)*.5
CAPA=(SMA*ST05)/ST01
ST04=ST01*ST01*ST01
ST04=ST04+2.*ST06*ST01*ST02
ST03=(ST08-ST09)*(ST08-ST09)
DCPDSM=(ST04-(ST07+ST06)*ST03)/(ST05*4.*(ST01*ST01))
RETURN
END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
FM	32542	77436	FM3	32539	77433	FM2	32540	77434	FM1	32541	77435	P1	32538	77432
XX	32562	77462												

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
128	00200	32537	77431

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
SQRT	0	00000								

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
ST01	127	00177	ST02	126	00176	ST03	125	00175	ST04	124	00174	ST05	123	00173
ST06	122	00172	ST07	121	00171	ST08	120	00170	ST09	119	00167			

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	115	00163	3)	106	00152	6)	109	00155	7)	114	00162			

SUBROUTINES NOT PUNCHED FROM LIBRARY

SQRT

```
      SUBROUTINE CRDPP(THTA12,THTA13,FNU,ALFPP,BETPP,GAMPP)
      DIMENSION ALFPP(3),BETPP(3),GAMPP(3)
      ALFPP(1)=0.
      BETPP(1)=0.
      GAMPP(1)=1.
      STO1=COSF(FNU)
      STO2=SINF(FNU)
      STO4=SINF(THTA12)
      STO6=SINF(THTA13)
      ALFPP(2)=-STO4*STO2
      ALFPP(3)=STO6*STO2
      BETPP(2)=STO4*STO1
      BETPP(3)=-STO6*STO1
      GAMPP(2)=COSF(THTA12)
      GAMPP(3)=COSF(THTA13)
      RETURN
      END (0,1,0,0,0)
```

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
89	00131	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
COS	1	00001	SIN	0	00000									

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
STO1	88	00130	STO2	87	00127	STO4	86	00126	STO6	85	00125			

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
9)	84	00124	3)	77	00115	6)	79	00117						

SUBROUTINES NOT PUNCHED FROM LIBRARY

COS SIN

```
      SUBROUTINE FOFPHI(CAPA,SMA,DCPDSM,PHI,FOFPHT,B)
      AITCHF(Z)=DCPDSM*CAPA**2*B**2*AC**1.5*(-Z/(ALFA**2*(ALFA**2-Z**2)*
     X*.5))+CAPA**3*SMA*B*B*AC**1.5/AA*((2.+ALFA**2)*Z**3-3.*ALFA**2*Z)/
     X(3.*ALFA**4*(ALFA**2-Z**2)**1.5)+CAPFL-FPRIME
      AA=SMA**2-B**2
      CAPFL=(DCPDSM*CAPA**2*AA*B**2*SMA+(2./3.)*CAPA**3*B**4)/SMA**3
      ALFA=SMA/B
      AC=ALFA**2-1.
      NBOX=0
      FOFPHT=0.
      FPRIME=RANF(1.)*2.*CAPFL
      Z1=AITCHF(-1.)
      Z2=AITCHF(0.)
      Z3=AITCHF(1.)
      ERROR=FPRIME*.001
      IF(AITCHF(0.))10,16,11
   10 X1=-1.
      X2=0.
      GO TO 12
   11 X1=0.
      X2=1.
   12 X=(X2*AITCHF(X1)-X1*AITCHF(X2))/(AITCHF(X1)-AITCHF(X2))
      Y=AITCHF(X)
      NBOX=NBOX+1
      IF(NBOX-50)18,18,19
   18 IF(ABSF(Y)-ERROR)17,13,13
   13 IF(Y)14,17,15
   14 X2=X
      GO TO 12
   15 X1=X
      GO TO 12
   16 PHI=3.14159265/2.
      GO TO 20
   17 PHI=ARCOS(X,SQRTF(1.-X**2))
      GO TO 20
   19 FOFPHT=-1.
   20 RETURN
      END (0,1,0,0,0)
```

NAMES OF ARITHMETIC STATEMENT FUNCTIONS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

	IFN	LOC		IFN	LOC		IFN	LOC		IFN	LOC		IFN	LOC
AITCH	2	00355												

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
10	15	00220	11	18	00225	12	20	00231	18	24	00266	13	25	00274
14	26	00277	15	28	00302	16	30	00305	17	32	00311	17	33	00317
19	35	00324	20	36	00326									

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
382	00576	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
EXP(3	3	00003	ARCOS	1	00001	RAN	2	00002	SQRT	0	00000			

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
FPRIME	381	00575	ERROR	380	00574	CAPFL	379	00573	ALFA	378	00572	AC	377	00571
AA	376	00570	NBOX	375	00567	X1	374	00566	X2	373	00565	X	372	00564
Y	371	00563	Z1	370	00562	Z2	369	00561	Z3	368	00560			

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	361	00551	1)1	342	00526	2)	218	00332	3)	224	00340	4)1	341	00525
6)	232	00350	7)	340	00524									

SUBROUTINES NOT PUNCHED FROM LIBRARY

EXP(3 RAN ARCOS SQRT

```
      SUBROUTINE GAMCNV
      COMMON NZZ,XX,GMCNVT,PCOORD,PG1X,PG1Y,PG1Z,PG2X,PG2Y,PG2Z,PIGAM1,
     1PIGAM2,FLGAM1,FLGAM2
      DIMENSION XX(53),PCOORD(3)
      ALF1=PG1X/PIGAM1
      BET1=PG1Y/PIGAM1
      GAM1=PG1Z/PIGAM1
      ALF2=PG2X/PIGAM2
      BET2=PG2Y/PIGAM2
      GAM2=PG2Z/PIGAM2
      GMCNVT=0.
      FLAMG1=5.1448+135.6738/PIGAM1-1292.6491/(PIGAM1*PIGAM1)+5508.5681/
     1(PIGAM1*PIGAM1*PIGAM1)
      FLAMG2=5.1448+135.6738/PIGAM2-1292.6491/(PIGAM2*PIGAM2)+5508.5681/
     1(PIGAM2*PIGAM2*PIGAM2)
      FLP1=POPATH(2.,23.,13.,ALF1,BET1,GAM1,PCOORD(1),PCOORD(2),PCOORD
     1(3))
      IF (FLP1) 15,100,15
   15 FLP2=POPATH(2.,23.,13.,ALF2,BET2,GAM2,PCOORD(1),PCOORD(2),PCOORD
     1(3))
      IF (FLP2) 30,100,30
   30 STO1=RANF(RANDMY)
      STO2=RANF(RANDMY)
      FLGAM1=-FLAMG1*LOGF(STO1)
      FLGAM2=-FLAMG2*LOGF(STO2)
      IF (FLGAM1-FLP1) 50,40,40
   60 IF (FLGAM2-FLP2) 80,70,70
   40 WRITE OUTPUT TAPE NZZ,2000
      GO TO 60
   50 WRITE OUTPUT TAPE NZZ,2001,FLGAM1
      GMCNVT=1.
      GO TO 60
   70 WRITE OUTPUT TAPE NZZ,2002
      GO TO 100
   80 WRITE OUTPUT TAPE NZZ,2003,FLGAM2
      GMCNVT=GMCNVT+2.
  100 RETURN
 2000 FORMAT(1H ,28X,21HNO. 1, NO CONVERSION.)
 2001 FORMAT(1H ,28X,15HNO. 1 CONVERTS, 2X,F9.4,3HCM.)
 2002 FORMAT(1H+,63X,21HNO. 2, NO CONVERSION.)
 2003 FORMAT(1H+,63X,15HNO. 2 CONVERTS, 2X,F9.4,3HCM.)
      END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GMCNVT	32508	77374	FLGAM2	32495	77357	FLGAM1	32496	77360	NZZ	32562	77462	PCOORD	32507	77373
PG1X	32504	77370	PG1Y	32503	77367	PG1Z	32502	77366	PG2X	32501	77365	PG2Y	32500	77364
PG2Z	32499	77363	PIGAM1	32498	77362	PIGAM2	32497	77361	XX	32561	77461			

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
15	16	00134	15	17	00134	30	19	00151	60	24	00202	40	25	00207
40	27	00220	50	29	00223	50	32	00237	70	35	00244	70	37	00255
80	39	00260	80	42	00274	100	44	00301	2000	46	00000	2001	47	00000
2002	48	00000	2003	49	00000									

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
257	00401	32494	77356

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
LOG	4	00004	POPATH	6	00006	RAN	5	00005	(FIL)	0	00000	(IOH)O	2	00002
(LEV)	3	00003	(STH)	1	00001									

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAM2	256	00400	GAM1	255	00377	FLP2	254	00376	FLP1	253	00375	FLAMG2	252	00374
FLAMG1	251	00373	BET2	250	00372	BET1	249	00371	ALF2	248	00370	ALF1	247	00367
RANDMY	246	00366	STO1	245	00365	STO2	244	00364						

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
8)1UJ	217	00331	8)1UI	223	00337	8)1UH	230	00346	8)1UG	236	00354	1)	238	00356
3)	197	00305	6)	206	00316	7)	237	00355						

SUBROUTINES NOT PUNCHED FROM LIBRARY

POPATH RAN LOG (LEV) (IOH)O (STH) (FIL)

```
SUBROUTINE GENPR(XI,ETA)
XI=RANF(RANDMY)
ETA=RANF(RANDMY)
RETURN
END (0,1,0,0,0)
```

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
27	00033	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
RAN	0	00000								

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
RANDMY	26	00032								

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
6)	21	00025								

SUBROUTINES NOT PUNCHED FROM LIBRARY

RAN

```
       SUBROUTINE GENTRP(COSLAM,FMU,FNU,LAMBDF)
       STO1=RANF(RANDMY)
       STO2=RANF(RANDMY)
       STO3=RANF(RANDMY)
       FNU=6.2831853*STO1
       FMU=6.2831853*STO2
       COSLAM=2.*STO3-1.
    30 SINLAM=SQRTF(1.-COSLAM*COSLAM)
       IF (COSLAM-1.) 40,200,40
    40 IF (COSLAM+1.) 100,200,100
   100 LAMBDF=1
   110 RETURN
   200 LAMBDF=0
       GO TO 110
       END (0,1,0,0,0)
```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
30	8	00050	40	10	00061	100	11	00064	110	12	00066	200	14	00072

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
76	00114	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
RAN	1	00001	SQRT	0	00000									

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
RANDMY	75	00113	SINLAM	74	00112	STO1	73	00111	STO2	72	00110	STO3	71	00107

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
2)	61	00075	3)	63	00077	6)	66	00102						

SUBROUTINES NOT PUNCHED FROM LIBRARY

RAN SQRT

```
SUBROUTINE GOFP(FM2,FM3,P1,SMA,GOFP1)
STO2=(FM2*FM2-FM3*FM3)
STO3=SMA*SMA-P1*P1
STO1=(STO2*STO2)/(STO3*STO3)
STO3=2.*(FM2*FM2+FM3*FM3)/(SMA*SMA-P1*P1)
STO2=1.-STO3+STO1
STO3=3.*SMA*SMA*(1.-STO1)-P1*P1*STO2
GOFP1=P1*P1*(SQRTF(STO2))*STO3
RETURN
END (0,1,0,0,0)
```

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
127	00177	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
SQRT	0	00000								

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT	DEC	OCT	DEC	OCT
STO1	126	00176	STO2	125	00175	STO3	124	00174				

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT	DEC	OCT
1)	120	00170	3)	111	00157	6)	114	00162	7)	119	00167		

SUBROUTINES NOT PUNCHED FROM LIBRARY

SQRT

```
      SUBROUTINE LBTRAN
      COMMON NZZ,LL,P1P,P2P,P3P,FM,FM1,FM2,FM3,YY,E,P1LP,P2LP,P3LP,ALFP,
     1BETP,GAMP,PL,PIDCAT,PDTPNT,P1L,P2L,P3L,EL1,EL2,EL3
      DIMENSION LL(10),P1P(3),P2P(3),P3P(3),YY(6),E(3),P1LP(3),P2LP(3),
     1P3LP(3)
      DIMENSION PIDOTU(3),STOP(3)
C
      EL=SQRTF(PL*PL+FM*FM)
      GAM=EL/FM
      GAMBET=PL/FM
      GAMMN1=GAM-1.
      P1PX=P1P(1)*ALFP
      P1PY=P1P(2)*BETP
      P1PZ=P1P(3)*GAMP
      P2PX=P2P(1)*ALFP
      P2PY=P2P(2)*BETP
      P2PZ=P2P(3)*GAMP
      P3PX=P3P(1)*ALFP
      P3PY=P3P(2)*BETP
      P3PZ=P3P(3)*GAMP
      PIDOTU(1)=P1PX+P1PY+P1PZ
      PIDOTU(2)=P2PX+P2PY+P2PZ
      PIDOTU(3)=P3PX+P3PY+P3PZ
      DO 10 I=1,3
      STOP(I)=GAMMN1*PIDOTU(I)+GAMBET*E(I)
   10 CONTINUE
      P1LP(1)=STOP(1)*ALFP+P1P(1)
      P2LP(1)=STOP(2)*ALFP+P2P(1)
      P3LP(1)=STOP(3)*ALFP+P3P(1)
      P1LP(2)=STOP(1)*BETP+P1P(2)
      P2LP(2)=STOP(2)*BETP+P2P(2)
      P3LP(2)=STOP(3)*BETP+P3P(2)
      P1LP(3)=STOP(1)*GAMP+P1P(3)
      P2LP(3)=STOP(2)*GAMP+P2P(3)
      P3LP(3)=STOP(3)*GAMP+P3P(3)
      P1L=SQRTF(P1LP(1)*P1LP(1)+P1LP(2)*P1LP(2)+P1LP(3)*P1LP(3))
      P2L=SQRTF(P2LP(1)*P2LP(1)+P2LP(2)*P2LP(2)+P2LP(3)*P2LP(3))
      P3L=SQRTF(P3LP(1)*P3LP(1)+P3LP(2)*P3LP(2)+P3LP(3)*P3LP(3))
      EL1=SQRTF(FM1*FM1+P1L*P1L)
      EL2=SQRTF(FM2*FM2+P2L*P2L)
      EL3=SQRTF(FM3*FM3+P3L*P3L)
      PLX=ALFP*PL
      PLY=BETP*PL
      PLZ=GAMP*PL
      WRITE OUTPUT TAPE NZZ,2000
      WRITE OUTPUT TAPE NZZ,2001,PL,PLX,PLY,PLZ,EL,FM,P1L,(P1LP(I),I=1,3
     1),EL1,FM1,P2L,(P2LP(I),I=1,3),EL2,FM2,P3L,(P3LP(I),I=1,3),EL3,FM3
  100 RETURN
 2000 FORMAT(1H0,38X,42HLORENTZ TRANSFORMATION TO LAB, FROM C.M.S.)
 2001 FORMAT(1H0,8HPRIMARY-,11X,2HP=F9.2,2X,8H PX,Y,Z=F9.2,2(1H,,F9.2),
     12X,7HENERGY=F9.2,2X,5HMASS=F9.2/1H ,21HSECONDARY NO. 1-  P1=F9.2,
     22X,8HP1X,Y,Z=F9.2,2(1H,,F9.2),2X,7HENERGY=F9.2,2X,5HMASS=F9.2/1H ,
     321HSECONDARY NO. 2-  P2=F9.2,2X,8HP2X,Y,Z=F9.2,2(1H,,F9.2),2X,7HEN
     4ERGY=F9.2,2X,5HMASS=F9.2/1H ,21HSECONDARY NO. 3-  P3=F9.2,2X,8HP3X
     5,Y,Z=F9.2,2(1H,,F9.2),2X,7HENERGY=F9.2,2X,5HMASS=F9.2)
      END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAMP	32518	77406	FM	32542	77436	FM3	32539	77433	FM2	32540	77434	FM1	32541	77435
E	32532	77424	EL3	32509	77375	EL2	32510	77376	EL1	32511	77377	BETP	32519	77407
ALFP	32520	77410	LL	32561	77461	NZZ	32562	77462	P1LP	32529	77421	P1L	32514	77402
P1P	32551	77447	P2LP	32526	77416	P2L	32513	77401	P2P	32548	77444	P3LP	32523	77413
P3L	32512	77400	P3P	32545	77441	PDTPNT	32515	77403	PIDCAT	32516	77404	PL	32517	77405
YY	32538	77432												

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
10	23	00113	100	61	00365	2000	63	00000	2001	64	00000			

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
355	00543	32508	77374

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
SQRT	4	00004	(FIL)	0	00000	(IOH)O	2	00002	(LEV)	3	00003	(STH)	1	00001

STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
PIDOTU	354	00542	STOP	351	00537									

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAM	348	00534	GAMMN1	347	00533	GAMBET	346	00532	EL	345	00531	P1PX	344	00530
P1PY	343	00527	P1PZ	342	00526	P2PX	341	00525	P2PY	340	00524	P2PZ	339	00523
P3PX	338	00522	P3PY	337	00521	P3PZ	336	00520	PLX	335	00517	PLY	334	00516
PLZ	333	00515												

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
8)1UH	319	00477	8)1UG	329	00511	1)	330	00512	2)	249	00371	3)	251	00373
6)	252	00374												

SUBROUTINES NOT PUNCHED FROM LIBRARY

SQRT	(LEV)	(IOH)O	(STH)	(FIL)

```
      SUBROUTINE MOMANG(PHI,CAPA,SMA)
      COMMON XX,FM1,FM2,FM3,P1,P2,P3,THTA12,THTA13,THTA23,E,YY
      DIMENSION XX(21),E(3),YY(20)
      STO1=.5*P1
      X10=STO1*((FM2*FM2-FM3*FM3)/(SMA*SMA-P1*P1))
      XP1=X10+STO1
      XP2=STO1-X10
      STO1=COSF(PHI)
      R=CAPA*SQRTF((SMA*SMA-P1*P1)/(SMA*SMA-P1*P1*STO1*STO1))
      STO2=R*R
      STO3=SINF(PHI)
      P2=SQRTF(XP1*XP1-2.*R*XP1*STO1+STO2)
      P3=SQRTF(XP2*XP2+2.*R*XP2*STO1+STO2)
      E(1)=SQRTF(P1*P1+FM1*FM1)
      E(2)=SQRTF(P2*P2+FM2*FM2)
      E(3)=SQRTF(P3*P3+FM3*FM3)
      THTA12=ATANF((R*STO3)/(R*STO1-XP1))
      THTA13=ATANF((-R*STO3)/(XP2+R*STO1))
      IF (THTA12) 10,30,30
   30 IF (THTA13) 20,40,40
   40 THTA23=6.2831853-THTA12-THTA13
      RETURN
   10 THTA12=THTA12+3.1415926
      GO TO 30
   20 THTA13=THTA13+3.1415926
      GO TO 40
      END (0,1,0,0,0)
```

```
              STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

       DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 FM3 32539 77433          FM2 32540 77434          FM1 32541 77435            E 32532 77424           P1 32538 77432
  P2 32537 77431           P3 32536 77430       THTA12 32535 77427       THTA13 32534 77426       THTA23 32533 77425
  XX 32562 77462           YY 32529 77421

        EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC          EFN   IFN   LOC
 30    20 00241           40    21 00245           10    24 00255           20    26 00261

                              STORAGE NOT USED BY PROGRAM

       DEC   OCT                DEC   OCT
       204 00314              32509 77375

                         LOCATIONS OF NAMES IN TRANSFER VECTOR

       DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
 COS     3 00003          ATAN    0 00000          SIN     1 00001          SQRT    2 00002

      STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

       DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
   R   203 00313          STO1  202 00312          STO2  201 00311          STO3  200 00310          X10  199 00307
 XP1   198 00306           XP2  197 00305

                  STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

       DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT                DEC   OCT
1)     191 00277       3)      181 00265       6)      185 00271       7)      190 00276

                              SUBROUTINES NOT PUNCHED FROM LIBRARY
 COS        SQRT        SIN        ATAN
```

```
SUBROUTINE PDTPUN
X=X
RETURN
END (0,1,0,0,0)
```

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
18	00022	32562	77462

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
X	17	00021								

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
6)	12	00014								

```
      SUBROUTINE PIDCAY
      COMMON NZZ,LBTRNT,ALFPP,BETPP,GAMPP,P1P,P2P,P3P,FM,FMP,PG1,THTA12,
     1THTA13,THTA23,E,P1LP,P2LP,P3LP,ALFP,BETP,GAMP,PL,PIDCAT,PDTPNT,
     2PLG,ELG,WW,PG1X,PG1Y,PG1Z,PG2X,PG2Y,PG2Z,PIGAM1,PIGAM2,FLGAM1,
     3FLGAM2
      DIMENSION ALFPP(3),BETPP(3),GAMPP(3),P1P(3),P2P(3),P3P(3),P1LP(3),
     1P2LP(3),P3LP(3),E(3),PCOORD(3),FMP(3),PG1(3),PLG(3),ELG(3),WW(4)
      DIMENSION PGAM1(3),PGAM2(3)
      IF (PIDCAT-1.) 1000,200,300
  200 DO 110 I=1,3
  201 ICOMP=1
      IF (FMP(I)-135.000) 110,100,110
  100 EPI=E(I)
      PPI=PG1(I)
      WRITE OUTPUT TAPE NZZ,2000
  102 STO1=RANF(RANDMY)
      COSTHT=2.*STO1-1.
      SINTHT=SQRTF(1.-COSTHT*COSTHT)
      STO1=RANF(RANDMY)
      FNUPI=6.28319*STO1
      COSNU=COSF(FNUPI)
      SINNU=SINF(FNUPI)
      STO1=SINTHT*SINNU
      STO2=SINTHT*COSNU
      STO3=EPI*COSTHT
      PGAM1(1)=67.500*STO1
      PGAM1(2)=-67.500*STO2
      PGAM1(3)=.5*(STO3+PPI)
      PGAM2(1)=-PGAM1(1)
      PGAM2(2)=-PGAM1(2)
      PGAM2(3)=.5*(-STO3+PPI)
      IF (GAMPP(I)+1.) 103,400,103
  103 IF (GAMPP(I)-1.) 104,500,104
  104 COSLAM=GAMPP(I)
      SINLAM=SQRTF(1.-COSLAM*COSLAM)
      COSMU=ALFPP(I)/SINLAM
      IF DIVIDE CHECK 600,106
  106 SINMU=BETPP(I)/SINLAM
      PG1X=PGAM1(1)*SINMU+PGAM1(2)*COSLAM*COSMU+PGAM1(3)*SINLAM*COSMU
      PG1Y=-PGAM1(1)*COSMU+PGAM1(2)*COSLAM*SINMU+PGAM1(3)*SINLAM*SINMU
      PG1Z=-PGAM1(2)*SINLAM+PGAM1(3)*COSLAM
      PG2X=PGAM2(1)*SINMU+PGAM2(2)*COSLAM*COSMU+PGAM2(3)*SINLAM*COSMU
      PG2Y=-PGAM2(1)*COSMU+PGAM2(2)*COSLAM*SINMU+PGAM2(3)*SINLAM*SINMU
      PG2Z=-PGAM2(2)*SINLAM+PGAM2(3)*COSLAM
  107 PIGAM1=SQRTF(PG1X*PG1X+PG1Y*PG1Y+PG1Z*PG1Z)
      PIGAM2=SQRTF(PG2X*PG2X+PG2Y*PG2Y+PG2Z*PG2Z)
      WRITE OUTPUT TAPE NZZ,2002,I,PIGAM1,PG1X,PG1Y,PG1Z,PIGAM2,PG2X,PG2
     1Y,PG2Z
      CALL GAMCNV
      GO TO (110,120), ICOMP
  110 CONTINUE
      GO TO 1000
  300 DO 120 I=1,3
  301 ICOMP=2
      IF (FMP(I)-135.0000) 120,101,120
  101 EPI=ELG(I)
      PPI=PLG(I)
      WRITE OUTPUT TAPE NZZ,2001
      GO TO 102
  120 CONTINUE
      GO TO 1000
```

```
 400 PG1X=PGAM1(1)
     PG1Y=-PGAM1(2)
     PG1Z=-PGAM1(3)
     PG2X=PGAM2(1)
     PG2Y=-PGAM2(2)
     PG2Z=-PGAM2(3)
     GO TO 107
 500 PG1X=PGAM1(1)
     PG1Y=PGAM1(2)
     PG1Z=PGAM1(3)
     PG2X=PGAM2(1)
     PG2Y=PGAM2(2)
     PG2Z=PGAM2(3)
     GO TO 107
1000 RETURN
 600 GO TO 1000
2000 FORMAT(1H0,35X,49HPI NAUGHT DECAY AND GAMMA CONVERSION, FROM C.M.S
    1.)
2001 FORMAT(1H0,32X,54HPI NAUGHT DECAY AND GAMMA CONVERSION, FROM LAB S
    1YSTEM.)
2002 FORMAT(1H0,6X,13HSECONDARY NO I1,8X,16HGAMMA NO. 1-  P=F9.2,2X,7HP
    1X,Y,Z=F9.2,2(1H,,F9.2)/1H ,28X,16HGAMMA NO. 2-  P=F9.2,2X,7HPX,Y,Z
    2=F9.2,2(1H,,F9.2))
     END (0,1,0,0,0,
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAMP	32518	77406	GAMPP	32554	77452	FM	32542	77436	FMP	32541	77435	FLGAM2	32495	77357
FLGAM1	32496	77360	E	32532	77424	ELG	32511	77377	BETP	32519	77407	BETPP	32557	77455
ALFP	32520	77410	ALFPP	32560	77460	LBTRNT	32561	77461	NZZ	32562	77462	P1LP	32529	77421
P1P	32551	77447	P2LP	32526	77416	P2P	32548	77444	P3LP	32523	77413	P3P	32545	77441
PDTPNT	32515	77403	PG1	32538	77432	PG1X	32504	77370	PG1Y	32503	77367	PG1Z	32502	77366
PG2X	32501	77365	PG2Y	32500	77364	PG2Z	32499	77363	PIDCAT	32516	77404	PIGAM1	32498	77362
PIGAM2	32497	77361	PLG	32514	77402	PL	32517	77405	THTA12	32535	77427	THTA13	32534	77426
THTA23	32533	77425	WW	32508	77374									

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
200	6	00024	201	7	00027	100	9	00041	102	14	00060	103	31	00166
104	32	00171	106	36	00210	107	43	00341	110	51	00430	300	53	00435
301	54	00436	101	56	00450	120	62	00472	400	64	00475	500	71	00512
1000	78	00530	600	80	00535	2000	81	00000	2001	82	00000	2002	83	00000

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
442	00672	32494	77356

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAMCNV	0	00000	COS	2	00002	RAN	4	00004	SIN	1	00001	SQRT	3	00003
(FIL)	5	00005	(IOH)O	7	00007	(LEV)	8	00010	(STH)	6	00006			

STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
PCOORD	441	00671	PGAM1	438	00666	PGAM2	435	00663						

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
I	432	00660	ICOMP	431	00657	FNUPI	430	00656	EPI	429	00655	COSTHT	428	00654
COSNU	427	00653	COSMU	426	00652	COSLAM	425	00651	PPI	424	00650	RANDMY	423	00647
SINLAM	422	00646	SINMU	421	00645	SINNU	420	00644	SINTHT	419	00643	STO1	418	00642
STO2	417	00641	STO3	416	00640									

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
E)D	294	00446	E)A	283	00433	E)3	31	00037	D)40I	343	00527	D)10J	348	00534
C)G2	415	00637	C)G0	414	00636	8)1UI	386	00602	8)1UH	398	00616	8)1UG	409	00631
1)	411	00633	2)	350	00536	3)	352	00540	6)	358	00546	7)	410	00632

SUBROUTINES NOT PUNCHED FROM LIBRARY

(LEV) (IOH)O (STH) (FIL) RAN SQRT COS SIN GAMCNV

```
SUBROUTINE PIPCMP(ALF,BET,GAM,P1,P2,P3,P1P,P2P,P3P)
DIMENSION ALF(3),BET(3),GAM(3),P1P(3),P2P(3),P3P(3)
P1P(1)=ALF(1)*P1
P1P(2)=BET(1)*P1
P1P(3)=GAM(1)*P1
P2P(1)=ALF(2)*P2
P2P(2)=BET(2)*P2
P2P(3)=GAM(2)*P2
P3P(1)=ALF(3)*P3
P3P(2)=BET(3)*P3
P3P(3)=GAM(3)*P3
RETURN
END (0,1,0,0,0)
```

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
91	00133	32562	77462

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT	DEC	OCT	DEC	OCT	DEC	OCT
9)	90	00132	6)	85	00125						

```
      SUBROUTINE PMAX(PMX)
      COMMON XX,FM,FM1,FM2,FM3,YY
      DIMENSION XX(20),YY(22)
      STO1=(FM1+FM2+FM3)*(FM1+FM2+FM3)
      STO2=(FM1-FM2-FM3)*(FM1-FM2-FM3)
      PMX=.5*(1./FM)*SQRTF((FM*FM-STO1)*(FM*FM-STO2))
      RETURN
      END (0,1,0,0,0)
```

```
                    STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
    FM 32542 77436        FM3 32539 77433        FM2 32540 77434        FM1 32541 77435        XX 32562 77462
    YY 32538 77432

                             STORAGE NOT USED BY PROGRAM

       DEC   OCT              DEC   OCT
        61 00075            32516 77404

                          LOCATIONS OF NAMES IN TRANSFER VECTOR

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
  SQRT   0 00000

   STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
  STO1  60 00074        STO2   59 00073

                 STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

       DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
 1)     54 00066      3)       46 00056      6)      48 00060      7)      53 00065

                            SUBROUTINES NOT PUNCHED FROM LIBRARY
  SQRT
```

```
      FUNCTION POPATH(ZF,ZB,R,A,B,C,X,Y,Z)
      COMMON NZZ
      FREQUENCY11(1,0,1),5(1,0,1),19(0,0,1),20(0,0,1),21(0,0,1)
   19 IF(Z-ZF)10,10,20
   20 IF(ZB-Z)10,10,21
   21 IF(R-SQRTF(X**2+Y**2))10,10,11
   10 POPATH = 0.0
      WRITE OUTPUT TAPE NZZ,105,ZF,ZB,R
  105 FORMAT(20X, 68HPOPATH SAYS THE ORIGIN IS OUTSIDE THE FIDUCIAL VOLU
     XME DEFINED BY ZF=,F6.2,5H  ZB=,F6.2,4H  R=,F6.2///)
      GO TO 7
   11 IF(C)12,6,3
   12 POPATH=(ZF-Z)/C
      GO TO 4
    3 POPATH=(ZB-Z)/C
    4 RO=SQRTF((X+A*POPATH)**2 + (Y+B*POPATH)**2)
    5 IF(RO-R)7,7,6
    6 POPATH=(-(A*X+B*Y)+SQRTF((A**2+B**2)*R**2-(B*X-A*Y)**2))/(A**2+B**
     X2)
    7 RETURN
      END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
NZZ	32562	77462												

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
19	4	00072	20	5	00077	21	6	00104	10	7	00117	105	12	00000
11	14	00142	12	15	00145	3	17	00152	4	18	00156	5	19	00176
6	20	00203	7	21	00246									

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
207	00317	32561	77461

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
SQRT	4	00004	(FIL)	0	00000	(IOH)O	2	00002	(LEV)	3	00003	(STH)	1	00001

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
POPATH	206	00316	RO	205	00315									

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
8)39	196	00304	1)	197	00305	2)	171	00253	3)	172	00254	6)	173	00255

SUBROUTINES NOT PUNCHED FROM LIBRARY

SQRT (LEV) (IOH)O (STH) (FIL)

```
      SUBROUTINE PSPDST(GMX,N,NSC,NFAIL,F1,F2)
      COMMON NZZ,LBTRNT,ALFPP,BETPP,GAMPP,P1P,P2P,P3P,FM,FM1,FM2,FM3,P1,
     1P2,P3,THTA12,THTA13,THTA23,E,P1LP,P2LP,P3LP,ALFP,BETP,GAMP,PL,
     2PIDCAT,PDTPNT,P1L,P2L,P3L,EL1,EL2,EL3,GMCNVT,PCOORD,PG1X,PG1Y,PG1Z
     3,PG2X,PG2Y,PG2Z,PIGAM1,PIGAM2,FLGAM1,FLGAM2,IPTTYP,IPTREM,XCOORD,
     4YCOORD,ZCOORD,GDIST,P1U,P1UP,G1UX,G1UY,G1UZ,G2UX,G2UY,G2UZ
      DIMENSION ALFPP(3),BETPP(3),GAMPP(3),P1P(3),P2P(3),P3P(3),P1LP(3),
     1P2LP(3),P3LP(3),E(3),PCOORD(3),IPTTYP(8),IPTREM(8),XCOORD(8),YCOOR
     2D(8),ZCOORD(8),GDIST(8),P1U(3),P1UP(3)
      WRITE OUTPUT TAPE NZZ,2000
      I=0
      NSCSFL=0
      CALL PMAX(PMX)
      GMXP=GMX/1.E08
      WRITE OUTPUT TAPE NZZ,2003,GMXP,PMX,N,NSC,NFAIL,F1,FM,FM1,FM2,FM3
  100 I=I+1
      IF (N-I) 1000,110,110
  110 NSCSFL=NSCSFL+1
      IF (NSC-NSCSFL) 1000,200,200
  200 CALL GENPR(XI,ETA)
      P1=XI*PMX
      CALL CAPSMA(CAPA,SMA,DCPDSM)
      CALL GOFP(FM2,FM3,P1,SMA,GOFP1)
      GOFPT=GOFP1-ETA*GMX
      IF (GOFPT) 3000,300,300
  300 CALL FOFPHI(CAPA,SMA,DCPDSM,PHI,FOFPHT,P1)
      IF (FOFPHT) 4000,310,310
  310 WRITE OUTPUT TAPE NZZ,2005,I,NSCSFL
      CALL MOMANG(PHI,CAPA,SMA)
      PHP=PHI*57.2957795
      THTP12=THTA12*57.2957795
      THTP13=THTA13*57.2957795
      THTP23=THTA23*57.2957795
      WRITE OUTPUT TAPE NZZ,2001,P1,P2,P3,THTP12,THTP13,THTP23,PHP
  400 CALL RNORNT(COSLAM,LAMBDF)
      WRITE OUTPUT TAPE NZZ,2002,(P1P(I),I=1,3),(P2P(I),I=1,3),(P3P(I),
     1I=1,3)
      WRITE OUTPUT TAPE NZZ,2006,(E(I),I=1,3)
      IF (LAMBDF) 500,430,500
  430 SINLAM=SQRTF(1.-COSLAM*COSLAM)
      FLAMBD=ARCOS(COSLAM,SINLAM)
      FLAMBP=FLAMBD*57.2957795
      WRITE OUTPUT TAPE NZZ,2004,FLAMBP
      GO TO 500
  500 IF (LBTRNT) 510,600,510
  510 CALL LBTRAN
      GO TO 600
  600 CALL PIDCAY
      CALL PDTPUN
      GO TO 100
 1000 RETURN
 3000 IF (I-NFAIL) 100,3010,3010
 3010 FNSCF=NSCSFL
      FI=I
      FINSC1=FNSCF/FI
      IF (FINSC1-F1) 3900,100,100
 3900 WRITE OUTPUT TAPE NZZ,2300,NSCSFL,I,FINSC1
      GO TO 1000
 4000 IF (I-NFAIL) 100,4010,4010
 4010 FNSCF=NSCSFL
      FI=I
      FINSC2=FNSCF/FI
```

```
      IF (FINSC2-F2) 4900,100,100
 4900 WRITE OUTPUT TAPE NZZ,2400,NSCSFL,I,FINSC2
      GO TO 1000
 2000 FORMAT(1H1,///1H0,31X,56HTHREE BODY DECAY BY MONTE CARLO PHASE SPA
     1CE DISTRIBUTION/1H ,47X,24HWITH RANDON ORIENTATION.)
 2001 FORMAT(1H ,3HP1=F9.2,2X,3HP2=F9.2,2X,3HP3=F9.2,2X,8HTHTA12=F9.4,
     12X,8HTHETA13=F9.4,2X,8HTHETA23=F9.4,2X,4HPHI=F9.4,1X,3HDEG)
 2002 FORMAT(1H ,8HP1X,Y,Z=F9.2,2(1H,,F9.2),2X,8HP2X,Y,Z=F9.2,2(1H,,F9.2
     1),2X,8HP3X,Y,Z=F9.2,2(1H,,F9.2))
 2003 FORMAT(1H0,17HMAX VAL FOR G(P)=F5.2,3HE08,2X,14HMAX VAL FOR P=F8.2
     1,2X,17HMAX NO OF TRIALS=I5,2X,20HMAX NO OF SUCCESSES=I4,2X,6HNFAIL
     2=I3,2X,3HF1=F4.3/1H ,6HKMASS=F9.2,2X,13HDECAY MASSES=F9.2,2(1H,,
     3F9.2))
 2004 FORMAT(1H ,7HLAMBDA=F9.4)
 2005 FORMAT(1H0,9HTRIAL NO.I5,1H/,I5)
 2006 FORMAT(1H ,21HENERGIES OF P1,P2,P3=F9.2,2(1H,,F9.2),1H.)
 2300 FORMAT(1H ,15HFAILURE IN GOFP,I4,1H/,I5,1H=,F5.3)
 2400 FORMAT(1 ,17HFAILURE IN FOFPHI,I4,1H/,I5,1H=,F5.3)
      END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
IPTTYP	32494	77356	IPTREM	32486	77346	GMCNVT	32508	77374	GDIST	32454	77306	GAMP	32518	77406
GAMPP	32554	77452	G2UZ	32435	77263	G2UY	32436	77264	G2UX	32437	77265	G1UZ	32438	77266
G1UY	32439	77267	G1UX	32440	77270	FM	32542	77436	FM3	32539	77433	FM2	32540	77434
FM1	32541	77435	FLGAM2	32495	77357	FLGAM1	32496	77360	E	32532	77424	EL3	32509	77375
EL2	32510	77376	EL1	32511	77377	BETP	32519	77407	BETPP	32557	77455	ALFP	32520	77410
ALFPP	32560	77460	LBTRNT	32561	77461	NZZ	32562	77462	P1LP	32529	77421	P1L	32514	77402
P1P	32551	77447	P1	32538	77432	P1UP	32443	77273	P1U	32446	77276	P2LP	32526	77416
P2L	32513	77401	P2P	32548	77444	P2	32537	77431	P3LP	32523	77413	P3L	32512	77400
P3P	32545	77441	P3	32536	77430	PCOORD	32507	77373	PDTPNT	32515	77403	PG1X	32504	77370
PG1Y	32503	77367	PG1Z	32502	77366	PG2X	32501	77365	PG2Y	32500	77364	PG2Z	32499	77363
PIDCAT	32516	77404	PIGAM1	32498	77362	PIGAM2	32497	77361	PL	32517	77405	THTA12	32535	77427
THTA13	32534	77426	THTA23	32533	77425	XCOORD	32478	77336	YCOORD	32470	77326	ZCOORD	32462	77316

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
100	16	00123	110	18	00133	200	20	00143	200	21	00143	300	29	00174
300	30	00174	310	32	00207	310	35	00224	400	46	00272	400	47	00272
430	69	00361	500	78	00415	510	79	00417	600	81	00421	1000	84	00424
3000	86	00430	3010	87	00435	3900	91	00456	3900	94	00474	4000	96	00477
4010	97	00504	4900	101	00525	4900	104	00543	2000	106	00000	2001	107	00000
2002	108	00000	2003	109	00000	2004	110	00000	2005	111	00000	2006	112	00000
2300	113	00000	2400	114	00000									

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
516	01004	32434	77262

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GOFP	8	00010	GENPR	10	00012	FOFPHI	7	00007	CAPSMA	9	00011	ARCOS	3	00003
LBTRAN	2	00002	MOMANG	6	00006	PDTPUN	0	00000	PIDCAY	1	00001	PMAX	11	00013
RNORNT	5	00005	SQRT	4	00004	(FIL)	12	00014	(IOH)O	14	00016	(LEV)	15	00017
(STH)	13	00015												

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
I	515	01003	GOFPT	514	01002	GOFP1	513	01001	GMXP	512	01000	FOFPHT	511	00777
FNSCF	510	00776	FLAMBP	509	00775	FLAMBD	508	00774	FI	507	00773	FINSC2	506	00772
FINSC1	505	00771	ETA	504	00770	DCPDSM	503	00767	COSLAM	502	00766	CAPA	501	00765
LAMBDF	500	00764	NSCSFL	499	00763	PHI	498	00762	PHP	497	00761	PMX	496	00760
SINLAM	495	00757	SMA	494	00756	THTP12	493	00755	THTP13	492	00754	THTP23	491	00753
XI	490	00752												

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
8)2B0	375	00567	8)27S	383	00577	8)1UM	392	00610	8)1UL	397	00615	8)1UK	401	00621
8)1UJ	434	00662	8)1UI	450	00702	8)1UH	470	00726	8)1UG	489	00751	2)	358	00546
3)	360	00550	6)	363	00553									

SUBROUTINES NOT PUNCHED FROM LIBRARY

(LEV) (IOH)O (STH) (FIL) PMAX GENPR CAPSMA GOFP FOFPHI MOMANG

RNORNT SQRT ARCOS LBTRAN PIDCAY PDTPUN

```
      SUBROUTINE RANLOD
      READ 1000,R
      IF (R) 10,100,10
   10 X=LODRANF(R)
  100 RETURN
 1000 FORMAT(1012)
      END (0,1,0,0,0)
```

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
10	6	00031	100	7	00034	1000	9	00000						

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
40	00050	32562	77462

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
LODRAN	0	00000	(CSH)	2	00002	(IOH)I	3	00003	(LEV)	4	00004	(RTN)	1	00001

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
R	39	00047	X	38	00046									

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
8)V8	37	00045	6)	32	00040									

SUBROUTINES NOT PUNCHED FROM LIBRARY

(LEV) (IOH)I (CSH) (RTN) LODRAN

```
      SUBROUTINE RANSTR
      Y=STORANF(R)
      PUNCH 1000,Y
  100 RETURN
 1000 FORMAT(1012)
      END (0,1,0,0,0)
```

```
 EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

 EFN   IFN  LOC      EFN   IFN  LOC      EFN   IFN  LOC      EFN   IFN  LOC      EFN   IFN  LOC
 100     6 00032    1000     8 00000

                                STORAGE NOT USED BY PROGRAM

       DEC   OCT               DEC   OCT
        38 00046             32562 77462

                              LOCATIONS OF NAMES IN TRANSFER VECTOR

         DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
STORAN     4 00004    (FIL)       0 00000    (IOH)O      2 00002    (LEV)       3 00003    (SCH)       1 00001

 STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

         DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
     R    37 00045        Y      36 00044

                    STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

         DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT              DEC   OCT
8)V8      35 00043    6)         30 00036

                               SUBROUTINES NOT PUNCHED FROM LIBRARY
STORAN      (LEV)     (IOH)O      (SCH)      (FIL)
```

```
      SUBROUTINE RNORNT(COSLAM,LAMBDF)
      COMMON NZZ,AA,ALFPP,BETPP,GAMPP,P1P,P2P,P3P,FM,FM1,FM2,FM3,P1,P2,
     1P3,THTA12,THTA13,THTA23,YY
      DIMENSION ALFPP(3),BETPP(3),GAMPP(3),P1P(3),P2P(3),P3P(3),YY(16)
      DIMENSION ALF(3),BET(3),GAM(3)
      CALL GENTRP(COSLAM,FMU,FNU,LAMBDF)
      CALL CRDPP(THTA12,THTA13,FNU,ALFPP,BETPP,GAMPP)
      CALL ABGCMP(ALFPP,BETPP,GAMPP,COSLAM,FMU,LAMBDF,ALF,BET,GAM)
      CALL PIPCMP(ALF,BET,GAM,P1,P2,P3,P1P,P2P,P3P)
      RETURN
      END (0,1,0,0,0)
```

STORAGE FOR VARIABLES APPEARING IN COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAMPP	32554	77452	FM	32542	77436	FM3	32539	77433	FM2	32540	77434	FM1	32541	77435
BETPP	32557	77455	ALFPP	32560	77460	AA	32561	77461	NZZ	32562	77462	P1P	32551	77447
P1	32538	77432	P2P	32548	77444	P2	32537	77431	P3P	32545	77441	P3	32536	77430
THTA12	32535	77427	THTA13	32534	77426	THTA23	32533	77425	YY	32532	77424			

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
68	00104	32516	77404

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GENTRP	3	00003	CRDPP	2	00002	ABGCMP	1	00001	PIPCMP	0	00000			

STORAGE LOCATIONS FOR VARIABLES APPEARING IN DIMENSION AND EQUIVALENCE SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
GAM	61	00075	BET	64	00100	ALF	67	00103						

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN DIMENSION,EQUIVALENCE OR COMMON SENTENCES

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
FNU	58	00072	FMU	57	00071									

STORAGE LOCATIONS FOR SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
6)	52	00064												

SUBROUTINES NOT PUNCHED FROM LIBRARY

GENTRP CRDPP ABGCMP PIPCMP

```
*0001                          00000      ORG 0                                                              *
*0002                                     ABS 4
  *    *    *    *    *    *    *    *    *    *    *    *    *    *    *    *    *    *  CARD    001
*0003       00000  0 00000 0 00104        PZE LAST,0,0
*0004       00001  0 00000 0 00000        PZE
*0005       00002  714443606060           BCD 1ZML
*0006       00003  0 00000 0 00000        PZE ZML
*0007                                     REL
*0008                          00000      ORG 0
*0009       00000  0 76200 0 00321 ZML    RCD
*0010       00001  0 70000 0 00020        CPY IN1
*0011       00002  0 70000 0 00021        CPY IN2
*0012       00003  0 50000 0 00020        CLA IN1
*0013       00004 -0 10000 0 00013        TNZ A2
*0014       00005  0 50000 0 00021        CLA IN2
*0015       00006  0 60100 0 00000 DOWN   STO **
*0016       00007  0 50000 0 00006        CLA DOWN
*0017       00010  0 40000 0 00017        ADD C1
*0018       00011  0 62100 0 00006 A1     STA DOWN
*0019       00012  0 02000 0 00000        TRA ZML
*0020       00013  0 50000 0 00021 A2     CLA IN2
*0021       00014 -0 10000 0 00021        TNZ IN2
*0022       00015  0 50000 0 00020        CLA IN1
*0023       00016  0 02000 0 00011        TRA A1
*0024       00017 +000000000001    C1     DEC 1
*0025       00020  0 00000 0 00000 IN1    HTR **
*0026       00021  0 00000 0 00000 IN2    HTR **
*0027                          00104 ENDZML BES 50
*0028                          00104 LAST   SYN *
  *    *    *    *    *    *    *    *    *    *    *    *    *    *    *    *    *    *  CARD    002
*0029 A                        00000      END

4AP ASSEMBLER STATISTICS

TAPE  TOTAL     1 FAIL    2 FAIL    3 FAIL    4 FAIL
INP     0         0         0         0         0
LIB     0         0         0         0         0
COL     1         0         0         0         0

NUMBER OF ON-LINE INPUT RECORDS        29

NUMBER OF OFF-LINE PRINT RECORDS       40

NUMBER OF SYMBOLS,  DEF     9,DEFOP     0,UNDEF     0


          ORG 0                                                  0001
```

APPENDIX: VALUES OF G_{max} FOR SOME DECAY PROCESSES

One item of the input data to the program is G_{max}, the maximum height of the distribution curve $G'(p_1)$ (Eq. 16). This can be easily found by trial and error. It should be remembered that the distribution generated by this program is not affected if too high a value is used for G_{max}, because only the ratio of successes to trials is reduced. Thus, it is sufficient to chose a value for G_{max} somewhat above the estimated value. If by accident too low a value has been chosen, an alarm is printed out whenever the computed $G'(p_1)$ exceeds the value of G_{max}. For convenience, some G_{max} values are listed for typical 3-body decays. Energy units of Mev and Mev/c are used throughout:

Masses:

$m(k^{\pm}) = 494.00$

$m(\pi^{\pm}) = 139.59$

$m(\mu^{\pm}) = 105.66$

$m(\pi^{0}) = 135.00$

$m(e^{\pm}) = .511$

$m(\nu) = 0$

$m(\gamma) = 0$

Types of Decays: (secondary masses are labelled in the sequence in which they occur as m_1, m_2, and m_3)

$K^{\pm} \rightarrow \mu^{\pm} + \pi^{0} + \nu$: $G_{max} = 44.366 \times 10^{8}$ at $p_1 = 158.5$

$K^{\pm} \rightarrow e^{\pm} + \pi^{0} + \nu$: $G_{max} = 61.633 \times 10^{8}$ at $p_1 = 167$

$K^{\pm} \rightarrow \pi^{\pm} + \pi^{+} + \pi^{-}$: $G_{max} = 12.78 \times 10^{8}$ at $p_1 = 101$

$K^{\pm} \rightarrow \pi^{\pm} + 2\pi^{0}$: $G_{max} = 14.954 \times 10^{8}$ at $p_1 = 107.5$

$K^{\pm} \rightarrow \pi^{\pm} + \pi^{0} + \gamma$: $G_{max} = 34.315 \times 10^{8}$ at $p_1 = 151$

$\mu^{\pm} \rightarrow e^{\pm} + 2\nu$: $G_{max} = 0.1836 \times 10^{8}$ at $p_1 = 43$

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Information Division
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