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Abstract: Disks, bubbles, and donuts have been observed in dynamical calculations of heavy ion collisions. These shapes are subject to a variety of surface and Coulomb instabilities. These instabilities are identified and analyzed in terms of their relevance to multifragmentation.

Introduction

Calculations of nucleus-nucleus collisions at intermediate energies performed by means of different transport codes, have shown the formation of rather peculiar shapes, like rods, disks, bubbles, and donuts, which then proceed to break up into several fragments (1, 2, 3).

These results are exciting for two reasons. On one hand they give new life to the old problem of stability for such shapes, on the other they suggest that multifragmentation may be due to the onset of easily describable static instabilities.

Much attention has been given to the spinodal instability as a possible cause of multifragmentation. Since the spinodal instability can occur in an infinite system, it can be called a bulk or volume instability. Rather strong manipulations, like compression followed by expansion, are necessary to bring about the spinodal instability. Here we are interested in the somewhat gentler physics of those instabilities that rely on surface, proximity, and Coulomb energies. We shall discuss these instabilities in the context of cylinders (rods), donuts, disks (sheets), bubbles, and spheres.

Surface Instabilities in Boltzmann-Nordheim-Vlasov Calculations

In head-on collisions of heavy-ions simulated using the Boltzmann-Nordheim-Vlasov (BNV) equation, a "disk" develops due to the side-squeezing of nuclear matter. When the disk becomes sufficiently thin, it breaks up into several fragments whose size is commensurate with the thickness of the disk.

Some of these features are shown in Fig. 1 for a head-on collisions of two ^{90}Mo nuclei. Similar calculations have been performed for a range of central impact parameters and entrance-channel mass asymmetries with similar results. At relatively small impact parameters one observes donut shapes, and at large impact parameters, rather long necks. Both donuts and necks break up into several fragments. Other calculations with different codes and for different systems show the formation of bubbles that break up in much the same manner as disks.

Metastability of a Sheet of Liquid

The characteristic way in which disk fragmentation occurs, strongly suggests that it is caused by surface instabilities. *The system appears to escape from the high surface energy of the disk by breaking up into a number of spherical fragments with less overall surface.*

One may be tempted to consider this instability as a case of the Rayleigh instability of a cylinder of liquid. The cylinder is unstable with respect to small a disk of liquid, or more generally, a sheet of liquid does not suffer the same kind of instability as a cylinder.

If we assume sharp surfaces (no surface thickness, no surface-surface interaction), a sheet can be metastable with respect to break-up into a layer of cylinders or spheres. The onset of metastability for both cases is easily calculated. On a sheet of thickness d let us identify stripes of width λ. These stripes can favorably collapse into cylinders when the surface area of a stripe (top + bottom) is greater than the surface area of the cylinder of equivalent volume.

Metastability and not necessarily instability is involved here, since there may be a barrier that prevents the sheet from reaching the more stable configurations illustrated above, and indeed there is. Clearly, any wave of infinitesimal amplitude A increases the surface area of the sheet, and thus its energy, independent of the sheet thickness, since, in the limit of infinitely sharp surfaces, the surfaces do not know of each other, until they touch. However, the systems portrayed in Fig.1 develop what appears to be a genuine instability. Perhaps, the system, which has plenty of energy, simply jumps the barrier. But, there is another, more likely possibility.

Instability of a sheet of liquid and surface-surface interactions

Nuclear surfaces are not sharp, but diffuse, and they interact with each other through an interaction of finite range called also the proximity force $\Phi(s)$, where s is the distance between surfaces. We can now calculate the incremental energy of a sheet subjected to a perturbation of wavelength λ and small amplitude A. The dimensionless proximity interaction is:

$$V_P = \frac{2}{\lambda}\int_0^\lambda \Phi(s)\,dx \sim \frac{2}{\lambda}\left(P(\lambda) + Q(\lambda)A^2\right) \qquad (1)$$

where

$$P(\lambda) = \int_0^\lambda \Phi_0(x)dx \quad \text{and} \quad Q(\lambda) = \int_0^\lambda \Phi_2(x)dx \qquad (2)$$

with $s = d + 2A \sin kx$, Φ_0 and Φ_2 being the zeroth and second order coefficients of the Taylor expansions of $\Phi(A,x)$ about $A = 0$, and $k = 2\pi/\lambda$.

The overall energy increase, including the surface energy term is:

$$\Delta V = A^2\left(\frac{2\pi^2}{\lambda^2} + \frac{Q(\lambda)}{\lambda}\right) \qquad (3)$$

Instability occurs when the coefficient of A^2 is zero or negative. Thus, the critical wavelength for the onset of the instability is given by the equation:

$$\lambda_c\, Q(\lambda_c) + 2\pi^2 = 0\,. \qquad (4)$$

Any perturbation with $\lambda > \lambda_c$ is then unstable, namely it will grow spontaneously and exponentially. Using for the proximity potential the expression in ref. 4, we obtain

$$\lambda_c = 1.10\, b \exp[2d/3b], \qquad (5)$$

where b is the range of the proximity interaction.

When the thickness of the sheet becomes much greater than the range of the proximity interaction, the critical wavelength tends to infinity. This is the trivial result for infinitely sharp surfaces that was mentioned above. However, when the thickness of the sheet becomes comparable to the proximity range b, the critical wave-length decreases very rapidly. This result is quite interesting, because it applies in general to all liquids. Specifically, thin disks and thin bubbles can be destabilized by the sheet instability.

Stability of a bubble

Nuclear bubbles and their stability have been discussed in a variety of contexts (5). Here we shall analyse the relevant degrees of freedom, and the physical quantities that affect their stability. The bubble degrees of freedom can be divided into two classes: the radial modes and the crispation modes. The radial modes are characterized by a constant thickness throughout the bubble, while the crispation modes modulate the thickness of the bubble's wall. The physical quantities that we shall consider here are the Coulomb force, the centrifugal force, and the pressure difference across the walls of the bubble. The most important degree of freedom, which defines the bubble itself, is the radial monopole mode.

The monopole mode

Let us consider a nucleus with radius R_0, fissility parameter X, and angular momentum I; let us allow a bubble of radius R_1 to grow at the center of the nucleus with an internal pressure p. If we define $x=R_1/R_0$, the energy of the system in units of twice the surface energy E_s of the unperturbed nucleus is:

$$E=\frac{1}{2}x^2+\frac{1}{2}\left(1+x^3\right)^{\frac{2}{3}}+5X\left[\frac{1}{5}\left(1+x^3\right)^{\frac{5}{3}}-\frac{1}{2}x^3\left(1+x^3\right)^{\frac{2}{3}}+\frac{3}{10}x^5\right]$$

$$-x^3P+\frac{R}{\left[\left(1+x^3\right)^{\frac{5}{3}}-x^5\right]}$$

$$\text{where: }\quad \begin{aligned} X &= E_c^0/2E_s^0; \\ P &= pV_0/2E_s^0; \\ R &= E_R^0/2E_s^0. \end{aligned} \qquad (6)$$

The first two terms of the equation give the surface energy of the inner and outer sphere, the third gives the Coulomb energy, the fourth the pressure energy, and the fifth the rotational energy. Here are the effects of each. If we neglect the Coulomb term, but we retain instead the pressure, we have a minimum at x=0, and a saddle at x=2/(3P). If the pressure does not depend on the radius, as in the case of nuclear vapor pressure, which just depends on temperature, the bubble will, for sufficiently large radii, become unstable towards indefinite expansion. On the other hand, the inclusion of both surface and Coulomb terms permits the formation of a secondary minimum for X>2.02. This is the Coulomb bubble. An example of the potential energy as a function of the inner radius is given in fig.(2). The rotational term mimics the Coulomb term, and generates a secondary minimum for R>.956. As the Coulomb (or rotational) energy increases, the secondary minimum becomes deeper and deeper, until it becomes the absolute minimum.

A word may be in order regarding the pressure. For T>0, a saturated vapor fills the bubble cavity. This vapor would have a pressure equal to the saturation pressure, which depends only upon the temperature, but not upon the volume. At first sight, one would not expect a pressure acting upon the outer surface, which is facing the vacuum. However, this is not the case. The inner surface has an outgoing flux of evaporated particles, and an ingoing flux of vapor particles. At equilibrium the two fluxes are equal: they impart the same impulse to the surface, and thus contribute equally to the pressure. The outer surface, however, has only the outgoing component, and thus feels a pressure equal to one half of the inner pressure.

The higher order modes

Before discussing the stability of the higher modes in a bubble, let us review the case of the liquid sphere.

Liquid sphere The eigen-frequencies of a liquid, incompressible non viscous sphere with irrotational flow are given by:

$$\omega'^2 = n(n-1)(n+2) \tag{7}$$

where: ω' is the frequency given in units of $(c_s/\rho R^3)^{1/2}$ (c_s is the surface energy coefficient, and ρ and R are the density and radius of the sphere respectively); and n is the order of the spherical harmonics under consideration. Since all these frequencies are real, all the modes are bound.

The introduction of a charge (uniformly distributed throughout the volume of the sphere) changes the frequencies as follows:

$$\omega'^2 = n(n-1)\,[(n+2) - 4X] \tag{8}$$

where X is the well known fissility parameter. The effect of the Coulomb field is that of destabilizing a number of modes.

For $X < 1$ all the modes are still stable. At $X = 1$, the frequency ω' goes to zero for $n = 2$. This is the onset of quadrupole instability, or of the fission instability. For $X > 1$ progressively higher modes are destabilized. The last unstable mode is:

$$n_{last} = 4X - 2.$$

One would think that when many modes are unstable, the most unstable mode would remain the lowest mode $n = 2$ or the fission mode. This is, curiously, not the case. For instance, for $X = 3$, $n_{last} = 10$, $n_{max} = 7$ and for $X = 4$, $n_{last} = 14$, $n_{max} = 10 - 9$. So, a highly charged sphere will not merely fission, but will break up in many droplets through an instability associated with a high multipole mode.

Bubbles. We consider first the radial modes. The stability of a Coulomb bubble towards monopole oscillations is not a sufficient condition for stability. The stability towards the higher order radial modes can be easily checked by means of a modification of eq.(8), All that one needs to do is to substitute in it for the parameter X the parameter X_{eff} containing the Coulomb and surface energy of the bubble. This is given by:

$$X_{eff}(x) = 5X\frac{\frac{1}{5}\left(1+x^3\right)^{\frac{5}{3}} - \frac{1}{2}x^3\left(1+x^3\right)^{\frac{2}{3}} + \frac{3}{10}x^5}{x^2 + \left(1+x^3\right)^{\frac{2}{3}}} \tag{9}$$

As the bubble expands, X_{eff} decreases. If the original nucleus is unstable up to the multipole of order n, as it develops into a bubble it starts stabilizing the higher multipoles. This is shown in fig.(3). At a sufficiently large value of X, the bubble will become stable even against the quadrupole deformation, and all the radial multipoles would be stable, with the possible exception of the monopole. However, at the value of x corresponding to the equilibrium of the Coulomb bubble, it turns out that the quadrupole and octupole modes are not yet stabilized. Therefore, no Coulomb bubble is absolutely stable. Of course, such bubble could exist as a transient. Radial instabilities can be escaped by a sufficiently rapid, dynamically driven expansion.

Yet, it may still be possible to have a stable nuclear bubble. If the bubble is warm, it fills up with vapor. The resulting pressure differential acts only upon the monopole mode, by displacing outwards the Coulomb minimum. The effect on the other radial modes is nil, since only changes in volume are relevant to pressure. Consequently the values of X corresponding to the stabilization of the various multipoles do not change. Thus, as shown in fig.(2), a sufficiently large pressure can move the bubble minimum sufficiently far out, beyond the onset of quadrupole stability.

The second class of modes consists of "crispation modes" involving a thickening and thinning of the liquid layer. In the limit of sharp surfaces (no surface-surface interaction) and in the absence of the Coulomb field all these modes are stable, excepting the dipole mode which is indifferent. The "inner" sphere of the bubble in this mode is free to drift with respect to the outer sphere, leading eventually to the puncturing of the bubble. The introduction of surface-surface interaction can make all the crispation modes unstable through the "sheet instability".

On the other hand, the Coulomb field tends to stabilize these modes. This is because the Coulomb force tends to resist the attempt to concentrate the charge in "clumps" distributed on the surface of the sphere as required by the higher modes.

Bubbles and the sheet instability

A bubble behaves much like a sheet, and is subject to the sheet instability. Since a bubble, like a sheet, must rely on the proximity interaction to become unstable, it will retain its surface stability until its thickness is of the order of the surface-surface interaction range. Thus, a rather thick-walled bubble is not susceptible to surface instabilities over a broad range of its inner sphere radius. This is shown in fig.(4). At a sufficiently large inner sphere radius, a neutral bubble becomes thin enough for the onset of quadrupole instability, and it becomes unstable with respect to higher order modes when it gets thinner. The same trend is observed for a coulomb bubble; however, the curve traces the loci of the critical inner radius stops at the mode below which the bubble becomes unstable at all inner sphere radii. BNV calculations for very heavy systems at low bombarding energies show the formation of a thin bubble that seems to burst under the action of the sheet instability.

Surface Instabilities in Rods and Donuts

Rayleigh showed that a cylinder is unstable with respect to perturbations of wave-length $\lambda \geqq 2\pi R$, where R is the radius of the cylinder. Notice that this instability has a purely geometric origin, and it does not depend upon the strength of the surface tension.

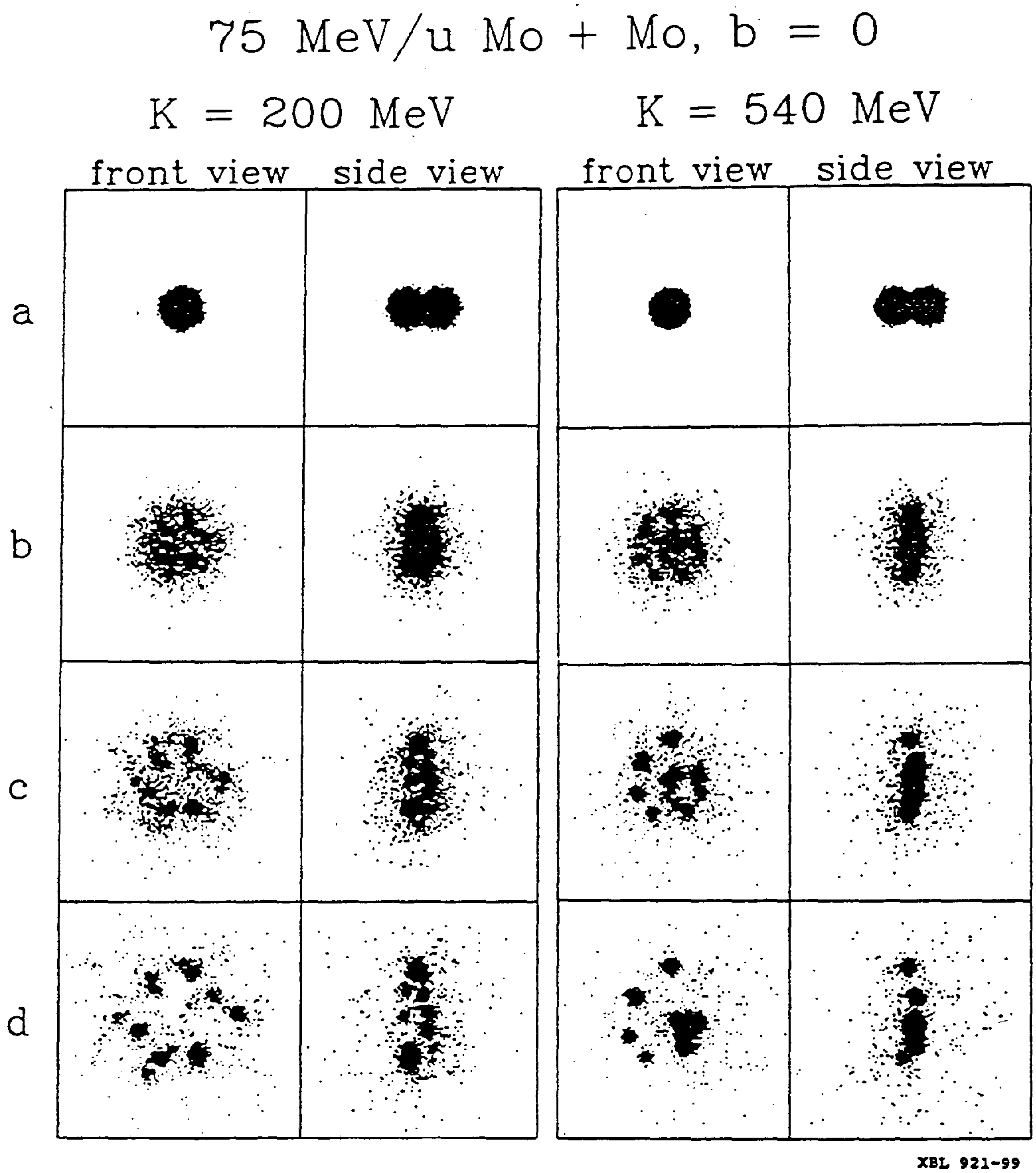
The breaking of the neck, either developing in a fissioning nucleus, or forming between the partners in a heavy ion collision, has been attributed to this instability. Long necks that break up producing a few little drops, are a common observation in BNV simulations of heavy ion collisions.

A torus can be imagined as a cylinder bent so that its two bases are united. Therefore it should manifest the classical Rayleigh instability, whose critical wave-length is given by the length of the circumference of the cylinder. A torus with a length equal to the critical wave-length has a very compact shape in which the inner circumpherence degenerates into a point. Any wider torus or donut with a finite hole should be unstable , and will spontaneously pinch off. This is a serious instability that can give rise to a number of fragments.

Conclusions

Missing in the discussion of these fancy theoretical constructs is the voice of experimental evidence. The unfortunate reason for this is that there is none. Despite a great deal of experimental effort, the mechanism of multifragmentation still remains a mystery and a challenge. There are some ominous indications that statistical effects may play a rather heavy role. On the other hand, there is still plenty of room to speculate, and one should take advantage of this opportunity while there is still time to do so.

References:
1) L.G. Moretto et al.., Phys. Rev. Lett. **69**, 1884, (1992).
2)W. Bauer, G.F. Bertsch, and H. Schultz, Phys. Rev. Lett. **69**, 1888, (1992).
3)B. Borderie, et al. Phys. Lett. B302, 15, (1992)
4)J. Blocki, J. Randrup, W. J. Swiatecki, and C.F. Tsang, Ann. Phys.(NY) **105**, 427 (1977).
5)C.Y. Wong, Phys. Rev. Lett. **55**, 1973, (1985) and references therein.



Firgure 1. BNV calculations for a head-on collision (b = 0) of the 75 MeV/u ^{90}Mo + ^{90}Mo reaction at time steps of (a) 20, (b) 60, (c) 120, and (d) 180 fm/c. The front and side-views of the colliding systems are given in columns 1 & 2, respectively for a value of the incompressibility constant, K = 200 MeV. Similar views are shown in columns 3 & 4 for K = 540 MeV.

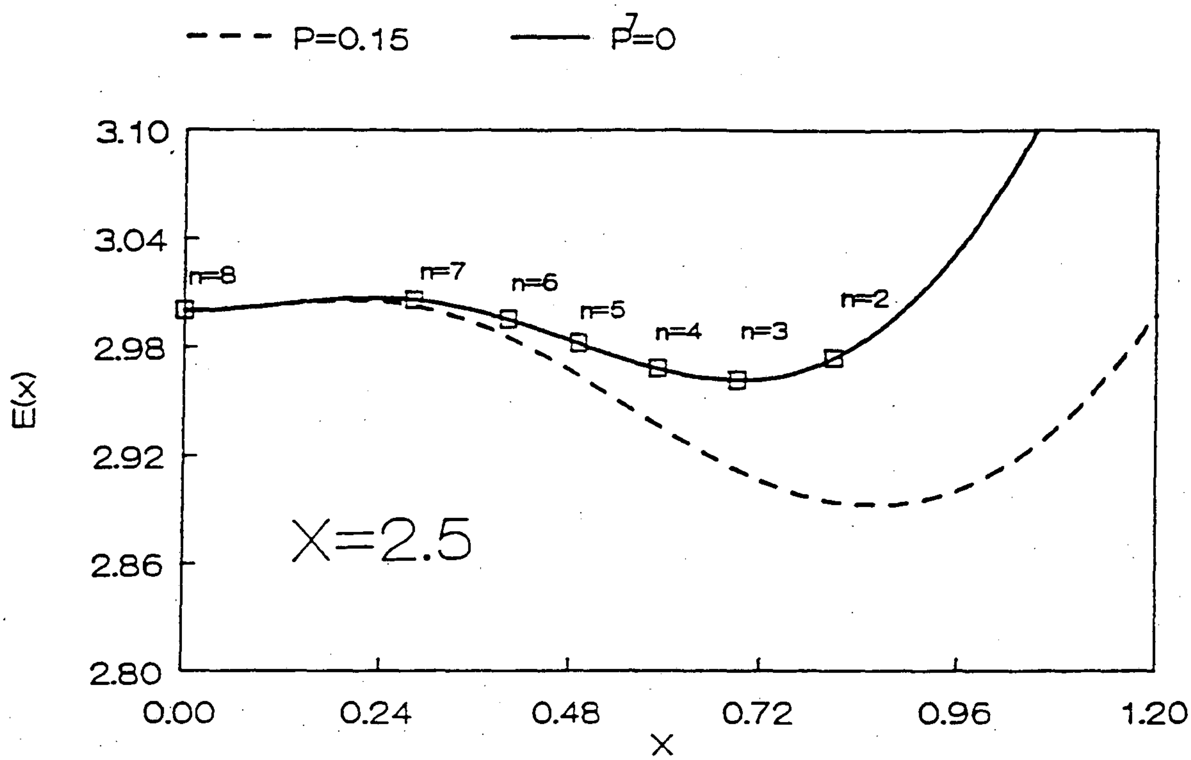


Figure 2. Potential energy of a bubble as a function of its inner radius. The fissility parameter of the equivalent sphere is X=2.5. The squares indicate the onset of instability for the various multipoles. The solid line corresponds to zero pressure, while the dashed line to the pressure that stabilizes all the multipoles.

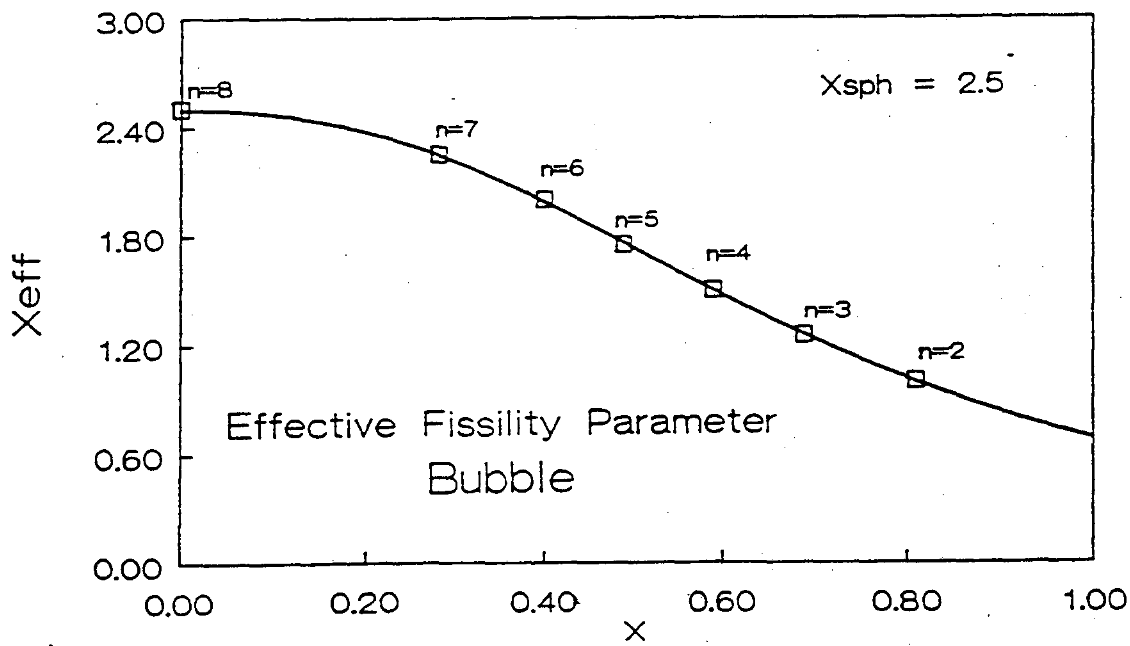


Figure 3. Effective fissility parameter of a bubble as a function of its inner radius. The fissility parameter of the equivalent sphere is X=2.5. The squares indicate the onset of instability for the various multipoles.

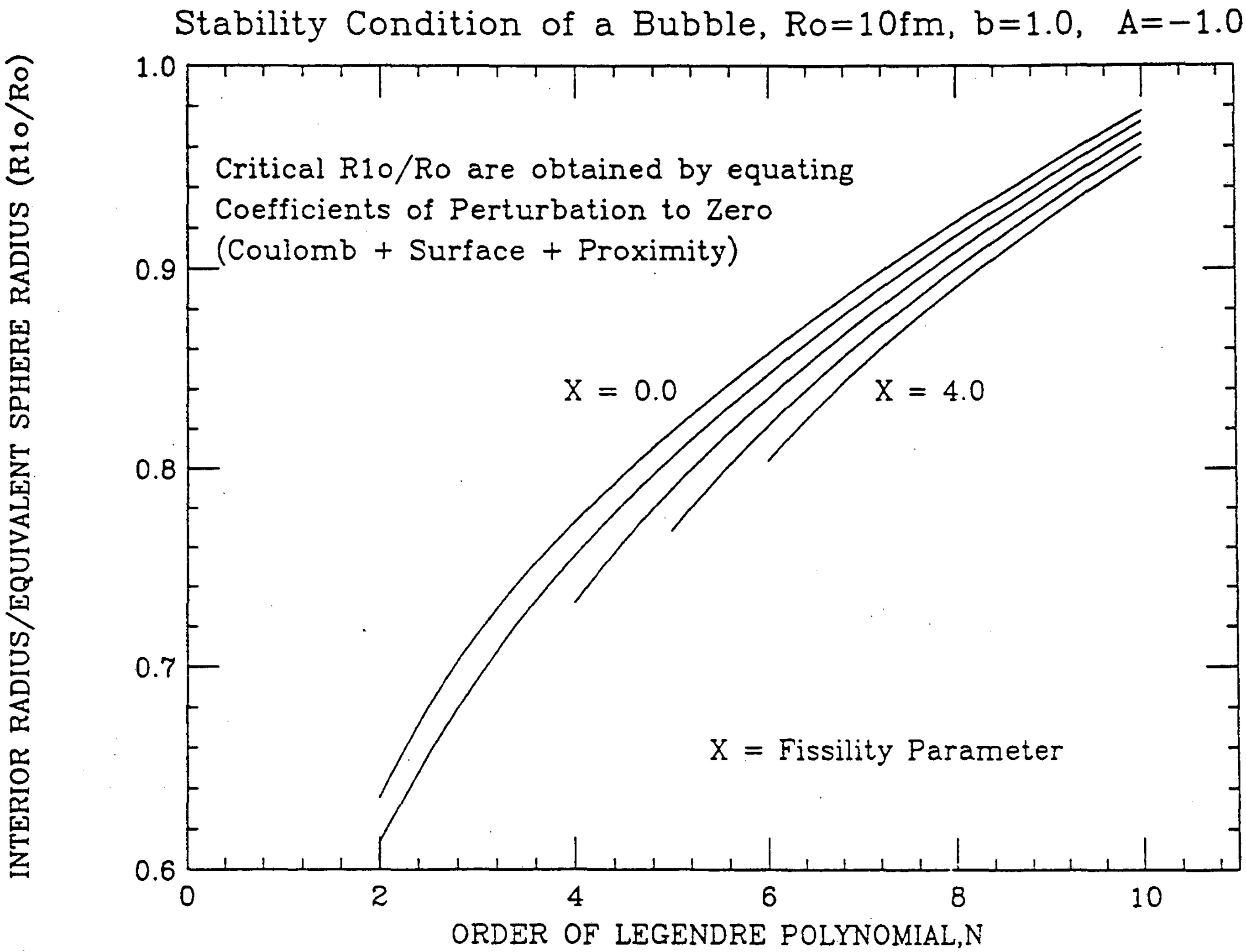


Figure 4. The critical inner radius of a bubble for the onset of the instability as a function of the order of Legendre Polynomial for various values of the fissility parameter. The perturbation on the two surfaces are equal in magnitude but opposite in phase. The radius of the equivalent sphere is Ro = 10 fm, and the range of proximity interaction is b = 1.0 fm.

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