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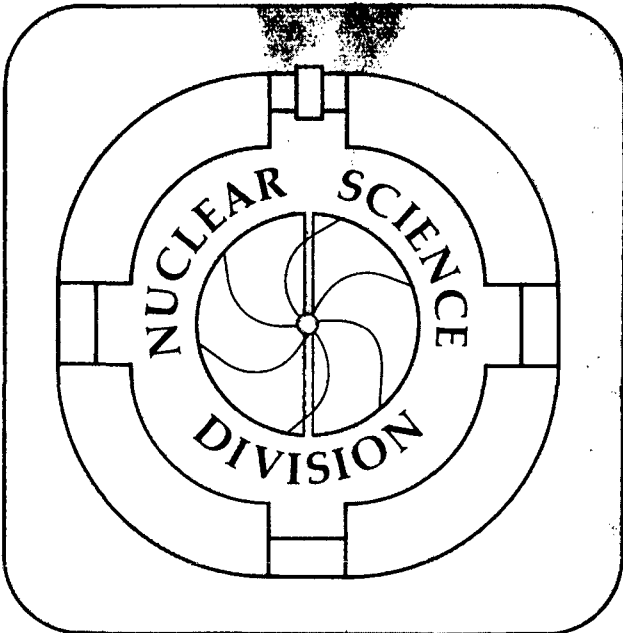
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SUBTHRESHOLD  $K^-$ -PRODUCTION BY COHERENTLY PRODUCED  
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Subthreshold  $K^-$ -Production by Coherently Produced  $\phi$ -Mesons  
in Heavy Ion Collisions\*

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We assume that the enhanced subthreshold  $K^-$ -production found recently in the reaction  $^{28}\text{Si}-^{28}\text{Si}$  at 2.1 GeV<sup>6)</sup> is due to the decay of coherently produced  $\phi$ -mesons. We calculate the differential  $K^-$ -production cross section by treating the source term of the  $\phi$ -meson field, which is the nucleon current, as an external c-number source. We parameterize this current by assuming a diving-, a compression-, and an expansion-stage during the nucleus-nucleus collision where the results of the intranuclear cascade calculations<sup>16)</sup> serve as a reference. Assuming a reasonable time for building up three times nuclear matter density we get agreement with the experimental data. We predict a differential cross section that is different from a thermal spectrum.

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## 1. Introduction

In a relativistic quantum field theory the strongly interacting nucleon system is described by a Lagrangian where the interaction Lagrangians are assumed to be due to  $\pi, \eta, \sigma, \rho, \omega, \phi \dots$  meson exchange.<sup>1-3)</sup> The equations describing the space and time behavior of the corresponding meson fields are inhomogeneous Klein-Gordon- or Proca-equations, respectively. In the case of a relativistic heavy ion collision<sup>4,5)</sup> where the nucleons are decelerated these equations do not only describe the virtual mesons inside of the colliding systems but also the production and emission of real mesons. While emitted  $\eta, \sigma, \rho$ , and  $\omega$  mesons decay mainly into photons or pions, which might be difficult to be detected in the background of photons produced by decaying  $\pi^0$ 's and pions produced via decaying  $\Delta$ 's, the  $\phi$ -meson has the advantage to decay into a  $K^+K^-$  pair. At bombarding energies far enough below the  $NN \rightarrow NNK^+K^-$  threshold, the  $K^-$ -meson produces a background-free signal from coherently created  $\phi$ -mesons assuming that the  $K^-$ -production due to the processes  $\pi\Lambda \rightarrow NK^-$  and  $\pi\Sigma \rightarrow NK^-$  are already sufficiently small.

Recently A. Shor et al.<sup>6)</sup> detected a surprisingly large number of subthreshold  $K^-$ -mesons at a c.m. momentum of 276 MeV/c at  $0^\circ$  in the reaction  $^{28}\text{Si}-^{28}\text{Si}$  at 2.1 GeV/nucleon. Here the  $K^-$  production threshold is 210 MeV above the available nucleon-nucleon centre of mass energy. If one tries to explain these data solely in terms of individual nucleon-nucleon  $K^-$ -production as it was done by the authors of ref. 6), taking Fermi motion in form of double Gaussian momentum distributions for projectile and target into account, one underestimates the  $K^-$ -yield by about a factor of 20.

The aim of this paper is to show that this surprisingly large number of subthreshold  $K^-$ -mesons might be due to coherently produced  $\phi$ -mesons during the nucleus-nucleus collision where the participant nucleons contribute

cooperatively<sup>7)</sup>. We propose the picture that the nuclear matter distribution of the participant nucleons is surrounded by (virtual and real)  $\phi$ -mesons in a similar way as a moving single nucleon. In analogy to photon-bremsstrahlung where a deceleration of a charged current produces real photons, in our model, the deceleration of the baryonic current during the nuclear collision leads to the creation of real  $\phi$ -mesons. A similar picture for the pion cloud surrounding the projectile nucleus was recently proposed by B. Hiller and H.J. Pirner<sup>8)</sup> to describe coherent pion production.

In sect. 2 the general formalism of our model is reviewed and the expression for the  $K^-$ -production cross section due to coherent  $\phi$ -meson creation is derived. In sect. 3 we describe a simple parametrization of the nucleon current during the nucleus-nucleus collision and represent in sect. 4 the results. Finally in sect. 5 we give a brief summary and discuss possibilities for future work that appears promising.

## 2. The Model

### 2a. The coherent $\phi$ -meson production mechanism

The interaction of the nucleon field  $\psi(\vec{x},t)$  with the neutral vector field  $\phi_\mu(\vec{x},t)$  of the  $\phi$ -mesons is described in the one meson exchange model by an interaction Lagrangian of the form

$$\mathcal{L}_I = -g_\phi \phi_\mu \bar{\psi} \gamma^\mu \psi \quad (1)$$

where  $\phi_\mu$  couples to the baryon current  $\bar{\psi} \gamma^\mu \psi$ <sup>9)</sup>.  $g_\phi$  is the Yukawa coupling constant,  $\gamma^\mu$  the standard  $\gamma$ -matrices. With the free Lagrangian for the  $\phi$ -meson field, this leads to a field equation for  $\phi_\mu$  of the form

$$\partial^\nu (\partial_\nu \phi_\mu - \partial_\mu \phi_\nu) + m_\phi^2 \phi_\mu = g_\phi \bar{\psi} \gamma_\mu \psi \quad (2)$$

Here  $m_\phi$  is the  $\phi$ -meson mass ( $m_\phi = 1020$  MeV). Since the baryon current is conserved, the Proca-equation (2) is equivalent to an inhomogeneous Klein-Gordon equation of the form

$$\square \phi_\mu + m_\phi^2 \phi_\mu = g_\phi \bar{\psi} \gamma_\mu \psi \quad (3)$$

with the Lorentz gauge

$$\partial_\mu \phi^\mu = 0 \quad (4)$$

In the case of subthreshold  $\phi$ -meson production one way of producing real  $\phi$ 's is via nucleons that act cooperatively. The coherent  $\phi$ -production is determined by substituting the current operator  $\bar{\psi} \gamma_\mu \psi$  by its expectation value. Therefore, from eq. (3) one obtains a Klein-Gordon equation with a pure c-number source.

$$\bar{\psi} \gamma_\mu \psi \rightarrow \langle \bar{\psi} \gamma_\mu \psi \rangle \quad (5)$$

In the  $\sigma + \omega + \phi$  model<sup>10)</sup> (the remaining mesons are neglected for simplicity), the equation that describes the nucleon field  $\psi(\vec{x}, t)$  is a Dirac equation of the form

$$(i \gamma_\mu \partial^\mu - m_N + g_S \sigma - g_\omega \gamma_\mu \omega^\mu - g_\phi \gamma_\mu \phi^\mu) \psi = 0 \quad (6)$$

$m_N$  is the nucleon mass and  $g_S, g_\omega$  coupling constants. The field eqs. (3) and (6) are coupled, which makes the inhomogeneous Klein-Gordon eq. (3) with substitution (5) at first sight intractable. We could get rid of this complication, if one could treat the source term (5) approximately as an "external" current, independent of  $\phi_\mu$  itself. When can one approximately neglect the recoupling of the  $\phi$ -field on the  $\psi$ -field? As far as virtual  $\phi$ -mesons are concerned, we can think that the effect of the  $\phi$ -field on  $\langle \bar{\psi} \gamma_\mu \psi \rangle$  is small compared to the effects of the  $\sigma$ - and  $\omega$ -fields. This is due to the small

coupling constant and the large mass of the  $\phi$ -field in comparison to those of the  $\sigma$ - and  $\omega$ -fields. In the case of emitted real  $\phi$ -mesons we can approximately neglect the recoupling on the expectation value of the nucleon current if the source acts as a huge energy and quantum number reservoir. This is the case if the available energy for particle production in the entire participant nucleon system is much larger than the energy carried away by the produced  $\phi$ -mesons. We will see later that in the cases we are considering, this is approximately the case. We therefore now approximate eq. (3) by

$$\square \phi_\mu + m_\phi^2 \phi_\mu = g_\phi j_\mu \quad (7)$$

where  $j_\mu$  is a fixed c-number current. How to choose this current  $j_\mu$  we will discuss in sect. 3.

The mean number of scalar mesons or photons produced by an external c-number current is derived in several textbooks<sup>11,12)</sup>. For the spin 1  $\phi$ -meson the derivation is similar and we will briefly sketch it here.

To determine the mean number of emitted  $\phi$ -mesons of momentum  $\vec{k}$  and polarization  $\alpha$  ( $\alpha = +1, 0, -1$ ), we have to ask for the probability  $P(n_{\vec{k}}^{(\alpha)})$  of finding after a heavy ion collision, where the dynamics is described by  $j_\mu(x,t)$ ,  $n_{\vec{k}}^{(\alpha)}$   $\phi$ -mesons. If  $|0, in\rangle$  is the Heisenberg in-state where there is no real  $\phi$ -quantum and  $|n_{\vec{k}}^{(\alpha)}, out\rangle$  the Heisenberg out-state with  $n_{\vec{k}}^{(\alpha)}$  real  $\phi$ -quanta one obtains for the probability

$$P(n_{\vec{k}}^{(\alpha)}) = |\langle n_{\vec{k}}^{(\alpha)}, out | 0, in \rangle|^2 \quad (8)$$

here

$$|n_{\vec{k}}^{(\alpha)}, out\rangle = \frac{1}{\sqrt{n_{\vec{k}}^{(\alpha)}!}} \left( a_{\vec{k}, \alpha}^{+out} \right)^{n_{\vec{k}}^{(\alpha)}} |0, out\rangle \quad (9)$$

where  $a_{\vec{k}, \alpha}^{+out}$  is an out- $\phi$ -meson creation operator. We now expand the in- and out-



$\phi$ -field into normal modes of in- and out-creation and annihilation operators

$$\phi_{\mu}^{\text{in/out}} = \sum_{\alpha} \sum_{\vec{K}} \frac{1}{\sqrt{2\omega\Omega}} \left\{ a_{\vec{K}\alpha}^{\text{in/out}} U_{\mu}^{(\alpha)} e^{i(\vec{K}\vec{X}-\omega t)} + a_{\vec{K}\alpha}^{\text{+in/out}} U_{\mu}^{(\alpha)*} e^{-i(\vec{K}\vec{X}-\omega t)} \right\} \quad (10)$$

$\Omega$  is an infinite normalization volume.  $U_{\mu}^{(\alpha)}$  is the polarization four vector of polarization  $\alpha$ . It is

$$U_{\mu}^{(\alpha)} U^{\mu(\beta)} = -\delta_{\alpha\beta} \quad (11)$$

Due to the Lorentz gauge of eq. (4) one obtains using eq. (10)

$$U_{\mu}^{(\alpha)} K^{\mu} = 0 \quad (12)$$

The solution of the field eq. (7) leads to the following dependence of the out- $\phi$ -field and the in- $\phi$ -field.

$$\phi_{\mu}^{\text{out}}(x) = \phi_{\mu}^{\text{in}}(x) + g_{\phi} \int d^4x' \left\{ G^{\text{ret.}}(x, x') - G^{\text{adv.}}(x, x') \right\} j_{\mu}(x') \quad (13)$$

$G^{\text{ret.}}$  and  $G^{\text{adv.}}$  are the retarded and advanced Green's functions that satisfy a Klein-Gordon equation of a point source.

Inserting in eq. (13) the explicit expression for  $G^{\text{ret.}}$  and  $G^{\text{adv.}}$  and introducing the space-time Fourier transformed current  $\tilde{j}_{\mu}(\vec{K}, \omega)$  we obtain

$$\phi_{\mu}^{(\text{out})}(\vec{x}, t) = \phi_{\mu}^{\text{in}}(\vec{x}, t) + g_{\phi} \frac{1}{(2\pi)^3} \int \frac{d^3K}{2i\omega} \left\{ e^{-i(\vec{K}\vec{x}-\omega t)} \tilde{j}_{\mu}^*(\vec{K}, \omega) - e^{i(\vec{K}\vec{x}-\omega t)} \tilde{j}_{\mu}(\vec{K}, \omega) \right\} \quad (14)$$

where  $\omega$  is the on-shell  $\phi$ -meson energy

$$\omega = \sqrt{\vec{K}^2 + m_{\phi}^2} \quad (15)$$

Eqs. (10) and (14) relate the in- and out-annihilation operators as

$$a_{\vec{K}\alpha}^{\text{out}} = a_{\vec{K}\alpha}^{\text{in}} + \frac{ig_{\phi}}{\sqrt{2\omega\Omega}} \left( U^{\mu(\alpha)} \tilde{j}_{\mu}(\vec{K}, \omega) \right)^* \quad (16)$$

With eqs. (8), (9), and (16) one finally obtains a Poisson distribution for  $P(n_{\vec{K}}^{(\alpha)})$  where the mean number of emitted  $\phi$ -quanta of momentum  $\vec{K}$  and polarization  $\alpha$  is

$$\langle n_{\vec{K}}^{(\alpha)} \rangle = \frac{g_{\phi}^2}{2\omega\Omega} \left| U^{\mu(\alpha)} \tilde{j}_{\mu}(\vec{K}, \omega) \right|^2 \quad (17)$$

2b. The  $\phi$ -decay into  $K^+K^-$

A  $\phi$ -meson decays outside of a medium with 47% probability into a  $K^+K^-$  pair. The decay time is 47 fm/c. A typical reaction time for a central heavy ion collision at 1-2 GeV/nucleon until the participant system goes over into a dilute nuclear matter system of density less than half normal nuclear matter density is about 15 fm/c. Therefore, assuming that the  $\phi$ -decay is not changed significantly by the nuclear medium, we can neglect the absorption of the produced  $K^-$ -mesons by the nucleons.

Since the  $\phi$ -meson is a spin 1 particle and the  $K^-$  mesons have spin 0 the produced  $K^+K^-$  pair is in a relative p-state in the rest frame of the  $\phi$ -meson.

The momentum distribution  $W_{\alpha}(\vec{p}, \vec{K})$  of a  $K^-$ -meson of momentum  $\vec{p}$  resulting from a decaying  $\phi$ -meson of momentum  $\vec{K}$  and polarization  $\alpha$  is given by an integral over the Lorentz invariant phase space where energy and momentum conservation is taken into account by  $\delta$ -functions<sup>13)</sup> and the relative p-state of the  $K^+K^-$  pair is considered by the spherical harmonics  $Y_{1\alpha}$ .

$$W_{\alpha}(\vec{p}, \vec{K}) = \frac{1}{I(m_{\phi}, m_K)} \int \frac{d^3p'}{\epsilon'} \left| Y_{1\alpha}(\vec{\vartheta}) \right|^2 \delta(\epsilon + \epsilon' - \omega) \delta^3(\vec{p} + \vec{p}' - \vec{K}) \quad (18)$$

$\vec{p}'$  is the momentum of the  $K^+$ -meson, which shall not be detected,  $m_K$  is the kaon mass.  $\epsilon$  and  $\epsilon'$  are the energies of the  $K^-$ - or  $K^+$ -meson, respectively.  $I(m_{\phi}, m_K)$  is the normalization constant.  $\vec{\vartheta}$  in eq. (18) is

the angle between the  $\phi$ -momentum  $\vec{K}$  and the relative momentum  $\vec{y}$  of the  $K^+K^-$  pair in the rest frame of the  $\phi$ -meson. It is

$$\cos \bar{\vartheta} = \frac{\vec{K}}{|\vec{K}|} \cdot \frac{\vec{y}}{|\vec{y}|} ; \quad \vec{y} = (\vec{p} - \vec{p}') - \vec{\beta} \gamma (\epsilon - \epsilon') + \vec{\beta} \frac{\gamma^2}{\gamma+1} \{ \vec{\beta}(\vec{p} - \vec{p}') \} \quad (19)$$

where

$$\vec{\beta} = \frac{\vec{K}}{\omega} \text{ and } \gamma = (1 - \beta^2)^{-1/2} \quad (20)$$

Expression (18) can easily be evaluated. We obtain

$$W_{\alpha}(\vec{p}, \vec{K}) = \frac{1}{I(m_{\phi}, m_K)} \left| Y_{12}(\bar{\vartheta}) \right|^2 \frac{\delta(\cos \bar{\vartheta} - \cos \vartheta_0)}{p \cdot K} \quad (21)$$

where  $\vartheta$  is the angle between the  $\phi$ -meson momentum  $\vec{K}$  and the  $K^-$ -meson momentum  $\vec{p}$ . And

$$\cos \vartheta_0 = \frac{2\omega\epsilon - m_{\phi}^2}{2pK} \quad (22)$$

One gets  $\cos \bar{\vartheta}$  by substituting each  $\vec{p}'$  in eq. (19) by  $\vec{K} - \vec{p}$ .

For the normalization constant we find

$$I(m_{\phi}, m_K) = \frac{\sqrt{m_{\phi}^2 - 4m_K^2}}{2m_{\phi}} \quad (23)$$

### 2c. The $K^-$ -production cross section

So far, we have determined the mean number of  $\phi$ -mesons of momentum  $\vec{K}$  and polarization  $\alpha$  and the probability that a decaying  $\phi$  produces a  $K^-$ -meson of momentum  $\vec{p}$  where  $\vec{p}$  lies in the Lorentz invariant phase space interval  $d^3p/\epsilon$ . In relativistic heavy ion collisions interference effects between different partial wave amplitudes in a differential cross section can be expected to be unimportant because of the enormous number of orthogonal final channels accessible<sup>14)</sup>. Therefore, it makes sense to sum simply over the

probabilities to produce  $K^-$ -mesons at different impact parameters. With this we obtain an expression for the invariant double differential  $K^-$ -production cross section of the form

$$\epsilon \frac{d^2\sigma}{p^2 dp d\Omega} = n 2\pi \int db b \frac{g_\phi^2}{(2\pi)^3} \int \frac{d^3K}{2\omega} \sum_\alpha W_\alpha(\vec{p}, \vec{K}) \left| U^\mu(\alpha) \tilde{j}_{\mu,b}(\vec{K}, \omega) \right|^2 \quad (24)$$

Here we summed over all possible polarizations of the  $\phi$ -meson, integrated over all possible momenta of the  $\phi$ , and furthermore integrated over all impact parameters  $b$ . Now, of course, the current depends on the impact parameter.  $n = 0.47$  is the branching fraction for the  $\phi$  to decay into a  $K^+K^-$  pair. To evaluate the inner integral in eq. (24), we choose polar coordinates  $(K, \varphi, \vartheta)$  where  $\vec{p}$  defines the  $z$ -direction. With  $\hat{z}$  being the ion beam direction the differential  $K^-$ -production cross section at an angle  $\vartheta$  with  $\cos\vartheta = \hat{z} \cdot \vec{P}/P$  reduces to the form

$$\epsilon \frac{d^2\sigma}{p^2 dp d\Omega} \Big|_{\vartheta} = \frac{ng_\phi^2}{(2\pi)^2} \frac{m_\phi}{\sqrt{m_\phi^2 - 4m_K^2}} \frac{1}{p} \int db b \int_{K_-}^{K_+} dK \frac{K}{\omega} \sum_\alpha \left| Y_{1\alpha}(\tilde{\vartheta}_0) \right|^2 \int d\varphi \left| U^\mu(\alpha) \tilde{j}_{\mu,b}(K, \varphi, \vartheta_0, \omega) \right|^2 \quad (25)$$

$\vartheta_0$  is defined by eq. (22).  $\tilde{\vartheta}_0$  is given by  $\tilde{\vartheta}$  as discussed in context with eq. (19), evaluated at the angle  $\vartheta = \vartheta_0$ . The integration limits for the momentum  $K$  are

$$K_\pm = \frac{1}{2} \frac{m_\phi}{m_K} \left| m_\phi p \pm \epsilon \sqrt{m_\phi^2 - 4m_K^2} \right| \quad (26)$$

Since the only data for subthreshold  $K^-$ -production in heavy ion collisions are those reported in ref. 6) where kaons were measured at an angle  $\vartheta = 0^\circ$ , we want to restrict our calculations to this particular angle, which further simplifies the evaluation of eq. (25).

The mean number  $\langle n_\phi \rangle$  of emitted  $\phi$ 's at a given impact parameter  $b$  is given by

$$\langle n_\phi \rangle(b) = \frac{g_\phi^2}{2(2\pi)^3} \int d^3K \frac{\sum_\alpha |U^{\mu(\alpha)} \tilde{j}_{\mu,b}(\vec{K}, \omega)|^2}{\omega} \quad (27)$$

where we used eq. (17).

If we choose in the rest frame of the  $\phi$ -meson for the spin basis spherical unit vectors, we derive from conditions (11) and (12) for the polarization four vector  $U^{\mu(\alpha)}$

$$U^{\mu(0)} = \left( \frac{K}{m_\phi}, \frac{\omega}{m_\phi} \vec{e}_K \right) ; \quad U^{\mu(\pm 1)} = (0, \vec{e}_\pm) \quad (28)$$

Since the continuity equation  $\partial_\mu j^\mu = 0$  is equivalent to

$$\vec{K} \tilde{j}(\vec{K}, \omega) = \omega \tilde{j}_0(\vec{K}, \omega) \quad (29)$$

we obtain in the case of polarization  $\alpha = 0$

$$\left| U^{\mu(0)} \tilde{j}_{\mu,b} \right|^2 = \frac{m_\phi^2}{K^2} \left| \tilde{j}_{0,b} \right|^2 \quad (30)$$

Here  $\tilde{j}_0$  as well as  $\tilde{j}$  contribute to give the right side of eq. (30). As we see from (28), in the case of polarizations  $\alpha = \pm 1$  only  $\tilde{j}$  contributes.

### 3. Parametrization of the nucleon current expectation value

Since we are not able to solve the coupled field eqs. (2) and (6) to determine the expectation value of the nucleon current in a relativistic heavy ion collision, we want to use a simple model parametrization for  $j_{\mu,b}(\vec{x}, t)$  and see if one can reproduce the  $K^-$ -yield reported in ref. 6) with some reasonable assumptions about the nuclear dynamics. We consider a projectile-target system of equal nucleon number  $A_0$  and discuss the

collision process seen from the nucleus-nucleus c.m. system. We assume the spectator-participant picture and use for the number of participants  $A(b)$  of target and projectile, respectively, the formula derived in ref. 15), which were used by the authors of ref. 16).

$$A(b) = A_0 \left[ \frac{3}{\sqrt{2}} (1 - n)^2 - \left( \frac{3}{\sqrt{2}} - 1 \right) (1 - n)^3 \right] \quad (31)$$

where

$$n = \frac{b}{2R} \quad ; \quad R = 1.2 A_0^{1/3} \quad (32)$$

We assume the collision dynamics for a given impact parameter essentially to be a three-step process, which is suggested by the results of intranuclear cascade calculations done by J. Cugnon et al.<sup>17)</sup>. For times  $t < 0$  the two participant distributions penetrate, approaching each other with the velocity  $\pm \vec{V}_{in}$  defined by the initial c.m. kinetic energy per nucleon. Each participant density distribution is assumed to be of Gaussian shape. This initial part of the reaction we call diving stage. At  $t = 0$  the two participant densities are assumed to overlap completely, having reached double normal nuclear matter density  $\rho_0$ . During the time interval from  $t = 0$  to  $t = t_0$  we assume a piling up of density where the spherical Gaussian shape becomes deformed. The width of the Gaussian in transverse direction is assumed to be unchanged while the width in z-direction becomes smaller corresponding to an increasing central density.

For times  $t > t_0$  a fireball expansion takes place. We assume that at time  $t_0$  equilibrium has been reached locally and from each point of the matter distribution local fireballs start to expand. The contribution to the  $\phi$ -production resulting from the spectator nucleon distribution is assumed to be negligible since the ablation of the spectators is a slow process compared to the violent compression stage of the participant density.

Therefore, we describe the time component of the current at impact parameter  $b$  by

$$\begin{aligned}
 j_{0,b}(\vec{x},t) = & A(b)a^3 \left[ e^{-\pi a^2(\vec{x}-\vec{v}_{in}t)^2} + e^{-\pi a^2(\vec{x}+\vec{v}_{in}t)^2} \right] \Theta(-t) \\
 & + 2A(b)a^2 \alpha_z e^{-\pi a^2 r_{\perp}^2} e^{-\pi \alpha_z^2 z^2} \Theta(t) \Theta(t_0 - t) \\
 & + 2A(b)\beta_{\perp}^2 \beta_z e^{-\pi \beta_{\perp}^2 r_{\perp}^2} e^{-\pi \beta_z^2 z^2} \Theta(t - t_0)
 \end{aligned} \tag{33}$$

Here the first part of the sum in eq. (33) is the diving stage, the second part the compression stage, and the last term the expansion stage, where

$$a = \left( \frac{\rho_0}{A(b)} \right)^{1/3}, \quad r_{\perp}^2 = x^2 + y^2 \tag{34}$$

$\Theta(t)$  is a step function.  $\alpha_z(t)$  determines the change of the longitudinal Gaussian width during the compression stage. We assume the following simple parametrization

$$\alpha_z(t) = \left[ \frac{1}{a} + \left( \frac{1}{\alpha_z(t_0)} - \frac{1}{a} \right) \frac{t}{t_0} - \frac{1}{2\pi} \left( \frac{1}{\alpha_z(t_0)} - \frac{1}{a} \right) \sin 2\pi \frac{t}{t_0} \right]^{-1} \tag{35}$$

$\alpha_z(t_0)$  determines the longitudinal width of the Gaussian participant distribution at the time when the fireball expansion starts. Therefore,  $\alpha_z(t_0)$  fixes the maximum density  $\rho_m$  reached at the end of the compression stage and equation (35) is completely determined for given values of  $\rho_m$  and  $t_0$ . With the assumption of local fireballs that start to expand at  $t = t_0$ , we find for  $\beta_{\perp}^2$  and  $\beta_z^2$  in eq. (33)

$$\beta_{\perp}^2 = \left[ \frac{1}{a^2} + \frac{2\pi kT}{m_N} (t - t_0)^2 \right]^{-1} \tag{36}$$

and

$$\beta_z^2 = \left[ \frac{1}{\alpha_z^2(t_0)} + \frac{2\pi kT}{m_N} (t - t_0)^2 \right]^{-1} \quad (37)$$

$m_N$  is the nucleon mass,  $kT$  the temperature corresponding to a nonrelativistic Maxwell distribution. For  $kT$  we used the experimentally determined temperature parameter of the proton spectrum at the corresponding bombarding energy.

The space component of the expectation value of the nucleon current in our model parametrization is given by

$$\begin{aligned} \vec{j}_b(\vec{x}, t) = & A(b) a^3 \left( e^{-\pi a^2 (\vec{x} - \vec{v}_{in} t)^2} - e^{-\pi a^2 (\vec{x} + \vec{v}_{in} t)^2} \right) \vec{v}_{in} \vartheta(-t) \\ & - 2A(b) a^2 \alpha_z \cdot e^{-\pi a^2 r_{\perp}^2} e^{-\pi \alpha_z^2 z^2} z \vec{e}_z \vartheta(t) \vartheta(t_0 - t) \\ & - 2A(b) \beta_{\perp} \beta_z \beta_{\perp} \cdot e^{-\pi \beta_{\perp}^2 r_{\perp}^2} e^{-\pi \beta_z^2 z^2} \begin{pmatrix} x \\ y \\ \beta_z^2 \\ \frac{\beta_z^2}{2} z \\ \beta_{\perp} \end{pmatrix} \vartheta(t - t_0) \end{aligned} \quad (38)$$

Figure 1 shows the density profiles during the diving stage (-4 fm/c-0 fm/c), the compression stage (here 0 fm/c-3.8 fm/c), and during the initial part of the expansion stage (3.8 fm/c-10 fm/c). The impact parameter was chosen such that the number of target or projectile participants is 14, respectively. The maximum density reached is  $3 \rho_0$ . The lab bombarding energy is 2.1 GeV/nucleon and  $kT = 120$  MeV.

One can see that this parametrization reflects the essential features of the dynamics one finds in an intranuclear cascade calculation<sup>17)</sup>. Figure 2 displays the current  $j_{z,b}$  in the three different stages. Since  $j_{z,b}(z) = -j_{z,b}(-z)$  only  $j_{z,b}(z)$  for  $z > 0$  is plotted. During the diving stage the



current decreases, reaching a value of zero at the moment of complete overlap. Then during the compression stage it increases at the beginning and drops to zero at  $t = t_0$ . The current changes its sign when the expansion stage starts. In the last column of fig. 2 the current  $j_{x,b}$  during the expansion stage is shown. Here  $j_{x,b}$  is similar to  $j_{z,b}$ . Figure 3 displays the time dependence of the longitudinal half Gaussian width  $\Delta z_L$  of the participant distribution during the compression and expansion stage as given by eqs. (35) and (37).

Our parametrization has the great advantage that we can easily perform the Fourier transformations to evaluate  $|U^\mu(\alpha) \tilde{j}_{u,b}|^2$  of eq. (25), using eqs. (28) and (30). In the case  $\alpha = 0$ , we obtain

$$\begin{aligned}
 U^\mu(0) \tilde{j}_{u,b} = \frac{m_0}{K} A(b) & \left[ i e^{-\frac{K^2}{4\pi a^2}} \left( \frac{1}{K_z V_{in} - \omega} - \frac{1}{K_z V_{in} + \omega} \right) \right. \\
 & + 2e^{-\frac{K_\perp^2}{4\pi a^2}} \int_0^{t_0} e^{-\frac{K_z^2}{4\pi \alpha_z^2}} e^{i\omega t} dt \\
 & \left. + 2e^{-\frac{K_\perp^2}{4\pi a^2}} e^{-\frac{K_z^2}{4\pi \alpha_z^2(t_0)}} e^{i\omega t_0} \left\{ e^{-\frac{\omega^2}{4b^2}} \frac{\sqrt{\pi}}{2b} + i \int_0^{\omega/2b^2} e^{\left(b^2 t^2 - \frac{\omega^2}{4b^2}\right)} dt \right\} \right] \quad (39)
 \end{aligned}$$

If we restrict ourselves to determine the double differential cross section (25) at an angle  $\theta = 0^\circ$  and neglect the fact that the current  $\tilde{j}$  in the expansion stage is not exactly radially symmetric (see fig. 2) we obtain in the cases of polarization  $\alpha = \pm 1$

$$U^{\mu(\pm 1)} \tilde{j}_{\mu,b} = \frac{-A(b)}{\sqrt{2}} \sin \vartheta \left[ e^{-\frac{K^2}{4\pi a^2}} V_{in} \left( \frac{1}{K_z V_{in} - \omega} + \frac{1}{K_z V_{in} + \omega} \right) + \frac{K_z}{\pi} e^{-\frac{K_1^2}{4\pi a^2}} \int_0^{t_0} \frac{\alpha_z}{\alpha_z^3} e^{-\frac{K_z^2}{4\pi \alpha_z^2}} e^{i\omega t} dt \right] \quad (40)$$

The above-mentioned approximation is justified, since the fireball expansion is slow compared to the compression process. To evaluate eqs. (39) and (40) we have further assumed that the source is switched on and off adiabatically.

#### 4. Results and Discussion

We calculated the double differential  $K^-$ -production cross section (25) at  $\vartheta = 0^\circ$  for the reaction  $^{28}\text{Si}-^{28}\text{Si}$  at 2.1 GeV/nucleon. The  $K^-$ -momenta  $\vec{p}$  are those in the c.m. frame. The Fourier transformations in eqs. (39) and (40) are performed numerically by using a 32 point Gauss quadrature formula<sup>18)</sup> for each time interval of length  $\sim \pi/\omega$ . For the coupling constant  $g_\phi$ , we choose the one given by the one boson exchange potential of Erkelenz, Holinde, and Machleidt<sup>19,3)</sup>; it is  $g_\phi^2/4\pi = 0.86$ . Taking the results of the intranuclear cascade calculations<sup>17,20)</sup> as a reference, we assumed a maximum density of  $3 \rho_0$  reached at the end of the compression stage. We varied the time parameter  $t_0$  in expression (35). The results for  $t_0 = 1.5, 2.0, 3.5$  fm/c and for the case where we switched off the compression stage completely are shown in fig. 4. A time parameter between 1.9-4 fm/c we found to be in agreement with the data<sup>6)</sup> at 276 MeV/c. During such a time a nucleon having still its initial velocity travels a distance of  $0.4R-0.8R$ , where  $R$  is the sharp sphere radius of  $^{28}\text{Si}$ . This time period seems to be

reasonable to build up three times normal nuclear matter density. Neglecting the compression stage completely underestimates the cross section at  $p = 276$  MeV/c by a factor of 23.

If one disentangles the contributions resulting from the different polarizations of the  $\phi$ -meson one finds that the  $\alpha = 0$  term mainly contributes for  $p < 200$  MeV/c and has a dip at  $p \approx 125$  MeV/c. This is due to the kinematical factor  $W_{\alpha=0}(\vec{K}, \vec{p})$  derived in sect. 2b.  $W_{\alpha=0}$  gets small at  $p \approx 125$  MeV/c since the corresponding  $\phi$ -mesons, which mainly contribute, are of small momenta  $K$ , where  $\vec{K} \perp \vec{p}$ , so that  $Y_{10} \approx 0$ . Plotting the double differential cross section for a fixed value of  $p$  versus the time parameter  $t_0$ , one finds not a smooth monotonic decrease but an oscillating cross section, which shows an overall decreasing behavior with increasing  $t_0$ . In reality, the maximum compression reached might decrease with increasing impact parameter. We did not take this into account, since it means a more complicated parametrization and we want to restrict our model on as few parameters as possible.

As far as the impact parameter integration is concerned, for  $b > 6.2$  fm there are fewer than 1.2 nucleons involved in the coherent  $\phi$ -production and the initial kinetic energy available in such a participant system is thus less than the rest mass of the  $\phi$ -meson. The calculation of the cross section (25) shows that such large impact parameter contributions can be neglected without noticeably changing the result. The integrand of the impact parameter integration peaks at  $b \approx 2$  fm. In this case  $A(b) \approx 20$  and therefore the initially available kinetic energy for particle production is about 17 GeV. One can think of such a system to act as an energy and quantum number reservoir for low-momentum  $\phi$ -meson production as long as one creates not more than one  $\phi$  in each collision. In our model this is the case. Figure 5 displays the mean

number  $\langle n_\phi \rangle$  of produced  $\phi$ 's in dependence of the impact parameter  $b$  for different time parameters  $t_0$ . In a central collision for  $t_0 = 2.0$  fm/c, there are on an average about 100 collisions necessary to produce one  $\phi$ -meson.

At a bombarding energy of 2.1 GeV/nucleon the  $K^+$ -meson contribution resulting from the decay of a  $\phi$  might be difficult to detect in the background of  $K^+$ -mesons produced by the reactions  $NN \rightarrow K^+ \Lambda N$  and  $NN \rightarrow K^+ \Sigma N$ , which leads to a thermal type of spectrum<sup>21,22</sup>).

We calculated the  $K^+$ -yield for the system Ne-Ne at 2.1 GeV/nucleon at  $\theta = 0^\circ$  and compared our prediction with the data of ref. 21). Figure 6 shows that a parameter of  $t_0 = 1.5$  fm/c is in contradiction to the data, while for  $t_0 = 2.0$  fm/c the  $K^+$ -yield resulting from a  $\phi$ -decay is compatible with the data. Here our  $K^+$ -yield is so small that it can not lead to a significant structure deviating from a thermal spectrum.

### Summary

The aim of this paper was to demonstrate the possibility of subthreshold  $K^-$ -production due to coherently created  $\phi$ -mesons during a heavy ion collision. Assuming a reasonable model parametrization for the global nucleus-nucleus current during the collision process we can explain the data of ref. 6). The model predicts a  $K^-$ -spectrum, which decreases much more rapidly than a thermal spectrum. The width of the spectrum is related to the size of the reaction zone; its height is due to the fastness in changes of the interacting matter distributions. Therefore, the  $K^-$ -cross section resulting from the decay of  $\phi$ 's contains direct information about the time-space development of the global nucleus-nucleus current during the reaction. We have shown that it is necessary to take a compression stage into account to explain the data. The model is not in contradiction to the  $K^+$ -data of ref. 21).

Up to now there are not enough data available to exclude or to confirm the possibility of coherent  $\phi$ -production in a heavy ion collision. To show experimentally that  $\phi$ 's are produced one should measure the  $K^-$ -yield at lower bombarding energies to decrease significantly the  $K^-$ -yield, which might be due to the reactions  $\pi\Lambda \rightarrow NK^-$  and  $\pi\Sigma \rightarrow NK^-$ . The subthreshold  $K^-$ -yield due to decaying  $\phi$ 's should decrease less rapidly with decreasing bombarding energies than the  $K^-$ -yield coming from the above elementary reactions. Up to now the parametrization of the nucleus-nucleus dynamics is crude and, of course, it should be improved if experimentally a clear signature of  $\phi$ -production should be detected.

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### Figure Captions

- Fig. 1. Density profiles  $j_{0,b}/\rho_0$  for  $A(b) = 14$ . The maximum density reached is assumed to be  $\rho_m = 3 \rho_0$ . The time parameter for the compression stage is chosen to be  $t_0 = 3.8$  fm/c. The bombarding energy is 2.1 GeV/nucleon and  $kT = 120$  MeV.
- Fig. 2. The current  $j_{z,b}$  in units of  $\text{fm}^{-3} c$ . The last column displays  $j_{x,b}$  during the expansion stage.  $A(b)$ ,  $\rho_m$ ,  $t_0$ ,  $E/A_0$ , and  $kT$  are chosen as in fig. 1.
- Fig. 3. Longitudinal half Gaussian width  $\Delta z_L$  of the participant distribution during the compression and expansion stage.
- Fig. 4. Invariant double differential  $K^-$ -production cross section at  $\theta = 0^\circ$  in the c.m. frame for compression-time parameters  $t_0 = 1.5, 2.0, 3.5$  fm/c for the reaction  $^{28}\text{Si}-^{28}\text{Si}$  at 2.1 GeV/nucleon. We choose  $\rho_m = 3 \rho_0$  and  $kT = 120$  MeV. N.C. means no compression stage.
- Fig. 5. The mean number of produced  $\phi$ 's during the reaction  $^{28}\text{Si}-^{28}\text{Si}$  at 2.1 GeV/nucleon versus the impact parameter  $b$  for different  $t_0$ . Again,  $\rho_m = 3 \rho_0$ ,  $kT = 120$  MeV.
- Fig. 6. Invariant double differential  $K^+$ -production cross section at  $\theta = 0^\circ$  in the c.m. frame for different compression time parameters  $t_0$  for the reaction  $^{20}\text{Ne}-^{20}\text{Ne}$  at 2.1 GeV/nucleon (solid curves). Again,  $\rho_m = 3 \rho_0$ . The data are taken from ref. 21). The dashed line corresponds to a thermal spectrum where  $kT = 142$  MeV.

$A(b) = 14, \rho_m = 3\rho_0, t_0 = 3.8 \text{ fm}/c, E/A_0 = 2.1 \text{ GeV}$

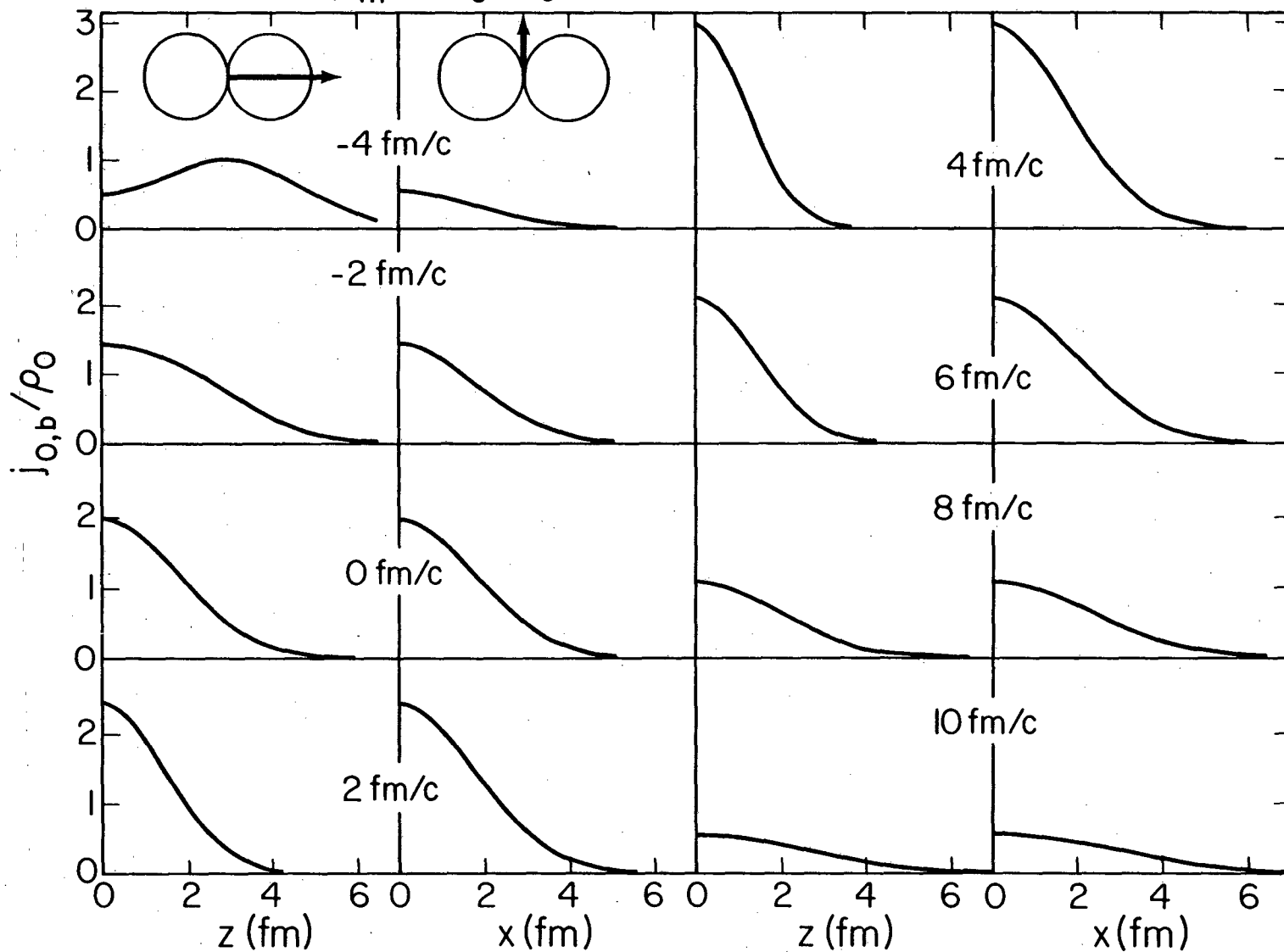
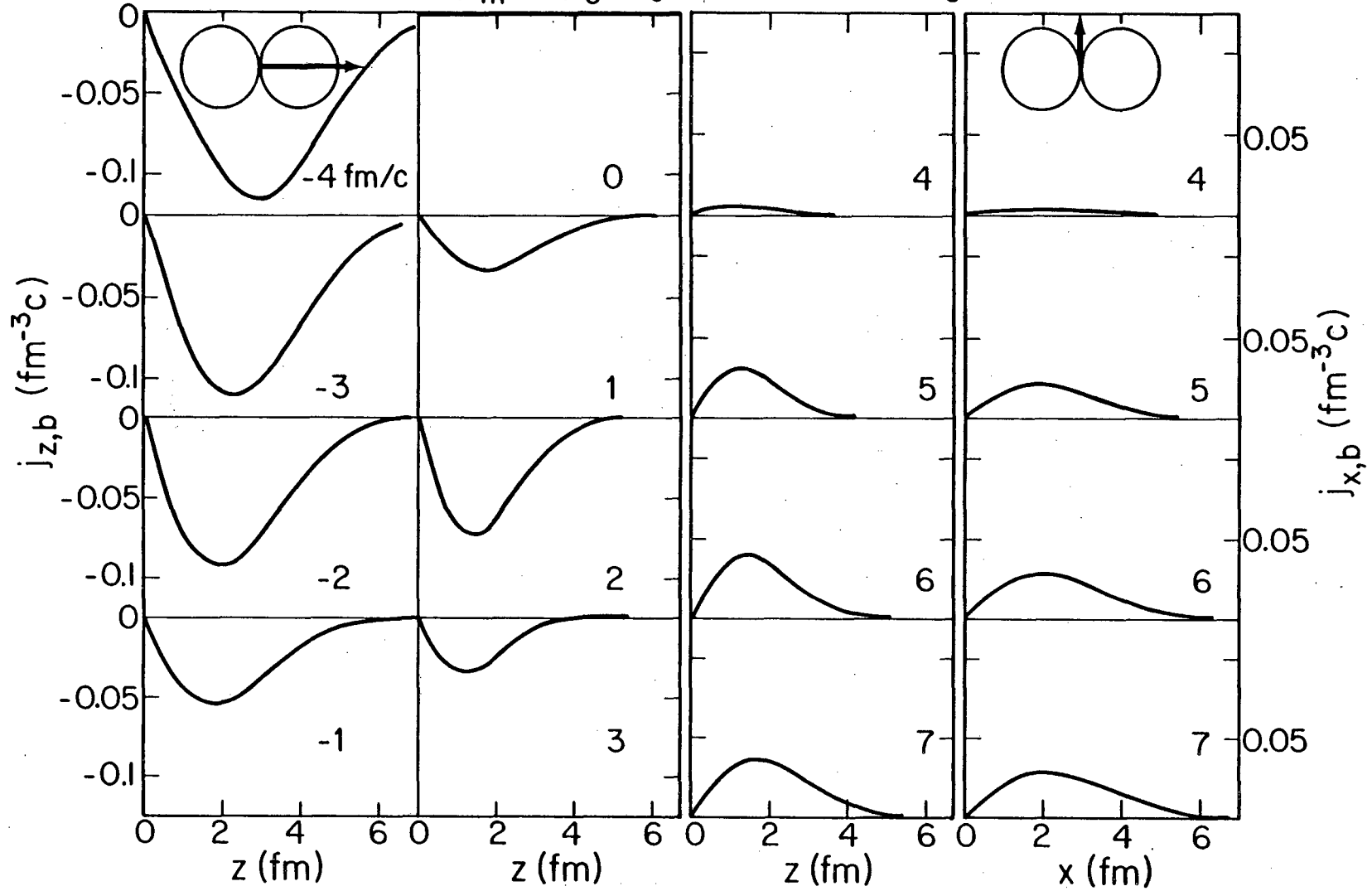


Fig. 1

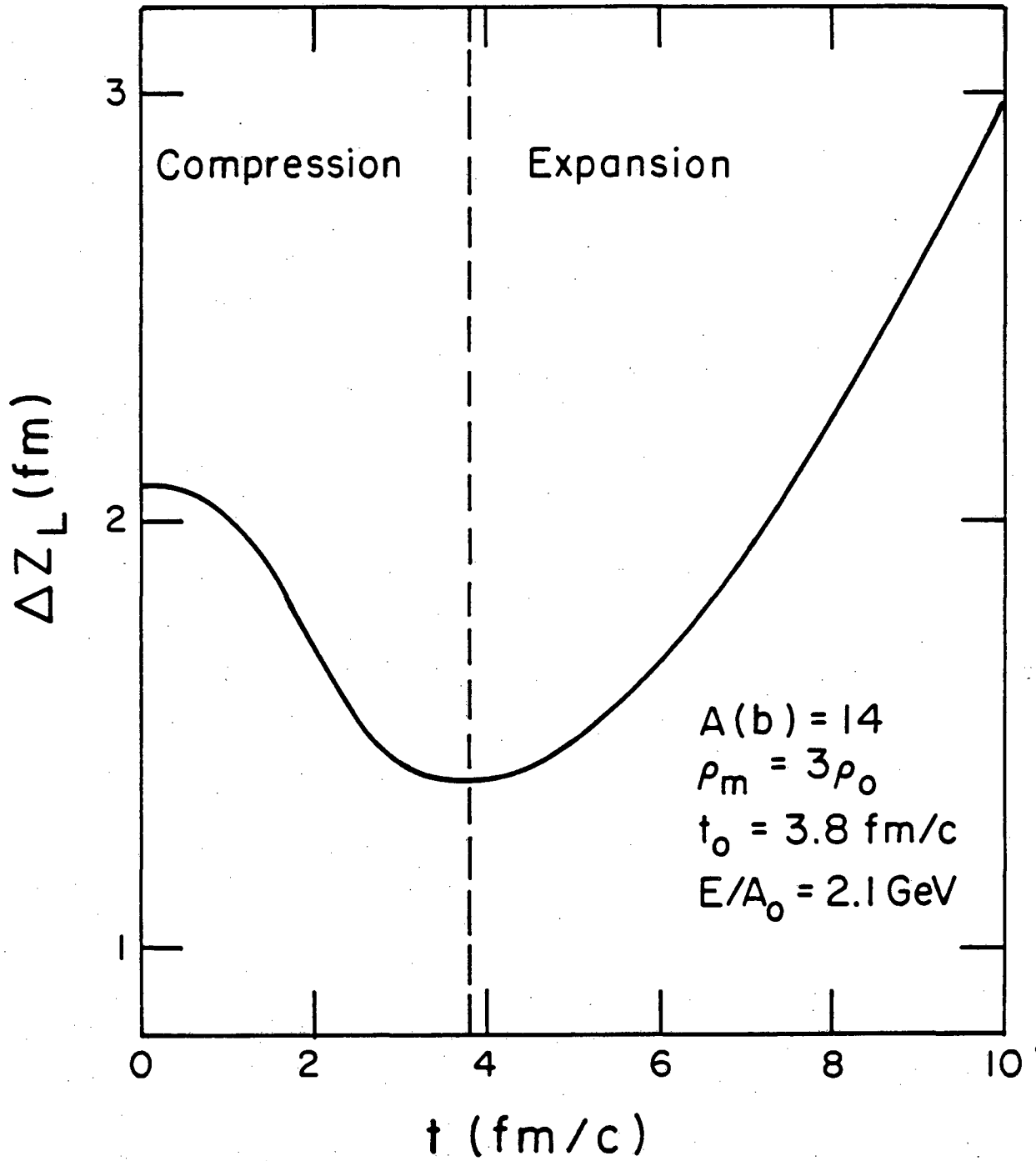


$A(b) = 14, \rho_m = 3\rho_0, t_0 = 3.8 \text{ fm}/c, E/A_0 = 2.1 \text{ GeV}$



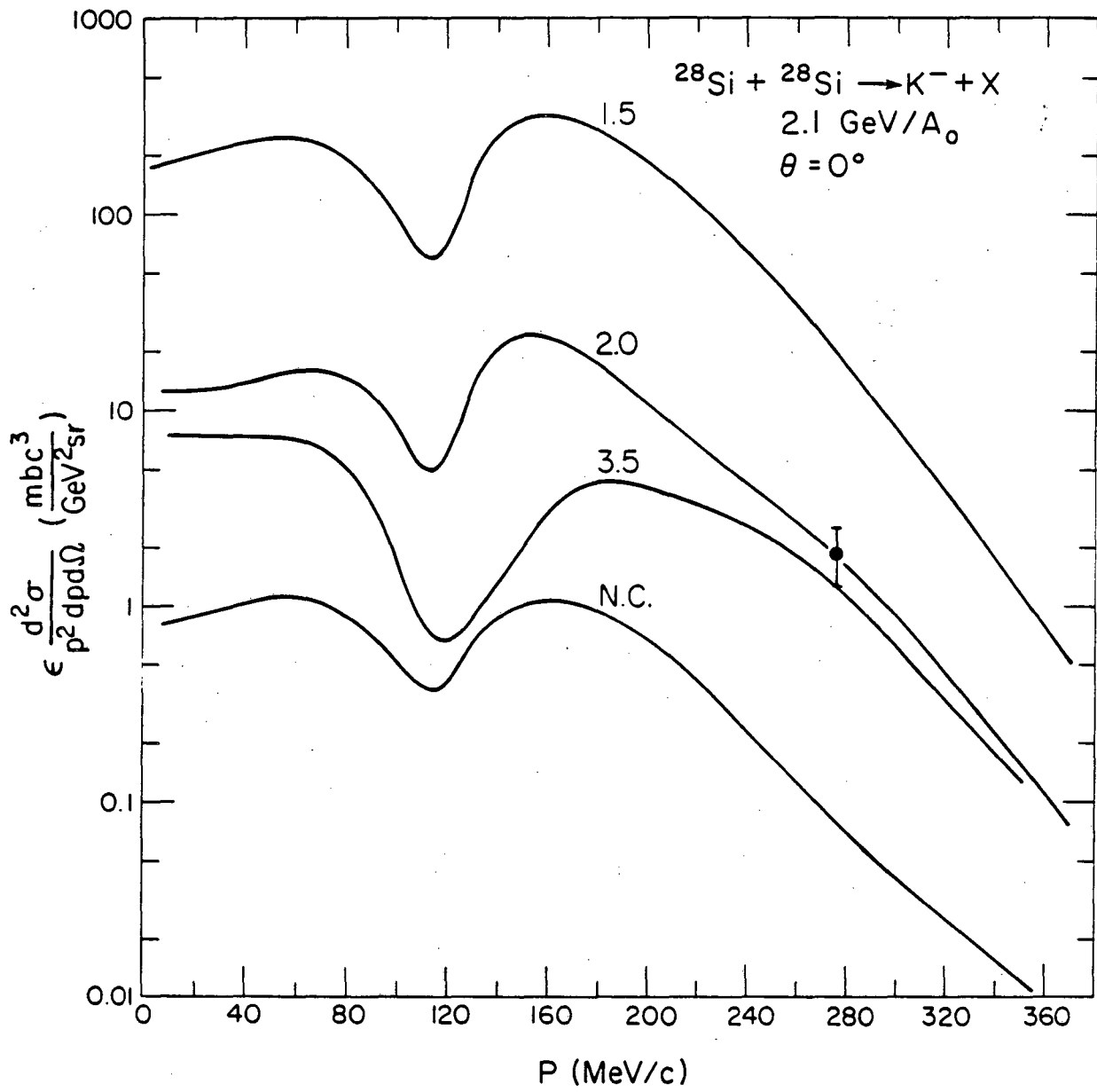
XBL 827 -966

Fig. 2



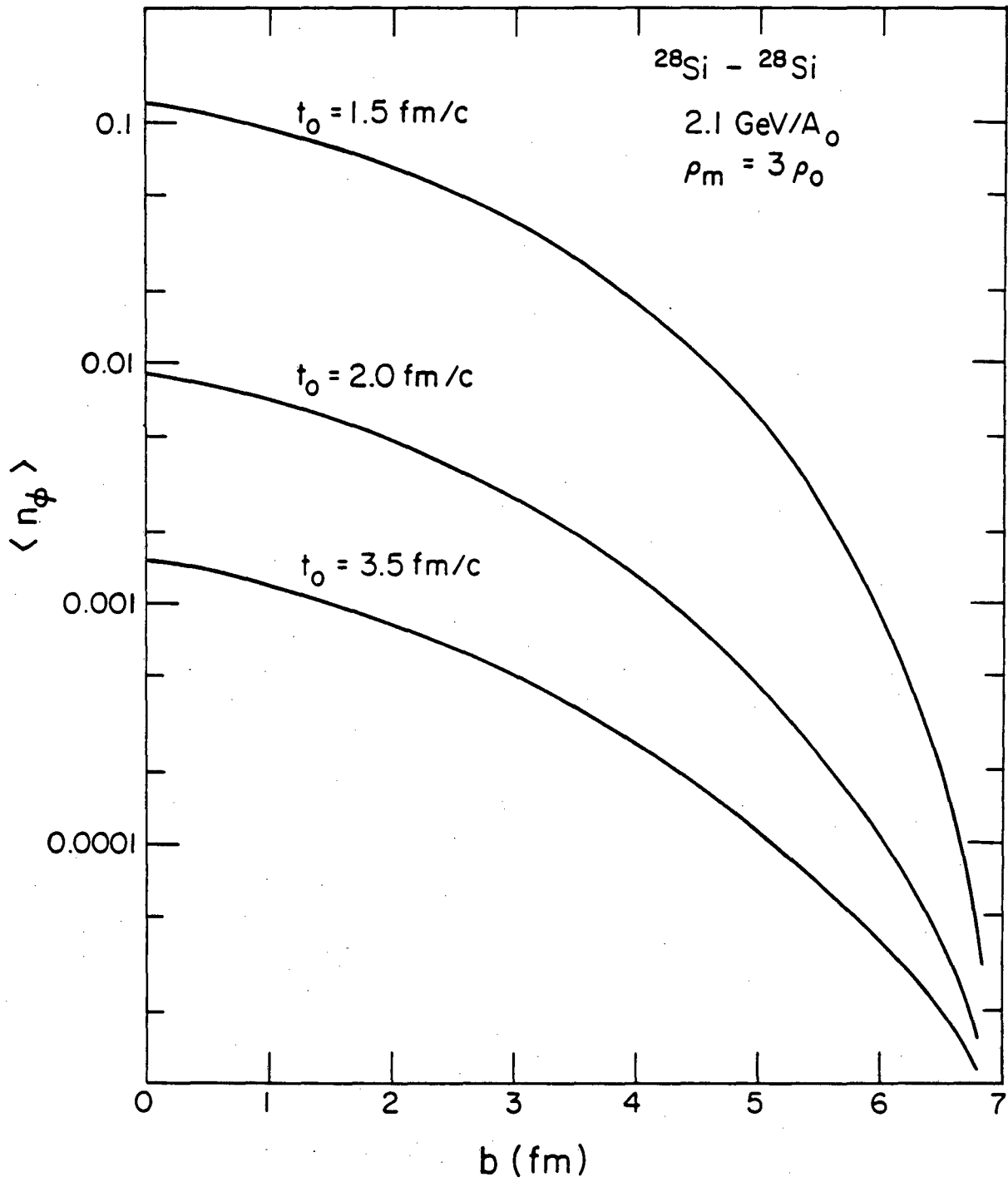
XBL 827 - 963

Fig. 3



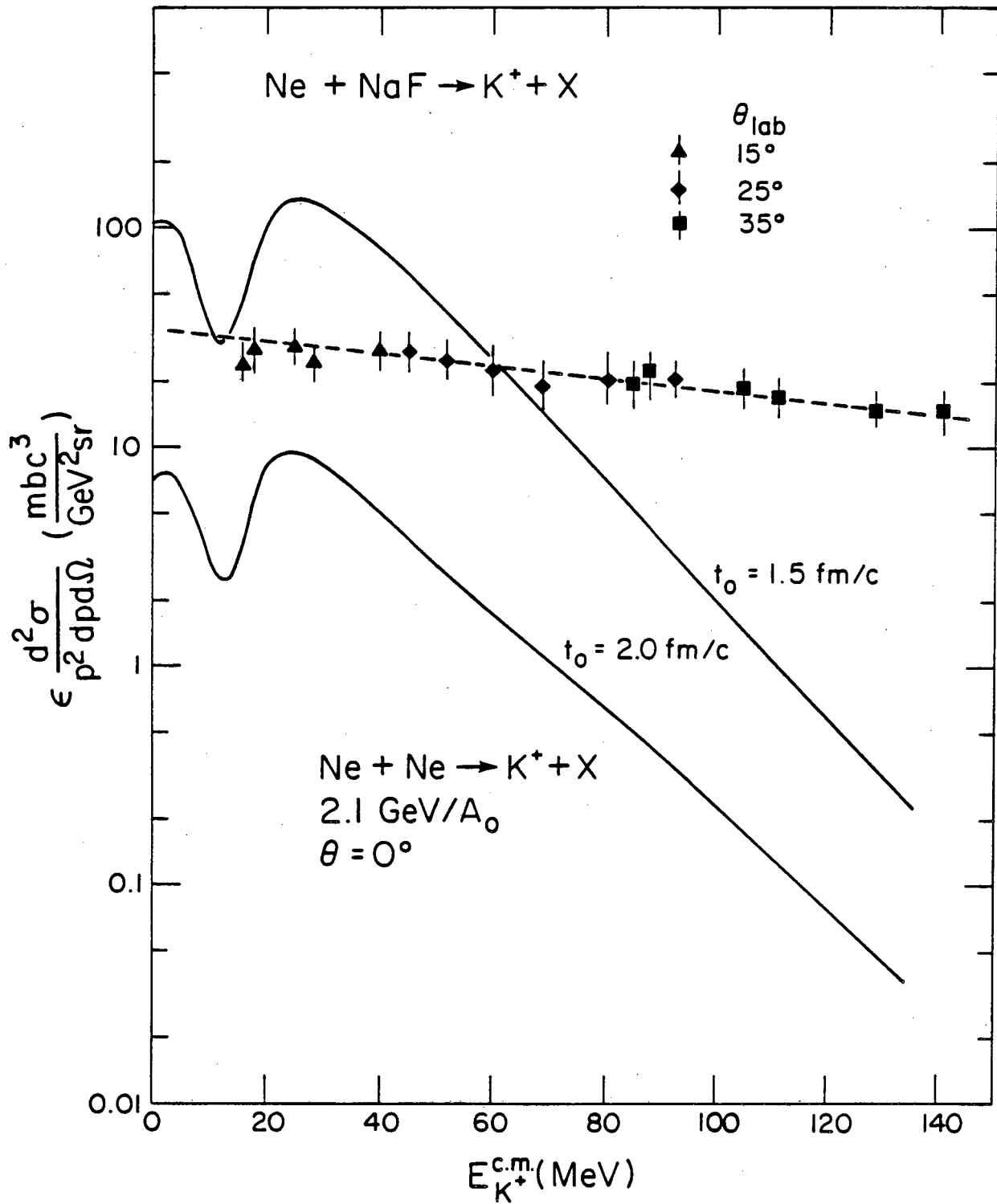
XBL 827 - 965

Fig. 4



XBL 827 -962

Fig. 5



XBL 827 - 964

Fig. 6

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