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On The Liquid Film Remaining in a Draining Circular Cylindrical Vessel

by

Paul Concus

It is of interest to have a theoretical estimate for the thickness of the film that remains on the wall of a right circular cylindrical vessel as it is being drained of a wetting liquid. Such an estimate, which includes the effects of surface tension and viscosity, has been given by Levich¹ for the case where the contact angle is zero and the static meniscus away from the wall becomes essentially a horizontal plane. In this note we describe how, with the aid of recently published graphs², Levich's technique can be extended to obtain estimates for the cases where the static meniscus away from the wall may be curved, such as in a vessel of small radius or in a reduced gravitational field.

The analysis presented for the flat plate film behavior on p.678, 679, and the top half of p.680 of Ref. (1) carries over unchanged for our problem. (The assumption that the film thickness be small compared with the radius of the vessel is, of course, needed.) The resulting expression for the constant asymptotic film thickness, h_o , for a withdrawal velocity that is not too large can be written as

$$h_o = 1.34 (\mu v_o / \sigma)^{2/3} / \kappa , \quad (1)$$

where μ is the liquid viscosity, v_o is the bulk velocity of the fluid, σ is the surface tension, and κ is the curvature at the wall of a meridian of the static meniscus. (The coefficient 1.34 in Eq. (1) was calculated using a computed value of 0.643 for α , the asymptotic dimensionless film

curvature, rather than the less accurate value of 0.63 given in Eq.(133.24) of Ref. (1).)

The appropriate value of κ to insert into Eq. (1) for any cylinder radius and gravitational field can be found from Ref. (2). From Eq. (1) of Ref. (2) one obtains that for a perfectly wetting liquid (zero-degree contact angle)

$$\kappa = \frac{1}{a}(Bf_w + \lambda - 1) ,$$

where a is the radius of the vessel, and $B = \rho g a^2 / \sigma$ is the Bond number (a dimensionless parameter) where ρ is the liquid density and g is the gravitational acceleration; the values of f_w , the dimensionless meniscus at the wall ($r=1$) height, and λ , twice the dimensionless mean curvature at the meniscus center ($r=0$), as a function of B (for zero contact angle) can be read directly from the graphs in figures 2 and 5 of Ref. (2), (see Figure 1).

Limiting values for κ and h_o for very large B (essentially horizontal meniscus away from the wall) and very small B (essentially hemispherical meniscus) can be obtained using the asymptotic expressions for f_w and λ given in Ref. (2). For very large B , Eqs. (15a), (18), and (23) of Ref. (2) yield

$$\kappa \approx \frac{1}{a} \sqrt{2B} ,$$

so that

$$h_o \approx 0.95 \frac{(\mu v_o)^{2/3}}{(\rho g)^{1/2} \sigma^{1/6}} ,$$

which is the same as the expression given by Eq. (133.26) of Ref. (1) (after the value of α is corrected). For very small B , Eq. (9) of Ref. (2) yields

$$\kappa \approx \frac{1}{a},$$

so that $h_o \approx 1.34 a (\mu v_o / \sigma)^{2/3}$

for this case. Note that for large B the film thickness is independent of the vessel radius and, as noted on p.681 of Ref. (1), is only weakly dependent on ^{the} surface tension. For small B , on the other hand, h_o is proportional to the vessel radius and depends more strongly on the surface tension.

This note is an outgrowth of a discussion with G. D. Bizzell, G. E. Crane, and H. M. Satterlee in which the author was made aware of the problem and the pertinent contents of Ref. (1). The work was performed in part under contract NAS 3-11526 of Lockheed Research Laboratory with NASA - Lewis Research Center and in part under the auspices of the U.S. Atomic Energy Commission.

References

- [1] V. G. Levich, Physicochemical Hydrodynamics, Prentice-Hall, Englewood Cliffs, N.J., (1962), p. 681, Eq. (133.26).
- [2] P. Concus, Static menisci in a vertical right circular cylinder, J. Fluid Mech. 34 (1968) 481-495.

Figure Caption

Figure 1. Static meniscus meridian.

The variables r and f , which are the ones used in Ref. (2), are dimensionless. The radius of the cylinder is a . The quantity κ denotes the meridian's curvature at $r = 1$, and $\lambda/2a$ its curvature at $r = 0$.

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Figure 1

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