Joint inversion for mapping subsurface hydrological parameters

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Summary

Using electromagnetic (EM) and seismic traveltime data and a least-square criteria, a two-dimensional joint inversion algorithm is under development to assess the feasibility of directly mapping subsurface hydrological properties in a crosswell setup. A simplified Archie's law combined with the time average equation relates the magnetic fields and seismic traveltime to two hydrological parameters; rock porosity and pore fluid electrical conductivity. For simplicity, the hydrological parameter distributions are assumed to be two-dimensional. Preliminary results show that joint inversion does have better resolving power for the interpretation than using the EM method alone. Various inversion scenarios have been tested, and it has been found that alternately perturbing just one of the two parameters at each iteration gives the best data fit.

Introduction

Geophysical inversion is frequently used to estimate subsurface geophysical parameters. The inversion can be formulated to handle multiple parameters, and at the same time multiple data sets may be used to improve resolution. Sena and Toksöz (1990) used high-frequency EM data to derive subsurface electrical conductivity and permittivity, and Zhang and Oldenburg (1999) used EM data to map magnetic susceptibility and electrical conductivity simultaneously. Hering et al. (1995) showed that improved results may be obtained through joint inversion if multitudes of data are available. For the resistivity problem Schlumberger, radial-dipole and two-electrode sounding data were jointly used to improve resistivity imaging, and for the seismic problem Love and Rayleigh group slowness data were used to obtain improved near-surface velocity structures. The objective of the joint inversion shown above is specifically to achieve better resolution of either the electrical parameters by jointly inverting available electrical/EM data, or the near surface seismic velocity by jointly inverting various seismic data, independently.

One of the main objectives of the geophysical inversion is to use the result to describe various subsurface processes involving fluid flow. Hydrological properties such as fluid electrical conductivity and porosity cannot be directly obtained with conventional inversion techniques. The EM field propagating through subsurface medium is a function of the bulk conductivity, which in turn may be empirically related to porosity, pore fluid conductivity, saturation, and sometimes the temperature. Seismic traveltime along a ray path in a medium depends on its propagation velocity, which may again be related to several factors such as porosity, density, elastic constants, temperature and pressure. The complex interrelationship among these variables renders direct mapping of these parameters very difficult. However, joint analysis of different geophysical data along with available information about the relationships between geophysical and hydrological parameters may allow us to achieve the objective.

To assess the feasibility of deriving hydrological properties directly, we have developed a joint inversion technique using EM and seismic traveltime data. The objective is to derive fluid conductivity and rock porosity of the medium between two boreholes. The Archie's law and the Wyllie time average relations are used to relate geophysical parameters to two of the hydrological variables: rock porosity and fluid conductivity. The inversion is based on a least-square criteria that minimizes the misfit between the observed data (synthetic data in this study) and that of the inverted hydrological model. A smoothness constraint is used to reduce the non-uniqueness. To begin with, the inversion is tested on a two-dimensional model. For the EM method, the model is axially symmetric about the transmitter borehole, and numerical simulation is carried out with the algorithm developed by Alumbaugh and Morrison (1995). Bulk electrical conductivity used for the EM simulation is estimated using Archie's law. Straight ray path is assumed for the seismic method and traveltime data is calculated based on the simplified Wyllie equation.

Theory

The magnetic field due to an EM transmitter is a function of the bulk formation conductivity (σ_b), which, in turn, is a function of porosity (ϕ), pore fluid conductivity (σ_{fl}), and a formation factor (m). They can be related with the empirical Archie's law:

$$\sigma_b = \frac{1}{a}\sigma_{fl} S^n \phi^m ,$$

where a is a constant specific to the certain formation. Assuming a fully saturated condition (S=1), with a = 1, and m = 2, the bulk conductivity now is expressed as

$$\sigma_b = \sigma_{fl}\phi^2 . \tag{1}$$

As a result, the magnetic field can now be related to the pore fluid conductivity and porosity. If the EM method is used alone, subsurface bulk conductivity can be inverted using the conventional inversion approach. However, in the inversion process it will be difficult to separate the fluid conductivity from the porosity since both are lumped together to give the value of the bulk conductivity. The uncertainty, or non-uniqueness, can be reduced either by applying heavy smoothness constraint in the inversion process or by incorporating other independent data. For this purpose we include the seismic traveltime data, which is a function of seismic wave velocity. The velocity depends on media porosity, density, pressure, etc. For simplicity, we chose the Wyllie equation to express seismic wave slowness (the reciprocal of velocity) as:

$$S = S_{ma} + (S_{fl} - S_{ma})\phi \,. \tag{2}$$

Consequently, seismic traveltime is now linked to the porosity, ϕ, the pore fluid slowness (S_{fl}), and rock matrix slowness (S_{ma}). If we further assume both S_{fl} and S_{ma} are known, then the traveltime data is a function of the porosity only. Using (1) and (2), one can jointly invert for the medium porosity and pore fluid conductivity using EM and seismic data.

The joint inversion scheme is developed based on a least-square criteria that minimizes a cost function, Φ, defined as:

$$\begin{aligned}\Phi = \Big\{ & \left\| \underline{\underline{\mathbf{W}}}_d \left(\underline{\mathbf{d}}(\underline{\sigma}_{fl}, \underline{\phi}) - \underline{\mathbf{d}}_{obs} \right) \right\|^2 - \chi^2 \\ & + \lambda_\phi \left\| \underline{\underline{\mathbf{W}}}_\phi \cdot \underline{\phi} \right\|^2 + \lambda_{fl} \left\| \underline{\underline{\mathbf{W}}}_{fl} \cdot \underline{\sigma}_{fl} \right\|^2 \end{aligned} \,. \tag{3}$$

Here, $\underline{\mathbf{d}}$ and $\underline{\mathbf{d}}_{obs}$ are one-dimensional vectors representing, respectively, the calculated and measured system response to the associated subsurface fluid conductivity and porosity; $\underline{\underline{\mathbf{W}}}_d$ is a square weighting matrix that assigns a relative importance to each data point. Usually, $\underline{\underline{\mathbf{W}}}_d$ is a diagonal matrix with the inverse of the standard deviation for the measurement. The number χ^2 is the estimated square-error in the observed data. To reduce the non-uniqueness to obtain a plausible solution, the inversion is constrained by the *a priori* of the model, which is specified by the two matrixes $\underline{\underline{\mathbf{W}}}_\phi$ and $\underline{\underline{\mathbf{W}}}_{fl}$ in the last two terms of (3). For this study, the smoothness of the model is applied and it is defined as differences in model parameters between adjacent discrete cells. The degree of smoothness is controlled by the two independent Lagrange multipliers λ_ϕ and λ_{fl}. The larger they are, the more the smoothness is emphasized, and, consequently, the lower the resolution. After minimizing the cost function with respect to the fluid conductivity and porosity, a system matrix equation is solved iteratively to derive the two unknowns: ϕ and σ_{fl}.

For a non-linear inversion problem, the derivation of the Jacobian matrix is vital. Since the magnetic field, **H**, is a function of rock conductivity, which in turn is function of porosity and fluid conductivity, its perturbation due to the constituent is:

$$\delta\mathbf{H} = \frac{\partial\mathbf{H}}{\partial\sigma_b}\frac{\partial\sigma_b}{\partial\phi}\delta\phi + \frac{\partial\mathbf{H}}{\partial\sigma_b}\frac{\partial\sigma_b}{\partial\sigma_{fl}}\delta\sigma_{fl} \,. \tag{4}$$

Applying the simplified Archie's law of (1), the sensitivity of the magnetic field to a small change in fluid conductivity and porosity becomes:

$$\frac{\partial\mathbf{H}}{\partial\phi} = 2\sqrt{\sigma_b\sigma_{fl}}\,\frac{\partial\mathbf{H}}{\partial\sigma_b}$$

$$\frac{\partial\mathbf{H}}{\partial\sigma_{fl}} = \frac{\sigma_b}{\sigma_{fl}}\frac{\partial\mathbf{H}}{\partial\sigma_b}.$$

Here, the term $\partial\mathbf{H}/\partial\sigma_b$ can be derived from traditional EM sensitivity. Similarly, the traveltime data, **T**, is affected by the change in porosity in a way that can be expressed as:

$$\frac{\partial\mathbf{T}}{\partial\phi} = l\left(S_{fl} - S_{ma}\right).$$

Here, l is the length of the ray passing through a certain model cross section.

Forward calculation is involved as part of the overall inversion process. For the EM method, a cylindrically symmetric geometry with the transmitter borehole located at the coordinate center is assumed. Vertical magnetic fields in the other borehole are calculated with an algorithm developed by Alumbaugh and Morrison (1995), based on the extended Born approximation (Habashy et al., 1993). For the seismic technique, straight-ray method is used for the traveltime and the resulting slowness is related to the porosity using the time average equation.

A synthetic crosshole EM and seismic data set is used to validate the newly developed algorithm. As sketched in Figure 1, two un-cased boreholes, 20 m apart and both 60 m deep, are located in a half space with a pore fluid conductivity of 0.1 S/m and a porosity of 0.1. The P-wave velocities are fixed at 5486 m/s and 1692 m/s for the matrix and fluid, respectively. A thin anomalous zone, with 1 S/m and 0.4 in fluid conductivity and porosity, correspondingly, is centered at the source borehole at 30 m depth and extended 10 m laterally toward the receiver borehole. Twenty-three source positions for both methods are distributed at 2.5 m intervals between the depths of 5 and 60 m. As many receivers are located in the receiver borehole. For the EM method, the source is a 10 kHz vertical magnetic dipole and vertical magnetic fields are calculated at the receivers using an algorithm SHEETS developed by Zhou (1989). Traveltime data is obtained with the time average equation for straight ray paths between the transmitter and the receiver locations.

The initial model used for the inversion is a half space with constant fluid conductivity and porosity the same as the background values for the forward simulation. We have been able to derive the bulk conductivity successfully with EM data only as presented in Figure 2(a). However, simultaneous reconstruction for both porosity and fluid conductivity with EM data only was not satisfactory. As shown in Figure 2(b), in addition to the expected anomaly at 30 m depth, two large areas with anomalous porosity show up in the background near the receiver borehole. The fluid conductivity as presented in Figure 2(c) is recovered but resolution is poor. As a result, the calculated bulk rock conductivity is erroneous compared to the simulated model. It is probably because both parameters are lumped together and equivalence problem could not be resolved.

Next, seismic traveltime data is added and the joint inversion is carried out to recover the two hydrological parameters simultaneously with two separate multipliers. However, we could not make the data misfit to converge. This may partly due to the fact that the two Lagrange multipliers in (3) are not weighted properly. Another scheme we have tried is to fixed the porosity first and use the fluid conductivity as the only varying parameter until the data misfit does not improve and then extend the inversion with the conductivity fixed and the porosity varying. The result is displayed in Figure 3 with the derived porosity and fluid conductivity at the two central panels and the calculated velocity and bulk conductivity at the sides. The anomalous volume is recovered but some extreme porosity values show up close to the transmitter borehole, and the conductivity is not well resolved either. In yet another scheme, the fluid porosity is fixed first to obtain the fluid conductivity distribution, and vice versa to obtain the final picture for the fluid conductivity. Data misfit has improved compared to the previous scheme. However, the best result is obtained by finding the appropriate porosity distribution while holding the values of the fluid conductivity and then vice versa at one iteration. The two central panels, (b) and (c), in Figure 4 present the inverted rock porosity and fluid conductivity, respectively, between the two boreholes. The location of the anomalous body is well retrieved. Figure 5 illustrates the data misfit vs. number of iteration for the two inversion schemes for deriving the results shown in Figures 3 and 4, respectively. The results correspond to the scheme for Figure 3 is labeled in circles while the one associated to Figure 4 is characterized by stars. Apparently, the alternating scheme shown by Figure 4 not only has better data matching, but also has faster convergence.

Conclusions

Based on the preliminary results of this research, we have demonstrated that hydrological parameters can be obtained directly with joint inversion of two geophysical survey methods. The optimum result of the joint inversion has been obtained by alternating the selection of inversion parameter while the other parameter is held fixed for each iteration. We have not been successful in simultaneously perturbing porosity and fluid conductivity employing two separate Lagrange multipliers. Further study in the use of multiple Lagrange multipliers is critically important for the success of the joint inversion. The empirical relationships chosen for the demonstration are so simple that the current algorithm may not be useful in handling the real data. Inclusion of more hydrological parameters in describing the relationships is desired and, as a result, more geophysical data may be required for the joint inversion to reduce the degree of non-uniqueness.

References

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Acknowledgments

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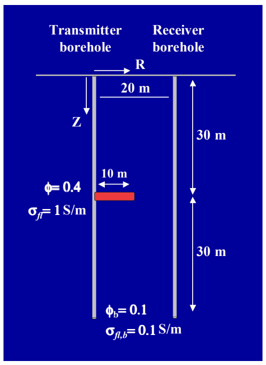
Joint inversion for subsurface hydrological mapping



Geophysical parameters:

	Bulk Velocity (m/s)	Bulk Conductivity (S/m)
Background	4500	0.001
Anomaly	2900	0.16

Fig. 1 Simulated crosshole model for validating the joint inversion algorithm.

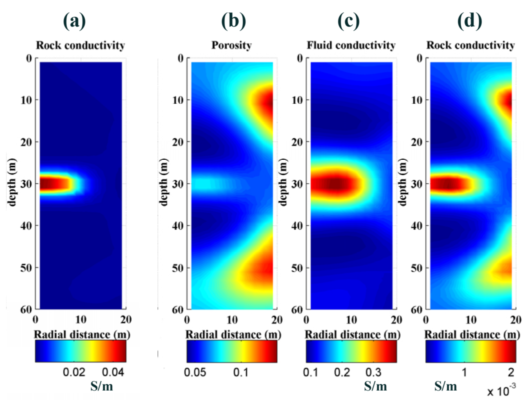


Fig.2 Inversion results with magnetic field only. (a) The bulk conductivity is the only unknown parameter, (b) and (c) correspond to, respectively, porosity and fluid conductivity inverted simultaneously, and (d) is the calculated bulk conductivity using the simplified Archie's law.

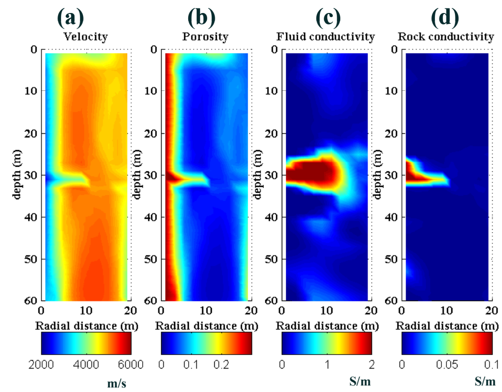


Fig. 3 Joint inversion results using both EM and seismic data. The porosity is fixed while fluid conductivity is varying till data misfit doesn't converge any more. Then vice versa to obtain the porosity.

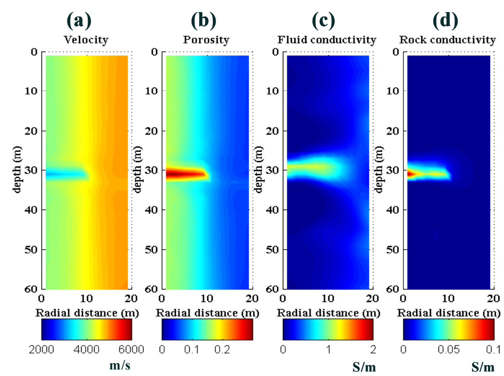


Fig. 4 Joint inversion results using both EM and seismic data. At each iteration, the conductivity is fixed first to obtain a better porosity model. Then vice versa.

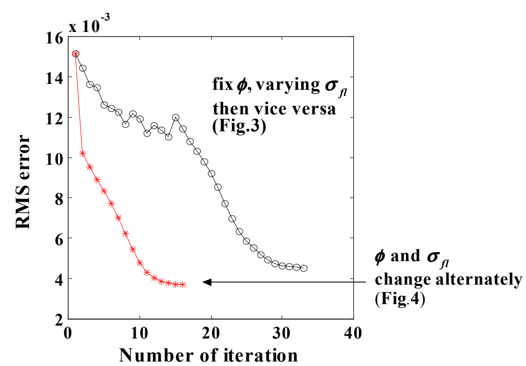


Fig. 5 RMS error vs. number of iteration for the two schemes used to derive the results illustrated in Figures 3 and 4.