Title
INTRODUCTION TO TOPOLOGICAL THEORY OF HADRONS

Author
Stapp, H.P.

Publication Date
1982

LBL-13940
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

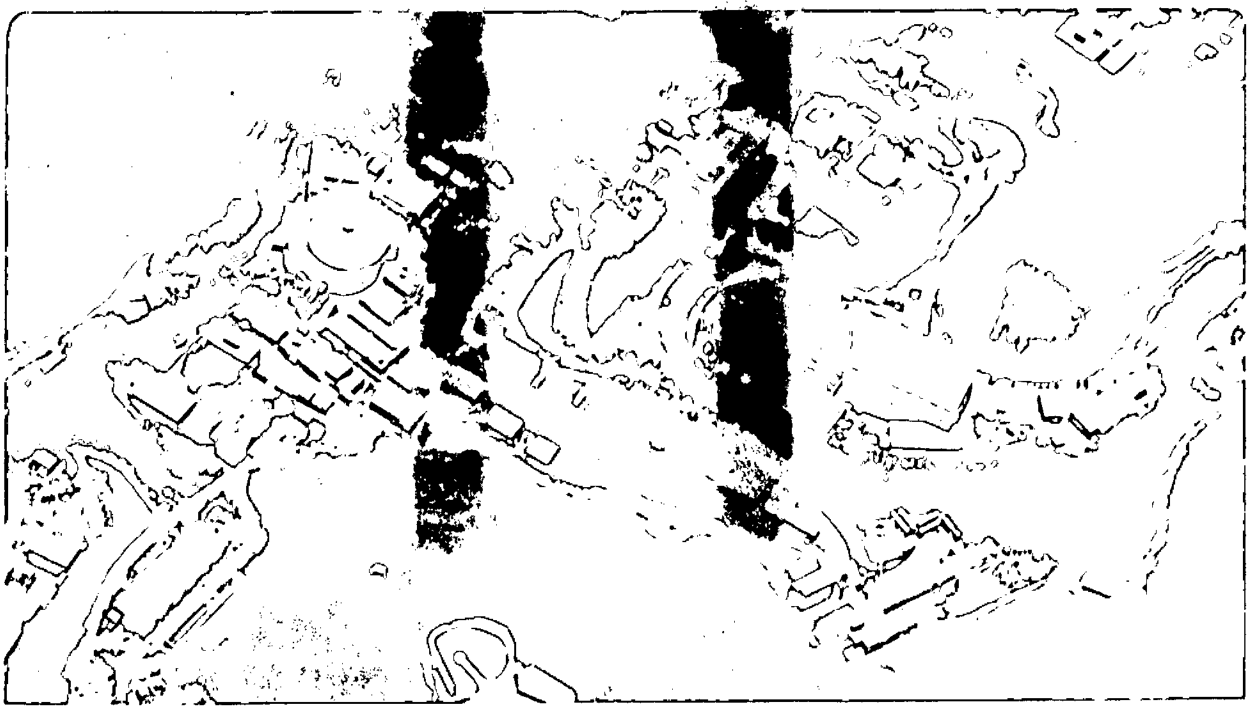
Physics, Computer Science & Mathematics Division

Submitted for publication

INTRODUCTION TO TOPOLOGICAL THEORY OF HADRONS

Henry P. Stapp

January 1982



Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

DISCLAIMER

January 1982 LBL-13940

INTRODUCTION TO TOPOLOGICAL THEORY OF HADRONS*

Henry P. Stapp

Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

ABSTRACT

A new and more informative introduction is provided for the author's recent paper Topological Theory of Hadrons.

* This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. W-7405-ENG-48.

1. INTRODUCTION

A scattering amplitude can be represented as a sum of contributions from all ways in which the process can occur. Each contribution has a phase factor, and the scattering amplitude between randomly chosen states tends to be small due to an averaging-out of these phase factors. The dominant transitions are between states in which the elements of order characterizing the initial state are carried into the final state in some "direct" way.

This tendency of the the dominant transitions to preserve order is particularly important in hadron physics, due to the inherent complexity of the hadrons and their interactions. Indeed, this order-preserving tendency has been made the basis of a successful approximation procedure for meson physics. This procedure is based not on the smallness of any coupling constant but rather on the smallness of contributions that do not preserve order. Order is defined so that it is preserved by contributions to the scattering amplitude that correspond to sequences of scattering events represented by graphs that can be drawn in a plane with no lines crossing. Contributions from non planar graphs generally have phase factors that tend to average to zero in high-energy regimes.

This topological approach to hadron dynamics, which originated in some works by Veneziano[1], and has been pursued by many workers, has been recently reviewed by Chew and Rosenzweig[2]. They show how the topological expansion procedure, combined with the requirements of unitarity, analyticity, duality, and Lorentz invariance, organizes and predicts many of the dominant features of meson physics.

The successes achieved in meson physics by this topological approach have motivated efforts to develop it into a comprehensive basis for particle physics. The most obvious deficiency of earlier work is its restriction to mesons. Accordingly, one major aim of the present two-part work is to extend the theory to baryons. Paper II[3] is devoted to that task.

But beyond this problem of baryons, there lie other problems of equal importance. To provide a satisfactory basis for particle physics, the theory must, first of all, provide a practical method of determining, through the nonlinear bootstrap conditions, the magnitudes of all coupling constants that occur in particle physics. Phenomenological analyses indicate that the ratios of the hadronic coupling constants satisfy $SU(6)_W$ symmetry to a degree unlikely to be purely accidental.[4] It is therefore probably essential to the practical viability of the topological approach, considered as a general basis for particle physics, that it be constructed so as to exhibit $SU(6)_W$ symmetry at a low level of topological complexity. Accordingly, a second major aim of this work is to construct a topological bootstrap framework that treats the spin degrees of freedom in a way that ensures $SU(6)_W$ symmetry at the lowest level of the topological

expansion.

Historically, this $SU(6)_W$ property emerged, within the present work, as an unexpected by-product of the effort to generalize the Chan-Paton factors[5] from isospin to ordinary spin. These original Chan-Paton factors enjoy the following important property: every product of amplitudes that contributes at the lowest topological level to any given amplitude has the same Chan-Paton factor. This product property ensures the existence, at the lowest topological level, of a solution to the isotopic spin part of the dual topological bootstrap dynamical conditions.

It is not obvious that this solution is unique. However, it is a simple solution that is quite possibly unique. For, a priori, the infinite number of dual bootstrap conditions need have no solution at all.

To ensure the solubility of the spin part of these dynamical conditions, the following requirement is here imposed: the spin-generalization of the Chan-Paton factors are required to satisfy the direct generalization of the product property enjoyed by the original Chan-Paton factors. This demand determines the basic character of the spin formalism described in this paper.

The problem of extending the Chan-Paton factors to ordinary spin was considered by Mandelstam in the late sixties. Mandelstam's work,[6] like the present work, was based on M functions.[7] These functions have the combined advantage of possessing simple crossing properties and a minimal number of spin components: crossing is represented by analytic continuation alone, and the redundant components that arise from describing spin-½ particles by four-component wave functions are avoided. Using these functions, one finds that the required product property (of the spin-generalizations of the Chan-Paton factor) cannot be reconciled with the demand of invariance under parity. Thus, Mandelstam, proceeding in the straight-forward way, summed two parity-reflected contributions, each of which individually satisfies the product property, in order to obtain a parity invariant form. However, this procedure of simply summing two separate terms, each of which enjoys also simple factorization properties, led first to a parity-doublet partner for each of the observed mesons π, ρ, η, ω, and then to a second doubling of this set of mesons.

The procedure followed here differs from that of Mandelstam by its strict enforcement of the product-property requirement described above. The present procedure, originally introduced to ensure the

the solubility of the spin part of the dynamical conditions, generates a number of important further consequences. First, by keeping the treatment of spin closely parallel to the successful Chan-Paton treatment of iso-spin, it leads automatically to $SU(6)_W$ symmetry of the hadronic coupling constants, at the lowest topological level. Second, it automatically produces a basic set of mesons that accords exactly to the phenomenologically observed set (π, ρ, η, ω): there is no parity doubling, or any other doubling, of the meson spectrum. Third, when combined with certain assumptions about the number of flavors, it leads to a super symmetry connection between the meson-meson-meson coupling constants and the meson-baryon-baryon coupling constants. This connection is in good agreement with experiment. Fourth, when supplemented by a plausible universality requirement, it leads to a value of the ratio of the strong-interaction coupling constants to the electromagnetic coupling constant. This ratio is also in satisfactory agreement with experiment.[8]

The technical basis of these achievements is the fact, recognized and exploited already in the work of Chew and Rosenzweig, that different levels of the topological expansion can act in different Hilbert spaces. In the paper of Chew and Rosenzweig, the lowest level of the topological expansion was the "ordered" level, and each ordered amplitude was associated with a corresponding ordered Hilbert space. These ordered amplitudes were summed to form "planar" amplitudes, which were associated with new "planar" Hilbert spaces. Approximate correspondence to physical amplitudes was possible only at the planar and higher levels of the topological expansion. Yet, the ordered amplitudes were important, for the topological expansion concentrated all nonlinear aspects of the original unitarity equations in the ordered unitary equations satisfied by the ordered amplitudes. These ordered unitarity equations were much simpler than the original unitarity equations because they involved only planar Landau diagrams. Thus, the critical problem of determining the overall strength of the couplings was greatly simplified. Moreover, the represention of physical amplitudes by low-order terms in the topological expansion allowed many aspects of meson physics to be understood even without solving the nonlinear equations.[2]

The product-property requirement on the spin-generalization of the Chan-Paton factors places severe conditions on the theoretical structure. These conditions can be satisfied by introducing a new topological level, called zero entropy, that

lies below the ordered level. The individual zero-entropy amplitudes are not invariant under parity, and, like the ordered amplitudes, they cannot be regarded as approximations to the physical amplitudes. It is the planar amplitudes, which do conserve parity, that are again to be considered as the first approximation to the physical amplitudes.

In the work of Chew and Rosenzweig, the planar amplitudes were formed as finite sums of the ordered amplitudes. In the present framework, the planar amplitudes are formed as infinite sums of zero-entropy amplitudes. These infinite sums create new technical problems. But they also create the possibility, and, in fact, the necessity, of calculating, for example, the $\pi - \rho$ mass difference. For at zero entropy, these two masses are equal.

The present work is part of a long-term collaborative effort with Geoffrey Chew to construct a practical basis for particle physics based on dual topological bootstrap dynamics. Chew's ideas are woven into it in many ways. However, this paper deals only with certain spinor, topological, and group theoretic aspects of the whole theory; other important aspects are left untouched.

One problem not considered in this paper itself is the extension to all hadrons of the formalism developed herein for mesons. An extension is described in paper II. It incorporates also the group-theoretic properties of the constituent-quark model. The whole work is formulated completely within the S matrix framework, and hence involves no microscopic description in terms of quark wave functions. Hence, it provides, in principle, the foundation of a Lorentz covariant approach to particle physics that has a basic set of particles that agrees, as far as spin, parity, and other group-theoretic properties are concerned, with those of the constituent-quark model, yet has no confinement problem. Moreover, it incorporates $SU(6)_W$ symmetry, at the lowest topological level.

The present paper is associated with a recent paper by Chew and Poenaru;[9] it describes technical results that have been used in the development of their ideas. However, the aims of Chew and Poenaru are broader than those of the present work, which simply accepts the group-theoretic structure of the constituent-quark model on the basis of its empirical success. Chew and Poenaru seek to derive the group-theoretic structures from topological considerations and consequently need a richer topological structure than the one used here. Their topological structure contains, in addition to the quark-particle graphs of the present theory, and surface upon which these graphs are imbedded, also a second surface, called the quantum

surface, in which the group-theoretic relations associated with flavor and other symmetries reside.

In the present work, flavor is an unconstrained variable. The flavor structure may, in fact, be determined by the nonlinear dynamical equations, but it is not determined within the present framework by topological considerations alone.

Proposals for extending the theory to electroweak interactions have been made recently by Chew, Finkelstein, McMurray and Poenaru[10,11,12]

A crucial problem not addressed in any of these works is the development of reliable methods for solving the nonlinear conditions. These conditions should determine the overall strength of the hadronic and electroweak interactions. However, several calculations have been performed,[13] and they all yield values that differ from the empirically observed overall strength of the hadronic-electroweak interaction by a factor of roughly two. This result seems significant, particularly because the spinor, topological, and group-theoretic considerations introduce as many as twenty different factors of two into this result. These theoretical factors were calculated <u>prior</u> to the calculation of the the approximate solutions to the nonlinear equations.

A second major problem not addressed in any of the published works is the development of reliable methods for constructing the planar amplitude from zero-entropy amplitudes. This construction must yield, for example, a first approximation to the $\pi - \rho$ mass difference. Some calculations of this difference have been made, with encouraging results, but the work is still in a developmental stage.

Much of the work contained in this paper was completed several years ago,[14] but was not submitted for publication because of the above-mentioned elements of incompleteness of the whole theory. However, a number of recently-published papers[8-12] are based directly on the spin formalism developed in that earlier work. This fact, in conjunction with the encouraging character of works in progress, makes publication of this expanded version now appropriate.

The theory developed here is based on the M-function formalism. Since the original description of that formalism[7] was very brief, the key points are described here in Section 2, with particular emphasis on the results that are important in the context of the present work. The main body of the paper is contained in Section 3. The results are summarized in Section 4. Appendix A shows that discontinuity equations, though usually considered in S-matrix theory as being derived from unitarity, are actually more basic than unitarity. Appendix B

explains the failure of unitarity at the zero-entropy level. The planar discontinuity equations nevertheless continue to hold. Appendix C describes the connection of the two-component formalism used in the body of the paper to the four-component formalism based on Dirac matrices.

2. SPIN

2.1. Lorentz Transformations in Spin Space

Let σ_μ represent the Pauli spin-matrix four-vector

$$\sigma_\mu = (\sigma_0, \sigma_1, \sigma_2, \sigma_3) = (1, \vec{\sigma}), \qquad (2.1)$$

where σ_0 is the two-by-two unit matrix and σ_1, σ_2, and σ_3 are

ACKNOWLEDGEMENT

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract No. W-7405-ENG-48.

REFERENCES

1. G. Veneziano, Nucl. Phys. B74, 365 (1974) Phys. Lett. 52B, 220 (1974).
2. G. F. Chew and C. Rosenzweig, Physics Reports 41C, 263 (1978).
3. H. P. Stapp, Topological theory of hadron II: Baryons, Lawrence Berkeley Laboratory Report 13310, Sept. 1981.
4. R. Delbourgo, M. A. Rashid, Abdus Salam, and I. Strathdee, in High-Energy Physics an Elementary Particles, IAEA 1965.
5. J. E. Paton and H. M. Chan Nuovo Cim. B10, 516 (1969).
6. K. Bardakci and M. Halpern, Phys. Rev. 183, 1456 (1969).
7. H. P. Stapp, Nuovo Cimento 66, 497 (1970) Eq. 30.
8. G. F. Chew, J. Finkelstein, and M. Levinson, Phys. Rev. Lett. 47, 767 (1981).
9. G. F. Chew, and. V. Poenaru, Z. Phys. C11, 59 (1981).
10. G. F. Chew, J. Finkelstein, R. E. McMurray Jr, V. Poenaru, Phys. Lett. 100B,53 (1981) and Phys. Rev. D24,2287 (1981).
11. G. F. Chew, Phys. Rev. Lett. 47,764 (1981).
12. G. F. Chew, J. Finkelstein, V. Poenaru, Topological theory of electroweak vector bosons, Lawrence Berkeley Laboratory Report LBL-12588 (1981) (to be published in Phys. Rev. D)
13. G. F. Chew, M. Levinson, J. Finkelstein, and R. Espinosa. Private Communication.
14. H. P. Stapp, Spins and baryons in the topological expansion, Lawrence Berkeley Laboratory Report LBL-10774 (1980).

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720