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We consider $PP \to PP$ scattering ($P = 0^-$ meson), and using Finite-Energy Sum Rules (FESR)[1] we compute the couplings PPV and PPT of the 1^- and 2^+ nonets. Taking degenerate masses for each J^P, we show that the existence of the observed resonances is consistent with our equations, and we prove that the observed 0^-, 1^-, and 2^+ multiplets must have SU(3) symmetric couplings. In addition we show that self-consistency requires a definite ratio of singlet/octet coupling for the 2^+ nonet. Next, using the observed masses we compute the broken couplings. In principle, we have here a bootstrap theory of SU(3) and its breaking. We obtain couplings reasonably in agreement with experiment.

The basic tools of the bootstrap are the finite-energy sum rules (FESR)[2]

$$\frac{1}{2} \int_{-N}^{+N} d\nu \, \nu^n \, \mathrm{Im}\, A(\nu,t) = \frac{\beta(t) N^{\alpha(t)+n+1}}{\alpha(t) + n + 1} . \qquad (1)$$

We work at fixed t, with definite isospin in the t channel. We assume the high-energy term on the right-hand side (RHS) of (1) is given by one Regge pole in the t channel. We therefore neglect secondary Regge poles (such as ρ'), cuts, and the background integral in the J plane. We approximate the low-energy integral on the left-hand side (LHS) of (1) by the dominant resonances in the s and u channels, neglecting nonresonant background. The validity of our approximations requires large N

on the RHS and small N on the LHS.[2] In order to check the dependence of our results on N, we consider the three choices: (I) N between $(m_1-)^2$ and $(m_2+)^2$; (II) N at $(m_2+)^2$; (III) N halfway between $(m_2+)^2$ and $(m_3-)^2$. We evaluate the FESR at $t = (m_{resonance})^2$ because that is the only point at which we know $\beta(t)$. We use the lowest moment sum rules only, S_0 for odd amplitudes (1^- resonances on the RHS), S_1 for even amplitudes (2^+ resonances on the RHS).[2] This gives one equation for each resonance, enabling us to compute coupling ratios. We do not attempt to compute masses here. We use the narrow resonance approximation on the LHS and therefore obtain algebraic equations in the β's. For each amplitude we have an independent system of n linear and homogeneous equations for the n resonances contributing. For example, the contributors in $K\pi \rightarrow K\pi$ are (K^*, ρ, K^{**}, ff'). Each system has a solution if and only if the determinant of the coefficients of the unknown β's is zero. This fixes the exact value of the limit of integration, s_N. A strong check on the self-consistency of our dynamical system is that the calculated s_N coincide approximately with the a priori choice for each of (I), (II), (III) described above.

In the mass degenerate case there is no distinction between f and f', and between ϕ and ω. The mixing angles are therefore indeterminate. Purely for convenience we also combine these particles in the numerical analysis.

For some quantum numbers, the t channel does not contain a known resonance and (1) becomes a superconvergence relation. We use these relations in our algebraic considerations for the mass degenerate case. We do not use them numerically because their sensitivity to N is much greater than that of the nonsuperconvergent sum rules.[2]

To exhibit the algebraic properties of (1) we make the particle labels explicit and write, assuming each multiplet is degenerate,

$$\sum_s \sum_{J=1}^{2} f^{(J)} K_n^{(J)} \left[g_{abs}^{(J)} \; g_{cds}^{(J)} + (-1)^{n+1} \, g_{ads}^{(J)} \, g_{cbs}^{(J)} \right] = \sum_s g_{acs}^{(n+1)} \; g_{bds}^{(n+1)} \; J_n \tag{2}$$

Again, (2) is evaluated at fixed t, and the s channel process is $a + b \to c + d$.

In (2) the $g_{abc}^{(J)}$ are effective dimensionless coupling constants.[3] The first two subscripts refer to 0^- states, the third subscript refers to a sum over intermediate spin J states. J_n and $K_n^{(J)}$ are positive kinematic factors depending on the masses (m_0, m_1, m_2) and on the limits of integration, N. $f^{(J)}$ is the fraction of the full contribution of the intermediate spin J states included on the LHS; $f^{(J)}$ is related to the choice of N. If we choose $f^{(2)} = 0$ and $f^{(1)} = 1$, N should be approximately halfway between the 1^- and 2^+ states; if $f^{(2)} = 1/2$, then $f^{(1)} = 1$, and N should be approximately at $m_2^2 +, \cdots$. If the term on the RHS of (2) is a t channel Regge pole, the terms on the left arise from resonances in s and u respectively. No crossing matrices appear[3] in (3); however, the signs of the kinematic factors are shown explicitly. (These signs arise from the signs of ν^n and of $\mathrm{Disc}(x - m^2 \pm i\epsilon)^{-1}$. We shall first evaluate (2), including only 1^- states on the LHS in the lower moment equations as in choice (I) above. In addition we evaluate the higher and lower moment equations including both 1^- and 2^+ on the LHS, as in choices (II), (III) above, but in our group theoretical discussion we leave $f^{(J)}$ and s_N free. The lower moment equations for case (I) give, introducing the convenient notation $F_{abc} = g^{(1)}_{abc}$

$$F_{abs}F_{cds} + F_{ads}F_{bcs} + \lambda_{00}F_{cas}F_{bds} = 0, \tag{3}$$

where $\lambda_{00} = J_0/f^{(1)} K^{(1)}_0$. Multiplying (3) by δ_{ad}, we have

$$(\lambda_{00} - 1) F_{acs}F_{abs} = 0. \tag{4}$$

The F's are real, and taking $b = c$ in (4), we get $\lambda_{00} = 1$, provided at least one F is nonzero. This gives[3]

$$F_{abs}F_{cds} + F_{ads}F_{bcs} + F_{cas}F_{bds} = 0 . \tag{5}$$

The three conditions: (A) equality of the 0^- and 1^- multiplicities; (B) total antisymmetry of F_{abc}; (C) orthonormality condition $F_{abs}F_{abs'} = \delta_{ss'}$, combined with (5), are the necessary and sufficient conditions for the F's to be the structure constants of a compact, semisimple, Lie algebra.[4] The condition (C) has the physical meaning that all V's have the same total reduced width, and that the V's couple to orthogonal combinations of PP states.[5]

Alternatively, we can include the 1^- and a fraction $f^{(2)}$ of the 2^+ contributions on the LHS of (2). Introducing $D_{abc} = g^{(2)}_{abc}$, we have

$$F_{abs}F_{cds} + F_{ads}F_{bcs} + \lambda_0 F_{cas}F_{bds} + \xi_0(D_{abs}D_{cds} - D_{ads}D_{cbs}) = 0, \tag{6}$$

$$D_{abs}D_{cds} + D_{ads}D_{cbs} - \lambda_1 D_{cas}D_{bds} + \xi_1(F_{abs}F_{cds} - F_{ads}F_{bcs}) = 0 . \tag{7}$$

Here $\lambda_0 = J_0/K_0^{(1)}$; $\lambda_1 = J_1/f^{(2)} K_1^{(2)}$; $\xi_0 = f^{(2)} K_0^{(2)}/K_0^{(1)}$; $\xi_1 = K_1^{(1)}/f^{(2)} K_1^{(2)}$.

Parity conservation requires $D_{abc} = +D_{bac}$, $F_{abc} = -F_{bac}$. Using (5), it is easy to show (6) and (7) reduce to the conditions $\lambda_1 = 2$, $\lambda_0 - 1 - \xi_0 \xi_1 = 0$, and

$$D_{ads} D_{cbs} - D_{dcs} D_{abs} + \xi_1 F_{cas} F_{dbs} = 0. \tag{8}$$

Assuming $\vec{I}$ and Y conservation, and inserting the known 0^-, 1^-, 2^+ particles, it is clear from (5) and (8) that our bootstrap allows an SU(3) symmetric solution for the couplings. Writing (5) and (8) out explicitly one verifies that this solution is unique. There is no consistent solution to our equations if one tries to use a pure SU(3) octet of 2^+ mesons, along with the known 0^- and 1^- particles. The singlet/octet coupling ratio for the 2^+ mesons is fixed by (5) and (8), and is equal to the one chosen by Glashow and Socolow.[6,7]

Equations similar in algebraic form to (3), (6), (7) have been derived by many authors[8] by drastically simplifying the assumed dynamics, for example the dynamics of the N/D equations. However, these methods have not yet yielded reliable quantitative, dynamical results. In contrast, we believe our approach yields reliable numerical predictions for such quantities as the $2^+/1^-$ coupling ratios.[9]

Because of the appearance of the D's, (5) and (8) lead to interesting restriction on possible solutions involving multiplets transforming like representations of Lie algebras.[10] For example, one cannot even define D_{abc} for the symmetric coupling of a pair of adjoint representations to a third adjoint representation, for a compact semisimple

Lie algebra other than SU(n).[11] Furthermore, we conjecture that a consistent solution to our equations does not exist if the 2^+ mesons transform irreducibly under an arbitrary Lie algebra.[12]

In the mass degenerate case, the superconvergent higher moment sum rule can only be satisfied for a particular value of $f^{(2)}$. In the lower moment case the 1^- and 2^+ contributions cancel among themselves, and using the SU(3) coupling ratios the equation is satisfied identically, independent of $f^{(2)}$. In the higher moment equation, however, 1^- cancels against a definite fraction, $f^{(2)}$, of 2^+. The remaining fraction of the 2^+ contribution presumably cancels against a piece of the 3^- contribution.

We can also make some interesting statements regarding exchange degeneracy.[13] By exchange degeneracy we mean for example, $\gamma_{\overline{K}K}^{\ \rho} : \gamma_{\overline{K}K}^{\ A_2} = \gamma_{\overline{K}K}^{\ \omega_8} : \gamma_{\overline{K}K}^{\ f_1+f_8}$, where the γ's are the reduced widths. Clearly a 1^- octet and a 2^+ octet cannot be exchange degenerate in this sense, since they have F and D type coupling, respectively. However, the addition of a 2^+ singlet of the proper strength enables exchange degeneracy to be realized for $\rho - A_2$, $\omega_8 - (f_1 + f_8)$, $K^* - K^{**}$, in $K\pi \to K\pi$, but it is not realized for $K^* - K^{**}$ in $K\eta \to K\eta$. $(\gamma_{K\eta}^{\ K^*} : \gamma_{K\eta}^{\ K^{**}} = 9\ \gamma_{K\pi}^{\ K^*} : \gamma_{K\pi}^{\ K^{**}})$

Physically, we expect, and we do obtain from our bootstrap, exchange degeneracy between resonances of even and odd J, in the $\overline{K}K$ system, because there are only direct forces and no exchange forces. By contrast the K^* and K^{**} need not be exchange degenerate since the $K\pi \to K\pi$ $(K\eta \to K\eta)$ channel contains not only direct forces: ρ, ff'(ff'), but also exchange forces: K^*, K^{**} (K^*, K^{**}).[13] Unless the exchange forces cancel there is no exchange degeneracy. The K^* and K^{**} forces are always opposite in sign because of the sign of $P_\ell(z)$. If the fraction $f^{(2)}$ is chosen equal to the one which produces superconvergence in the appropriate channels, the exchange forces in the higher moment equation cancel in the $K\pi$, but not in the $K\eta$, channel.

In Table 1, we show our numerical results for the three independent systems: $\{\pi\pi \to \pi\pi\}$; $\{K\pi \to K\pi; \bar{K}K \to \pi\pi\}$; $\{KK \to KK; \bar{K}K \to KK\}$. While the mass degenerate, symmetric case requires $f^{(2)} \simeq 1/4$, the numerical work has been performed with the choices $f^{(2)} = 0, 1/2, 1$, in order to test the sensitivity of the results with respect to the choice of the limit of integration. The calculated self-consistent limits of integration, s_N, for the cases (I), (II), (III), (see above) can be compared with the a priori values estimated from the experimental masses. For example, for $K\pi \to K\pi$ the a priori values are $s_N(I) \approx 1.3$, $s_N(II) \approx 1.9$, $s_N(III) \approx 2.4$. The tabulated values differ from these by 10-15%. This is a strong check on the quantitative self-consistency of our equations.

The calculated self-consistent β's are seen to agree reasonably with experiment. The theoretical uncertainty in the β's arising from different choices of N is shown explicitly, in columns E(I) - E(III). The average deviation of the β's from 1 shows the deviation of the couplings from their symmetric values, as induced by the mass splittings.

In principle, factorization could lead to an internal contradiction between various channels; this does not happen in our system.[14] In Table I we have assumed all $\alpha'(t) = 1$.[15] We have combined (ω, ϕ) and (f, f') into two effective particles for numerical convenience.

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FOOTNOTES AND REFERENCES

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1. R. Dolen, D. Horn, and C. Schmid, Phys. Rev. 166, 1768 (1968); K. Igi and S. Matsuda, Phys. Rev. Letters 18, 625 (1967); A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Rev. Letters 24B, 181 (1967).

2. For details of notation and a discussion of approximations and errors, see C. Schmid, Phys. Rev. Letters 20, 628 (1968).

3. $g_{abc}^{(J)}$ can be thought of as the coupling constant of three real fields, as opposed to fields of definite charge. In Eqs. (2) - (7), there is no a priori reason that the third index of the $g_{abc}^{(J)}$ should run over the same range as the first two. Neither is the interchange of c with a or b necessarily defined.

4. See for example, N. Jacobson, Lie Algebras, (Interscience, London, 1962). We are unable to derive these conditions from $P + P \to P + P$.

5. For an alternative interpretation of the orthonormality condition see R. E. Cutkosky, Phys. Rev. 131, 1888 (1963).

6. S. L. Glashow and R. Socolow, Phys. Rev. Letters 15, 329 (1965).

7. M. Ademollo et al, Phys. Rev. Letters 19, 1402 (1967) and unpublished, have also derived this result considering the process $P + P \to P + V$; See also D. Gross, Phys. Rev. Letters 19, 1303 (1967).

8. R. E. Cutkosky, Phys. Rev. 131, 1888 (1963); Chan Hong-Mo, P. DeCelles, and J. Paton, Phys. Rev. Letters 11, 521 (1963); R. H. Capps, Phys. Rev. Letters 10, 312 (1963).

9. In contrast, symmetry breaking of SU(3) couplings has been studied quantitatively by R. F. Dashen et al, Phys. Rev. 143, 1185 (1966); ibid. 151, 1127 (1966).

10. See also R. H. Capps (Purdue University preprint, unpublished).

11. For B_n, C_{n+1}, D_{n+2} $(n \geq 2)$, adjoint $\otimes$ adjoint contains adjoint only once. In that case the Clebsch-Gordan coefficients are the (antisymmetric) structure constants. The same statement is true, by explicit calculation, for G_2, and we conjecture it also works for F_4, $E_{6,7,8}$. See R. E. Behrends et al, Rev. Mod. Phys. 34, 1 (1962).

12. This is equivalent to the statement that the crossing matrix has at least one nonzero nondiagonal element in the row and column corresponding to a symmetric coupling.

13. R. C. Arnold, Phys. Rev. Letters 14, 657 (1965).

14. For the SU(3) symmetric case, a contradiction with factorization is impossible, since the amplitudes explicitly factorize in the symmetric limit.

15. This slope is our energy scaling factor.

16. In Table I, we have not employed factorization.

17. Experimental reduced widths taken from A. H. Rosenfeld et al, Rev. Mod. Phys. 40, 77 (1968).

TABLE CAPTIONS

(A) Independent groups of reactions corresponding to one amplitude.

(B) Resonances occurring in (A) as intermediate states.

(C) (Isoscalar factor)^2 for SU(3), assuming magic mixing angles $\tan^2\theta = 1/2$, and the 2^+ singlet/octet coupling ratio $G^2/F^2 = 8$. (2^+ isoscalars are multiplied by 5/9.)

(D) Reduced experimental widths, dividing by the corresponding entries in column (C).

(E) Our predicted theoretical reduced widths divided by the corresponding entries in column (C). Shown are results for cases (I), (II), (III), with the corresponding self-consistent limits of integration.[16]

(F) Experimental partial widths taken from Reference 17. In the $\pi\pi \to K\bar{K}$ case: $\Gamma_{exp}^{(i,j)} = (\Gamma_{\pi\pi} \cdot \Gamma_{K\bar{K}})^{\frac{1}{2}}$ is the effective width.

TABLE I.

A Channel	B Resonance	C Iso-Scalars	D $\frac{\beta(\text{exp})}{\text{Isoscalars}}$	E β theory/isoscalars I	 II	 III	F $\Gamma\text{exp}^{(i,j)}$ in Bev
$\pi\pi \to \pi\pi$	ρ	2/3	0.73 ± 0.18	1	0.91	1.04	0.124
	f } $f_1 + f_8$	1	1 (± 0.14)	-	1	1	0.137
	f'	0	not observed				<0.010
	limit of integration, s_N (Bev)^2			1.05	1.57	1.87	
$K\pi \to K\pi$ $\pi\pi \to K\bar{K}$	ρ	1/3	-	1.04	0.91	0.85	-
	K^*	1/4	1 (± 0.02)	1	1	1	0.049
	f } $f_1 + f_8$	$1/\sqrt{6}$	0.83 ± 0.32	-	1.30	1.13	0.021
	f'	0	not observed				<0.023
	K^{**}	1/4	0.85 ± 0.05	-	0.92	0.76	0.045
	limit of integration, s_N (Bev)^2			1.17	1.75	2.08	
$K\bar{K} \to K\bar{K}$ $KK \to KK$	ρ	1/6	-	1.06	0.94	0.92	-
	ω } ω_8	1/6	-	1	1	1	-
	ϕ	1/3	0.85 ± 0.20				0.0029
	A_2	1/6	0.54 ± 0.14	-	1.59	1.37	0.0036
	f } $f_1 + f_8$	1/6	0.69 ± 0.24	-	1.36	1.22	0.0032
	f'	1/3	1.20 ± 0.44				0.053
	limit of integration, s_N (Bev)^2			1.27	2.00	2.40	