LBL-35266
UC-414
Preprint

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Submitted to Physical Review Letters

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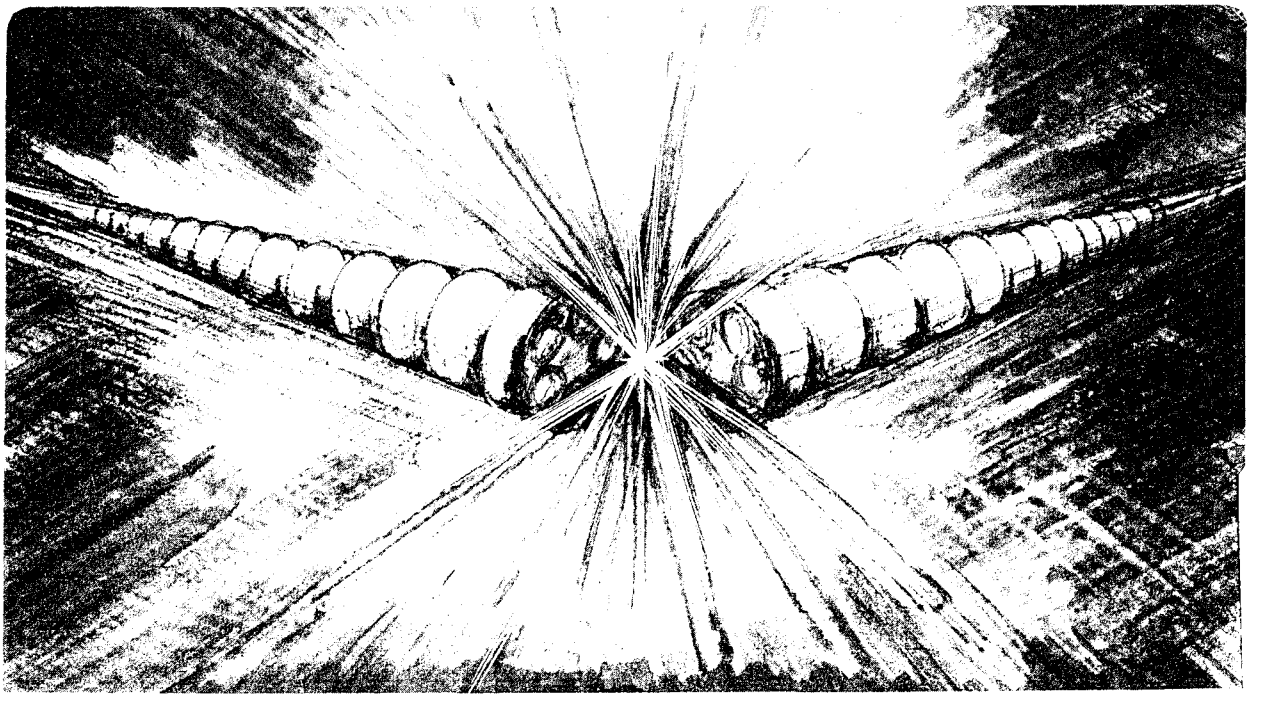
February 1994



Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098

LBL-35266
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THREE DIMENSIONAL LASER COOLING OF STORED AND CIRCULATING ION BEAMS BY MEANS OF A COUPLING CAVITY*

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16 February 1994

Abstract: It is shown, theoretically, that a coupling cavity; namely an rf cavity operating in the TM_{210} mode, when inserted in a storage ring will ehance the coupling between longitudinal and transverse degrees of freedom. As a result, it is shown that the demonstrated very effective laser cooling of the longitudinal motion, can now be extended to transverse motion; i.e., employed to cool a beam in all three directions.

PACS numbers: 29.20.Dh, 29.27.Fh, 84.40.Sr

* Work supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

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Laser cooling [1] of stored, circulating ion beams is remarkably effective [2,3]. Longitudinal temperatures in the mK range have been achieved with a beam of 100 keV $^7Li^+$ ions[3]. The transverse temperature is, however, on the order of 1000 K. To date, no effective method has been developed to realize simultaneous cooling in both the longitudinal and transverse directions, although a possible transverse cooling method was suggested some time ago [4]. We propose, in this Letter, the use of "coupling cavities" to couple the transverse and longitudinal degrees of freedom and thus to allow laser cooling in the longitudinal direction to simultaneously cool the transverse motion. The use of such a cavity was suggested by previous work, where "conditioner cavities" were developed to condition a beam, and therefore make it much more suitable, for free-electron lasers [5].

The idea is based upon developing a forced synchro-betatron resonance where the transverse tune ν_T and the longitudinal tune ν_L satisfy the condition ν_T-ν_L=integer. The coupling is induced by a coupling rf cavity set on a storage ring. The cavity is excited with a specific mode whose longitudinal field component has a transverse-coordinate dependence; here we consider the TM_{210} mode which gives very effective coupling. In principle, it should be possible to cool transverse beam temperatures to the same order as the longitudinal temperature, whose achieved level now is below about 1 mK, as mentioned above. If this kind of ultra-low temperature beam becomes available, we might then consider some important applications of such a beam. First, especially for nuclear physics applications, we could use the cooled ion beam to cool another beam just as in the electron cooling scheme [6]. Second, the achievable level of beam temperature should be theoretically sufficient to observe beam crystallization [7].

For the coupling cavity, consider a rectangular rf cavity which has a width of 2a and the height of 2b. For the TM_{210} mode, the longitudinal electric field component is obtained from the Maxwell's equations as

$$E_z = -V_c \sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{2b}\right)\cos(\omega_c t + \phi_c). \tag{1}$$

Here, V_c corresponds to the maximum voltage of excited field and ϕ_c is the initial rf phase. The oscillation angular frequency ω_c is given by $(\pi/a)^2+(\pi/2b)^2=(\omega_c/c)^2$, where c is the speed of light. Note that, in the coupling cavity, the longitudinal electric field is proportional to transverse displacement (and zero on axis). The transverse electric field is zero, but there are transverse magnetic fields. These electromagnetic fields are derivable from the vector potential

$$\mathbf{A}_c = \left(0,\quad 0,\quad \frac{V_c}{\omega_c}\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{\pi y}{2b}\right)\sin(\omega_c t + \phi_c)\right). \tag{2}$$

In addition, we also have an rf bunching cavity whose vector potential is

$$\mathbf{A}_b = \left(0,\quad 0,\quad \frac{V_b}{\omega_b}\sin(\omega_b t + \phi_b)\right), \tag{3}$$

where V_b and ω_b are, respectively, the voltage amplitude and angular frequency of the bunching cavity, and ϕ_b is the initial rf phase.

The Hamiltonian for the coupled motion caused by the coupling cavity can be readily obtained. Since the derivation including effect of an rf cavity has been presented

previously [8], we only recall the result here. For simplicity, it is assumed that the storage ring has a single coupling cavity and a single bunching cavity. Taking the distance s along the reference particle orbit in a storage ring as the independent variable, instead of time, and considering only dipole and quadrupole magnets installed on the ring, we obtain, together with the vector potentials in Eqs.(2) and (3), the approximate Hamiltonian

$$H_1 = -p + (p_0 - p)\frac{x}{\rho} + \frac{{p_x}^2}{2p} + \frac{p_0 K(s) x^2}{2} - \frac{qV_b}{\omega_b}\sin(\omega_b t + \phi_b)\delta_p(s - s_b) - \frac{\pi q V_c}{\omega_c}\frac{x}{a}\sin(\omega_c t + \phi_c)\delta_p(s - s_c), \tag{4}$$

where q and ρ are, respectively, the charge state of stored ions and the local curvature of the orbit, K(s) corresponds to the quadrupole field strength, $\delta_p(s)$ denotes a periodic delta function, and we have assumed that the bunching and coupling cavity are located at the position s_b and s_c respectively. Writing the total energy of a particle as W, the total momentum p is expressed as $p=[(W/c)^2-m_0{}^2c^2]^{1/2}$ where m_0 is the rest mass of the ions. In the following analysis, quantities with the subscript 0 are used to represent those corresponding to the reference particle. Note that, since the vertical motion is decoupled under the approximation adopted here, we have neglected the vertical variables. To accomplish effective three-dimensional cooling, we will finally need some coupling between horizontal and vertical motion and, as is well-known, this simply requires employing, for example, a skew quadrupole or a solenoid.

Applying several canonical transformations [8] and scalings to Eq.(4), we eventually find, changing the independent variable to $\theta=s / R$ (R=average ring radius),

$$\tilde{H}_1 = \frac{\tilde{p}_x{}^2}{2} + \frac{\nu_T{}^2\tilde{x}^2}{2} - \frac{\xi_0\tilde{W}^2}{2} + \frac{2\pi\bar{\nu}_L{}^2}{\xi_0}\sin(\tilde{\psi} + \psi_b)\delta_p(\theta - \theta_b) - \frac{2\pi h_b \Gamma_c}{h_c}\tilde{x}\cdot\sin\left(\frac{h_c}{h_b}\tilde{\psi} + \psi_c\right)\delta_p(\theta - \theta_c), \tag{5}$$

where $\theta_b=s_b/R$, $\theta_c=s_c/R$, $\xi_0=\alpha-1/\gamma_0{}^2$ where α is the momentum compaction factor, and ψ_b and ψ_c are the so-called synchronous phase at the bunching and coupling cavity whose harmonic numbers are, respectively, h_b and h_c. The coupling constant Γ_c has been introduced as

$$\Gamma_c \equiv \frac{qV_c}{2\beta_0 c p_0}\frac{R}{a}, \tag{6}$$

and we have simply assumed that the storage ring studied here has been designed such that the dispersion η, and $d\eta/ds$, vanish at the rf cavity positions. In addition, the betatron motion has been smoothed out introducing the transverse tune ν_T, while $\bar{\nu}_L$ is a constant which roughly corresponds to the longitudinal tune ν_L, and is given by the relation $\cos(2\pi\nu_L) = 1 - 2\pi^2\bar{\nu}_L{}^2$.

The ions susceptible to laser cooling are heavy particles for which the synchrotron radiation loss is negligible and, therefore, it is unnecessary to accelerate to compensate for energy loss. However, we need the ordinary rf cavity as a bunching cavity. The energy of stored heavy-ion beams is, in general, below transition, i.e. $\xi_0<0$, and ψ_b must then be positive in the definition introduced here. Then, to have the maximum bunching effect, we choose the synchronous phase $\psi_b=\pi/2$. Similarly, ψ_c is chosen to be zero, so that the coupling effect becomes maximum. Under these simplifications, Eq.(5) can be re-written as

$$H=\frac{1}{2}(p_x{}^2+\nu_T{}^2x^2)-\frac{\xi_0 W^2}{2}-\frac{\pi\bar{\nu}_L{}^2\psi^2}{\xi_0}\delta_p(\theta-\theta_b)-2\pi\Gamma_c x\psi\delta_p(\theta-\theta_c), \qquad (7)$$

where the higher order terms in x and ψ have been neglected, and the tilde has been dropped. This Hamiltonian leads to the equations of motion

$$\begin{cases}\dfrac{d^2x}{d\theta^2}+\nu_T{}^2x=2\pi\Gamma_c\psi\delta_p(\theta-\theta_c),\\ \dfrac{d^2\psi}{d\theta^2}+2\pi\bar{\nu}_L{}^2\psi\delta_p(\theta-\theta_b)=-2\pi\xi_0\Gamma_c x\delta_p(\theta-\theta_c).\end{cases} \qquad (8)$$

These linear equations can be solved by employing matrix methods. Before doing that, we add a term which replicates the laser cooling; namely a term on the l.h.s. of the ψ-equation of Eq.(8), which is $\Lambda(d\psi/d\theta)$ over the laser cooling section.

The 4 X 4 matrix equations can be solved numerically but, with a few further assumptions, it is straightforward to obtain an analytic result which gives us good insight into the beam behavior. We put the coupling cavity and the bunching cavity at the same azimuthal position and, at the opposite side of the storage ring, we also put the laser cooling section for which we take thin lens approximation with $\Lambda_D=\Lambda\cdot\Delta\theta$ where $\Delta\theta$ is the extent of the section. Further, we assume that we are exactly on a coupling resonance; i.e., $\nu_T-\nu_L=\text{integer}$, because the transverse damping rate due to the coupling is most enhanced under this situation. Writing the eigenvalues of the one-turn matrix as $e^{i2\pi\nu}$ and applying perturbation analysis to the dispersion relation derived from the one-turn matrix, we obtain for the imaginary part of ν in the small Γ_c region:

$$\mathrm{Im}(\nu)\approx\begin{cases}-\dfrac{1}{4\pi}\ln\left[1+\dfrac{(2\pi)^3\xi_0\Gamma_c{}^2}{2\nu_T}\coth\left(\dfrac{\Lambda_D}{2}\right)\right],\\ \dfrac{\Lambda_D}{4\pi}+\dfrac{1}{4\pi}\ln\left[1+\dfrac{(2\pi)^3\xi_0\Gamma_c{}^2}{2\nu_T}\coth\left(\dfrac{\Lambda_D}{2}\right)\right].\end{cases} \qquad (9)$$

In the large Γ_c region, the value of $\mathrm{Im}(\nu)$ is saturated at the level $\Lambda_D/8\pi$, which is just 1/2 of the longitudinal damping rate without coupling. Of note is the fact that $\mathrm{Im}(\nu)$ remains positive in both transverse and longitudinal modes unless the coupling strength is too big. Therefore, we can always, more or less, realize damping in the both directions. To get the most effective three-dimensional damping, it is preferable to make the $\mathrm{Im}(\nu)$ values of

both modes approximately equal to each other so that we can have the same damping rate $\Lambda_D/8\pi$ in both directions. As briefly mentioned above, and also seen from Figs.1 and 2, this situation can be realized by driving the parameters onto a difference resonance. In this case, all we need to do is to design the value of Γ_c larger than that at which the damping rate in the two modes comes to the saturation level. The required minimum value of Γ_c can be evaluated from Eq.(9), leading to

$$\Gamma_c \to \Gamma_0 = \frac{1}{2\pi}\sqrt{\frac{\nu_T(e^{-\Lambda_D/2}-1)}{\pi\xi_0}\tanh\left(\frac{\Lambda_D}{2}\right)}, \tag{10}$$

and, for $\Lambda_D \ll 1$ (weak damping rate), Eq.(10) is approximated as

$$\Gamma_0 \approx \frac{\Lambda_D}{4\pi^{3/2}}\sqrt{\frac{\nu_T}{-\xi_0}}. \tag{11}$$

Fig.3 shows the results of tracking; i.e. actually solving Eq.(8) with the laser damping term. In these cases, we have Γ_c=0.015. It can be observed, from this figure, how close we must be to the coupling resonance. We see that even an error of $\Delta\nu_L \approx \pm 0.015$ looks acceptable. The acceptable amount of the tune error can be somewhat increased by raising the voltage of the coupling cavity; i.e. employing a larger Γ_c. Finally, in Table.1, we present the parameters of the ASTRID ring and examples of the bunching and coupling cavity parameters that would allow three-dimensional cooling. After the cooling is accomplished, one might, for various reasons such as obtaining a crystalline beam, turn off the coupling and the bunching cavities.

For some purposes, it may be desirable to employ specially-designed cavities. For example, if the stored ions have very low energy, the circulation frequency will also be very low. Hence, the desired frequency of a bunching and a coupling cavity will be very low and a simple rectangular structure will have large physical dimensions. In such a case, an extremely re-entrant cavity with lumped impedances supplied by a coil may be desirable.

In summary, we have shown that a coupling cavity in a storage ring will allow the cooling of transverse degrees of freedom although the cooling-laser need only operate on the longitudinal degree of freedom.

The authors would like to thank Dr. Jeffrey S. Hangst and Dr. Jie Wei for helpful discussions and for kindly providing the design parameters of the storage ring ASTRID in Denmark and TSR in Heidelberg.

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FIGURE CAPTIONS

Fig.1. Imaginary part of the eigenvalues describing longitudinal and transverse motion as a function of the coupling strength between the modes. The bunching and coupling cavities are next to each other and 180^o from the laser cooling section. The longitudinal and transverse tunes are varied in the four figures, keeping the resonance condition satisfied. The damping rate was held fixed such that $\Lambda_D/2\pi=0.01$. The solid curves are obtained from solving the 4 X 4 determinant while the dotted curves come from Eq.(9). One can see that the agreement is quite good.

Fig.2. The same as Fig 1, but now the position of the bunching and damping cavities are varied fixing the tunes as $\nu_T=2.29$ and $\nu_L=0.29$. The laser cooling is at $\theta = 0^o$.

Fig.3. Tracking results, i.e. solutions of Eq.(8), in which 500 particles are followed and, from them, transverse (solid line) and longitudinal (dotted line) scaled rms emittances are evaluated. The results are in accord with the eigenvalue analysis, but easily could (in future work) include non-linear effects, etc. The effective damping of both degrees of freedom is seen in all the figures, but most dramatically in Fig.3(c) where the operating point is exactly on resonance. The transverse tune and coupling constant are fixed, respectively, at $\nu_T=2.29$ and $\Gamma_c=0.015$ in all cases and the damping rate is taken as $\Lambda_D/2\pi=0.01$.

Table. 1

ASTRID main parameters

Stored beam of ions	: $^{7}Li^{+}$, $^{24}Mg^{+}$, etc.
Total energy of stored ions	: 100 [keV] ~ 7 [MeV] for $^{24}Mg^{+}$. 100 [keV] ~ 13 [MeV] for $^{7}Li^{+}$.
Ring circumference	: 40 [m]
Tunes	: 2.29 (hor.), 2.73 (vert.)
Momentum compaction	: 0.053
Transition energy γ	: 4.58

Example of the cavity parameters for 3D cooling

Stored Ions	13 MeV $^{7}Li^{+}$	100 keV $^{24}Mg^{+}$
Bunching Cavity		
Frequency :	400.2 [MHz]	580.7 [kHz]
Voltage amplitude:	12.8 [kV]	3.2 [kV]
Coupling Cavity		
Frequency :	400.2 [MHz]	580.7 [kHz]
Voltage amplitude:	~50 [kV]#1	less than ~1 [kV]#2
Cavity type :	Pill box type excited with TM_{210} mode	Lumped circuit type

#1*This voltage corresponds to* $\Gamma_c \approx 0.015$.

#2*This value would strongly depend on the cavity design*.

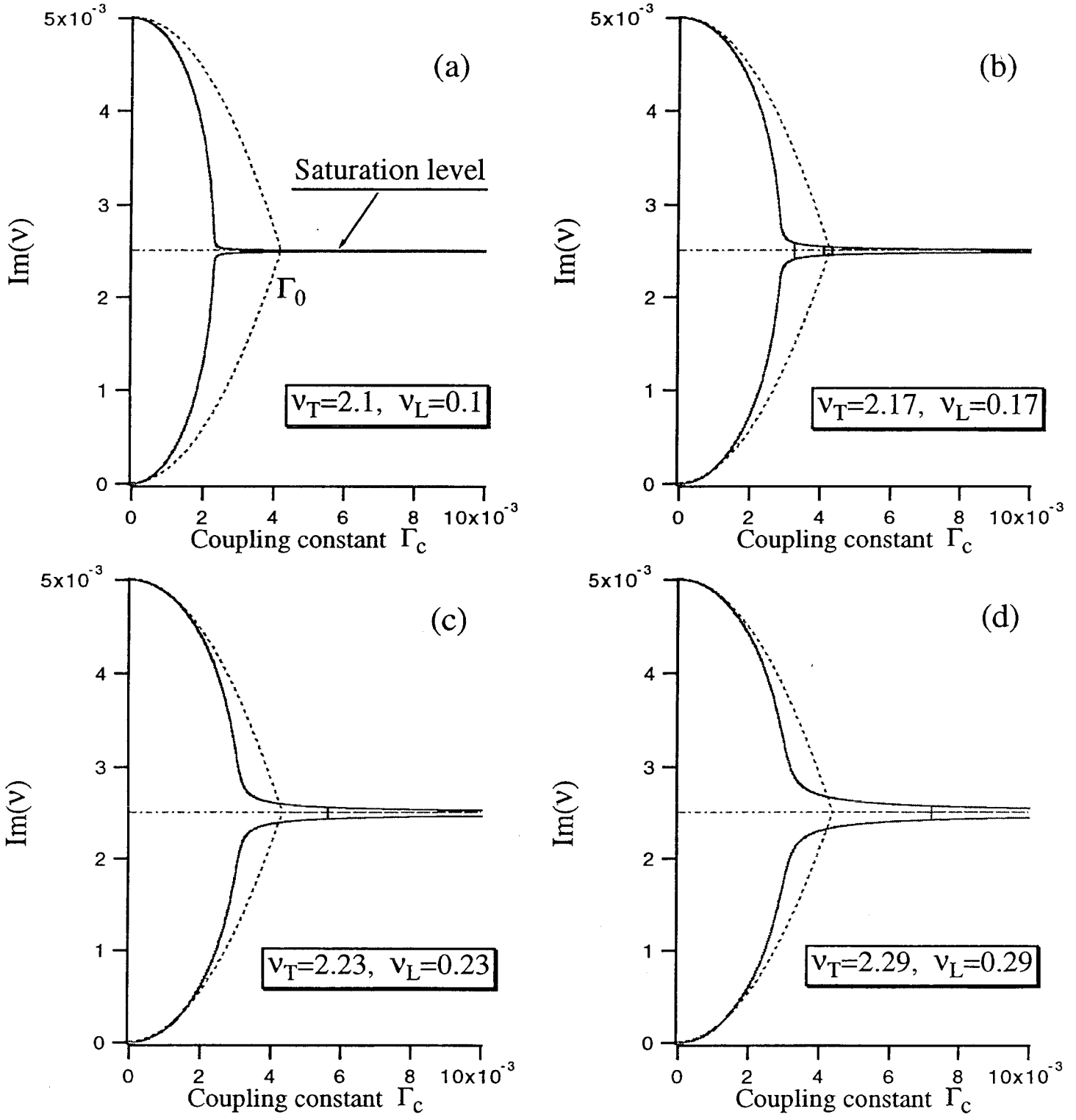
5x10^-3
4
3
2
1
0
Im(ν)
(a)
Saturation level
Γ0
νT=2.1, νL=0.1
0
2
4
6
8
10x10^-3
Coupling constant Γc
(b)
νT=2.17, νL=0.17
(c)
νT=2.23, νL=0.23
(d)
νT=2.29, νL=0.29

Fig.1

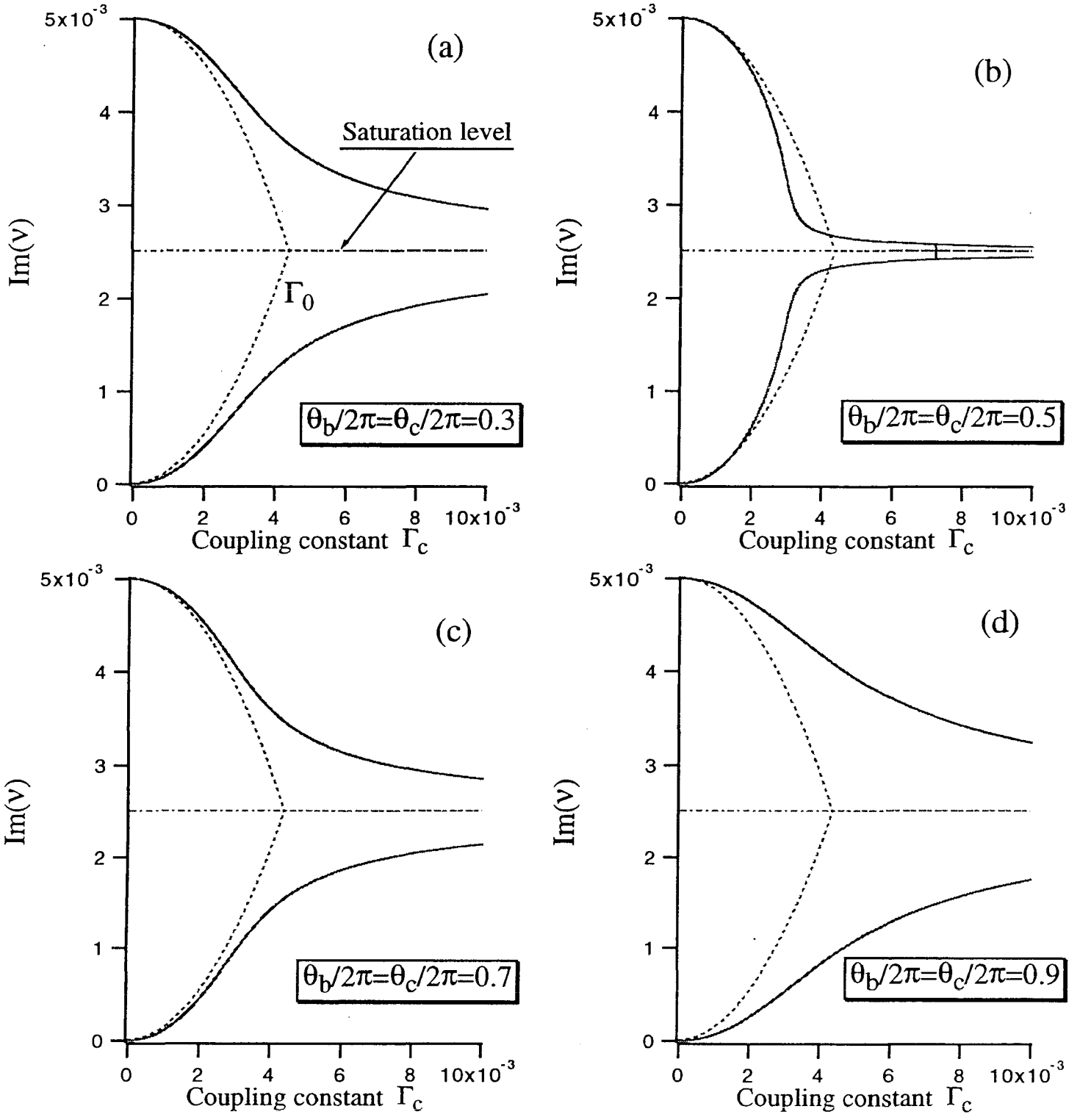
(a)
(b)
(c)
(d)
Saturation level
Γ_0
$\theta_b/2\pi=\theta_c/2\pi=0.3$
$\theta_b/2\pi=\theta_c/2\pi=0.5$
$\theta_b/2\pi=\theta_c/2\pi=0.7$
$\theta_b/2\pi=\theta_c/2\pi=0.9$
Im(ν)
Coupling constant Γ_c
5x10^{-3}
10x10^{-3}
0
1
2
3
4
6
8

Fig.2

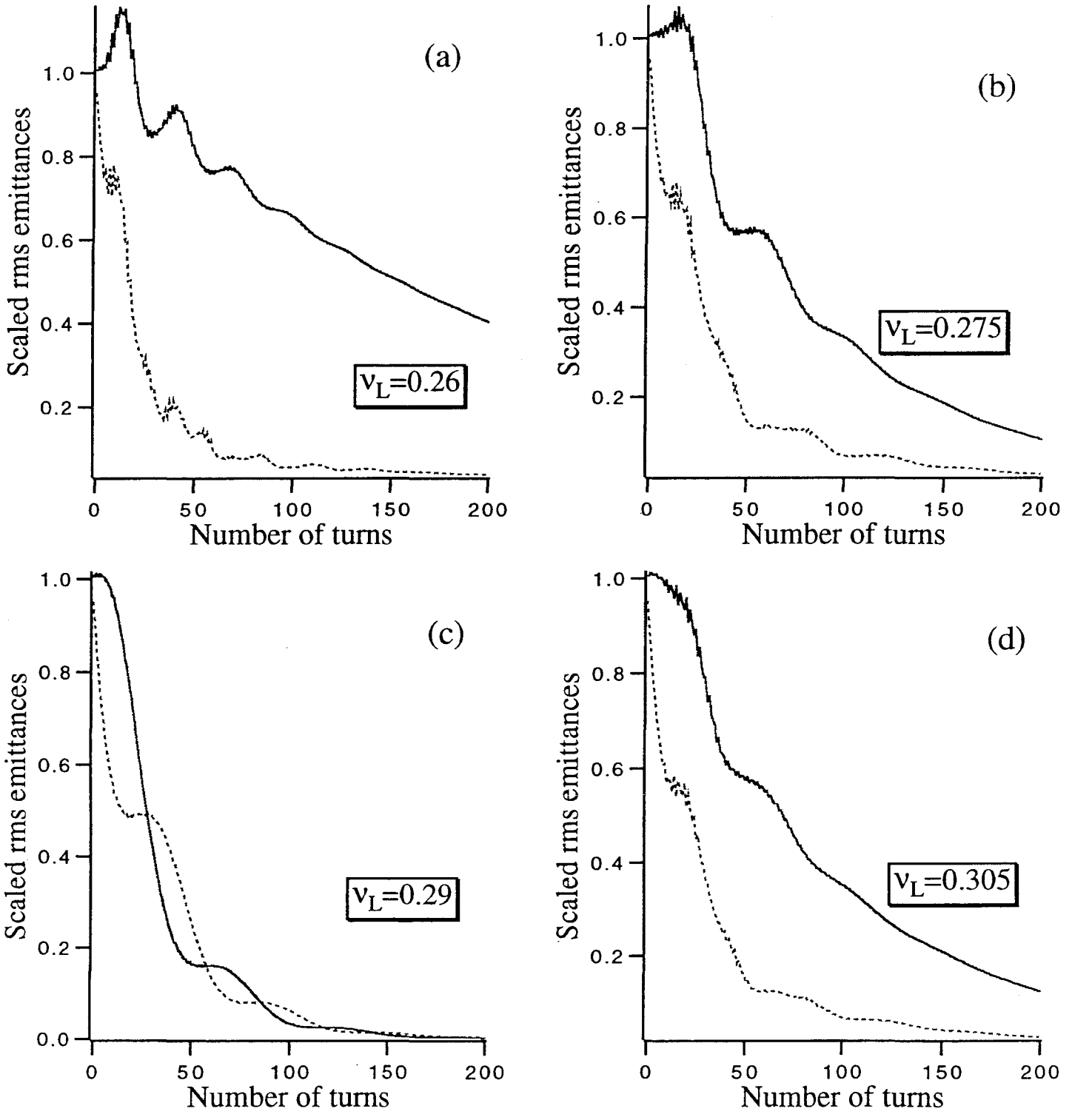
(a)
(b)
(c)
(d)
Scaled rms emittances
Number of turns
ν_L=0.26
ν_L=0.275
ν_L=0.29
ν_L=0.305
0
50
100
150
200
0.0
0.2
0.4
0.6
0.8
1.0

Fig.3