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# HIGH Sc LIMIT OF FREE CONVECTION AT A VERTICAL PLATE WITH UNIFORM FLUX CONDITION

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In the course of an investigation of mass transfer by free convection in electrolytic solutions,[1,2] numerical solutions were obtained for the problem of laminar free-convection heat (or mass) transfer to a vertical plate in the limit of very high Prandtl (or Schmidt) number. Free convection at a vertical plate under conditions of uniform temperature (or concentration) at the plate is treated extensively in standard text-books.[3] The condition of uniform flux has been dealt with by Sparrow and Gregg,[4] with the Prandtl number as a parameter. No solution is available in the literature for the case of infinitely high Prandtl (or Schmidt) number.

In the limit of high Prandtl numbers the equation of motion:*

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(t-t_\infty) \quad , \tag{1}$$

after being brought in the form:

$$F''' + 4FF'' - 3F'F' + \theta = 0 \tag{2}$$

by the transformation[4]

$$\eta = y\left(\frac{g\beta q}{5k\nu^2 x}\right)^{1/5}$$

*Notation is that of reference 4

$$F = \frac{\psi}{5x\nu}\left(\frac{5k\nu^2 x}{g\beta q}\right)^{1/5} \tag{3}$$

$$\theta = \frac{k(t_\infty - t)}{q}\left(\frac{g\beta q}{5k\nu^2 x}\right)^{1/5} ,$$

can be further simplified by stretching the variables

$$\zeta = \eta \Pr^{1/5} , \quad f = F \Pr^{4/5} , \quad \Theta = \theta \Pr^{1/5} \tag{4}$$

In the limit $\Pr \to \infty$ the inertial terms in

$$f''' + \frac{1}{\Pr}(4ff'' - 3f'f') + \Theta = 0 \tag{5}$$

become negligible, i.e., viscous friction alone balances the buoyancy force in the thin region where density variations occur.

The set of coupled equations

$$f''' + \Theta = 0 \tag{6}$$

$$\Theta'' + 4f\Theta' - f'\Theta = 0 \tag{7}$$

have been solved with the boundary conditions:

$$\eta = 0 \quad , f = f' = 0 \quad , \quad \Theta' = 1 \tag{8}$$

$$\eta = \infty \quad , f'' = 0 \quad , \Theta = 0 \tag{9}$$

The results of interest are the local temperature difference $t_\infty - t_o$ and the local shear stress $\tau_o$ at the plate:

$$t_\infty - t_o = \frac{q}{k}\left(\frac{5k^2\nu x}{g\beta q\rho C_p}\right)^{1/5} \Theta(0) \tag{10}$$

$$\tau_o = -5\mu k x \left(\frac{g\beta q\rho C_p}{5k^2\nu x}\right)^{3/5} f''(0) \tag{11}$$

The values of $\Theta(0)$ and $f''(0)$ are:

$\Theta(0) = -\,1.14747$

$f''(0) = \quad 0.83789$

Table 1 shows that these results are in good agreement with the trend of the values $\Theta(0)$ and $F''(0)$ reported by Sparrow and Gregg.[4]

Table 1. Dimensionless temperature difference and shear stress as reported by Sparrow and Gregg[4] and in this work.

| Pr | $\theta(0)$ | $\Theta(0)$ | $F''(0)$ | $f''(0)$ | Ref. |
|---|---|---|---|---|---|
| 0.1 | -2.7507 | -1.7356 | 1.6434 | 0.65425 | 4 |
| 1 | -1.3574 | -1.3574 | 0.72196 | 0.72196 | 4 |
| 10 | -0.76746 | -1.2163 | 0.30639 | 0.76962 | 4 |
| 100 | -0.46566 | -1.1697 | 0.12620 | 0.79628 | 4 |
| $\infty$ | | -1.14747 | | 0.83789 | 1 |

References

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4. E. M. Sparrow and J. L. Gregg, "Laminar Free Convection from a Vertical Plate with Uniform Surface Heat Flux," Transactions A.S.M.E. 78, 435-440 (1956).

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