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HIGH Sc LIMIT OF FREE CONVECTION AT A VERTICAL
PLATE WITH UNIFORM FLUX CONDITION

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October 1970

In the course of an investigation of mass transfer by free convection in electrolytic solutions,^{1,2} numerical solutions were obtained for the problem of laminar free-convection heat (or mass) transfer to a vertical plate in the limit of very high Prandtl (or Schmidt) number. Free convection at a vertical plate under conditions of uniform temperature (or concentration) at the plate is treated extensively in standard textbooks.³ The condition of uniform flux has been dealt with by Sparrow and Gregg,⁴ with the Prandtl number as a parameter. No solution is available in the literature for the case of infinitely high Prandtl (or Schmidt) number.

In the limit of high Prandtl numbers the equation of motion:^{*}

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(t-t_\infty) \quad , \quad (1)$$

after being brought in the form:

$$F''' + 4FF'' - 3F'F' + \theta = 0 \quad (2)$$

by the transformation⁴

$$\eta = y \left(\frac{g\beta q}{5k\nu^2 x} \right)^{1/5}$$

* Notation is that of reference 4

$$F = \frac{\psi}{5xv} \left(\frac{5kv^2x}{g\beta q} \right)^{1/5} \quad (3)$$

$$\theta = \frac{k(t_\infty - t)}{q} \left(\frac{g\beta q}{5kv^2x} \right)^{1/5} ,$$

can be further simplified by stretching the variables

$$\zeta = \eta \text{Pr}^{1/5} , \quad f = F \text{Pr}^{4/5} , \quad \theta = \theta \text{Pr}^{1/5} \quad (4)$$

In the limit $\text{Pr} \rightarrow \infty$ the inertial terms in

$$f''' + \frac{1}{\text{Pr}} (4ff'' - 3f'f') + \theta = 0 \quad (5)$$

become negligible, i.e., viscous friction alone balances the buoyancy force in the thin region where density variations occur.

The set of coupled equations

$$f''' + \theta = 0 \quad (6)$$

$$\theta'' + 4f\theta' - f'\theta = 0 \quad (7)$$

have been solved with the boundary conditions:

$$\eta = 0 , \quad f = f' = 0 , \quad \theta = 1 \quad (8)$$

$$\eta = \infty , \quad f' = 0 , \quad \theta = 0 \quad (9)$$

The results of interest are the local temperature difference $t_{\infty} - t_0$ and the local shear stress τ_0 at the plate:

$$t_{\infty} - t_0 = \frac{q}{k} \left(\frac{5k^2 \nu x}{g\beta q \rho C_p} \right)^{1/5} \theta(0) \quad (10)$$

$$\tau_0 = -5\mu k x \left(\frac{g\beta q \rho C_p}{5k^2 \nu x} \right)^{3/5} f''(0) \quad (11)$$

The values of $\theta(0)$ and $f''(0)$ are:

$$\theta(0) = -1.14747$$

$$f''(0) = 0.83789$$

Table 1 shows that these results are in good agreement with the trend of the values $\theta(0)$ and $F''(0)$ reported by Sparrow and Gregg.⁴

Table 1. Dimensionless temperature difference and shear stress as reported by Sparrow and Gregg⁴ and in this work.

| Pr | $\theta(0)$ | $\theta(0)$ | $F''(0)$ | $f''(0)$ | Ref. |
|----------|-------------|-------------|----------|----------|------|
| 0.1 | -2.7507 | -1.7356 | 1.6434 | 0.65425 | 4 |
| 1 | -1.3574 | -1.3574 | 0.72196 | 0.72196 | 4 |
| 10 | -0.76746 | -1.2163 | 0.30639 | 0.76962 | 4 |
| 100 | -0.46566 | -1.1697 | 0.12620 | 0.79628 | 4 |
| ∞ | | -1.14747 | | 0.83789 | 1 |

References

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4. E. M. Sparrow and J. L. Gregg, "Laminar Free Convection from a Vertical Plate with Uniform Surface Heat Flux," Transactions A.S.M.E. 78, 435-440 (1956).

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