SUMMARY OF REMARKS MADE AT SHERWOOD CONFERENCE IK BERKELEY, FEBRUARY 1955 END LEAKAGE LOSSES FROM THE MIRROR MACHINE

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SUMMARY OF REMARKS MADE AT SHERWOOD CONFERENCE IN BERKELEY, FEBRUARY 1955

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In the mirror machine of Richard Post, escape from the ends is possible for charged ions whose orbits at the midway point make an angle smaller than θ_c with the magnetic field direction. The expression for sin θ_c in terms of the mirror ratio[1] is

$$\sin \vartheta_c = \sqrt{\frac{H_0}{H_m}} \tag{1}$$

where H_o is the average field at the midpoint along the axis and H_m is the field at the mirror (ends). This leakage is a continuous process throughout the operating cycle, since an ion for which escape is impossible at one time in the cycle may interact with another ion through their Coulomb fields and have its orbit perturbed so that escape becomes possible. We propose to calculate this leakage produced by the Coulomb interaction between ions by the spatially independent Boltzmann equation.[2]

$$\frac{\partial f_1}{\partial t} = \int (f_0' f_1' - f_0 f_1)\, v \frac{d\sigma}{d\Omega}\, d\vec{c}_0 \tag{2}$$

and using for $\frac{d\sigma}{d\Omega}$ the Rutherford differential cross section. Spatial variation in f can be taken into account approximately by averaging over the plasma volume and no great error will be introduced, since both the rate of energy loss and the rate of energy production are proportional to n^2, the squared number density.

Using the fact that small-angle collisions produce a given mean squared deviation of pitch angle θ with approximately ten times the frequency of a single encounter resulting in a large-angle deflection, we perform a Taylor expansion in velocity space of the integrand in Eq. (1). The coefficients of the derivatives of the distribution function will be the components of the vector increments of velocity $\vec{\delta c}_0 = \vec{c}_0{}' - \vec{c}_0$ and $\vec{\delta c}_1 = \vec{c}_1{}' - \vec{c}_1$ and will be functions of the coordinates in velocity space of the two colliding particles $c_0, c_1, \mu_0 = \cos\vartheta_0$ and $\mu_1 = \cos\vartheta_1$; of the angle between their velocity vectors μ_ξ and of the apsidal

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[1] Richard Post, Sixteen Lectures on Controlled Thermonuclear Reactions, University of California Radiation Laboratory, Report No. UCRL-4231 (1954).

[2] Chapman and Cowling, Mathematical Theory of Non-Uniform Gases, Cambridge University Press.

coordinates of the collision. We shall incorporate losses into the Boltzmann equation by assuming that particles whose pitch angles fall within the escape "cone" are immediately lost from the system, and express this by the boundary condition on f that

$$f(c^2, \mu) = 0 \qquad |\mu| \geq \mu_c \tag{3}$$

where $\mu = \cos\theta$, the cosine of the pitch angle between the velocity vector and the field.

As a result of the divergence of the integrated Coulomb cross section, logarithmically divergent terms containing the minimum scattering angle for a collision appear when the integration over the apsidal coordinates is performed. If a cutoff on the maximum impact parameter at the Debye length is assumed, the terms containing a logarithm are found to be ~20 times as large as the convergent terms. Consequently we follow Chapman and Cowling in retaining only the logarithmic terms, and expect to introduce no more than 5 % error into the calculation.

In order to perform the resulting integration on c_0, μ_0 and μ_ξ, we assume that factorization of the distribution function into a radial function $h(c^2, t)$ and an angular function $g(\mu)$ is possible,

$$f(c^2, \mu, t) = h(c^2, t)\, g(\mu) \tag{4}$$

This factorization is not inconsistent with the boundary condition on the angular function $g(\mu)$ since Eq. (3) is not a function of the velocities of the particles. We now proceed to linearize the equation for $g(\mu_1)$ by making an assumption for $g(\mu_0)$ in the spirit of an iterative procedure for getting $g(\mu)$.

$$g(\mu_0) = 1 \qquad |\mu_0| \leq 1$$

After integrating on c_0, μ_0 and μ_ξ, we now find the equation for $h(c_1^2, t) = h_1$

$$\begin{aligned}
\frac{\partial h_1}{\partial t} = {} & 8\pi^2 \frac{e^4}{m^2} \ln \frac{1}{\sin\vartheta_m} \Bigg\{ \int_0^{c_1} dc_0\, c_0^2 \Bigg[\frac{4}{3c_0c_1} h_1 \frac{\partial h_0}{\partial c_0} - \left(\frac{2}{c_1^2} + \frac{2c_0^2}{3c_1^4} \right) h_0 \frac{\partial h_1}{\partial c_1} \\
& + \frac{2}{3c_1^3} \left(c_1^2 h_1 \frac{\partial^2 h_0}{\partial c_0^2} - 2c_0c_1 \frac{\partial h_0}{\partial c_0} \frac{\partial h_1}{\partial c_1} + c_0^2 h_0 \frac{\partial^2 h_1}{\partial c_1^2} \right) \Bigg] \\
& + \int_{c_1}^{\infty} dc_0\, c_0^2 \Bigg[\frac{4}{3c_0c_1} h_0 \frac{\partial h_1}{\partial c_1} - \left(\frac{2}{c_0^2} + \frac{2c_0^2}{3c_1^4} \right) h_1 \frac{\partial h_0}{\partial c_0} \\
& + \frac{2}{3c_0^3} \left(c_0^2 h_0 \frac{\partial^2 h_1}{\partial c_1^2} - 2c_0c_1 \frac{\partial h_1}{\partial c_1} \frac{\partial h_0}{\partial c_0} + c_1^2 h_1 \frac{\partial^2 h_0}{\partial c_0^2} \right) \Bigg] \Bigg\} \\
& + \left\{ \frac{16\pi^2}{3} \frac{e^4}{m^2} \lambda \ln \frac{1}{\sin\vartheta_m} \frac{h_1}{c_1^2} \left[\int_0^{\infty} dc_0\, c_0 h_0 - \int_0^{c_1} dc_0\, c_0 h_0 \left(1 - \frac{c_0}{c_1}\right)^2 \left(1 + \frac{c_0}{2c_1}\right) \right] \right\} \quad \ldots
\end{aligned} \tag{5}$$

$$(1-\mu_1^2)\frac{\partial^2 g_1}{\partial \mu_1^2} - 2\mu_1\frac{\partial g_1}{\partial \mu_1} + \lambda g_1 = 0 \qquad \ldots(6)$$

In this equation θ_m is the minimum scattering angle in the laboratory system. The equation for $g_0(\mu)$ is just Legendre's equation so that $g(\mu)$ is a linear combination of the regular and irregular solutions satisfying

$$g(\mu) = g(\pi - \mu) \qquad g(\mu_c) = 0$$

A very convenient and rather good approximation for the lowest eigenvalue λ_0 is

$$\lambda_0 \approx \frac{1}{\log_{10}\frac{H_m}{H_0}} \qquad (7)$$

For values of $\theta_c > 0.5^\circ$.

From the equation for h_1 we can find the loss rate of particles and energy,

$$\dot{n} \approx -n^2\frac{4}{3}\frac{e^4}{m^2}\pi\lambda_0 \ln\frac{1}{\sin\vartheta_m}\left(\overline{c^{-1}}\right)\left(\overline{c^{-2}}\right)$$

$$\dot{\mathcal{E}} \approx -n^2\frac{2}{3}\frac{e^4}{m}\pi\lambda_0 \ln\frac{1}{\sin\vartheta_m}\left(\overline{c^{-1}}\right) \qquad (8)$$

Although $(\overline{c^{-2}})$ is sensitive to the distribution $h(c^2, t)$, $(\overline{c^{-1}})$ is not, so that one can calculate $\dot{\mathcal{E}}$ with some assurance by taking some particular distribution, say a Maxwell-Boltzmann distribution, to determine $(\overline{c^{-1}})$.

The rate of energy production can be calculated by including all the energy-producting reactions, but a simple estimate will be made by considering only the DT reactions,

$$D + T \rightarrow He^4 + n + 17.6\ \text{Mev} \qquad (9)$$

and using for $\dot{\mathcal{E}}_p$ therefore

$$\dot{\mathcal{E}}_p = \frac{1}{4}n^2\,\overline{(\sigma v)}_{DT}\,Q \qquad Q = 18\ \text{Mev} \qquad (10)$$

where n is the number of charged ions; deuterons, and tritons. The ratio of the total energy produced to the energy lost by leakage in a cycle is then given by

$$\frac{\mathcal{E}_p}{\mathcal{E}_l} = \frac{Q\oint\frac{n^2}{4}\,\overline{(\sigma v)}_{DT}\,dt}{\oint n^2\frac{2}{3}\frac{e^4}{m}\pi\lambda_0 \ln\frac{1}{\sin\vartheta_m}\left(\overline{c^{-1}}\right)d\tau} \qquad (11)$$

Although the value of this ratio ~~cannot be determined unless~~ $h(c^2, t)$ is known as a function of time, the maximum of this ratio is

$$\frac{\dot{\mathcal{E}}_P}{\dot{\mathcal{E}}_x} \leq \frac{\frac{1}{4}\,\overline{(\sigma v)}_{DT}\quad Q}{\frac{2}{3}\frac{e^4}{m}\,\pi\lambda_0\,\ln\frac{1}{\sin\vartheta_m}\,\overline{(c^{-1})}} \tag{12}$$

For a Maxwell-Boltzmann distribution the average $\overline{c^{-1}} = \frac{2}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{\frac{1}{2}}$ and this maximum is

$$\frac{\dot{\mathcal{E}}_P}{\dot{\mathcal{E}}_x} \leq \frac{\frac{1}{4}\,\overline{(\sigma v)}_{DT}\quad Q}{\frac{1}{2\tau}\;kT} \tag{13}$$

where τ is a characteristic time for the Maxwell-Boltzmann distribution,

$$\tau = \frac{4}{3}\sqrt{2\pi}\,\lambda_0\,\ln\frac{1}{\sin\vartheta_m}\left(\frac{e^2}{mc^2}\right)^2\left(\frac{mc^2}{kT}\right)^{\frac{3}{2}} \tag{14}$$

(In this expression c is the velocity of light.) This ratio has a maximum value at a temperature of ~ 50 kilovolts and is equal to ~ 200%. Although the average of the ratio $\frac{\mathcal{E}_P}{\mathcal{E}_x}$ over a cycle must be less than this maximum value, the ratio could be attained if steady-state operation at the maximum value could be maintained.

We emphasize that the Maxwell-Boltzmann distribution cannot be the equilibrium solution of Eq. (5) and cannot even be maintained by a source, since the decay of $\frac{\partial h_1}{\partial t}$ is infinite at $c_1 = 0$ unless $\left(\frac{h_1}{c_1}\right)_{c_1=0} = 0$ or $\lambda_0 = 0$. An integro-differential equation for $h(c^2, t)$ can be obtained from Eq. (5) by integration by parts of all derivatives of $h(c_0^2, t)$. This will be a nonlinear differential equation for $h(c_1^2, t)$ with coefficients which involve integrations on $h(c_0^2, t)$. With this equation the progressive development in time of any assumed initial radial distribution function can be found. This is being done for several assumed initial distributions.

The question of validity of applying the Boltzmann equation to systems of particles interacting through a long-range potential has also been studied in detail by Maurice Neuman, Robert J. Riddell, and Stephen Gasiorowicz. The results of their analysis are precisely those which we have obtained with the use of the Boltzmann equation.

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