Title
PROPOGATION VELOCITY OF HeII IN HeI AND VICE VERSA

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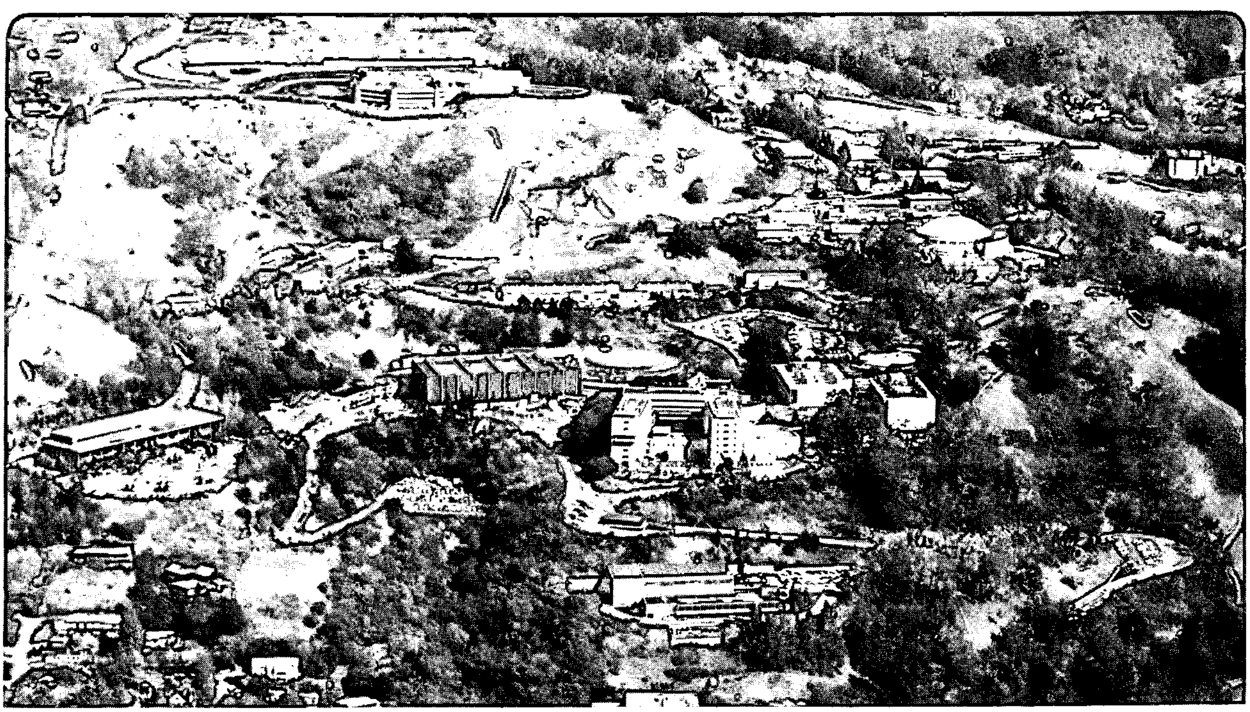

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ENGINEERING NOTE

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PROGRAM – PROJECT – JOB: High-Field Magnet Development

Analysis

TITLE: Propagation Velocity of He II in He I and vice versa.

ABSTRACT

During the course of superconducting magnet study a cryogenic He II support system has recently been built. He II at 1 atm is being produced by cooling He I from its NBP.

The laws of thermodynamics are used to express the relationship between the velocity of one phase (e.g He II) propagating into another (e.g He I), and the velocity of second sound in He II. An upper limit is set for the propagation velocity $W \leq C_2$.

A - unit area

C_2 - velocity of second sound

C_p - specific heat at constant p

H - Enthalpy

I - Entropy change in universe

K_E - kinetic energy

Q - Heat of interaction

S - Entropy

V - velocity

Sub n, s - normal, super

* Δ ≡ Difference between final and initial states.

Consider the case of a He I pool at constant pressure (e.g 1 atm) and a temperature just above $T_\lambda(P)$. At time $\tau = 0$ the He I makes thermal contact with a reservoir which is at $T_R < T_\lambda$ as shown in Fig 1 (one dimension)

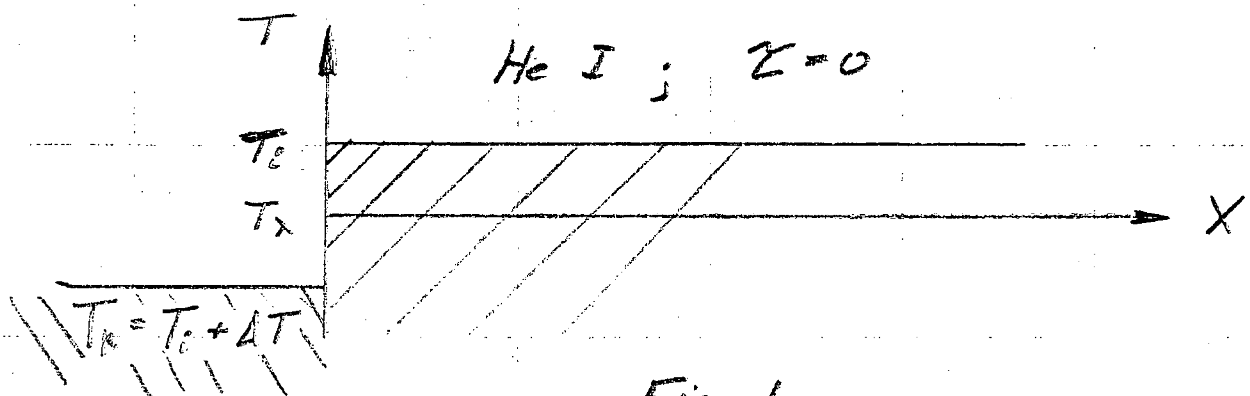


Fig 1

It is assumed that the temperature difference ΔT around T_λ causes a spontaneous transformation from He I to He II.

At time $\tau = \tau_0$ He II is formed (Fig 2). It is assumed that the boundary separating the two states is moving in +X direction with a constant velocity $\underline{w}$

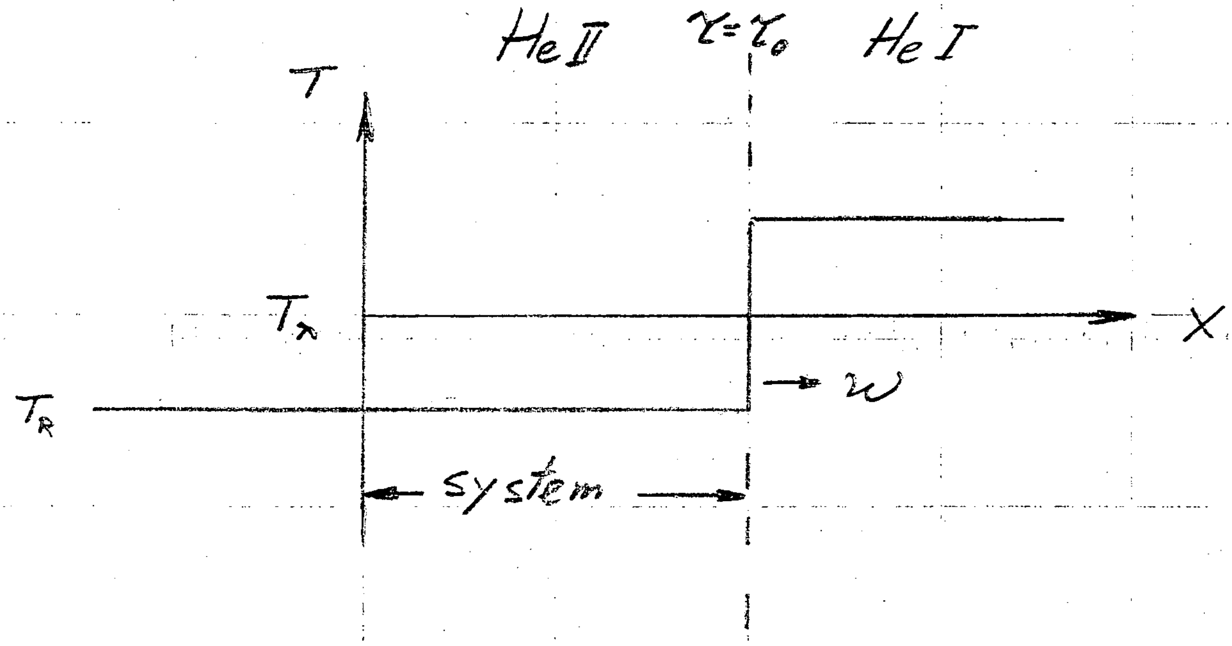


Fig 2

The thermodynamic system is defined as the volume occupied by the newly created He II $AW\tau_0$, (state 2) and is compared with the same volume of He I (state 1).

The total change in enthalpy is

$$\Delta H = (AW\tau_0)\rho C_p \Delta T \qquad (1)$$

$$dH = T\,ds + v\,dp \qquad (\text{I and II law})$$

since p = constant $\quad ds = \frac{1}{T}\,dH$

Substituting Eq. (1)

$$dS = (A w \tau_0)\,\rho C_p \frac{dT}{T}$$

$$\Delta S = (A w \tau_0)\,\overline{\rho C_p} \int_T^{T+\Delta T} \frac{dT}{T}$$

$$\Delta S = (A w \tau_0)\,\rho C_p \left[\frac{\Delta T}{T} - \frac{1}{2}\left(\frac{\Delta T}{T}\right)^2 \right]$$ up to second order in $\frac{\Delta T}{T}$

introducing Eq. (1) again

$$\Delta S = \frac{\Delta H}{T}\left[1 - \frac{1}{2}\frac{\Delta T}{T}\right] \qquad (2)$$

note: $\Delta T < 0 \rightarrow \Delta H < 0 \rightarrow \Delta S < 0$

An amount of entropy ΔS has been removed from the system by the reservoir.

The mechanism which "convects" the entropy from any place in the system towards the reservoir is the normal fluid component of He II

Fig 3.

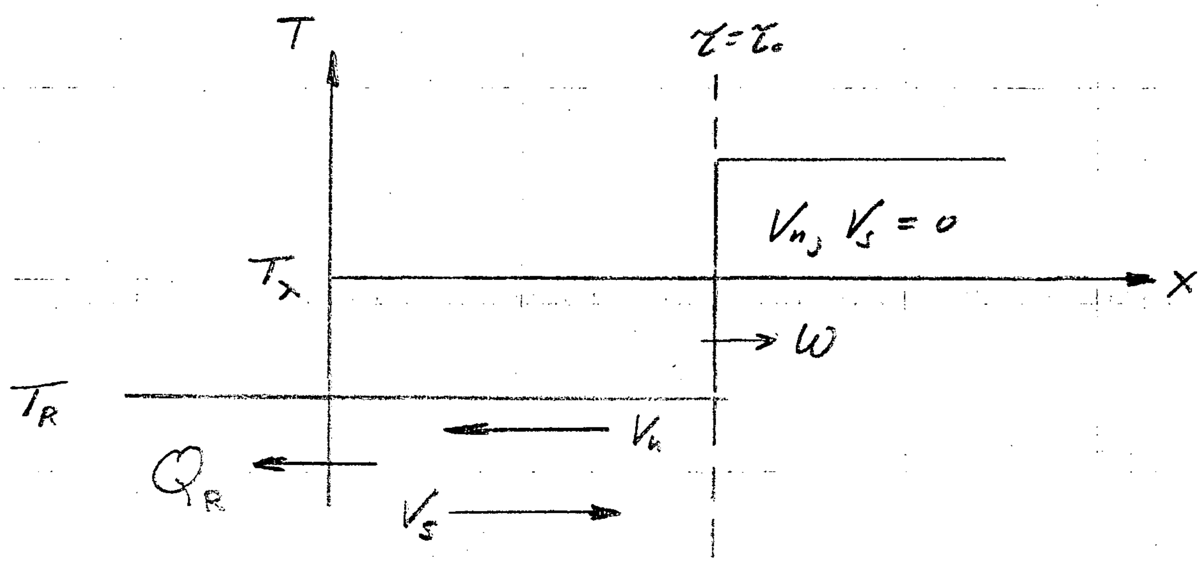


Fig 3

The entropy current density in He II is written as

$$q = \rho \Delta T V_n \qquad (\Delta = S/m)$$

$$(A\tau_0)\, q = \Delta H$$

$$(A\tau_0)\rho \Delta T V_n = (A W \tau_0)\rho C_p \Delta T$$

$$\frac{\Delta T}{T} = \frac{\Delta V_n}{W C_p}$$

multiplying by ΔH

$$\underline{\frac{\Delta H \Delta T}{T} = (A\tau_0)\rho T \frac{\Delta^2 V_n^2}{W C_p}} \qquad (3)$$

The kinetic energy of the system

$$\frac{K_E}{AWZ_0} = \frac{1}{2}\rho_n V_n^2 + \frac{1}{2}\rho_s V_s^2$$

conservation of momentum.

$$\rho_n \vec{V_n} + \rho_s \vec{V_s} = 0 \implies |V_s| = \left(\frac{\rho_n}{\rho_s}\right)|V_n|$$

sub. into the kinetic energy and introducing

$$\rho = \rho_s + \rho_n$$

$$\underline{K_E = (AWZ_0)\,\frac{1}{2}\rho\left(\frac{\rho_n}{\rho_s}\right)V_n^2} \qquad (4)$$

First law (conservation of energy)

$$Q_R = -(\Delta H + K_E)$$

The total entropy change of the system was calculated in Eq. (2)

The total entropy change in the reservoir is

$$\Delta S_R = \frac{Q_R}{T_R} = \frac{Q_R}{T+\Delta T} = -\frac{\Delta H + K_E}{T+\Delta T}$$

expending $\Delta T/T$ up to second order

$$\Delta S_R = -(\Delta H + K_E)\left(\frac{1}{T} - \frac{\Delta T}{T^2}\right) \qquad (5)$$

The total change of entropy I in the universe is

$$I = \Delta S_R + \Delta S_{sy}$$

The process is always $I \geq 0$

(Equality stays for the reversible one only)

introducing Eq. (2) and (5)

$$I = \frac{1}{T}\left[\frac{1}{2}\frac{\Delta H \Delta T}{T} - K_E\left(1 + \frac{\Delta T}{T}\right)\right]$$

Sub. Eq. (3) , neglecting second terms in K_E

and introducing the definition for the velocity of second sound C_2

$$C_2^2 = \frac{\rho_s}{\rho_n} \frac{s^2 T}{C_p}$$

$$\dot{i} = \frac{I}{A\tau_0} = \frac{1}{T} \frac{1}{2} \rho \left(\frac{\rho_n}{\rho_s}\right) V_n^2 \left[\frac{C_2^2 - W^2}{W}\right] \qquad (6)$$

$\dot{i}$ is the entropy production flux density.

note that the coefficient to the term in parenthesis is the kinetic energy per unit volume

since $\dot{i} \geq 0$

$$W^2 \leq C_2^2$$

The propagation velocity W is always less than C_2 except in the reversible case where $W = C_2$

since at $T > 1.2\,K$ $C_2 < 20\ m/sec \rightarrow W < 20\ m/sec$

Although $\lim_{T \to T_\lambda} C_2 \to 0$ in a real case ΔT is of the order of $\Delta T \simeq 10^{-3}$ K. (e.g $T_\lambda - T \simeq 10$ mK $C_2 \simeq 5$ m/sec).

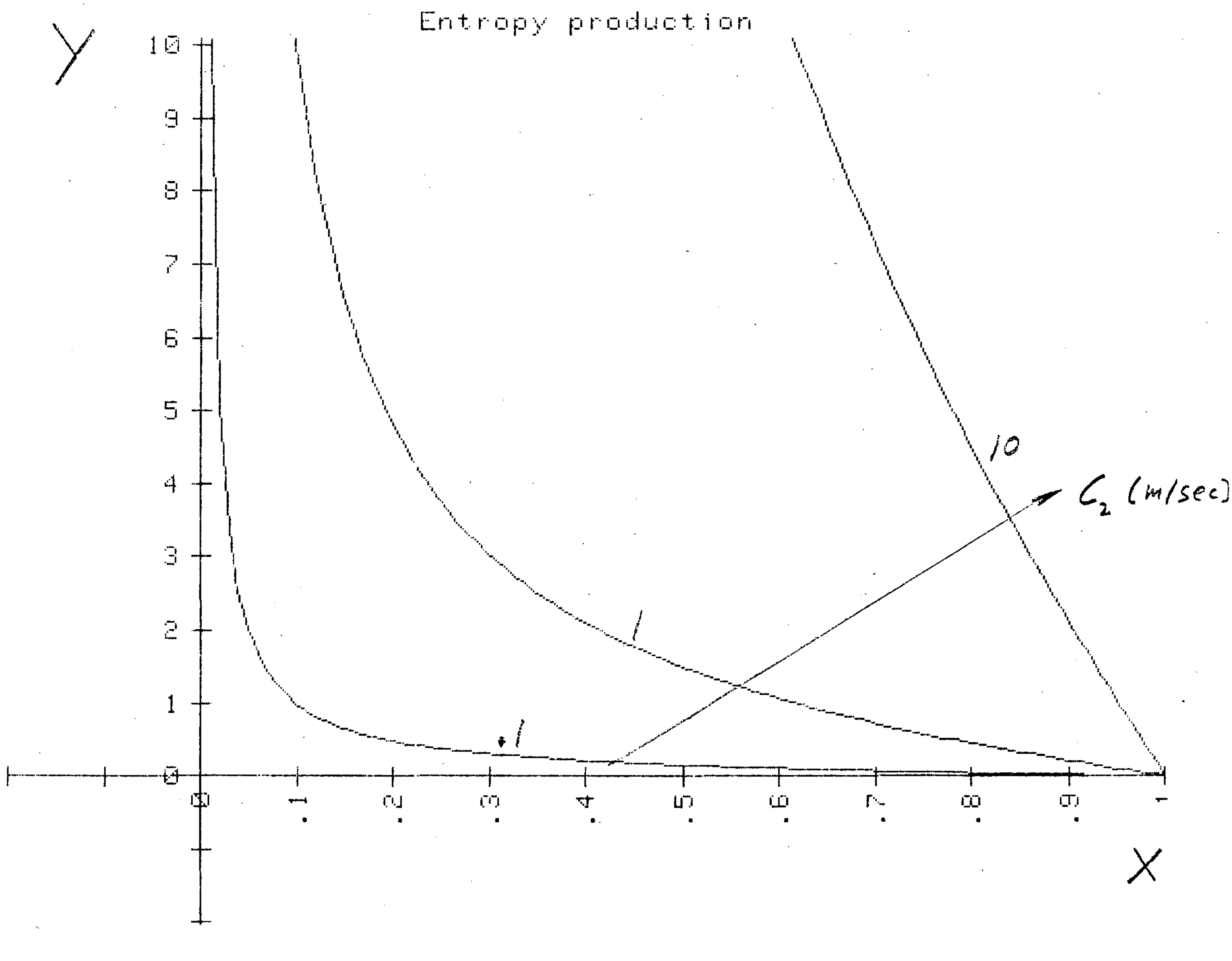
Eq (6) can be written as

$$Y = C_2 \frac{1 - X^2}{X}$$

where $Y = \frac{\dot{q} \cdot T}{(KE/V)} \equiv \frac{\text{Heat of production flux density}}{\text{system kinetic energy per unit volume}}$

$$X = \frac{W}{C_2}$$

Figures 4 and 5 plots Y vs X using C_2 as a parameter. Those curves are only meaningfull at the vicinity of $X = 1$. It is interesting to note however that for example if 20% of the system energy is wasted the propagation velocity will be reduced by 10%.



Entropy production
Y
10
9
8
7
6
5
4
3
2
1
0
.0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1
.1
1
10
C_2 (m/sec)
X

Fig 4

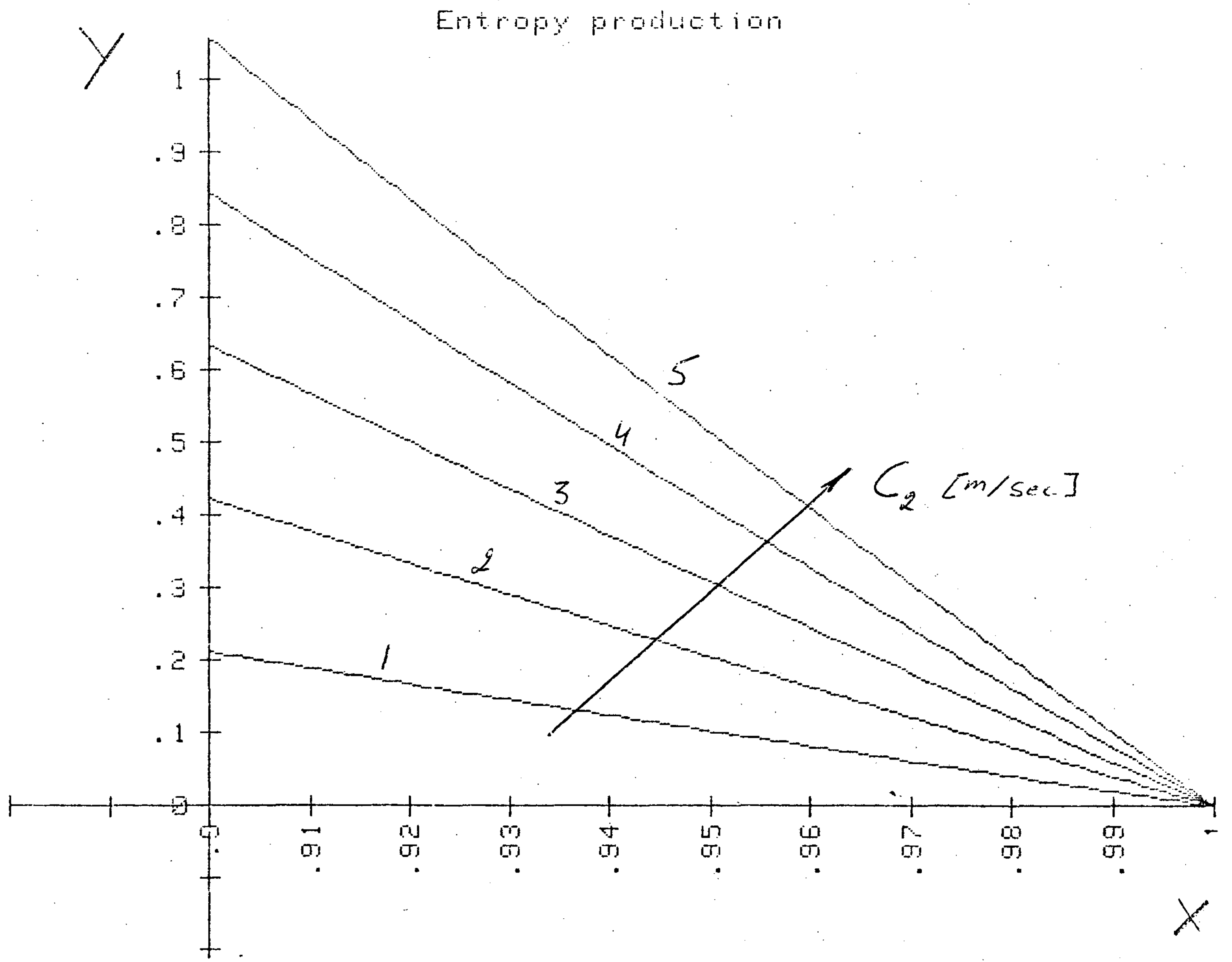

Entropy production
Y
1
.9
.8
.7
.6
.5
.4
.3
.2
.1
0
.9
.91
.92
.93
.94
.95
.96
.97
.98
.99
1
X
5
4
3
2
1
C2 [m/sec]

Fig 5

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