Selecting Building Predictive Control Based on Model Uncertainty

Mehdi Maasoumy[1], Meysam Razmara[2], Mahdi Shahbakhti[2], and Alberto Sangiovanni Vincentelli[3]

Abstract— **Model uncertainty limits the utilization of Model Predictive Controllers (MPC) to minimize building energy consumption. We propose a new Robust Model Predictive Control (RMPC) structure to make a building controller robust to model uncertainty. The results from RMPC are compared with those from a nominal MPC and a common building Rule Based Control (RBC). The results are then used to develop a methodology for selecting a controller type (i.e. RMPC, MPC, and RBC) as a function of building model uncertainty. RMPC is found to be the desirable controller for the cases with an intermediate level (30%-67%) of model uncertainty, while MPC is preferred for the cases with a low level (0-30%) of model uncertainty. A common RBC is found to outperform MPC or RMPC if the model uncertainty goes beyond a certain threshold (e.g. 67%).**

I. INTRODUCTION

Total primary energy consumption in the United States increased from 78.3 quads in 1980 to 97.8 quads in 2010, of which the building sector accounts for 41% [1]. The building sector is also responsible for about 40% of greenhouse gas emissions and 70% of electricity use in the United States. 41.4% of energy consumption in buildings is directly related to the space heating, ventilation and air conditioning (HVAC) [1]. Therefore, reducing the energy consumption of buildings by designing smart control systems to operate the HVAC system in a more efficient way is critically important to address energy and environmental concerns.

Design and validation practices in the building industry show the importance of a model-based design flow for building controls. To attain energy efficiency, control algorithms need to be tailored to the physical properties of the building at hand rather than being an adaptation of a standard sequence designed for a typical building.

MPC provides a potential building energy saving of 16%-41% compared to the commonly used rule-based HVAC controllers in the market as shown in [2]–[6]. At each iteration of MPC, a prediction of future states is obtained through forward evolution of the system dynamics. The predictive cost is then calculated and constraints on future states and inputs are applied, leading to a large optimization problem, which is solved to obtain the optimal control action. MPC for building HVAC systems has desirable characteristics such as

robustness, tunability, and flexibility [4]. Application of MPC for building energy control has been reported in [4]–[13]. Different variations of nominal MPC such as distributed [12], [13], robust [9], [14] and stochastic [8], [15] model predictive control strategies have been also reported. All these MPC techniques rely on accurate building models to achieve optimal performance of HVAC system and provide sufficient comfort level.

Hence, reducing *model uncertainty* is crucial for successful realization of building model-based controllers. However, uncertainty is an inevitable attribute of every model. For building models, model uncertainty is even more remarkable due to the substantial contribution of outside and inside thermal heats (realized as disturbance in our model) in the climate of the building, and unpredictability of the environment conditions such as wind speed and direction, ambient temperature, solar radiation and cloudiness of sky, and to some extent indoor factors such as human behavior.

In this paper we provide a systematic approach to:

- Design model-based building controllers which are robust to model uncertainties;
- Assess the overall performance of different control type over a range of model uncertainty in terms of both energy consumption and provided comfort level and
- Propose a guideline to pick the right type of controller based on the level of model uncertainty.

This paper builds on top of our previous work on developing a parameter adaptive building (PAB) model presented in [16] and centers on designing and analyzing the performance of three types of controllers: 1) a controller which is insensitive to model uncertainty (i.e. a typical on-off control), 2) a controller which is somewhat sensitive to model uncertainty (i.e. nominal MPC) and 3) a controller which is robust against model uncertainty (i.e. Robust MPC) for building HVAC system.

II. PARAMETER-ADAPTIVE BUILDING MODEL

A. System Dynamics

1) Heat Transfer: A building is modeled as a graph in which there are two types of nodes: walls and rooms. n is the total number of nodes, m of which represent rooms and the remaining $n - m$ nodes represent walls. We denote the temperature of room r_i with T_{r_i}. The wall node and temperature of the wall between room i and j are denoted by (i, j) and $T_{w_{i,j}}$, respectively, and is governed by the following equation:

$$C_{i,j}^w \frac{dT_{w_{i,j}}}{dt} = \sum_{k \in \mathcal{N}_{w_{i,j}}} \frac{T_{r_k} - T_{w_{i,j}}}{R_{i,j_k}} + r_{i,j}\alpha_{i,j}A_{w_{i,j}}Q_{rad_{i,j}}$$

(1)

[1]Mehdi Maasoumy is with Mechanical Engineering Department of University of California, Berkeley mehdi@me.berkeley.edu

[2]Meysam Razmara and Mahdi Shahbakhti are with Mechanical Engineering Department of Michigan Technological University {mrazmara, mahdish}@mtu.edu

[3]Alberto Sangiovanni Vincentelli is with the Department of Electrical Engineering and Computer Science, University of California, Berkeley alberto@eecs.berkeley.edu

where $C_{i,j}^w$, $\alpha_{i,j}$ and $A_{w_{i,j}}$ are heat capacity, radiative heat absorption coefficient and area of wall between room i and j, respectively. R_{i,j_k} is the total thermal resistance between the centerline of wall (i,j) and the side of the wall where node k is located. $Q_{rad_{i,j}}$ is the radiative heat flux density on wall (i,j). $\mathcal{N}_{w_{i,j}}$ is the set of all of neighboring nodes to node $w_{i,j}$. $r_{i,j}$ is wall identifier which is equal to 0 for internal walls, and equal to 1 for peripheral walls (*i.e.* either i or j is the outside node). Temperature of the ith room is governed by the following equation:

$$C_i^r \frac{dT_{r_i}}{dt} = \sum_{k \in \mathcal{N}_{r_i}} \frac{T_k - T_{r_i}}{R_{i,k_i}} + \dot{m}_{r_i} c_a (T_{s_i} - T_{r_i}) + \\ w_i \tau_{w_i} A_{win_i} Q_{rad_i} + \dot{Q}_{int_i} \quad (2)$$

where T_{r_i}, C_i^r and $\dot{m}_{r_i}$ are the temperature, heat capacity and air mass flow into the room i, respectively. c_a is the specific heat capacity of air, and T_{s_i} is the temperature of the supply air to room i. π_i is window identifier which is equal to 0 if none of the walls surrounding room i have window, and is equal to 1 if at least one of them has a window. τ_{w_i} is the transmissivity of glass of window i, A_{win_i} is the total area of window on walls surrounding room i, Q_{rad_i} is the radiative heat flux density per unit area radiated to room i, and $\dot{Q}_{int_i}$ is the internal heat generation in room i. $\mathcal{N}_{r_i}$ is the set of all of the neighboring *room* nodes to room i. The details of building thermal modeling and estimation of the unmodelled dynamics is presented in [4], [6], [7].

The heat transfer equations for each wall and room yield the following system dynamics:

$$\dot{x}_t = f(x_t, u_t, d_t, t) \\ y_t = C x_t \quad (3)$$

where $x_t \in \mathbb{R}^n$ is the state vector representing the temperature of the nodes in the thermal network, $u_t \in \mathbb{R}^{lm}$ is the input vector representing the air mass flow rate and discharge air temperature of conditioned air into each thermal zone, and $y_t \in \mathbb{R}^m$ is the output vector of the system which represents the temperature of the thermal zones. l is the number of inputs to each thermal zone (e.g. air mass flow and supply air temperature). C is a matrix of proper dimension and the disturbance vector is given by

$$d_t = g(Q_{rad_i}(t), \dot{Q}_{int}(t), T_{out}(t)) \quad (4)$$

2) Disturbance: Following the intuitive linear relation between T_{out}, $\dot{Q}_{int}$, Q_{rad} and the building temperature rise we approximate g with an affine function of these quantities, leading to:

$$d_t = a Q_{rad_i}(t) + b \dot{Q}_{int}(t) + c T_{out}(t) + e \quad (5)$$

where a, b, c, e are a constant to be estimated. We approximate the values of $Q_{rad_i}(t)$ and $\dot{Q}_{int}(t)$ using $Q_{rad_i}(t) = \tau T_{out}(t) + \zeta$ and $\dot{Q}_{int}(t) = \mu \Psi(t) + \nu$. By substituting for $Q_{rad_i}(t)$ and $\dot{Q}_{int}(t)$, and rearranging the terms, we obtain:

$$d_t = (a\tau + c)T_{out}(t) + b\mu\Psi(t) + a\zeta + b\nu + e \\ = \bar{a} T_{out}(t) + \bar{b} \Psi(t) + \bar{e} \quad (6)$$
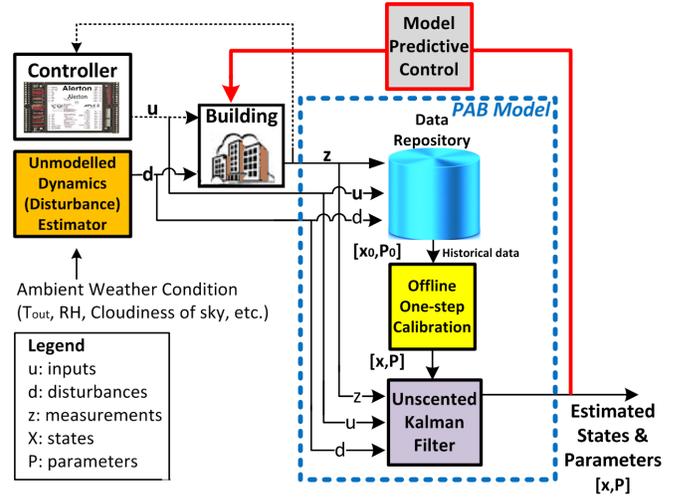

Fig. 1. Architecture of the building control system utilizing the PAB model from [16]. Updated model parameters are obtained from UKF estimation process at each time step. At the next time step, MPC uses the model with updated parameters to calculate the optimal inputs. Inputs are implemented on the system and at the next sampling time new states (temperatures) are measured and sent to the PAB model, and this process repeats. Black dotted lines connecting the traditional control system to the building are replaced by the red solid lines connecting the estimation module to the MPC block and the MPC block to the building.

where $\bar{a} = a\tau + c$, $\bar{b} = b\mu$, and $\bar{e} = a\zeta + b\nu + e$. Therefore, only measurements of outside air temperature and CO_2 concentration levels are needed to determine the disturbance. The values of $\bar{a}$, $\bar{b}$, and $\bar{e}$ are estimated along with other parameters of the model.

3) Additive uncertainty: To account for model uncertainties, we use an uncertain system dynamics, based on the following state update model:

$$x_k = f(x_{k-1}, u_{k-1}, d_{k-1}, w_{k-1}) \\ z_k = h(x_k) + v_k \quad (7)$$

where w_k and v_k are the process and measurement noise.

B. Architecture

We utilize the building thermal model that was developed in [16]. The architecture of the PAB model along with control structure from this work is shown in Fig. 1. Measurement data from various sensors such as temperature and airflow throughout the building are stored in a data repository. Historical data is used to perform off-line, one-step model calibration. The obtained parameters from model calibration is sent to the Kalman filter algorithm as an initial set of parameters. As the new measurements arrive, Kalman filter updates the parameters. Updated parameters are then used to update the mathematical model used in the MPC algorithm.

C. Estimation Algorithm

In order to estimate the unknown parameters of the system we augment the states of the system with a vector p_k which stores the parameters of the system, with a time evolution dynamics of $p_{k+1} = p_k$. The dynamics of the system is nonlinear, thus a nonlinear Kalman filter, namely unscented
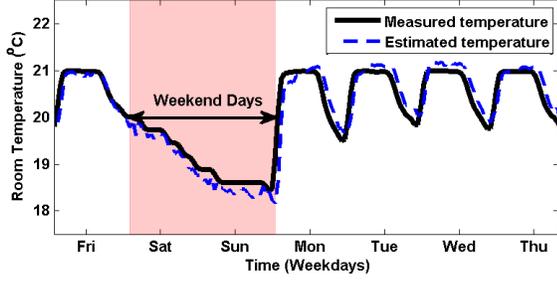
Fig. 2. Estimated room temperature using UKF in the PAB model.

Fig. 3. Required information for designed RBC, MPC and RMPC controllers in this study.

Kalman filter (UKF), is used as detailed in [16]. The PAB model with updated parameters using UKF is validated against historical data [16] of an office building as shown in Fig. 2.

III. CONTROLLER DESIGN

For control design, we use the linearized version of the state update equation (7), with sampling time of 1 hour and using the Euler linearization method, as given by

$$x_{k+1} = Ax_k + Bu_k + E(d_k + w_k) \qquad (8)$$

where the uncertainty $w_k \in \mathbb{R}^r$ is a stochastic additive disturbance. The set of possible disturbance uncertainties is denoted by $\mathcal{W}_k$ and $w_k \in \mathcal{W}_k$, $\forall k = 0, 1, ..., N-1$. For this study, we consider box-constrained disturbance uncertainties with uniform distribution, given by

$$\mathcal{W}_k = \{w : ||w||_\infty \le \lambda_k\} \qquad (9)$$

We study a traditional rule-based control (RBC), i.e. on-off control, a nominal MPC and a robust MPC (RMPC). Fig. 3 shows the required information realizing each of these three controllers. The MPC assumes that the model is perfect (no uncertainty), and the RMPC assumes that the model is uncertain and hence computes a robust control action for the specified class of uncertainty given as constraint on w_k. The goal of RMPC is to satisfy constraints on states and inputs for all the uncertainties within a specified class. The results from MPC and RMPC are compared to a conventional rule based control (RBC) for a typical building. To be consistent and to perform a fair comparison, we use the same time constants Δt for all controller implementations.

A. Rule-Based Control (RBC)

The rule based controller use in this paper is a conventional on-off HVAC controller. The time constant of the control implementation is Δt. The controller opens the dampers of conditioned air flow to the thermal zones when heating is required and keeps it fully open for the duration of Δt. In the next time step the controller checks the temperature again and adjusts the damper position if the room temperature is within the comfort zone, or keeps it open if the room air temperature is still outside the comfort zone. In on-off control, position of the dampers can be either the *min* value or the *max* value. When system goes to the cooling mode, supply air temperature changes accordingly. The experimental data presented here is for the heating mode only.

B. Model Predictive Control (MPC)

A model predictive control problem is formulated with the objective of minimizing a linear combination of total (l_1 norm) and peak (l_∞ norm) control input (i.e. airflow). Fan energy consumption is proportional to the *cubic* of the airflow. Hence minimizing the peak airflow would dramatically reduce fan energy consumption. We implement the control inputs obtained from the MPC using the linearized system dynamics of the model on the original nonlinear model for forward simulation.

The alternative would be to use the actual nonlinear function of fan energy consumption. However, this approach would lead to nonlinear MPC which is much slower than linear MPC to solve. Hence, we use the proposed cost function (10a) to achieve better computational properties. Also in order to guarantee feasibility (constraint satisfaction) at all times, we implement soft constraints. The predictive controller solves at each time step t, the following optimization problem:

$$\min_{U_t, \bar{\epsilon}, \underline{\epsilon}} \{|U_t|_1 + \kappa |U_t|_\infty + \rho(|\bar{\epsilon}_t|_1 + |\underline{\epsilon}_t|_1)\} \qquad (10a)$$

subject to:

$$x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} + Ed_{t+k|t} \qquad (10b)$$

$$y_{t+k|t} = Cx_{t+k|t} \qquad (10c)$$

$$\underline{\mathcal{U}}_{t+k|t} \le u_{t+k|t} \le \overline{\mathcal{U}} \qquad (10d)$$

$$\underline{T}_{t+k|t} - \underline{\varepsilon}_{t+k|t} \le y_{t+k|t} \le \overline{T}_{t+k|t} + \overline{\varepsilon}_{t+k|t} \qquad (10e)$$

$$\underline{\varepsilon}_{t+k|t}, \overline{\varepsilon}_{t+k|t} \ge 0 \qquad (10f)$$

where constraints (10b) and (10d) should hold for all $k = 0, 1, ..., N-1$ and (10c), (10e) and (10f) should hold for all $k = 1, 2, ..., N$. $U_t = [u_{t|t}, u_{t+1|t}, \cdots, u_{t+N-1|t}]$ is vector of control inputs, $\underline{\epsilon}_t = [\underline{\varepsilon}_{t+1|t}, \cdots, \underline{\varepsilon}_{t+N|t}]$ and $\overline{\epsilon}_t = [\overline{\varepsilon}_{t+1|t}, \cdots, \overline{\varepsilon}_{t+N|t}]$ are the slack variables used to take into account the soft constraints on room temperature to avoid feasibility issues. $y_{t+k|t}$ is the thermal zone temperature vector, $d_{t+k|t}$ is the disturbance load prediction, and $\underline{T}_{t+k|t}$ and $\overline{T}_{t+k|t}$ for $k = 1, \cdots, N$ are the lower and upper bounds on the zone temperature, respectively. $\underline{\mathcal{U}}_{t+k|t}$ and $\overline{\mathcal{U}}$ are the lower and upper limits on the airflow input by
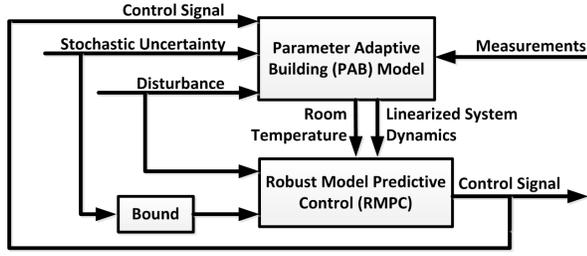
Fig. 4. Schematic of the robust model predictive control implementation.

the variable air volume (VAV) damper, respectively. Physical limit on maximum airflow generated by fans is not time varying, hence time invariant constraint $\overline{\mathcal{U}}$. Note that based on ASHRAE requirements for *Air Change per Hour* (ACH) of rooms, there has to be a minimum non-zero airflow during occupied hours for ventilation purposes, hence $\overline{\mathcal{U}} > 0$. ρ is the penalty on the comfort constraint violations, and κ is the penalty on peak power consumption.

At each time step only the first entry of U_t is implemented on the model. At the next time step the prediction horizon N is shifted leading to a new optimization problem. This process is repeated over and over until the total time span of interest is covered.

C. Robust Model Predictive Control (RMPC)

The crucial question in robust control is how to exploit knowledge about uncertainty. Typical knowledge can be bounds on uncertain parameters in the system, or bounds on external disturbances, such as the disturbance load to the building. In this paper we consider additive uncertainty to the system model as previously described in (8). A schematic of the robust optimal control implementation on the nonlinear building model is depicted in Fig. 4. In RMPC algorithm the cost function is the same as in the MPC case (10a), with input constraints similar to (10c), (10d), (10e), (10f), but with the following update state equation:

$$x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} + E(d_{t+k|t} + w_{t+k|t}) \quad (11)$$

to be evaluated for all $k = 0, 1, ..., N-1$, and the uncertainty variable constrained to

$$w_{t+k|t} \in \mathcal{W}_{t+k} \quad \forall \; k = 0, 1, ..., N-1 \quad (12)$$

The only difference with respect to the MPC algorithm is the introduction of the additive uncertainty term w in the state update equation. The disturbance set $\mathcal{W}$ is one of the ingredients that determines the type of optimization problem we end up with. When the uncertainty set $\mathcal{W}$ in a *Linear Programming* (LP) is a polyhedron, then the robust counterpart is also an LP [17].

1) Feedback predictions: Ideally the closed-loop min-max problem should be solved as given by

$$\min_{u_{k|k}} \max_{w_{k|k}} \cdots \min_{u_{k+N-1|k}} \max_{w_{k+N-1|k}} \sum_{j=0}^{N-1} p(x_{k+j|k}, u_{k+j|k})$$
$$(13)$$

where we incorporate the notion that measurements will be obtained in future times. $p(.)$ is the performance index. Instead of solving this intractable problem, the idea in feedback prediction, is to introduce new decision variables $n_{k+j|k}$, and parameterize the future control sequences in the future disturbances and $n_{k+j|k}$ such as $u_{k+j|k} = m_{k,j}w_{k+j|k} + n_{k+j|k}$. To incorporate feedback predictions, we write the feedback predictions in a vectorized form $U = \mathbf{M}\mathbf{w} + \mathbf{n}$ where $\mathbf{n}$ and $\mathbf{w}$ are given by $\mathbf{n} = [n'_{k|k}, n'_{k+1|k}, \cdots, n'_{k+N-1|k}]'$, and $\mathbf{w} = [w'_{k|k}, w'_{k+1|k}, \cdots, w'_{k+N-1|k}]'$. Where $(.)'$ represents the transpose of a matrix or vector. The only requirement for matrix $\mathbf{M}$ is that this matrix is *causal* in the sense that $u_{k+j|k}$ only depends on $w_{k+i|k}$, $i \leq j$. The choice of $\mathbf{M}$ is not obvious. In [18] it is shown through simulation examples that the choice of $\mathbf{M}$ is crucial for good performance of the min-max controller. However, $\mathbf{M}$ can be incorporated as a decision variable in the online optimization problem. We propose a method denoted as Two Lower Diagonal Structure (TLDS):

2) Two Lower Diagonal Structure (TLDS): By analyzing the structure of the optimal matrix $\mathbf{M}$, it was observed that the parameterization of the input does not need to consider feedback of more than past *two* values of $\mathbf{w}$ at each time, therefore we propose the following disturbance feedback:

$$u_i := m_{i,i-2}w_{i-2} + m_{i,i-1}w_{i-1} + n_i$$
$$= \sum_{j=i-2}^{i-1} m_{i,j}\omega_j + n_i \qquad \forall i = 2, 3, ..., N-1 \quad (14)$$

and the corresponding parameterization matrix $\mathbf{M}$ is an $N \times N$ matrix that has the entries on the second and third diagonal of $\mathbf{M}$ below its main diagonal as decision variables and 0 elsewhere as in (15).

$M_{i,j}$ and n_i are stored in the two following matrices as follows:

$$\mathbf{M} := \begin{bmatrix} 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ m_{3,0} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ m_{4,0} & m_{4,1} & \ddots & & & & 0 \\ 0 & \ddots & \ddots & \ddots & & & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & & \vdots \\ 0 & \cdots & \cdots & & m_{\mathrm{I}} & 0 & 0 \\ 0 & \cdots & \cdots & 0 & m_{\mathrm{II}} & m_{\mathrm{III}} & 0 \end{bmatrix} \quad (15)$$

where $m_{\mathrm{I}} = m_{N-2,N-3}$, $m_{\mathrm{II}} = m_{N-1,N-3}$, and $m_{\mathrm{III}} = m_{N-1,N-2}$, and:

$$\mathbf{n} := \begin{bmatrix} n_0 & n_1 & n_2 & \cdots & \cdots & \cdots & n_{N-1} \end{bmatrix} \quad (16)$$

The main problem with the min-max formulations based on existing parameterization in the literature is the excessive number of decision variables and constraints [18]. The reason is the high-dimensional parameterization of matrix $\mathbf{M}$. In this proposed structure, we have tried to overcome this problem by exploiting the sparsity of the feedback gain matrix to enhance the computational characteristics of the optimal problem.

IV. PERFORMANCE METRICS

To compare the overall performance of the proposed controllers we define indices to measure the energy consumption and comfort level provided by each controller. In addition, we define a new index to evaluate the overall performance of each controller considering both the energy and comfort indices.

A. Energy Index

The energy index I_e in (kWh) is defined as:

$$I_e = \int_{t=0}^{24} \left[P_c(t) + P_h(t) + P_f(t) \right] dt \qquad (17)$$

where cooling power P_c, heating power P_h and fan power P_f are determined by:

$$P_c(t) = \dot{m}_c(t) c_p [T_{out}(t) - T_c(t)] \qquad (18a)$$
$$P_h(t) = \dot{m}_h(t) c_p [T_h(t) - T_{out}(t)] \qquad (18b)$$
$$P_f(t) = \alpha \dot{m}^3(t) \qquad (18c)$$

where $c_p = 1.012 (\text{kJ/kg.}^\circ\text{C})$ is the specific heat capacity of air and $\alpha = 0.5 (\text{kW.s}^3/\text{kg}^3)$ is the fan power constant [19]. Using these constants, the fan power values, in (kW), can be calculated. T_c and T_h are output temperature of cooling and heating system, respectively.

B. Discomfort Index

The discomfort index I_d in degree Celsius hour ($^\circ$Ch) is defined as the integral of all the temperature violations over the course of a day:

$$I_d = \int_{t=0}^{24} \left[\hat{T} . \mathbf{1}_{\mathcal{B}(t)^c}(T(t)) \right] dt \qquad (19)$$

where $\hat{T} = \min \left\{ \left| T(t) - \overline{T}(t) \right|, \left| T(t) - \underline{T}(t) \right| \right\}$ and $\mathcal{B}(t) = [\underline{T}(t), \overline{T}(t)]$ is the allowable temperature boundary at time t and $\mathbf{1}$ is the indicator function.

C. Overall Performance Index

A good control performance means not only low energy consumption, but also low resulting discomfort. To assess the overall performance of the controllers, we need to examine both I_e and I_d at the same time. Using the two indices defined above we define a third index called Overall Performance Index (I_{OP}). The intuition behind this new index is to take into account the energy and discomfort index in one single term. I_{OP} is defined as:

$$I_{OP} = \frac{(I_d^* - I_d)/||I_d||_\infty}{I_e/||I_e||_\infty} \qquad (20)$$

where I_d^* is the maximum allowed discomfort which is selected according to the required probability of maintaining room temperature within the comfort zone, and $||.||_\infty$ denotes infinity norm or the maximum value of energy indices among all three controllers. Negative value of I_{OP} means that the discomfort index is not within the preferred range. The lower the I_d and I_e are, the higher the I_{OP} will be. Therefore, the higher the I_{OP}, the better the overall performance. In
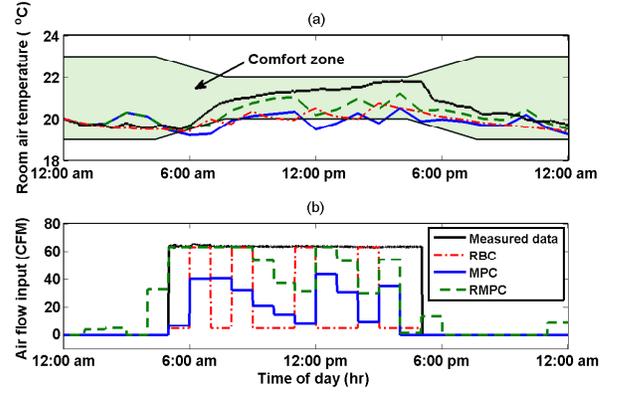

Fig. 5. Control input and resulting temperature profile for the existing controller on the building (denoted as Measurements in the figure), RBC, MPC, and robust MPC controllers.($\delta = 60\%$)

this study, the limit on the allowed discomfort index is heuristically chosen to be $I_d^* = 0.5 (^\circ\text{Ch})$ to ensure adequate comfort level.

V. COMPUTATIONAL RESULTS

To illustrate the effectiveness of the controllers proposed in Section III, we assess their performances for different *model uncertainty* values denoted by δ:

$$\delta = \frac{\lambda}{||\mathbf{d}||_\infty} \qquad (21)$$

where λ is the l_∞ norm bound of the uncertainty and $\mathbf{d} = [d_1', d_2', ..., d_N']'$ is the disturbance realization vector. In general, model uncertainty can be obtained by comparing the simulation results of a given model of a building for a period in the past, with historical data of same building over the same time period.

Time constant $\Delta t = 1$ (hr) is used for all the following simulations. We implement the introduced model predictive controllers with a prediction horizon of $N = 24$. The choice of $N = 24$ is to provide a good balance between performance and computational cost for the MPC framework.

We use the following numerical values for parameters in (10). $\overline{\mathcal{U}} = 63$ CFM (0.03 m^3/s), and $\underline{\mathcal{U}} = 5$ CFM (0.002 m^3/s) are the lower and higher limit on air mass flow during occupied hours, $[\underline{T}_{.|t} \ \overline{T}_{.|t}] = [20 \ 22]^\circ$C during occupied hours, and $[\underline{T}_{.|t} \ \overline{T}_{.|t}] = [19 \ 23]^\circ$C is used during unoccupied hours. For the simulations we use $\kappa = 0.75$ and $\rho = 50$.

Optimal controller and the resulting room temperature with the presence of a box-constrained uncertainty in four cases are depicted in Fig. 5. Measurements, as shown in black, shows the air mass flow and temperature recording for the room using a simple existing control policy of the building HVAC system which controls the fan speed by turning it on and off by the start and end time of occupancy hours. RBC, MPC, and RMPC refer to algorithms described in Section III-A, III-B, and III-C.

Controller performances are evaluated based on indices introduced in Section IV. We use YALMIP [20] to set up the
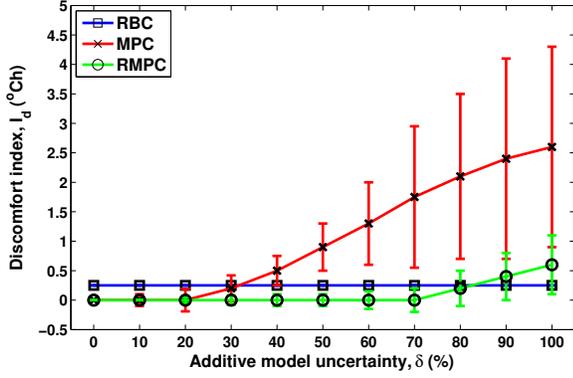
Fig. 6. Discomfort index I_d versus additive model uncertainty (δ). We generate a uniform random sequences based on the disturbance prediction error value δ. The generated random sequences are used in the Monte Carlo simulations for making this graph. The mean value (marker position) and standard deviation (error bar length) of discomfort are shown in the figure.
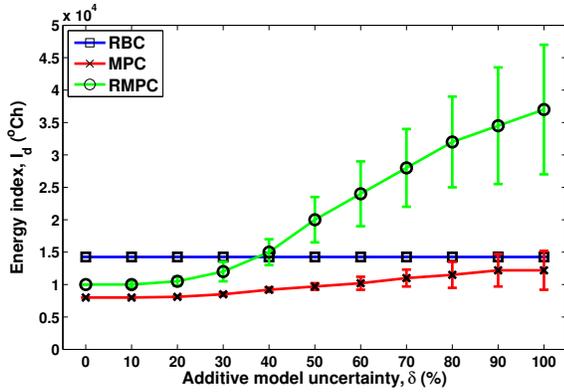

Fig. 7. Energy index I_e versus additive model uncertainty (δ). We generate data with similar approach as in Fig. 6.

MPC problem in MATLAB. Problem is solved using CPLEX 12.2 [21]. We performed mass simulations for different values of δ. Fig. 6 and Fig. 7 depict the results from these simulations. Monte Carlo simulations were performed to obtain the mean and standard deviation of energy and discomfort index for various values of model uncertainty.

Comfort: It can be observed from Fig. 5 and Fig. 6 that the RMPC is the only controller that is able to keep the temperature within the comfort zone, at all times, i.e. maintaining minimum level of discomfort ($I_d \leq I_d^*$) for all $\delta \leq 80\%$, while RBC still performs well, MPC fails to do so, resulting to $I_d > I_d^*$ for all $\delta \geq 40\%$. Fig. 6 depicts how *discomfort index* I_d, varies with additive model uncertainty δ. As shown in Fig. 6, RMPC manages to keep the perfect comfort level ($I_d = 0$), for additive model uncertainty up to $\delta = 70\%$, while the MPC maintains the perfect comfort level for uncertainty bounds up to $\delta = 20\%$. Since RBC is not a model-based control technique, its performance does not depend on values of δ, hence the straight horizontal line in Fig. 6 ($I_d = 0.25°$Ch).

Energy Consumption: Fig. 7 depicts the variations of *energy index* I_e, versus model uncertainty. It is clear that the
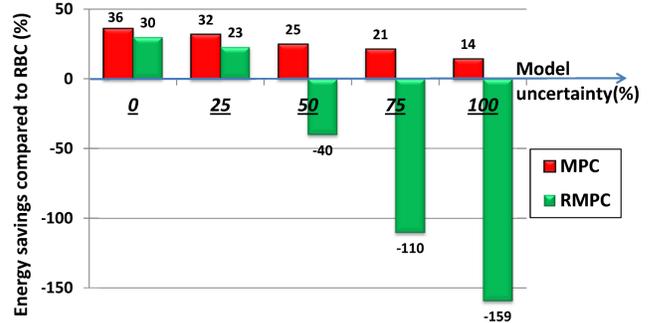

Fig. 8. Energy saving of MPC and RMPC compared to RBC as a function of model uncertainty.

energy index for RMPC increases dramatically with δ, while the energy index for MPC only changes slightly. However, this comes with the drawback of increased discomfort index for MPC. Fig. 7 also shows energy consumption of RBC ($I_e = 1.43 \times 10^4$ kWh). MPC for all values of δ leads to a lower amount of energy consumption than RBC, but RMPC leads to more energy consumption than RBC soon after $\delta = 40\%$.

Consider the case where $\delta = 70\%$. MPC will lead to a discomfort index of $1.7°$Ch on average, while the RMPC is able to maintain the temperature below a discomfort index of 0.016^oCh on average. However this level of comfort provided by the RMPC comes at a cost of energy consumption of more than 2 times of the MPC case.

Due to the trade-off between comfort and energy consumption, the choice of which controller to pick is not obvious, and depends on various factors such as criticality of meeting the temperature constraints for the considered thermal zone in the building, and price of energy at that time of the day/year, as well as uncertainty level of the model.

MPC and RMPC versus RBC: Fig. 8 demonstrates savings of MPC and RMPC versus RBC. As shown, the maximum theoretical energy saving of MPC compared to RBC is 36%, and that of RMPC is 30% for the building studied. These saving values decrease as model uncertainty increases. Energy saving of MPC versus RBC stays positive for all values of model uncertainty, while energy saving of RMPC versus RBC is positive only for model uncertainty values up to about 34%, and is negative for larger model uncertainties (i.e. RMPC consumes more energy than RBC).

The result of an extensive study in [22] shows that MPC HVAC control can potentially provide 16%-41% building energy saving compared to rule-based controllers, which complies with our findings. The saving also depends on various factors including climate zone, insulation level, and construction type.

For evaluation of energy consumption and provided comfort level, we have compared the overall performance of the three controllers using the overall performance index, I_{OP}. The results, as shown in Fig. 9, suggest that for model uncertainties less than 30% MPC performs best among the three controllers studied here. For model uncertainties between
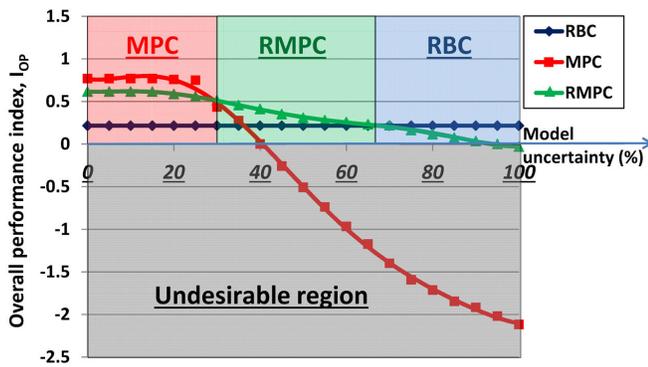
Fig. 9. Overall performance index for RBC, MPC and RMPC as a function of model uncertainty. The red zone demonstrates the region which MPC outperforms RMPC and RBC as it yields a higher I_{OP}. The green zone represents the region that I_{OP} of RMPC is higher than that of MPC and RBC. RBC dominates in terms of I_{OP} in the blue zone. In the gray zone the resulting discomfort index is not acceptable.

30% and 67% RMPC is the best, and for model uncertainties larger than 67%, RBC leads to better overall performance than model-based control techniques. This information can be of utility for choosing a controller type for building HVAC system based on how detailed and accurate the building model is, in capturing time-varying dynamics of a building.

VI. CONCLUSION

Model uncertainty is unavoidable for building HVAC system. In this paper, we characterized the impact of model uncertainty on model-based controllers, i.e. model predictive control (MPC), and robust model predictive control (RMPC). Closed-loop RMPC uses uncertainty knowledge to enhance the nominal MPC. The RMPC is shown capable of maintaining the temperature within the comfort zone for model uncertainty up to 70%. Closed loop RMPC outperforms nominal MPC controllers considering the provided level of comfort. However, higher comfort comes at the cost of dramatically higher energy consumption for RMPC. For the case study considered in this paper, we found the best choice for controller type, taking into consideration overall performance, ranges from MPC (up to 30% uncertainty) to RMPC (between 30% and 67% uncertainty), and then finally to RBC (above 67% uncertainty) controllers as the model uncertainty increases.

ACKNOWLEDGMENT

REFERENCES

[1] "DOE Building Energy Data Book". *http://buildingsdatabook.eren.doe.gov/*. 2013.
[2] F. Oldewurtel et al. "Use of Model Predictive Control and Weather Forecasts for Energy Efficient Building Climate Control". *Energy and Buildings*, vol. 45, pp. 15-27, 2012.
[3] Y. Ma et al. "Model Predictive Control for the Operation of Building Cooling Systems". In *ACC*, pp. 5106-5111, Baltimore, MD, 2010.
[4] M. Maasoumy and A. Sangiovanni-Vincentelli. "Total and Peak Energy Consumption minimization of Building HVAC Systems Using Model Predictive Control". In *Design and Test of Computers*, 2012.
[5] S. Prívara et al. "Building Modeling as a Crucial Part for Building Predictive Control". *Energy and Buildings*, vol. 56, pp. 8-22, 2013.
[6] M. Maasoumy and A. Sangiovanni-Vincentelli. "Modeling and Optimal Control Algorithm Design for HVAC Systems in Energy Efficient Buildings". In *MSc Thesis, EECS Department, University of California, Berkeley*, 2011.
[7] M. Maasoumy et al. "Model-Based Hierarchical Optimal Control Design for HVAC Systems". In *ASME, DSCC*, Arlington, VA, 2011.
[8] F. Oldewurtel et al. "Energy Efficient Building Climate Control Using Stochastic Model Predictive Control and Weather Predictions". In *ACC*, pp. 5100-5105, 2010.
[9] X. Xu et al. "Robust MPC for Temperature Control of Air-Conditioning Systems Concerning on Constraints and Multiple Uncertainties". In *Building Services Engineering Research and Technology*, vol. 31, no. 1, pp. 39-55, 2010.
[10] J. Širokỳ et al. "Experimental Analysis of Model Predictive Control for an Energy Efficient Building Heating System". In *Applied Energy*, vol. 88, no. 9, pp. 3079 3087, 2011.
[11] J. Ma et al. "Economic Model Predictive Control for Building Energy Systems". In *Innovative Smart Grid Technologies (ISGT)*, Detroit, MI, 2011.
[12] P.D. Morosan et al. "Building Temperature Regulation Using a Distributed Model Predictive Control". In *Energy and Buildings*, vol. 42, no. 9, pp. 1445-1452, 2010.
[13] Y. Ma et al. "A Distributed Predictive Control Approach to Building Temperature Regulation". In *ACC*, San Francisco, CA, 2011.
[14] M. Maasoumy and A. Sangiovanni-Vincentelli. "Optimal Control of HVAC Systems in the Presence of Imperfect Predictions". In *DSCC*, Fort Lauderdale, FL, 2012.
[15] F. Oldewurtel et al. "A Tractable Approximation of Chance Constrained Stochastic MPC Based on Affine Disturbance Feedback". In *CDC*, pp. 4731-4736, 2008.
[16] M. Maasoumy et al. "Online Simultaneous State Estimation and Parameter Adaptation for Building Predictive Control". In *DSCC*, Stanford, CA, Accepted in May 2013.
[17] D. Bertsimas and M. Sim. "Tractable Approximations to Robust Conic Optimization Problems". *Mathematical Programming*, vol. 107, no. 1-2, pp. 5-36, 2006.
[18] J. Löfberg. "Minimax Approaches to Robust Model Predictive Control". *Dept. of Electrical Eng. Linkoping University*, 2003.
[19] Y. Ma et al. "Model Predictive Control of Thermal Energy Storage in Building Cooling Systems". In *Control System Magzine*, pp. 1-65, 2011.
[20] J. Löfberg. "YALMIP : A Toolbox for Modeling and Optimization in MATLAB". In *Proceedings of the CACSD Conference, http://users.isy.liu.se/johanl/yalmip*, Taipei, Taiwan, 2004.
[21] "IBM ILOG CPLEX Optimizer". In *www.ibm.com/ (cplex-optimizer)*.
[22] D. Gyalistras and M. Gwerder. "Use of Weather and Occupancy Forecasts for Optimal Building Climate Control (OptiControl): Two Years Progress Report". In *ETH Zurich*, Zug, Switzerland, 2010.