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A PROGRAM FOR CONSTRUCTION OF A REGGE THEORY WITH CROSSING SYMMETRY AND UNITARITY*

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ABSTRACT

A scheme for construction of a crossing-symmetric, unitary Regge theory of meson-meson scattering is reviewed. The construction proceeds by solution of the non-linear crossing-unitarity equations in a new formulation based entirely on partial waves. The high-energy divergences encountered in earlier formulations are avoided.

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1. INTRODUCTION

Analytic S-matrix theory[1]) has certain advantages not enjoyed by other approaches to the strong interactions. A continuing development of the theory will be rewarding, I think, even if a quark field theory prevails as a view of subhadronic physics. The calculation of hadronic scattering, say under ordinary conditions at low energy, is currently beyond the aspirations of field theory, and probably will remain so for a long time. By contrast, there are opportunities to calculate scattering in S-matrix theory, with increasing prospects of success. There have been substantial clarifications of the mathematical structure of the theory since its first heyday in the nineteen-sixties, and relevant experimental information is also much more abundant.

The idea is to begin with a scheme containing certain phenomenological elements as needed, and to remove those elements in a step-wise fashion as understanding increases, adhering as much as possible to the principles of analyticity, crossing-symmetry, and unitarity, and the demands of mathematical consistency. Such a program is best begun in the domain of "old" physics, where the experimental data are more plentiful; namely, spectroscopy and scattering of ordinary hadrons in the low-energy resonance region, and scattering at high energy but low momentum-transfer. An extension to "new" physics should be possible when the time is ripe.

S-matrix theory is unique in providing a suitable setting for Regge theory. That is an important advantage, since by now it seems clear that the ideas of Regge theory do relate to the real world, even if many points in Regge phenomenology remain obscure.[2] Attempts to carry Regge theory beyond phenomenology[3,4] lead to an interesting new definition of Reggeon exchange, i.e., exchange of composite particles. The definition bears the closest possible analogy to elementary particle exchange, and is meaningful at all energies. A semi-empirical study of the Reggeon exchange terms should throw light on the ideas of exchange degeneracy, duality, and planarity, which are pivotal in identifying the role of quarks in a S-matrix view.[5]

The frustration with ambitious S-matrix schemes encountered in the late sixties was due in part to technical difficulties. Attempts to make a theory of π-π scattering, incorporating crossing symmetry, unitarity, and Regge behavior, led to integral equations with apparent high-energy divergences.[6] Nevertheless, the physical ideas of the model seemed reasonable, and with some modifications might be acceptable today. It has recently been shown that divergences in fact do not appear in a new, more appropriate form of the crossing-unitarity equations.[3,4] This result was obtained after systematic studies of various crossing-unitarity equations of ascending difficulty using methods of non-linear functional analysis (fixed point theorems and implicit function theorems). In proving existence theorems by the fixed point method, one seeks an appropriate Banach space B of scattering amplitudes, and a subset of B mapped into itself by the non-linear

crossing-unitarity operator. The choice of amplitudes, of the space B, and of the form in which the operator is represented, are all not obvious at the start. The search for appropriate choices has led to a seemingly unique satisfactory form for the crossing-unitarity equation in Regge theory.[3]

2. PROGRESS TOWARD A REGGE THEORY

The integral equations of Ref.3 are the essential tool for construction of a scattering amplitude $A(s,t)$ [for neutral, unit-mass, pseudoscalar mesons] with the following properties:

(i) Mandelstam analyticity[1,2] in s and t, with the correct support of double spectral regions.

(ii) Exact crossing symmetry: $A(s,t) = A(t,s)$.

(iii) Exact unitarity in the elastic region, $4 \leq s \leq 16$.

(iv) Inelasticity of the multi-peripheral type for $s > 16$ (namely, the crossed two-particle processes required by crossing symmetry[1]), plus any desired additional inelasticity. The latter is obtained from a model external to the scheme, and is introduced in a way such as to preserve crossing symmetry, through the central spectral function $v(s,t)$.

(v) Partial wave amplitudes $a(\ell,s)$, meromorphic in ℓ for $\mathrm{Re}\,\ell > -\epsilon$, $0 < \epsilon < 1/2$, with Regge trajectories $\alpha(s)$ such that $\alpha(0) \leq 1$, $\max\ \mathrm{Re}\,\alpha(s) \leq L < \infty$, where L may be arbitrarily large. Correspondingly, $A(s,t)$ has the usual Regge behavior for large s and fixed t.

The following remarks are in order:

(a) The existence of a solution of the equations, yielding an amplitude with the properties listed, has been proved for the case of "weak coupling", in which no Regge pole enters the right-hand half plane, $\mathrm{Re}\,\ell > -\epsilon$. Weak coupling means that the externally prescribed central spectral function $v(s,t)$ is sufficiently small and smooth. The central spectral function is defined by the equation

$$\rho(s,t) \;=\; \rho^{e\ell}(s,t) + \rho^{e\ell}(t,s) + v(s,t),$$

where $\rho^{e\ell}(s,t)$ is the part of the total spectral function $\rho(s,t)$ having only s-channel elastic contributions. In one version of the theory, a subtraction constant and CDD poles are allowed in the s-wave. For weak coupling, the constant and the CDD pole residues must be sufficiently small.

(b) The inelastic unitarity constraint, $0 \leq \eta(\ell,s) \leq 1$, is not automatically guaranteed by the equations, but one should be able to adjust $v(s,t)$, if necessary, so as to meet that constraint. In any case, the partial waves are bounded at high energy. Such a bound was not assured in earlier formulations.

(c) The equations may be generalized to allow arbitrarily many coupled two-body channels, still with exact crossing symmetry. One then has n-channel unitarity, plus the

multi-peripheral processes obtained by crossing all of the two-particle contributions. As more and more channels are included, more and more of the interior part of the double spectral function is built up dynamically, and there is less reliance on the externally prescribed central spectral function.

(d) An extension to include spin should be straightforward, and is on the schedule for future work.

(e) The issue of how to suppress ghost poles on Regge trajectories must be faced.[2]) One method is to make the physical s-wave different from the continuation of the ℓ-analytic amplitude $a(\ell,s)$ to $\ell = 0$, by introduction of an s-wave subtraction constant.[3]) This method has implications for high energy scattering, which might be tested experimentally.

3. A VERY QUICK SKETCH OF THE METHOD

The equations are formulated entirely in terms of partial waves. In a primitive form the crossing-unitarity equation is just the Froissart-Gribov formula construed as an integral equation:

$$a(\ell,s) \;=\; \frac{4}{\pi(s-4)} \int_4^\infty dt\,Q_\ell\!\left(1 + \frac{2t}{s-4}\right) A_t(s,t). \qquad (1)$$

The t-channel absorptive part, A_t, is written as a non-linear functional of the partial waves $a(\ell,s)$ with complex ℓ, in such a way that elastic unitarity and crossing symmetry are built in. The necessary analytic continuation of A_t outside the t-channel physical region is accomplished by means of a Watson-Sommerfeld transformation. Thus, Eq. (1) is an integral equation for $a(\ell,s)$, but it is not in a useful form as it stands. There are two related reasons for transforming the equation into a non-linear functional equation based on the N/D method. First, the transformation results in a well-posed mathematical problem involving a compact operator, suitable for numerical computation as well as analysis. Second, the N/D system is physically reasonable as a basis for dynamics: it allows for dynamical generation of Regge trajectories as zeros of D, and reveals the ambiguities of CDD type which are possibly important for the correct construction of the S-matrix. Formal expressions for the inputs to the N/D equation (the left-hand cut part of $a(\ell,s)$, and a piece of the right-hand cut part) may be read off from (1), but those expressions seem to entail divergences at large s. By a somewhat intricate rearrangement of integrals through contour deformations, one can show that the divergences actually are absent. Some minimal analyticity properties of the trajectory $\alpha(t)$ must be assumed. In fact, Regge theory without analyticity of α is nearly inconceivable, since analyticity seems necessary for definition of the Froissart-Gribov amplitude at small values of $\mathrm{Re}\,\ell$.

4. A NEW DEFINITION OF REGGEON EXCHANGE, MEANINGFUL AT ALL ENERGIES

Let $A_t^{e\ell}$ be the elastic part of the t-channel absorptive part, and take $t > 4$. If there is one Regge pole in the right-hand

half-plane, a Watson-Sommerfeld transformation of the Legendre series for $A_t^{e\ell}$ yields

$$A_t^{e\ell}(s,t) = -\pi\theta(s_1-t)\Delta_t\left\{\frac{[2\alpha(t)+1]\beta(t)}{\sin\pi\alpha(t)}\,P^{(e)}_{\alpha(t)}\left(1+\frac{2s}{t-4}\right)\right\}$$
$$+\frac{i}{2}\int_{\mathrm{Re}\,\ell=-\epsilon} d\ell\;\frac{q(t)\,a(\ell,t_+)a(\ell,t_-)}{\sin\pi\ell}\,P^{(e)}_{\ell}\left(1+\frac{2s}{t-4}\right), \qquad (2)$$

where $P^{(e)}_\ell$ is the even part of the Legendre function, s_1 is the squared energy at which the Regge pole leaves the half-plane, $q(t)$ is a phase space factor, $\beta(t)$ is the residue in the pole of $a(\ell,t)$ at $\ell=\alpha(t)$, and Δ_t denotes the discontinuity over the real t-axis: $\Delta_t f(t) = [f(t_+) - f(t_-)]/2i$. When the first term of (2) is introduced in place of A_t in (1), the resulting amplitude is the Reggeon exchange term in the s-channel partial wave. Actually, one should take a small piece of the background integral in (2) along with the pole term to get a function with the proper spectral support. The corrected Reggeon exchange term is then

$$a^R(\ell,s) = -\frac{4}{s-4}\int_4^{s_1} dt\, Q_\ell\left(1+\frac{2t}{s-4}\right)\Delta_t\left\{\frac{[2\alpha(t)+1]\beta(t)}{\sin\pi\alpha(t)}\right.$$
$$\left.\times\left[P^{(e)}_{\alpha(t)}\left(1+\frac{2s}{t-4}\right)+\frac{\sin\pi\alpha(t)}{2\pi}\int_0^{\tau(s)}\frac{ds'\,P_{\alpha(t)}\left(1+\frac{2s'}{t-4}\right)}{s'-s}\right]\right\} \qquad (3)$$

where $\tau(s) = 16s/(s-4)$. Notice that the integral is over positive t, the "spectroscopic region" of the trajectory, and that there is no reference to a high-s limit. The amplitude appears to behave as $(\ell n\ s)\, s^{\max(\mathrm{Re}\,\alpha)-1}$ at large s, which would be a disastrous violation of unitarity. An appropriate deformation of the t contour shows, however, that the amplitude is actually bounded.

It is interesting to search for experimental evidence of Reggeon exchange in low-energy processes for which phase shifts have been determined. Studies to look for such evidence, by computing the left-cut parts of partial waves from phase shifts, are under way. It is likely that Reggeon exchange accounts for a large part of the left-cut term in many non-exotic channels. In that case one has an important key to a qualitative understanding of the complicated equations.

5. NUMERICAL SOLUTION OF THE FULL EQUATIONS

Numerical solution is feasible in the weak-coupling case. The operator involved is contractive, and simple iteration will yield a locally unique solution. Numerical treatment of the strong-coupling case should also be possible, but the choice of method for a realistic solution may require experimentation with different methods, so as to improve one's understanding of the dynamical genesis of Regge trajectories. It may happen that trajectories are generated from a basic short-range interaction, analogous to the interaction in field theory. For instance, a meson-meson interaction, analogous to that of a $\lambda\phi^4$ theory, may be introduced through a subtraction constant in the s-wave N/D equation. At the other extreme, one might have a bootstrap theory with homogeneous equations, in which no subtraction constant would be allowed. The existence of non-trivial solutions of homogeneous bootstrap equations is an

entirely open question, however. A third possibility, conjectured but not yet established, is that rather arbitrary trajectories can be put into the theory from the outside, by choosing a $v(s,t)$ such as to give Regge poles in the overlap function. This possibility is not necessarily incompatible, in principle, with the first two. The arbitrariness of the externally specified trajectories may be understood as a symptom of failure to take all possible channels into account. Only in a complete theory with all channels will the trajectories be uniquely determined, whether by specification of basic interactions, or by the bootstrap condition.

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