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Frequency- and Time-Domain Green's Functions for a Phased Semi-Infinite Periodic Line Array of Dipoles

Filippo Capolino, *Member, IEEE*, and Leopold B. Felsen, *Life Fellow, IEEE*

Abstract—In this paper, we extend a previous prototype study [1] of frequency-domain (FD) and time-domain (TD) Green's functions for an infinite periodic phased line array of dipoles to account for the effects of truncation, as modeled by a semi-infinite array. These canonical problems are to be used eventually for the systematic analysis and synthesis of actual rectangular TD plane phased arrays. In the presentation, we rely on the analytic results and phenomenologies pertaining to the infinite array, which are reviewed. Major emphasis is then placed on the modifications introduced by the truncation. Finite Poisson summation is used to convert the individual dipole radiations into collective truncated wavefields, the FD and TD Floquet waves (FW). In the TD, exact closed-form solutions are obtained, and are examined asymptotically to extract FD and TD periodicity-matched conical truncated FW fields (both propagating and nonpropagating), corresponding tip-diffracted periodicity-matched spherical waves, and uniform transition functions connecting both across the FD and TD-FW truncation boundaries. These new effects can again be incorporated in a FW-modulated geometrical theory of diffraction. A numerical example of radiation from a finite phased TD dipole array with band-limited excitation demonstrates the accuracy and efficiency of the FW-(diffracted field) asymptotic algorithm when compared with an element-by-element summation reference solution.

Index Terms—Antenna arrays, antenna transient analysis, arrays, Floquet waves (FW), periodic structures, time-domain analysis, time-domain Green's functions, transient analysis.

I. INTRODUCTION

IN a planned sequence of physics-based prototype studies aimed toward the construction of time-domain (TD) Green's functions for truncated planar phased periodic arrays, we have first explored an *infinite* periodic *line array* of sequentially pulsed axial dipole radiators. That problem has been solved via a variety of alternative techniques which have granted different insights into the collective behavior of the dipole-excited wavefield as expressed by TD Floquet waves (FW) in the propagating *and* nonpropagating parameter regimes. In

Fig. 1. Semi-infinite periodic line array of axial electric current dipoles, distributed along z -axis locations $z' = nd, 0 \leq n < \infty$. d = interelement spacing; $k = \omega/c$ = free space wavenumber; c = ambient wavespeed; $k\eta = \omega\gamma_z$ = interelement phasing; $\gamma_z = \eta/c = 1/v_z^{(p)}$ = interelement "slowness" (normalized wavenumber); $v_z^{(p)} = c/\eta$ = phase speed along z . In cylindrical (ρ, z) coordinates, $R(nd)$ is the distance from the n -th element to the observer; $R_d \equiv R(0)$ and $\beta_d \equiv \beta(0)$ highlight spherical wave diffraction from the array tip later on.

the present paper, we explore truncation effects associated with a *semi-infinite* periodic line array of sequentially pulsed axial dipoles arranged along the z -axis of the cylindrical (ρ, z) coordinate system in Fig. 1. This modification of the infinite line dipole array leads to new truncation-induced TD phenomena. Because these new phenomena are parameterized in terms of the TD-FW behavior of the infinite array, we shall employ in the presentation below the same layout and notation as in [1]. This implies inclusion of frequency-domain (FD) formulations as preliminaries to, and explanations of, the TD phenomenologies. After the problem statement in Section II, for convenience of the reader, the key results required from [1] are summarized (Section III), with references to equations, sections, figures, etc., in [1]. The truncation problem, in the FD and TD, is addressed thereafter in Sections IV–VI, with conclusions in Section VII.

II. PROBLEM FORMULATION AND STRATEGY

The truncated dipole line array geometry with period d is shown in Fig. 1. As in [1, Sec. II], the radiated electric field E is related to the z -directed magnetic scalar potential A in the frequency domain and to $\hat{A}$ in the time domain, with the caret $\hat{\cdot}$ denoting time-dependent fields throughout. Frequency-domain ($\exp(j\omega t)$ dependence) and time-domain potential fields are related by the Fourier transform pair [1, eq. (1)]

$$\begin{aligned} A(\mathbf{r}, \omega) &= \int_{-\infty}^{\infty} \hat{A}(\mathbf{r}, t) e^{-j\omega t} dt \\ \hat{A}(\mathbf{r}, t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\mathbf{r}, \omega) e^{j\omega t} d\omega. \end{aligned} \quad (1)$$

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Similarly, the phased array FD and TD dipole currents $J(\omega)$ and $\hat{J}(t)$, respectively, are given by

$$\left. \begin{matrix} J(\omega) \\ \hat{J}(t) \end{matrix} \right\} = \sum_{n=0}^{\infty} \delta(z' - nd) \left\{ \begin{matrix} e^{-j\eta knd} \\ \delta(t - \eta nd/c) \end{matrix} \right\}, \quad k = \frac{\omega}{c} \quad (2)$$

where $z' = nd$ is the n th dipole location, $k = \omega/c$ denotes the ambient wavenumber, c denotes the ambient wavespeed, and where we have introduced the important nondimensional parameter

$$\eta = c\gamma_z = c/v_z^{(p)}. \quad (3)$$

Here, $\gamma_z = \eta/c = 1/v_z^{(p)}$ is the normalized (with respect to ω) interelement phase gradient, and $v_z^{(p)} = c/\eta$ is the corresponding impressed phase speed along the array. Thus, in the FD portion of (2), $\omega\gamma_z d = \eta kd$ accounts for an assumed (linear) phase difference between adjacent elements. In the sequentially pulsed TD portion, the dipole element at $z' = nd$ is turned on at time $t_n = \eta nd/c$. The collective effect of the truncated periodic array of *individual* phased dipole radiators is obtained by application of the truncated Poisson sum formula for a series with n -indexed elements [2, pp. 47]

$$\sum_{n=0}^{\infty} F_n = \frac{F_0}{2} + \sum_{q=-\infty}^{\infty} \tilde{F}_q, \quad \tilde{F}_q = \int_0^{\infty} F(\nu) e^{-j2\pi q\nu} d\nu \quad (4)$$

which converts the semi-infinite FD or TD n -series into an infinite FD or TD q -series whose summands $\tilde{F}_q$ are the truncated Fourier transforms of the smoothed-out function $F(\nu)$, sampled at integer multiples of 2π . Referring to [1], for the n -sum of sources in (2), the $\tilde{F}_q$ functions represent *truncated* linearly *smoothly* phased equivalent *line source* distributions along the axis of the array. The wavefields excited by these initial conditions comprise propagating and nonpropagating FW, as well as an end point contribution that behaves like a tip diffracted field. Instead of using the designation “evanescent” FW for the nonpropagating case as in [1], which commonly implies exponential decay, we prefer to change the terminology to “nonpropagating” or “nonradiating” FW because this better describes the associated FW behavior in the *time domain* [see discussion following (11)].

The quantitative treatment of these fields and phenomenologies is the subject of the remainder of this paper.

III. PRELIMINARIES: FD- AND TD-FW FOR THE INFINITE LINE ARRAY

Results pertaining to the *infinite* phased (sequentially excited) periodic line array of dipoles in [1] are now summarized. In this case the n -sums in (2), (4), and Fig. 1 extend from $-\infty$ to $+\infty$, the truncation term $F_0/2$ in (4) is omitted, and the $\tilde{F}_q$ integral, now from $\nu = -\infty$ to $+\infty$, becomes the complete Fourier transform (FT). References to the “infinite version” of equations, etc., are denoted by $(\dots\infty)$, i.e., (4 ∞) is (4) modified so as to pertain to the infinite array.

Fig. 2. FD-FW phenomenology for the truncated case (planar cut through conical wavefields). The truncated continuous Floquet-type line source distribution is schematized by the thick line for $z > 0$. Conical propagating FWs with $q = 0$ and $q = -1$ arrive from directions β_0 and β_{-1} , respectively. Spherical q th tip diffracted field arrives from direction β_d . Conical ray of a propagating FW $_q$ and its truncation shadow boundary SB_q are parallel. Each SB_q cone is surrounded by a conical parabolic transition region (shaded).

A. Frequency-Domain FW

Applying (4 ∞) to the radiated spherical FD fields $F_n \equiv A_n(\mathbf{r}, \omega)$ from each dipole [1, eq. (6)], with

$$A_n(\mathbf{r}, \omega) = \frac{e^{-jkR(nd)}}{4\pi R(nd)} e^{-j\eta knd} \quad R(z') = \sqrt{\rho^2 + (z - z')^2} \quad (5)$$

yields for $\tilde{F}_q$ at $\mathbf{r} = (\rho, z)$ the FW $\tilde{F}_q \equiv A_q^{\text{FW}}(\mathbf{r}, \omega)$ which is the FD-FW $_q$ smoothly phased line source field expressed by [3, pp. 493]

$$A_q^{\text{FW}}(\mathbf{r}, \omega) = \frac{e^{-jk_{zq}z}}{4jd} H_0^{(2)}(k_{\rho q}\rho) \sim \frac{e^{-j(k_{\rho q}\rho + k_{zq}z + \pi/4)}}{2d\sqrt{2\pi k_{\rho q}\rho}}. \quad (6)$$

The FW $_q$ wavenumber along z

$$k_{zq}(\omega) = k\eta + \alpha_q, \quad \alpha_q = 2\pi q/d, \quad q = 0, \pm 1, \pm 2, \dots \quad (7)$$

represents the Floquet-type dispersion relation. The radial FW $_q$ wavenumber $k_{\rho q}$ is given by

$$k_{\rho q}(\omega) = \sqrt{k^2 - k_{zq}^2} \quad (8)$$

where (consistent with the radiation condition at $\rho = \infty$) $\Im m k_{\rho q} \leq 0$ on the top Riemann sheet, while $\Re e k_{\rho q} \geq 0$ or ≤ 0 for $\omega > 0$ or < 0 , respectively. In (6), $|k_{zq}| < |k|$ characterizes radially propagating FW while $|k_{zq}| > |k|$ characterizes radially nonpropagating FW (which in the FD are also evanescent). Each propagating FW contributes at $\mathbf{r}$ a ray-asymptotic field lying on a ray cone with semiangle (see Fig. 2 ∞) [1, Fig. 2]

$$\beta_q = \cos^{-1}(k_{zq}/k), \quad \omega > 0 \text{ or } \omega < 0. \quad (9)$$

When $|k_{zq}| > |k|$, the cone angle becomes complex, with $k_{\rho q} = -j|k_{\rho q}|$, yielding a corresponding evanescent field along ρ ; accordingly, the evanescent FW portion of $\sum_q A_q^{\text{FW}}$ converges rapidly away from the array axis.

B. Time-Domain FW: Closed-Form Solutions

Applying (4 ∞) to the TD impulsive individually radiated fields $F_n \equiv \hat{A}_n(\mathbf{r}, t)$, with [see (1) and (5)]

$$\hat{A}_n(\mathbf{r}, t) = \delta[t - \eta nd/c - R(nd)/c]/(4\pi R(nd)) \quad (10)$$

Fig. 3. TD-FW phenomenology for the truncated radiating case $|\eta| < 1$ (planar cut through conical wavefields). (a) Turn-on time $t = t_0(\rho, z)$; all TD-FW $_q$ arrive from the direction β_0 , independent of q . Current impulse location: $z' = t_0 c/\eta$; first signal arrival originates at z'_0 . (b) $t > t_0$: $z'_1(t)$ and $z'_2(t)$ are the two real (propagating) solutions $z'_i(t) > 0$ for $i = 1, 2$ of the TD-FW dispersion (11), arriving simultaneously from the q -independent directions $\beta_1(t)$ and $\beta_2(t)$. (c) Later instant: contribution from $z'_1(t)$ only because $z'_2(t)$ is now off the array.

yields a closed form for the ν -integral $\tilde{F}_q \equiv \hat{A}_q^{\text{FW}}(\mathbf{r}, t)$ in (4 ∞). The δ function in $\hat{A}_\nu(\mathbf{r}, t)$ [see (10)] contributes to the integral only for those ν -values in $-\infty < \nu < \infty$ that satisfy

$$c\tau + \eta(z - z') - R(z') = 0, \quad \tau = t - \eta z/c \quad (11)$$

where the new variable $z' = \nu d$ identifies the source location in the smoothed out equivalent line source distribution. The radiating case is parameterized by $|\eta| < 1$, the nonradiating case by $|\eta| > 1$, and the transition between the two by $|\eta| = 1$. The “radiating” case is characterized by the presence of a wavefront and its turn-on time. The “nonradiating” case is characterized by the absence of a wavefront (i.e., no turn-on-time).

1) *Radiating Case* $|\eta| < 1$: With $\tau > 0$, (11) has the two solutions [1, eq. (14)]

$$z'_i(t) = z - \frac{c}{1 - \eta^2}(\tau\eta + (-1)^i \sqrt{\tau^2 - \tau_0^2}), \quad i = 1, 2 \quad (12)$$

$$\tau_0 \equiv \sqrt{1 - \eta^2} \rho / c = t_0 - \eta z / c \quad (13)$$

with $\Im \sqrt{1 - \eta^2} \leq 0$ in accord with $\Im m k_{\rho q} \leq 0$ in (8) [since $k_{\rho 0} = \sqrt{k^2(1 - \eta^2)}$; see (7) and (8)]. One observes that $z'_i(t)$ does not depend on q , is real for $\tau > \tau_0$, and that z'_1 and z'_2 coincide at the causal turn-on time $\tau = \tau_0$, i.e., from (11), $t_0 = \tau_0 + \eta z/c = (\eta z + \sqrt{1 - \eta^2} \rho)/c$, as in Fig. 3(a) [1, Fig. 3]. For $\tau > \tau_0$, i.e., $t > t_0$, these solutions move toward $z'_1(t) \rightarrow +\infty$ and $z'_2(t) \rightarrow -\infty$ (Fig. 3(b)), as modified by extension (not shown) of the dipole array to $z = -\infty$. For $\tau < \tau_0$, i.e., $t < t_0$, the integral in (4 ∞) vanishes (causality) since the two solutions are conjugate complex so that the delta function in (10) does not contribute. The two causal wave fields corresponding to $z'_1(t)$ and $z'_2(t)$, which we have referred to as *fundamental* solutions, are [1, eq. (16)]

$$\hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t) = \frac{e^{-j\alpha_q z'_i(t)}}{4\pi d \sqrt{\tau^2 - \tau_0^2}} U(\tau - \tau_0) \quad (14)$$

where $U(\tau - \tau_0) \equiv U(t - t_0)$, with $U(\tau) = 1$ or 0 if $\tau > 0$ or $\tau < 0$, respectively, and $U(0) = 1/2$. A stationary observer at $P(\rho, z)$ is reached by the two $i = 1, 2$ FW wavefields in (14), as shown in Figs. 3(a), (b). The conical wavefront reaches P in Fig. 3(a) at the turn-on time t_0 (i.e., $\tau = \tau_0$), being excited by the current impulse located at $z' = t_0 c/\eta$. The radiation cone angle β_0 defined by $\cos \beta_0 = (z - z'_0)/R(z'_0)$ coincides with that in (9) for $q = 0$ (i.e., $\alpha_q = 0$). Thus, at the turn-on time t_0 , the wavefront arrives at P from a direction coincident with that of the FD-FW $_0$ wave vector $\mathbf{k}_0^{\text{FW}} = (k_{z0}, k_{\rho 0})$.

At times $t > t_0$ ($\tau > \tau_0$), the current impulse is located at $z' = tc/\eta = tv_z^{(p)}$, the wavefront has moved beyond P to the location shown in Fig. 3(b) [1, Fig. 3(b)], and the observer is reached by two distinct wavefields that arrive simultaneously but are launched from points $z'_i(t)$, $i = 1, 2$ at times $t'_i = t - R(z'_i(t))/c$, respectively, with launch angles $\beta_i(t)$ [Fig. 3(b)]

$$\cos \beta_i = (z - z'_i(t))/R(z'_i(t)), \quad i = 1, 2. \quad (15)$$

For $|q| > 0$ ($\alpha_q \neq 0$), where the result in (14) is complex due to the $\exp(-j\alpha_q z'_i)$ term, we take

$$\Re \hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t) = \frac{\cos(\alpha_q z'_i(t))}{4\pi d \sqrt{\tau^2 - \tau_0^2}} U(\tau - \tau_0) \quad (16)$$

with $i = 1, 2$, to obtain a real causal wavefield which is the same for $(+q)$ and $(-q)$ since $\alpha_{-q} = -\alpha_q$. However, adding the $(+q)$ and $(-q)$ contributions for $i = 1$ or 2 directly from (14) yields *a priori* the real field

$$\hat{A}_i^{q|\text{FW}} \equiv \hat{A}_{+q,i}^{\text{FW}} + \hat{A}_{-q,i}^{\text{FW}} = 2\Re \hat{A}_{q,i}^{\text{FW}} \quad (17)$$

which we have referred to as physically *observable*, in contrast to the complex “fundamental” solution in (14) and its corresponding real version in (16). For a more detailed discussion, see [1].

2) *Nonradiating Case* $|\eta| > 1$: Now, from (13), we have $\sqrt{1 - \eta^2} = -j\sqrt{\eta^2 - 1}$ and thus $\tau_0 = -j|\tau_0|$. In this case, as shown in [1, Appendix A], (11) has only a single real solution $z'_i(t)$, valid for $\tau > 0$ and $\tau < 0$ provided that $i = 1$ for $\eta > 0$ and $i = 2$ for $\eta < 0$. Thus, the FW wavefield corresponding to (14) is

$$\hat{A}_q^{\text{FW}}(\mathbf{r}, t) = \frac{e^{-j\alpha_q z'_i(t)}}{4\pi d \sqrt{\tau^2 + |\tau_0|^2}} \quad i = \begin{cases} 1, & \eta > 0 \\ 2, & \eta < 0 \end{cases} \quad (18)$$

for all τ . This FW field exists in *all of space at all times*, and *there is no wavefront* (a wavefront characterizes a *causal radiating* TD field). Again, for $\alpha_q \neq 0$, the result in (18) is complex, and a real field is obtained from (18) by taking $\Re \hat{A}_q^{\text{FW}}$; but also in this case, adding the $(+q)$ and $(-q)$ FW $_q$ directly from (18) yields *a priori* a real wavefield

$$\hat{A}^{q|\text{FW}} \equiv \hat{A}_q^{\text{FW}} + \hat{A}_{-q}^{\text{FW}} = 2\Re \hat{A}_q^{\text{FW}}. \quad (19)$$

For a “moving particle” interpretation of the FW field, see [1].

C. TD-FW: Asymptotic Inversion From the FD

Using (1), with (6), yields $\hat{A}_q^{\text{FW}} \sim \int_{-\infty}^{\infty} F(\omega) e^{-j\hat{\psi}(\omega)} d\omega$, $F(\omega) = \exp(-j\pi/4)(4\pi d\sqrt{2\pi k_{\rho q}\rho})^{-1}$, where

$$\hat{\psi}(\omega) = k_{zq}z + k_{\rho q}\rho - \omega t \quad (20)$$

with $k_{zq}(\omega)$ and $k_{\rho q}(\omega)$ given in (7) and (8), respectively.

1) $q \neq 0$ *Asymptotic TD-FW*: By high-frequency asymptotics based on the stationary (saddle) points ω_q of $\hat{\psi}(\omega)$, defined by $(d\hat{\psi}/d\omega)|_{\omega_q} = 0$, one finds for the radiating case ($|\eta| < 1$) with $q \neq 0$ the real saddle point frequencies (see [1, Appendix A])

$$\omega_{q,i}(\mathbf{r}, t) = \frac{\alpha_q c}{1 - \eta^2} \left(\eta + \frac{(-1)^i \tau}{\sqrt{\tau^2 - \tau_0^2}} \right), \quad i = 1, 2 \quad (21)$$

for $\tau > \tau_0$, with τ and τ_0 defined in (11) and (13), respectively. The two solutions in (21) identify the *local instantaneous* frequencies of oscillation of the TD-FW_q at a given point $\mathbf{r}$ and a given instant τ in the causal domain $\tau > \tau_0$ ($t > t_0 = \eta z/c + \tau_0$). The frequencies increase with FW mode index q , decrease with time t , and approach their cutoff value $\omega_{q,i}(t \rightarrow \infty) = \omega_{q,i}^{\text{cutoff}} = \alpha_q c / [(-1)^i - \eta]$, in accord with $|k_{zq}| = |k|$ in (8). The corresponding instantaneous wavenumbers $k_{q,i}(t)$, $k_{zq,i}(t)$, and $k_{\rho q,i}(t)$, are obtained from (7) and (8). The instantaneous FW propagation angles $\beta_{q,i}(t)$, $i = 1, 2$, defined from (9) as $\cos \beta_{q,i}(t) = k_{zq,i}(t)/k_{q,i}(t)$, $i = 1, 2$, are independent of q and equal to $\beta_i(t)$ in (15) [1].

The standard asymptotic evaluation of the q th TD-FW integral yields [3, pp. 382]

$$\hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t) \sim F(\omega_{q,i}) \frac{\sqrt{2\pi} e^{-j\hat{\psi}(\omega_{q,i}(t))}}{\sqrt{j(d^2/d\omega^2)\hat{\psi}|_{\omega_{q,i}}}} U(\tau - \tau_0), \quad i = 1, 2. \quad (22)$$

From (20) and (21), one finds $\hat{\psi}(\omega_{q,i}(t)) = \alpha_q z'_i(t)$ and $(d^2/d\omega^2)\hat{\psi}|_{\omega_{q,i}} = -\rho \alpha_q^2 / (c^2 k_{\rho q,i}^3(t))$, which yields for the TD-FW wavefield in (22) the same result as in (14). The causal $\tau > \tau_0$ condition, $U(\tau - \tau_0)$, conforms with the domain of existence of real saddle point frequencies $\omega_{q,i}$. In the nonradiating case $|\eta| > 1$ there is only one single real solution of $(d\hat{\psi}/d\omega)|_{\omega_q} = 0$ given by $\omega_{q,1}$ when $\eta > 0$ and by $\omega_{q,2}$ when $\eta < 0$, for all times $-\infty < \tau < \infty$. The corresponding saddle point evaluation via (22) (without the U function) for the q th TD-FW yields the same result as in (18). Agreement between the lowest order asymptotic and exact solutions here indicates that the instantaneous frequency parameterization captures the essential features of the TD infinite array problem.

2) $q = 0$ *Asymptotic TD-FW (New Addition to [1])*: For $q = 0$, the FD-FW phase in (20) is linearly dependent on ω [see (7)] and therefore not amenable to saddle point asymptotics. However, the ω -integral (1) of the asymptotic expression of the FD-FW in (6) can be inverted exactly. Since $\hat{A}_0^{\text{FW}}(\mathbf{r}, -\omega) = (\hat{A}_0^{\text{FW}}(\mathbf{r}, \omega))^*$, where the asterisk $*$ denotes the complex conjugate, (1) can be performed by integration over positive frequencies. Taking the real part yields

$$\hat{A}_0^{\text{FW}}(\mathbf{r}, t) \sim (2\pi d\sqrt{2\pi})^{-1} \Re \int_0^\infty d\omega \exp(j\omega(t-t_0)) / \sqrt{j\tau_0\omega}$$

which includes both the $i = 1$ and $i = 2$ contributions, and where $t_0 = \tau_0 + \eta z/c$ and $\tau_0 = \sqrt{1 - \eta^2} \rho/c$. The integral is evaluated in closed form using the change of variable $u = \sqrt{\omega}$, leading to $\hat{A}_0^{\text{FW}} \sim (2\pi d)^{-1} \Re e(2\tau_0(t - t_0))^{-1/2}$. For the radiating case, the result

$$\hat{A}_0^{\text{FW}}(\mathbf{r}, t) \sim \frac{1}{2\pi d\sqrt{2\tau_0}} \frac{U(\tau - \tau_0)}{\sqrt{\tau - \tau_0}} \quad (23)$$

is the wavefront approximation (for $\tau \approx \tau_0$) of the exact TD-FW $\hat{A}_0^{\text{FW}} = \hat{A}_{0,1}^{\text{FW}} + \hat{A}_{0,2}^{\text{FW}}$ in (14). The expression in (23) is restricted to *early observation times* because it is obtained from the inversion of the high-frequency representation of the FD-FW on the right side of (6) [3, pp. 157–158].

This review of the infinite line array problem lays the foundation for the truncated array problem to follow.

IV. TRUNCATED FLOQUET WAVES: FREQUENCY DOMAIN

From here on, the Poisson sum formula in (4) and all figures apply as shown. Accordingly, from (4), the FD potential field $A^{\text{tot}}(\mathbf{r}, \omega)$ at $\mathbf{r} = (\rho, z)$, synthesized by the semi-infinite sum of $A_n(\mathbf{r}, \omega)$ fields generated by the linearly phased dipole array element currents, can be expressed collectively as

$$A^{\text{tot}}(\mathbf{r}, \omega) = \sum_{n=0}^{\infty} A_n(\mathbf{r}, \omega) = \frac{A_0(\mathbf{r}, \omega)}{2} + \sum_{q=-\infty}^{\infty} A_q^T(\mathbf{r}, \omega). \quad (24)$$

Here, A_n is the n th-element free space Green's function in (5), and from (4), $\hat{F}_q \equiv A_q^T$ is the truncated integral (using $\nu = z'/d$) in (25), which can be transformed into the wavenumber spectral integral in (27) via the following series of steps:

$$A_q^T(\mathbf{r}, \omega) = \frac{1}{d} \int_0^\infty \frac{e^{-jkR(z')}}{4\pi R(z')} e^{-jk_{zq}z'} dz' \quad (25)$$

$$= \frac{1}{d} \int_0^\infty dz' \frac{e^{-jk_{zq}z'}}{8\pi j} \int_{-\infty}^\infty dk_z H_0^{(2)}(k_\rho \rho) e^{-jk_z(z-z')} \quad (26)$$

$$= \frac{1}{8\pi d} \int_{-\infty}^\infty \frac{H_0^{(2)}(k_\rho \rho)}{k_z - k_{zq}} e^{-jk_z z} dk_z. \quad (27)$$

In (26), we have inserted the spectral integral representation [3, p. 481] for $[4\pi R(z')]^{-1} \exp(-jkR(z'))$, which is then reduced by interchanging the order of integration and performing the z' integration to obtain the result in (27). The poles at $k_z = k_{zq}$ are located below the k_z -integration path; the radial wavenumber is $k_\rho = \sqrt{k^2 - k_z^2}$, with the top Riemann sheet chosen to render $\Im m(k_\rho) \leq 0$ as in (8).

A. Asymptotic Evaluation of A_q^T

To evaluate the integral in (27) asymptotically, first, the Hankel function is approximated by

$$H_0^{(2)}(x) \sim \sqrt{2/(\pi x)} \exp(j(\pi/4 - x))$$

so that

$$A_q^T(\mathbf{r}, \omega) \sim \frac{e^{j\pi/4}}{4\pi d\sqrt{2\pi\rho}} \int_{-\infty}^\infty \frac{e^{-j(k_z z + k_\rho \rho)}}{k_z - k_{zq}} \frac{dk_z}{\sqrt{k_\rho}}. \quad (28)$$

Fig. 4. Complex θ -plane, showing original integration path C_θ and steepest descent path (SDP) through saddle point $\theta = \beta_d$. In the shaded zones, the integrand in (29), (30) converges exponentially for large $\Im m(\theta)$. A pole at $\theta = \beta_q$ identifies a propagating FW when real and a nonpropagating FW if $\Im m(\beta_q) \neq 0$.

Changing from the cylindrical to spherical coordinates in the spectral and spatial domains via $k_z = k \cos \theta$, $k_\rho = k \sin \theta$ and $z = R_d \cos \beta_d$, $\rho = R_d \sin \beta_d$, transforms (28) into

$$A_q^T(\mathbf{r}, \omega) \sim \frac{e^{j\pi/4}}{4\pi d \sqrt{2\pi k \rho}} \int_{C_\theta} \frac{\sqrt{\sin \theta} e^{-jk R_d \cos(\theta - \beta_d)}}{\cos \theta - \cos \beta_q} d\theta \quad (29)$$

with the real k_z -axis mapped into the complex θ -plane path C_θ along the contour from $-j\infty$ to $\pi + j\infty$ shown in Fig. 4, intended to avoid the pole at $\theta = \beta_q$ in the counterclockwise direction. The poles represent the angular spectra of FWs on the infinite array. The high-frequency asymptotic evaluation of the integral in (29) is carried out by deforming the integration contour into the steepest descent path (SDP) through the pertinent saddle point $\theta = \beta_d$ of the phase (see Fig. 4). When intercepted, the pole residue contributes the FW_q field in (6), i.e., $A_q^{\text{FW}}(\mathbf{r}, \omega) U(\beta_q^{\text{SB}} - \beta_d)$. The SDP integral through the saddle point yields the q th diffracted FW field $A_q^d(\mathbf{r}, \omega)$ emanating from the tip of the array, which is given by the right side of (29) with C_θ replaced by SDP. Rearranging the integrand so that

$$A_q^d(\mathbf{r}, \omega) \sim \frac{e^{j\pi/4}}{4\pi d \sqrt{2\pi k \rho}} \int_{\text{SDP}} Q(\theta) \frac{e^{-jk R_d \cos(\theta - \beta_d)}}{\sin\left(\frac{\beta_q - \theta}{2}\right)} d\theta \quad (30)$$

allows the regular function

$$Q(\theta) = \sqrt{\sin \theta} \sin((\beta_q - \theta)/2) (\cos \theta - \cos \beta_q)^{-1}$$

to be approximated by its saddle point value $Q(\beta_d)$ and to be removed from the integrand (this approximation fails for the limiting case $\beta_d = \beta_q = 0$ or π , where the conical wavefronts collapse onto the z -axis). The remaining integral can be evaluated in closed form [4]

$$\begin{aligned} & \int_{\text{SDP}} \frac{e^{-jk R_d \cos(\theta - \beta_d)}}{\sin\left(\frac{\beta_q - \theta}{2}\right)} d\theta \\ &= \frac{\sqrt{2\pi j}}{\sqrt{k R_d}} \frac{e^{-jk R_d}}{\sin\left(\frac{\beta_q - \beta_d}{2}\right)} F\left(2k R_d \sin^2\left(\frac{\beta_q - \beta_d}{2}\right)\right) \end{aligned} \quad (31)$$

yielding for A_q^d in (30) a locally (pole β_q near saddle point β_d) uniform approximation. The error function $F(x)$ is the transition function of the uniform theory of diffraction (UTD) [5]

$$F(x) = 2j\sqrt{x} e^{jx} \int_{\sqrt{x}}^{\infty} e^{-jt^2} dt, \quad \text{with } -\frac{3\pi}{2} < \arg(x) \leq \frac{\pi}{2}. \quad (32)$$

Summarizing these results we obtain for A_q^T in (27) the following locally uniform asymptotic representation:

$$A_q^T(\mathbf{r}, \omega) \sim A_q^{\text{FW}}(\mathbf{r}, \omega) U(\beta_q^{\text{SB}} - \beta_d) + A_q^d(\mathbf{r}, \omega) \quad (33)$$

where the FW_q field $A_q^{\text{FW}}(\mathbf{r}, \omega)$ is given in (6), and the q th diffracted FW emanating from the tip truncation of the array is given by

$$A_q^d(\mathbf{r}, \omega) \sim \frac{e^{-jk R_d}}{j4\pi d k R_d \cos \beta_q - \cos \beta_d} \frac{F(\delta_q^2)}{\delta_q} \quad (34)$$

$$\delta_q = \sqrt{2k R_d} \sin((\beta_q - \beta_d)/2). \quad (35)$$

The Heaviside unit function $U(\alpha)$ is defined in (14). The physical interpretation of (33) is as follows. Every FW_q in (33) is the same as for the infinite array in (6), but confined by the truncation function $U(\beta_q^{\text{SB}} - \beta_d)$ to the region $\beta_d < \beta_q^{\text{SB}}$ (Fig. 2), with β_q^{SB} locating the conical FW_q shadow boundary (SB). For propagating FWs, one has $\beta_q^{\text{SB}} = \beta_q$ in (9), while for nonpropagating (evanescent) FWs, β_q^{SB} has a more complicated form (see [6]) which is not given here since evanescent FWs can be neglected when observing sufficiently far away from the array axis. However, evanescent FWs excite *propagating diffracted fields* that have to be taken into account and can be evaluated by nonuniform asymptotics that leads to (34) but with $F \approx 1$. In this case, the pole lies on the vertical portions of C_θ in Fig. 4 and (except near “cutoff”, $\beta_q \approx 0, \pi$, from the imaginary side) is never in the vicinity of the saddle point β_d ; therefore the nonuniform evaluation is quite accurate (for the uniform evaluation of these diffracted waves see [6]).

For positive and negative frequencies, δ_q^2 in (35) is positive and negative, respectively, and the Fresnel function F can also be calculated using the property $F(-x) = F^*(x)$ for real x . We now show that the high-frequency FD field $A_q^T(\mathbf{r}, \omega)$ in (33) is continuous everywhere, i.e., the discontinuity of the propagating FW_q at the shadow boundary β_q^{SB} is compensated by an opposite discontinuity of the diffracted field. Fig. 2 shows the transition region surrounding the conical SB which, as in related canonical FW diffraction problems (see [7]), has a parabolic cross section in any plane containing the z -axis. On the SB of a propagating FW, $\delta_q^2 = 0$ and the integral in (32) equals $\sqrt{-j\pi}/2$. Near the SB, F reduces to $F(\delta_q^2) \approx \sqrt{\pi j} |\delta_q|$; using the limit $|\delta_q| (\cos \beta_q - \cos \beta_d)^{-1} \rightarrow \text{sgn}(\beta_d - \beta_q^{\text{SB}}) \sqrt{k R_d / 2} (\sin \beta_q)^{-1}$ for $\beta_d \rightarrow \beta_q$, the diffracted field is given approximately by $A_q^d \approx (1/2) \text{sgn}(\beta_d - \beta_q^{\text{SB}}) A_q^{\text{FW}}$. Adding this transitional field to that of $A_q^{\text{FW}} U(\beta_q^{\text{SB}} - \beta_d)$ restores the continuity of $A_q^T(\mathbf{r}, \omega)$ in (33) throughout.

B. Total FD Field

The total field A^{tot} in (24) is obtained by adding to the continuous FW-modulated truncated asymptotic contributions A_q^T in (33) the $A_0/2$ contribution arising from the weighted $n = 0$ element radiation in (5). The combination of the q -sum over all diffracted fields A_q^d , $q = 0, \pm 1, \pm 2, \dots$, and the $A_0/2$ term in (24) can be expressed compactly (see [8], [9]) as a function A^d , thereby rendering the total field equal to $A^{\text{tot}} = \sum_{q=-\infty}^{\infty} A_q^{\text{FW}} U(\beta_q^{\text{SB}} - \beta_d) + A^d$. The restructuring is accomplished by first expressing the q sum $\sum_{q=-\infty}^{+\infty}$ over the *nonuniform* expressions for A_q^d in (34) (i.e., when $F \approx 1$) in closed form via the formula $\sum_{q=-\infty}^{\infty} (a - 2\pi q)^{-1} = (1/2) \cot(a/2)$ with $a = kd(\cos \beta_d - \cos \beta_0)$, and $k \cos \beta_0 = k \cos \beta_q - \alpha_q$; adding the term $A_0/2$ yields $[(1/2) - (1/2j) \cot(a/2)] = [1 - \exp(ja)]^{-1}$. Uniform asymptotic results for any propagating FW pole β_q in the vicinity of β_d are obtained by regularization implemented through adding and subtracting the nonuniform ($F \approx 1$) expressions in the full A_q^d expressions in (34). The outcome for the *total spherical wave diffracted field* is

$$A^d(\mathbf{r}, \omega) = \frac{A_0}{2} + \sum_{q=-\infty}^{\infty} A_q^d \sim \frac{e^{-jkR_d}}{4\pi R_d} \cdot \left[\frac{1}{1 - e^{jk d(\cos \beta_d - \cos \beta_0)}} + \sum_q' \frac{F(\delta_q^2) - 1}{jkd(\cos \beta_q - \cos \beta_d)} \right] \quad (36)$$

in which the sum $\sum_q'$ extends over all regularized pole contributions with $|k_{zq}| < |k|$. For edge and tip diffractions in different truncated array configurations, see [6], [7], [10], and [11].

V. TRUNCATED FW: TD

A. Direct Solutions: Exact Closed Forms

Applying the truncated Poisson summation (4) to the total field $\hat{A}^{\text{tot}}(\mathbf{r}, t)$ excited by the impulsive dipole currents leads to

$$\hat{A}^{\text{tot}}(\mathbf{r}, t) = \sum_{n=0}^{\infty} \hat{A}_n(\mathbf{r}, t) = \frac{\hat{A}_0(\mathbf{r}, t)}{2} + \sum_{q=-\infty}^{\infty} \hat{A}_q^T(\mathbf{r}, t) \quad (37)$$

where $F_n \equiv \hat{A}_n$ is the n -th-element free space impulsive Green's function in (10) and $\hat{F}_q \equiv \hat{A}_q^T$, is the truncated integral (using $\nu = z'/d$)

$$\hat{A}_q^T(\mathbf{r}, t) = \frac{1}{4\pi d} \int_0^{\infty} \frac{e^{-j\alpha_q z'}}{R(z')} \delta\left(t - \frac{\eta z'}{c} - \frac{R(z')}{c}\right) dz' \quad (38)$$

with $R(z')$ defined in (5). The integrand in (38) contributes only for those real z' -values on the integration domain $z' \geq 0$, which satisfy (11). If the point z' that satisfies (11) coincides with the endpoint $z' = 0$, its contribution is multiplied by 1/2, using the definition $\int_0^{\infty} \delta(x) dx = 1/2$. As in [1, Sec. III], the solutions of (11) behave differently for $|\eta| \gtrless 1$.

1) *Radiating Case* $|\eta| < 1$: For $|\eta| < 1$, with $\tau > 0$, (11) has the two solutions $z'_1(t)$ and $z'_2(t)$ in (12) but now subject to the constraint $z'_i(t) > 0$. Thus, *causal* field contributions corresponding to $z'_1(t)$ and $z'_2(t)$ are given by

$$\hat{A}_{q,i}^T(\mathbf{r}, t) = \hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t) U(z'_i(t)), \quad i = 1, 2 \quad (39)$$

where $U(x)$ is defined in (14), and $\hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t)$ is the infinite array wavefield in (14) (for comparison with the high frequency asymptotic approximation, see Section V.B.1). In analogy to (14), we refer to (39) as the *fundamental solution* for the (q, i) -indexed truncated FW. Thus, the wavefield $\hat{A}_{q,i}^T$ is equal to the infinite array $\hat{A}_{q,i}^{\text{FW}}$ when $z'_i(t)$ lies inside the array domain ($z'_i(t) > 0$), to $(1/2)\hat{A}_{q,i}^{\text{FW}}$ when $z'_i(t) = 0$, and to zero when $z'_i(t) < 0$. Typical behavior is depicted in Fig. 3, where the array occupies the region $z > 0$. The $\hat{A}_{q,i}^{\text{FW}}$ space-time phenomenology has already been explained in connection with (14), except that due to the truncation, at later times [Fig. 3(c)] than those in Figs. 3(a), (b), the observer at P is reached only by the contribution arriving from $z'_1(t)$. The (dashed) contribution from $z'_2(t)$ is off the array and therefore not excited. Since $z'_i(t) = 0$ corresponds to $\beta_i(t) = \beta_d$, the FW existence condition can be parameterized in terms of the instantaneous propagation angles derived via FD inversion asymptotics in Section V-B

$$U(z'_i(t)) = U(\beta_i(t) - \beta_d) \quad i = 1, 2. \quad (40)$$

Like the TD-FW $_{q,i}$ in (14) for the infinite array, the wavefield in (39) is complex. Therefore, as in Section III-B, to obtain a real causal field from (39), we take the real part as in (16). Alternatively, adding the $(+q)$ and $(-q)$ contributions for $i = 1$ or 2 from (39) *directly* yields the *real* field

$$\hat{A}_i^{q|T} \equiv \hat{A}_{+q,i}^T + \hat{A}_{-q,i}^T = 2\Re \hat{A}_{q,i}^{\text{FW}} U(z'_i(t)) \quad (41)$$

Thus, $(+q)$ and $(-q)$ pairing yields an *observable* truncated FW $_{q,i}$ field, corresponding to the U function in (41).

2) *Nonradiating Case* $|\eta| > 1$: For $|\eta| > 1$, the same considerations apply as for the infinite case in (18), the only difference being the truncation of the integration domain through the multiplicative U function in (40)

$$\hat{A}_q^T(\mathbf{r}, t) = \hat{A}_q^{\text{FW}}(\mathbf{r}, t) U(z'_i(t)) \quad i = \begin{cases} 1, & \eta > 0 \\ 2, & \eta < 0 \end{cases} \quad (42)$$

with $\hat{A}_q^{\text{FW}}(\mathbf{r}, t)$ taken from (18). The physical interpretation of the FW truncation effect in (42) is directly analogous to that discussed in connection with (40). The *observable* truncated FW $_q$ field obtained by $(+q), (-q)$ pairing is

$$\hat{A}^{q|T} = \hat{A}_q^T + \hat{A}_{-q}^T = 2\Re \hat{A}_q^{\text{FW}} U(z'_i(t)). \quad (43)$$

3) *Total TD Field*: The total field in (37) is obtained by adding the $n = 0$ element spherical wave field contribution $\hat{A}_0/2$ to the q -sum of truncated TD-FW contributions $\hat{A}_q^T(\mathbf{r}, t)$ given by $\hat{A}_q^T = \hat{A}_{q,1}^T + \hat{A}_{q,2}^T$, with $\hat{A}_{q,i}^T(\mathbf{r}, t)$ in (39) for the radiating case, and in (42) for the nonradiating case. For the radiating case, the global conical TD-FW wavefields are synthesized by contributions from the moving points $z'_1(t)$ and $z'_2(t)$ on the array axis in Fig. 3. Note, from Fig. 3 and the discussion following (38) that for $z'_0 > 0$ ($z > \rho\eta/\sqrt{1-\eta^2}$), the $(q = 0, \pm 1, \pm 2, \dots)$ contributions from z'_2 disappear when $z'_2(t) < 0$, and that the sum $\sum_{q=-\infty}^{\infty} \hat{A}_{q,2}^T$ of the limiting contributions due to $z'_2 = 0$ equals $\hat{A}_0/2$. When added to the explicit $\hat{A}_0/2$ term in (37), this restores the full $n = 0$ element contribution in the original element-by-element n -sum

in (37). Analogous considerations, with the role of z'_1 and z'_2 interchanged, apply when $z'_0 < 0$.

B. Inversion From the FD: Asymptotics

To parameterize the truncated FW phenomenologies in Section V-A in terms of instantaneous frequencies and wavenumbers, we access the time domain through Fourier inversion [see (1)]

$$\hat{A}_q^T(\mathbf{r}, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_q^T(\mathbf{r}, \omega) e^{j\omega t} d\omega \quad (44)$$

but approximate $A_q^T(\mathbf{r}, \omega)$ in the integrand by its high-frequency asymptotic value in (33). This restricts the validity of the truncated TD solution in (44) to *early observation times* near the wavefronts, far from the asymptotically induced spurious branch point at $\omega = 0$ in (29). Each of the $A_q^{\text{FW}}(\mathbf{r}, \omega)$ and its corresponding diffracted field $A_q^d(\mathbf{r}, \omega)$ has a particular arrival time, near which the asymptotics will be most accurate. As already seen for the infinite case, we need to distinguish between the dispersive $q \neq 0$ and the nondispersive $q = 0$ cases.

1) $q \neq 0$, *Close to the Moving SB*: For $q \neq 0$, we perform the asymptotic inversion of $A_q^T(\mathbf{r}, \omega)$ with an approximated form of the *diffracted* field valid when the space-time position of the observer is in the vicinity of the SB. While each term in (33) is ω -discontinuous on the SB, their sum is a continuous function of ω there. The corresponding inverted TD-FW, valid away from and near the SB, is obtained as in Section III-C for the infinite case, using the dominant instantaneous frequencies $\omega_{q,i}(t)$ in the shadow boundary truncation functions $U(\beta_q(\omega_{q,i}(t)) - \beta_d) \equiv U(\beta_i(t) - \beta_d)$, $i = 1, 2$. For the diffracted field A_q^d in (33), when $\beta_d \approx \beta_q(\omega)$, the argument δ_q in (34) of the transition function F in (32) tends to zero, allowing the F function to be approximated as discussed in the paragraph after (32). The resulting inversion of (33) near the SB becomes

$$\begin{aligned} \hat{A}_q^T(\mathbf{r}, t) \sim & \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{-j\hat{\psi}(\omega)} \left[\frac{e^{-j\pi/4}}{2d\sqrt{2\pi k_\rho \rho}} \right. \\ & \times U(\beta_q(\omega) - \beta_d) + \left. \frac{j e^{j\pi/4}}{4\sqrt{\pi} dk R_d} \frac{\pm \delta_q \operatorname{erfc}(\pm e^{j\pi/4} \delta_q)}{\cos \beta_d - \cos \beta_q} \right] \end{aligned} \quad (45)$$

with $\hat{\psi}(\omega)$ given in (20). In the integrand, A_q^{FW} in the first term has been approximated as in (6), and the approximated form of A_q^d in the second term contains the complementary error function $\operatorname{erfc}(z) = (2/\sqrt{\pi}) \int_z^\infty e^{-u^2} du$, where the upper/lower sign in (45) applies for $\Re(e^{j\pi/4} \delta_q) \gtrless 0$ (see [6]).

For $q \neq 0$, the dominant contributions to the integral in the high-frequency range arise from the stationary (saddle) points $\omega_{q,i}(t)$ of $\hat{\psi}(\omega)$ given in (21). In (45), erfc is considered as an amplitude function as in [8], [9] because over its *effective* range $\beta_q(\omega) \approx \beta_d$ (in the proximity of the SB), its phase varies slowly. (Away from the SB, the phase varies rapidly; there, however, the FW field is not discontinuous and the diffracted field may either be neglected without appreciable loss of accuracy or evaluated by other techniques. Better approximations for the diffracted field away from the shadow boundaries are beyond the scope

Fig. 5. Moving shadow boundaries (SB). At the instant t_0 , the SBs of all FWs coincide with $SB(t_0)$. At later instants $t > t_0$, they separate into $SB_i(t)$, $i = 1, 2$, forming angles $\beta_i(t) \equiv \beta_q(\omega_{q,i}(t))$ with the z -axis. According to (46), the FW $_{q,i}$ are confined to the right side of $SB_i(t)$ for $i = 1, 2$, respectively. Recalling (40), the observer is reached only by those FWs $\hat{A}_{q,i}^{\text{FW}}$ that originate at the q -independent points $z'_1(t) > 0$ and $z'_2(t) > 0$.

of the present investigation). The asymptotic evaluation of (45) is carried out via the formula in (22) where $F(\omega)$ now accounts for the slowly varying portion (in brackets) in the integrand. This leads to

$$\hat{A}_{q,i}^T(\mathbf{r}, t) \sim \hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t) U(\beta_i(t) - \beta_d) + \hat{A}_{q,i}^d(\mathbf{r}, t) \quad (46)$$

in which $\hat{A}_{q,i}^{\text{FW}}(\mathbf{r}, t)$ is the same as in (14). The Heaviside function U terminates the TD-FW $_{q,i}$ domain of existence at the shadow boundary cone $\beta_d = \beta_i(t) \equiv \beta_q(\omega_{q,i}(t))$. The diffracted field is

$$\begin{aligned} \hat{A}_{q,i}^d(\mathbf{r}, t) = & \frac{\sqrt{-j\omega_{q,i}} e^{j\omega_{q,i}(t-t_d)}}{4\pi d \sqrt{2\pi} R_d} \frac{\sin^{3/2} \beta_i}{\cos \beta_i - \cos \beta_d} \\ & \cdot F(\delta_{q,i}^2) \frac{\sqrt{c}}{|\alpha_q| \sqrt{\rho}} U(t - t_0), \quad t_d = R_d/c \end{aligned} \quad (47)$$

where $\omega_{q,i}$, and therefore β_i and $\delta_{q,i}$, are functions of t , and $\alpha_q = 2\pi q/d$. Evidently, the (q, i) wavefields in (46) are complex. Since the FWs are the same as those found via the exact inversion in Section V.A., the same properties apply here; i.e., $(+q), (-q)$ pairing $\hat{A}_i^{[q]\text{FW}} = \hat{A}_{q,i}^{\text{FW}} + \hat{A}_{-q,i}^{\text{FW}}$ is a *real TD observable*. The same property is found for the diffracted field. To show this, we recall that $\omega_{-q,i} = -\omega_{q,i}$ [see (21)] whence the argument $\delta_{q,i}^2$ of the Fresnel function F in (34) can be negative. Based on the property $F(-x) = F^*(x)$ for x real, adding the $(+q)$ and $(-q)$ contributions for $i = 1$ or 2 yields the *real observable* diffracted field

$$\hat{A}_i^{[q]d} = \hat{A}_{q,i}^d + \hat{A}_{-q,i}^d = 2\Re \hat{A}_{q,i}^d. \quad (48)$$

Every paired diffracted field $\hat{A}_i^{[q]d}$, $i = 1, 2$, compensates for the discontinuity of the TD-FW $\hat{A}_i^{[q]\text{FW}}$ at the moving shadow boundaries $\beta_d = \beta_1(t)$ and $\beta_d = \beta_2(t)$, respectively. At $t = t_0$, the two SBs coincide and are located at $\beta_1(t_0) = \beta_2(t_0) = \beta_0$, as shown in Fig. 5. For $t > t_0$, they are distinct such that $\beta_1(t) > \beta_0 > \beta_2(t)$. Also in Fig. 5, at a time t , the observer is always reached by the diffracted fields $\hat{A}_i^{[q]d}$, $i = 1, 2$ but only by those FWs $\hat{A}_i^{[q]\text{FW}}$ that originate at the q -independent points $z'_1(t) > 0$ and $z'_2(t) > 0$.

It is important to note that the asymptotic result for $\hat{A}_{q,i}^T$ with $q \neq 0$ in (46) differs from that in (39) obtained by the exact formulation. Since, from (40), the two Heaviside functions in (46) [with (47)] coincide, the asymptotic solution (46) is equal to the exact solution in (39), plus the diffracted field term $\hat{A}_{q,i}^d$.

that does not appear in (39). The discrepancy is due to the high-frequency approximation which happens not to affect the FW saddle point contribution but does affect the diffracted endpoint contribution. The exact result in (39) has no high frequency constraints, and therefore no *localized* endpoint contribution.

2) $q = 0$: The asymptotic FD-FW and diffracted waves with $q = 0$ in (33) have a phase linearly dependent on ω and can be inverted exactly into their TD counterparts through (44) without distinguishing between $i = 1$ or $i = 2$. In this case, the shadow boundary β_q^{SB} is independent of ω and the two contributions (FW and diffracted field) can be inverted individually. The inversion of the asymptotic FD-FW has already been obtained in (23). The inversion of the asymptotic FD-diffracted field in (34) can likewise be obtained in closed form. Inserting (34) into the inverse transform in (1) and separating the ω -dependent from the ω -independent terms, one obtains

$$\hat{A}_0^d(\mathbf{r}, t) = \frac{c}{4\pi d R_d} \frac{\hat{I}(t)}{\cos \beta_0 - \cos \beta_d} \quad (49)$$

with [using (35)]

$$\hat{I}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{j\omega(t-t_d)} \frac{F(\omega t_\delta)}{j\omega} \quad (50)$$

$$t_\delta = 2t_d \sin^2((\beta_0 - \beta_d)/2) \equiv \delta_0^2/\omega \quad t_d = R_d/c. \quad (51)$$

Furthermore, $t_\delta = t_d(1 - \cos(\beta_0 - \beta_d)) = t_d - t_0$, where $t_0 = t_d \cos(\beta_0 - \beta_d) = R_d/c \cos \beta_0 \cos \beta_d + \sin \beta_0 \sin \beta_d = (\eta z + \sqrt{1 - \eta^2} \rho)/c$; the last equality is obtained from $z = R_d \cos \beta_d$, $\rho = R_d \sin \beta_d$ (see Fig. 1), and from $\cos \beta_q = k_{zq}/k = \eta$ for $q = 0$ [see (9) and (7)]. To evaluate $I(t)$ in (50), we introduce the integral representation [3, pp. 402]

$$F(x) = \frac{jx}{\sqrt{\pi}} \int_{-\infty}^{\infty} ds \frac{e^{-s^2}}{s^2 + jx}, \quad x = \omega t_\delta \quad (52)$$

insert into (50) and change the order of integration to obtain

$$\hat{I}(t) = \frac{t_\delta}{2\pi\sqrt{\pi}} \int_{-\infty}^{\infty} ds e^{-s^2} \int_{-\infty}^{\infty} d\omega \frac{e^{j\omega(t-t_d)}}{jt_\delta(\omega - js^2/t_\delta)}. \quad (53)$$

In the ω -integral, the pole at $\omega = js^2/t_\delta$ is located in the upper half of the complex ω -plane. For $(t - t_d) < 0$, the integration contour can be closed by a semicircle at infinity in the lower half plane; since no singularities are enclosed, the integral vanishes by Cauchy's theorem. For $(t - t_d) > 0$, the integration contour can be closed by a semicircle in the upper half plane; the enclosed pole contributes a residue and (53) becomes

$$\begin{aligned} \hat{I}(t) &= \frac{U(t - t_d)}{\sqrt{\pi}} \int_{-\infty}^{\infty} ds e^{-s^2} ((t - t_d)/t_\delta + 1) \\ &= \frac{\sqrt{t_d - t_0}}{\sqrt{t - t_0}} U(t - t_d). \end{aligned} \quad (54)$$

Combining (49) and (54) yields the desired closed-form expression for the asymptotic $q = 0$ spherically spreading diffracted field.

The total asymptotic $q = 0$ wavefield of the truncated array can now be expressed as

$$\hat{A}_0^T(\mathbf{r}, t) \sim \hat{A}_0^{\text{FW}}(\mathbf{r}, t) U(\beta_0^{\text{SB}} - \beta_d) + \hat{A}_0^d(\mathbf{r}, t) \quad (55)$$

Fig. 6. TD-FW and diffracted wavefront phenomenologies. The conical TD-FW wavefront of $\hat{A}_0^{\text{FW}}$ and the spherical diffracted field wavefront of $\hat{A}_0^d$ reach the observer at $t = t_0$ and $t = t_d \geq t_0$, respectively. Since the $q = 0$ FW is not dispersive, its shadow boundary SB_0 is fixed at $\beta_d = \beta_0$. Adding the TD-diffracted field to the truncated TD-FW restores continuity along and near the wavefront.

where $\hat{A}_0^{\text{FW}}$ is given in (23), and $\hat{A}_0^d$ in (49) with the TD transition function in (54). The wavefronts of both $\hat{A}_0^{\text{FW}}$ and $\hat{A}_0^d$ are depicted in Fig. 6. Since $t_0 = t_d \cos(\beta_0 - \beta_d) \leq t_d$, the TD-FW wavefront reaches the observer at $t = t_0$ from the direction β_0 before the diffracted wavefront arrival at $t = t_d$ from the direction β_d see (Figs. 3(a) and 6). For $t \approx t_d$, the transition function becomes $\hat{I}(t) \approx U(t - t_d)$, implying that the diffracted spherical wavefront field in (49) behaves like a step function. On its wavefront, the diffracted field $\hat{A}_0^d$ has a finite amplitude for $\beta_d \neq \beta_0$. However, when $\beta_d \rightarrow \beta_0$ on the wavefront, $\hat{A}_0^d$ becomes infinite because of the denominator in (49). This is not surprising since at $\beta_d = \beta_0$, the diffracted field has to match the TD-FW in (55) with (23), which likewise is infinite on its wavefront.

Because the TD-FW is abruptly truncated at the SB $\beta_d = \beta_0^{\text{SB}} \equiv \beta_0$ by the U function in (55), we need to show that adding the TD diffracted field restores continuity along and near the wavefront. Near the shadow boundary $\beta_d \approx \beta_0$, one has from (51) that $t_d \approx t_0$, whence $\hat{A}_0^{\text{FW}}$ and $\hat{A}_0^d$ both have approximately the same turn-on time. At $\beta_d = \beta_0$, the spherical diffracted wavefront is tangent to the cylindrically spreading TD-FW wavefront (see Fig. 6). From (51), it is seen that $\sqrt{t_d - t_0} = \sqrt{2t_d} |\sin((\beta_0 - \beta_d)/2)|$, so that close to the SB ($\beta_d \approx \beta_0$)

$$\frac{\hat{I}(t)}{\cos \beta_0 - \cos \beta_d} \approx \frac{\text{sgn}(\beta_d - \beta_0)}{2 \sin \beta_0} \frac{\sqrt{2t_d} U(t - t_d)}{\sqrt{t - t_0}} \quad (56)$$

has a finite value except at the wavefront $t = t_d$ where $t_d = t_0$ ($\beta_d = \beta_0$). Recalling that $t_d = R_d/c$, $R_d \sin \beta_d = \rho$, $\sin \beta_0 = \sqrt{1 - \eta^2}$, and $\tau_0 = \sqrt{1 - \eta^2} \rho/c$, the diffracted wave at $\beta_d \approx \beta_0$ is [see (23)]

$$\hat{A}_0^d(\mathbf{r}, t) \approx \frac{1}{2} \text{sgn}(\beta_d - \beta_0) \hat{A}_0^{\text{FW}}(\mathbf{r}, t). \quad (57)$$

This demonstrates that at $\beta_d = \beta_0$, $\hat{A}_0^d$ and $\hat{A}_0^{\text{FW}}$ have the same arrival time, and that the angular discontinuity at $\beta_d = \beta_0$ of the truncated FW $\hat{A}_0^{\text{FW}} U(\beta_0^{\text{SB}} - \beta_d)$ is compensated, via $\text{sgn}()$, by an opposite discontinuity of the the diffracted field $\hat{A}_0^d$, thereby rendering $\hat{A}_0^T$ in (55) continuous anywhere behind the wavefront, at any t .

VI. BAND-LIMITED PULSE EXCITATION

In this Section we analyze the effects of physically realizable band-limited (BL) pulsed dipole excitation on the field radiated by the truncated array. The pulse excitation function is represented as $\hat{G}(t)$ with spectrum $G(\omega)$. Accordingly, the factor

multiplying $\delta(z' - nd)$ in (2) becomes $G(\omega) \exp(-j\eta knd)$ for the FD dipole currents and $\hat{G}(t - \eta nd/c)$ for the TD dipole currents.

The total BL response $\hat{A}^{\text{tot},\text{BL}}$ of the *truncated* array is then obtained by convolving the total TD impulse response in (37) with the BL signal $\hat{G}(t)$, yielding (see (10), with $R(0) \equiv R_d$, and $t_d = R_d/c$)

$$\hat{A}^{\text{tot},\text{BL}}(\mathbf{r}, t) = \frac{\hat{A}_0^{\text{BL}}(\mathbf{r}, t)}{2} + \sum_{q=-\infty}^{\infty} \hat{A}_q^{T,\text{BL}}(\mathbf{r}, t) \quad (58)$$

$$\hat{A}_q^{T,\text{BL}}(\mathbf{r}, t) = \int_{-\infty}^{\infty} \hat{G}(t') \hat{A}_q^T(\mathbf{r}, t - t') dt' \quad (59)$$

and $\hat{A}_0^{\text{BL}}(\mathbf{r}, t) = \hat{G}(t - t_d)/(4\pi R_d)$. The BL Floquet-modulated signal $\hat{A}_q^{T,\text{BL}}$ due to the truncated array can be calculated either by convolution with the exact truncated FW or by inversion of FD asymptotics.

A. Convolution With the Exact Truncated FW

Here, the exact FW field in (38) with (39) or (42) for the radiating or nonradiating cases, respectively, is used in (59). For the radiating case ($|\eta| < 1$), $\hat{A}_q^T$ can be expressed in terms of its $i = 1, 2$ constituents $\hat{A}_{q,i}^T$ in (39) which, when $(\pm q)$ -paired and convolved with $\hat{G}(t)$, provide the real *fundamental* BL solutions $\hat{A}_i^{q|T,\text{BL}} = \hat{A}_i^{q|T} \otimes \hat{G}$, with $\hat{A}_i^{q|T}$ given in (41) and $\otimes$ denoting the convolution operation in (59).

B. Inversion of FD Asymptotics

Avoiding the convolution in (59), the q th BL field $\hat{A}_q^{T,\text{BL}}$ can be calculated as the inverse Fourier transform of $G(\omega) \hat{A}_q^T(\mathbf{r}, \omega)$. Therefore, for $q \neq 0$, using the high-frequency asymptotics in Section V-B, $\hat{A}_q^{T,\text{BL}}$ can be evaluated by including the pulse spectrum $G(\omega)$ in the inversion integral (44). For wideband (short duration) pulses, $G(\omega)$ can be considered a slowly varying function with respect to the phase in the integrand of (45) [9], and can therefore be approximated by its value at the saddle point frequencies $\omega_{q,i}(t)$, $i = 1, 2$. Thus, near the wavefronts

$$\hat{A}_q^{T,\text{BL}} = \sum_{i=1}^2 \hat{A}_{q,i}^{T,\text{BL}} \quad (60)$$

with

$$\hat{A}_{q,i}^{T,\text{BL}}(\mathbf{r}, t) \sim G(\omega_{q,i}(t)) \hat{A}_{q,i}^T(\mathbf{r}, t) \quad (61)$$

and $\hat{A}_{q,i}^T$ approximated asymptotically as in (46). Again, $(+q), (-q)$ pairing gives the *real fundamental* BL solution

$$\hat{A}_i^{q|T,\text{BL}} = \hat{A}_{q,i}^{T,\text{BL}} + \hat{A}_{-q,i}^{T,\text{BL}} = 2\text{Re} \hat{A}_{q,i}^{T,\text{BL}} \quad (62)$$

since $G(-\omega) = G^*(\omega)$, $\omega_{-q,i}(t) = -\omega_{q,i}(t)$ [see (21)], and $\hat{A}_{-q,i}^T = (\hat{A}_{q,i}^T)^*$. For $q = 0$, the high-frequency FD field in (33) has a phase linear in ω , and therefore cannot be inverted by asymptotics. The pulsed response $\hat{A}_0^{T,\text{BL}}$ is calculated by convolving $\hat{G}(t)$ with the $q = 0$ locally uniform wavefront approximation (55), with (23), (49), and (54).

Fig. 7. Field radiated by a 20 element array observed at a radial distance $\rho = 15d$. (a) $z = 17d$. (b) $z = -15d$. (Observation points a) and b) are shown in Fig. 6). The fields are plotted versus normalized time t/T , with $T = d/c$. Parameters and physical interpretations are provided in the text.

The impulsively excited asymptotic wavefields in Section V-B are valid only for *early times*, on and close to (behind) the wavefronts. However, convolution with a waveform whose frequency spectrum $G(\omega)$ has no low-frequency components and is thus dominated by high frequencies may enlarge the range of validity of the resulting pulsed response to later observation times behind the wavefront. For $q \neq 0$, the relevant TD-FW $_q$ fields are those with $\omega_{q,i}$ in the $\hat{G}(t)$ signal bandwidth.

C. Numerical Examples

Numerical experiments have been carried out in order to test the asymptotic solution for $\hat{A}^{\text{tot},\text{BL}}$ in (58) via the TD asymptotic solution (60), and to compare the result with a reference solution obtained by an element-by-element summation over the pulsed radiation from all dipoles, i.e., $\hat{G}(t - \eta nd/c - R(nd)/c)[4\pi R(nd)]^{-1}$, $n = 0, 1, 2, \dots$

Fig. 7 shows plots for an array of 20 elements with interelement phasing $\eta = 0.2(\beta_0 = 78^\circ)$. Both left and right truncation effects have been accounted for, treating the actual line array as the difference between two semi-infinite line arrays. The BL excitation is a normalized Rayleigh pulse $\hat{G}(t) = \text{Re}[j/(j + \omega_M t/4)^5]$ (i.e., $\hat{G}(0) = 1$) [12], with FD spectrum $G(\omega) = \pi(6\omega_M)^{-1}(j4\omega/\omega_M)^4 \exp(-4|\omega|/\omega_M)$, and central radian frequency $\omega_M = 4\pi c/d(\lambda_M = 2\pi c/\omega_M = d/2)$; d is the interelement spacing. The fields are plotted vs. normalized time t/T , with $T = d/c$. In this example, the relevant range of $\omega_{q,i}(t)$ is $|q| \leq 5$, and in all cases the

asymptotic $q \neq 0$ TD diffracted field (47) has been included only for a time interval t such that $|\delta_{q,i}(t)| < 0.04$, i.e., in the transition region, see (35); elsewhere, it has been neglected. Therefore, the TD diffracted field compensates for the abrupt FW-discontinuities at the moving shadow boundaries (Fig. 5) when they cross the observer. The results in Fig. 7 show that neglecting that contribution away from the moving SB does not substantially affect the accuracy.

The observer is located at (a) $z = 17d$ and (b) $z = -15d$ (as shown in Fig. 6) at the same radial distance $\rho = 15d$. At location $z = 17d$ [Fig. 7(a)], the field in the first plot includes the term $\hat{A}_0^{\text{BL}}/2$ and all terms with $|q| = 0, 1, 2$ (solid curve) in the series (58), which suffice to give excellent agreement with the reference solution (dotted curve). It is also shown (second plot) that the BL truncated FW₀, $G(t) \otimes \hat{A}_{q=0}^{\text{FW}} U(\beta_0 - \beta_d)$, without the diffracted term $\hat{A}_{q=0}^d$, gives the dominant contribution. Note that the BL truncated FW with $|q| = 1$ arrives after the wavefront and reproduces the major features of the reference solution. Moreover, the BL FW with $|q| = 2$ (third plot), though small, is important in establishing the the total reference field; its instantaneous oscillation frequency is higher than that of the FW with $|q| = 1$. The diffracted fields, given by $\hat{A}_0^{\text{BL}}/2 + \sum_{q=-2}^2 [\hat{A}_{q,1}^d + \hat{A}_{q,2}^d]$ for both the left and right truncations, arrive at the observer later than the FW wavefront. In contrast, at location $z = -15d$, i.e., off the array, the contributions from each element of the array can be distinctly be resolved [there are 20 peaks in Fig. 7(b)]. Now the field obtained from $|q| = 0, 1, 2$ (as before) in (58), plus $\hat{A}_0^{\text{BL}}/2$, does not replicate the reference solution (dashed); FWs (and their diffracted fields) with $|q| = 0, 1, \dots, 5$ (solid curve) are necessary to achieve agreement. Thus, construction of the field at $z = -15d$ necessitates inclusion of higher order FWs to synthesize the individual spikes which are not observed at the $z = 17d$ location in front of the array. In both cases, the included asymptotic terms are $|q| \leq 5$, thereby demonstrating good convergence of the TD-FW field representation.

VII. CONCLUSION

In this second of our planned series of prototype studies, we have explored the behavior of a semi-infinite periodic sequentially pulsed line array of dipoles, by again exploiting (as in [1]) their collective behavior in terms of radiating and nonradiating TD-FW. Also, as in [1], we have used the frequency domain FW footprints (instantaneous frequencies and wavenumbers) and uniform asymptotics to parameterize and physically interpret the time domain closed-form exact solutions. The outcome has been a periodicity-matched new TD diffraction model that incorporates truncated radiating and nonradiating conical FW fields and their corresponding tip-diffracted spherical waves, progressing along smoothly connected conical-spherical wavefronts. The FW-modulated diffracted fields exhibit all of the “anomalies”—concerning

physically “observable” wave objects [see (41) and (43)]—that have been discussed in detail in our previous prototype study of the infinite array [see (17) and (19)]. In addition, the interplay of uniform and nonuniform FD or TD asymptotic approximations furnishes a rich variety of new periodicity-modulated compensations in the vicinity of truncation-induced shadow boundaries so as to ensure the required continuity of the total FD or TD wavefield [see discussion after (35) and (55)]. Particularly noteworthy is the exact TD-FW result in (39): it consists only of the truncated TD-FW *without* a tip diffraction contribution! This can be attributed to the infinite bandwidth of the impulsive excitation, whose low frequency components render high-frequency diffraction concepts inapplicable; band-limited signals without these components do give rise to tip diffraction [see (46) and results in Fig. 7(a)]. A preliminary numerical example involving a bilaterally truncated finite line array of band-limited pulsed dipoles suggests that the asymptotic algorithm works satisfactorily. Further numerical tests are in progress. The results obtained here are a further step toward the construction of TD dipole-excited impulsive or band-limited Green’s functions pertaining to two-dimensional planar rectangular phased arrays.

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