Title
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During Disruptions and Emergency Evacuations

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ABSTRACT
The events of hurricane seasons and the threat of terrorist attacks have made evacuation during disruptions a leading management issue. Transportation networks, which form the backbone of any emergency management plan, should be able to respond to disruptions by ensuring safe, organized and quick movement of people at the time of crisis. This work proposes two models to capture the highly uncertain and time-dependent nature of transportation systems in the face of an emergency. The first model addresses the location of safety shelters. It uses risk management tools, the classical facility location model and traffic assignment techniques with Wardrop’s principle to determine the optimal location of shelters. The second model deals with real time decision-making during evacuations. It incorporates a simulation algorithm with the successive shortest path algorithm to model evacuation. Detailed traffic information in the network can be obtained from this algorithm to facilitate the evacuation.
INTRODUCTION
The nature of evacuations in the United States has changed in recent years. Hurricanes Georges in 1998 and Floyd in 1999 saw two of the largest evacuation efforts as well as two of the largest traffic jams in U.S. history. Numerous explanations for the jams have been proposed including the threat of these storms, an over-reaction to the need to evacuate, insufficient planning, and limited coordination between the agencies responsible for evacuation. Whatever the causes, it is clear that evacuation planning deserves significant attention. Proper preparedness and response are important for improved traffic flow and unhindered movement during evacuations.

Early work primarily focused on emergency management during nuclear power plant malfunctions and natural disasters such as hurricanes. Since September 11, 2001, more attention has been given to dealing with terrorist attacks. Researchers have used simulation and mathematical modeling to assist in the design and evaluation of evacuation plans: the most notable being the use of contra flow operations to increase the capacity of evacuation routes and the application of intelligent transportation systems (ITS) to collect and communicate up-to-date traffic information.

Disaster Management is defined as “the range of activities designed to maintain control over disaster and emergency situations and to provide a framework for helping at-risk persons to avoid or recover from the impact of the disaster” (Disaster Management Center, 1982). Disaster management considers the situations before, during, and after disasters. Thus, preparedness and planning before disruptions and dynamic decision making during disruptions are both critical to effective evacuation.

This work will apply tools from financial engineering and supply chain management to this problem. Since the planning phase precedes the occurrence of the disaster, uncertainty about future events is unavoidable. A widely used method to deal with uncertainty is to define a number of possible future scenarios and identify solutions that minimize the total regret over all scenarios. The regret of a scenario is defined as the difference between the objective function value given by the overall compromise solution and the optimal solution for the particular scenario. This work uses the $\alpha$-reliable mean-excess regret model in the context of the $p$-median problem suggested by Chen, Daskin, Shen and Uryasev [1], which minimizes the expected regret with respect to an endogenously selected subset of worst-case scenarios whose collective probability of occurrence is no more that $1 - \alpha$. This model is tractable and provides meaningful evaluations of different advanced response plans. These results will give the optimal locations of safety shelters, and the results of the location problem will be used in dynamic real-time decision making during actual evacuations.

The real-time component addresses the critical issue of managing an emergency evacuation and quickly guiding the traffic to safety. Compared with the already complicated traffic operations problem under normal traffic conditions, operations during emergency evacuation can be much more difficult. Significantly heavier flows throughout the network result from a rapid rise in demand. Excessive queues and delays are expected under such scenarios, as the demand volume far exceeds the capacity of the network. Improper evacuation plans will cause some areas to be more congested than others, reducing the opportunity for escape from these areas. We propose a routing strategy that minimizes evacuation time to the safety shelter locations determined as above.
PROBLEM DESCRIPTION
A transportation network consists of nodes and links. Nodes are access points to the road network and links are the connections between the nodes. A disruption may make some links unfit for use by traffic while increasing the number of people trying to access the network to get to safety shelters. Safety shelters must be located to minimize evacuation time under each possible scenario. Further, the routing strategy must guide traffic to safety shelters in the least possible time.

The two major components of emergency preparedness are the
- Optimal location of safety shelters and the
- Optimal routing of traffic following a disruption

LITERATURE REVIEW
Sherali, H.D. et al. [2] considered the impact of safety shelter locations on evacuation plans for hurricanes and floods. They proposed a location-allocation model to select a set of shelters from a set of candidate sites and created a plan to minimize total evacuation time. El Dessouki [3] further studied pre- and post-disaster management issues. The pre-disaster evacuation plan was modeled as a combined trip distribution and assignment (CDA) problem. This model considers shelter capacity constraints, but it is supposed that the link travel time is independent of the flow on the links. The post-disaster emergency management plan was treated as a special case of the multi-period network design problem. The optimal path problem has been studied in both stochastic and time-dependent networks. Miller-Hooks and Mahmassani [4][5] proposed a label-correcting based procedure for determining the least expected travel time (LET) paths in stochastic time-varying (STV) networks. Miller-Hooks and Mahmassani [6] and Miller-Hooks and Yang [7][8] address further topics on STV networks. They consider the delay time at intermediate nodes due to traffic signal control in determining the optimal path in an STV network. Fewer works address solving STV network problems than deterministic and time-
invariant ones. Moreover, no work considers the possible waiting time (delay) due to insufficient link capacity, which is common in evacuations.

Feng and Wen [9] studied different traffic control strategies for post-earthquake response. They proposed multiple models to maximize the traffic volume entering the safety zone and to minimize the rescue time in the disaster zone. Murray and Mahmassani [10] considered household behavior, how households act as a single unit, in evacuation modeling. Opasanon [11] addressed two classes of evacuation problems in STV networks. The first seeks to provide routing guidance as information regarding travel time becomes available. The second aims to determine a set of prior optimal evacuation paths in a capacitated network.

SOLUTION METHODOLOGY

Model I

Description

We first address the safety shelter location problem by proposing the $\alpha$-reliable mean-excess regret model in the context of the $p$-median problem developed by Chen, Daskin, Shen and Uryasev [7]. Here, the distance between demand nodes and safety shelters as well as the demand at these nodes is stochastic. Scenario analysis is used to address the stochastic $p$-median problem in this context. However, there are significant differences between the traditional $p$-median problem and the one embedded in the $\alpha$-reliable mean-excess regret model used in this project.

First, a transportation network is assumed to exist between nodes and safety shelters instead of a single route with a fixed distance. During evacuation large demand will be generated from a node, overwhelming any single traffic route. Considering the distribution of vehicles over a network is more efficient and realistic for evacuation modeling. Therefore, multiple routes are supposed to exist between each node-shelter pair. Link performance functions, which relate flow and travel time, are associated with each link in the network.

Second, the objective is to minimize the travel time in the network instead of the aggregate distance traveled. The travel time is taken to mean the maximum user equilibrium (UE) travel time among all node-shelter pairs. Demand at one node can be assigned to multiple safety shelters to achieve the minimum maximum UE travel time, whether the safety shelters are capacitated or not. However, we assume shelters with unlimited capacity in this project.

The UE condition used for traffic assignment is adopted for the reason of fairness. The system optimal (SO) condition would minimize the total travel time in the network, but it would be at the expense of some drivers who have far longer than others. The UE condition ensures that everyone achieves his minimum travel time without laterally increasing the travel time of others. In addition, link performance functions require a demand rate but not the total evacuation population as input. The demand rate chosen would affect the total evacuation duration for the entire population. However, for the purpose of planning, the demand rate can be estimated from past experience. The notification to evacuate is done in phases, and it is unlikely that the entire affected population would evacuate all at once.

Notation

Indices:

$i = 1, \ldots, M$ - Index of demand nodes subject to evacuation
\[ j = 1, \ldots, N \] - Index of candidate safety shelters
\[ s = 1, \ldots, S \] - Index of possible scenarios
\[ a = 1, \ldots, A \] - Index of links
\[ r = 1, \ldots, R \] - Index of routes

Parameters and decision variables:
\[ t^{as} = t(x^{as}) \] - Link performance function of link \( a \) under scenario \( s \), as a function of \( x \)
\[ f_{ij}^{rs} \] - Flow on route \( r \) connecting nodes \( i \) and \( j \) under scenario \( s \)
\[ \delta_{ij}^{ars} \] - Path-arc incidence relationship indicator
\[ \begin{cases} 1 & \text{if link } a \text{ is on route } r \text{ linking nodes } i \text{ and } j \text{ under scenario } s \\ 0 & \text{otherwise} \end{cases} \]
\[ x^{as} = \sum_{i,j,r} f_{ij}^{rs} \delta_{ij}^{ars} \] - Flow on link \( a \) under scenario \( s \)
\[ C_{ij}^{rs} = \sum_{a} t^{as} \delta_{ij}^{ars} \] - Travel time on route \( r \) connecting nodes \( i \) and \( j \) under scenario \( s \)
\[ h_{i}^{s} \] - Demand originating from node \( i \) under scenario \( s \)
\( p \) - Number of facilities to locate
\( \psi_{s} \) - Best \( p \)-median value (maximum UE travel time) that can be obtained under scenario \( s \)
\( q^{s} \) - Probability of occurrence of scenario \( s \)
\( \alpha \) - Desired probability level
\( M \) - A large number
\[ \pi_{ij}^{s} \] - Equilibrium travel time between nodes \( i \) and \( j \) under scenario \( s \)
\[ \pi_{ij}^{s} = \min \{ C_{ij}^{rs} \} \forall r \text{ between } i \text{ and } j \text{ under scenario } s \text{ in UE condition} \]
\( R^{s} \) - Regret associated with scenario \( s \) and the current solution
\[ R^{s} = \max \{ \pi_{ij}^{s} \} - \psi_{s} \]
\( \zeta \) - A free variable
\( U^{s} \) - The excess regret over \( \zeta \)
\( \chi \) - decision variable \((X, Y)\)
\( R(\chi, s) \) - Regret as a function of \( \chi \) and scenario \( s \)
\[ f(\chi, \zeta) = P\{s \mid R(\chi, s) \leq \zeta \} \] - With \( \chi \) fixed, the collective probability of those scenarios in which the regret does not exceed \( \zeta \)
\[ f(\chi, \zeta^{-}) = P\{s \mid R(\chi, s) < \zeta \} \] - With \( \chi \) fixed, the collective probability of those scenarios in which the regret is strictly less than \( \zeta \)
\[ \zeta_{a}(\chi) = \min \{ \zeta \mid f(\chi, \zeta) \geq \alpha \} \] - With \( \chi \) fixed, the minimum value \( \zeta \) such that \( f(\chi, \zeta) \geq \alpha \), namely, the \( \alpha \)-quantile of the regrets of the \( S \) scenarios
\[ \zeta_{a}^{+}(\chi) = \inf \{ \zeta \mid f(\chi, \zeta) > \alpha \} \] - With \( \chi \) fixed, the infimum value \( \zeta \) such that \( f(\chi, \zeta) > \alpha \)
\( X_j \) - Binary for locating a safety shelter at node \( j \)

\[
X_j = \begin{cases} 
1 & \text{if we locate at candidate node } j \\
0 & \text{otherwise}
\end{cases}
\]

\( Y^s_{ij} \) - Fraction of demand from node \( i \) assigned to safety shelter at node \( j \) under each scenario \( s \)

**Formulation**

The \( \alpha \)-reliable mean-excess regret model for emergency preparedness and planning can be formulated as follows:

\[
\begin{align*}
\text{Minimize} & \quad F_\alpha^s \left( (X,Y), \zeta \right) = \zeta + \frac{1}{1-\alpha} \sum_{s=1}^{S} q^s U^s \\
\text{Subject to} & \quad \sum_{j=1}^{N} X_j = p \\
& \quad \sum_{j=1}^{N} Y^s_{ij} = 1 \quad \forall i, s \\
& \quad Y^s_{ij} - X_j \leq 0 \quad \forall i, j, s \\
& \quad f^s_{ij} \left( C^s_{ij} - \pi^s_{ij} \right) = 0 \quad \forall i, j, r, s \\
& \quad C^s_{ij} - \pi^s_{ij} \geq 0 \quad \forall i, j, r, s \\
& \quad \sum_{r} f^s_{ij} - h^s_i Y^s_{ij} = 0 \quad \forall i, j, s \\
& \quad \pi^s_{ij} \leq W^s \quad \forall i, j, s \\
& \quad R^s - (W^s - V^s) = 0 \quad \forall s \\
& \quad U^s \geq R^s - \zeta \quad \forall s \\
& \quad X_j \in \{0,1\} \quad \forall j \\
& \quad 0 \leq Y^s_{ij} \leq 1 \quad \forall i, j, s \\
& \quad f^s_{ij} \geq 0 \quad \forall i, j, r, s \\
& \quad \pi^s_{ij} \geq 0 \quad \forall i, j, r, s \\
& \quad U^s \geq 0 \quad \forall s
\end{align*}
\]

The objective function \((1a)\) minimizes the \( \alpha \)-reliable mean-excess regret. The first term \( \zeta \) satisfies \( f(\chi, \zeta^-) \leq \alpha \leq f(\chi, \zeta^+) \), with the lowest value \( \zeta^-_{\alpha}(\chi) \) and the highest \( \zeta^+_{\alpha}(\chi) \); while the second term is the expected value of regrets conditioned on the above \( \zeta \) \([12]\). Constraint \((1b)\) ensures that there are exactly \( p \) shelters. Constraint \((1c)\) states that all the demand at each node must be assigned to shelters under each scenario. Constraint \((1d)\) restricts those assignments to only the open shelters in each scenario. Constraints \((1e)\) to \((1g)\) are the first order conditions of the UE minimization program by the Beckmann transformation \([13]\). Constraint \((1e)\) forces the travel time along every route between each demand node (origin) and safety shelter (destination) (O-D) pair must be equal to the equilibrium travel time of that O-D pair if there is flow.
Constraint (1f) makes certain that any unused route between each O-D pair has travel time greater than or equal to the equilibrium travel time of that O-D pair. Constraint (1g) is the flow conservation equation - it requires the total flow along all routes between each O-D pair to equal the total demand assigned to the O-D pair. Constraint (1h) selects the maximum of the equilibrium travel times under each scenario as an input to calculate the regret. Constraint (1i) defines the regret for each scenario. Constraint (1j) calculates the excess regret for each scenario over the value \( \zeta \). Together with constraint (1o), (1j) makes sure that only those regrets greater than or equal to \( \zeta \) are considered as excess regrets. Constraints (1k) to (1o) are simply non-negativity and binary constraints. This formulation of the \( p \)-median location problem includes all the constraints necessary to achieve UE under each scenario while the mean-excess regret is minimized.

For each scenario \( s \), the best obtainable \( p \)-median value is found through:

\[
\text{Minimize} \quad V
\]
\[
\text{Subject to} \quad \sum_{j=1}^{N} X_j = P
\]
\[
\sum_{j=1}^{N} Y_{ij} = 1 \quad \forall i
\]
\[
Y_{ij} - X_j \leq 0 \quad \forall i, j
\]
\[
f^r_{ij} \left( C^r_{ij} - \pi^r_{ij} \right) = 0 \quad \forall i, j, r
\]
\[
C^r_{ij} - \pi^r_{ij} \geq 0 \quad \forall i, j, r
\]
\[
\sum_r f^r_{ij} - h_i Y_{ij} = 0 \quad \forall i, j
\]
\[
\pi^r_{ij} \leq V \quad \forall i, j
\]
\[
X_j \in \{0, 1\} \quad \forall j
\]
\[
0 \leq Y_{ij} \leq 1 \quad \forall i, j
\]
\[
f^r_{ij} \geq 0 \quad \forall i, j, r
\]
\[
\pi^r_{ij} \geq 0 \quad \forall i, j, r
\]

The objective function (2) minimizes the maximum UE travel time with scenario-specific choices of shelter locations. The constraints for formulation (2) are similar to those for formulation (1) except that they are all scenario-specific.

**Reformulation**

Both formulations (1) and (2) have too many decision variables. However, the situation can be ameliorated with a reformulation as a Stackelberg game. Different locations \( \chi \) selected by the leader will lead to different UE solutions and responses by the followers. Thus formulations (1) and (2) can be reconstructed as bilevel programming models - algorithms for solving these bilevel programming models are proposed below. In the upper level, the shelter locations are chosen to either minimize the mean-excess regret in the regret model or the maximum UE travel time in the scenario-specific model. The lower level uses a UE formulation depending on the shelter locations chosen in the upper level. However, the UE solution depends on the assignment
of demand to shelters, \( Y_{ij} \), since the UE formulation requires specific O-D assignments as input. To eliminate the decision variables \( Y_{ij} \) from this formulation, a super-destination (SD) is created which links to all selected shelters with zero travel time, and demand from all demand nodes will be sent to the SD. With this construction, constraints (1c) and (1d) will automatically be satisfied.

The \( \alpha \)-reliable mean-excess regret model is thus reformulated as:

**Upper level:**

\[
\text{Minimize} \quad F_{\alpha}((X,Y),\zeta) = \zeta + \frac{1}{1-\alpha} \sum_{s=1}^{S} q^s U^s
\]

\[
\text{Subject to} \quad \sum_{j=1}^{N} X_j = p
\]

\[
\pi_{i,SD}^{s} \leq W^s \quad \forall i, s
\]

\[
R^s - (W^s - V^s) = 0 \quad \forall s
\]

\[
U^s \geq R^s - \zeta \quad \forall s
\]

\[
X_j \in \{0,1\} \quad \forall j
\]

\[
U^s \geq 0 \quad \forall s
\]

**Lower level, for all scenarios \( s \):**

\[
\text{Minimize} \quad \sum_{a} \int_{0}^{r_a} t(x) dx
\]

\[
\text{Subject to} \quad \sum_{r} f_{i,SD}^r - h_i = 0 \quad \forall i
\]

\[
f_{i,SD}^r \geq 0 \quad \forall i, r
\]

\[
x^a = \sum_{i,r} f_{i,SD}^r \delta_{i,SD}^{ar} \quad \forall a
\]

The lower level formulation (3b) is the UE formulation according to the Beckmann transformation [13]. The path-arc incidence relationship indicator \( \delta_{i,SD}^{ar} \) in the constraint (3c) defines the configuration of the whole network and is determined by the shelter location choices from the upper level formulation (3a).

The scenario-specific model used to obtain the best \( p \)-median value is reformulated as follows, for each scenarios \( s \):

**Upper level:**

\[
\text{Minimize} \quad V
\]

\[
\text{Subject to} \quad \sum_{j=1}^{N} X_j = P
\]

\[
\pi_j \leq V \quad \forall i, j
\]

\[
X_j \in \{0,1\} \quad \forall j
\]
Lower level:

Minimize

\[ \sum_a \int_0^x t(x) dx \]  \hspace{1cm} (4b)  

Subject to

\[ \sum_r f_{i,SD}^r - h_i = 0 \]  \hspace{1cm} \forall i 
\[ f_{i,SD}^r \geq 0 \]  \hspace{1cm} \forall i, r 
\[ x^a = \sum_{i,r} f_{i,SD}^{r} \delta_{i,SD}^a \]  \hspace{1cm} \forall a 

Solution Algorithm

Bilevel programming models are intrinsically nonconvex and hence difficult to solve for the global optimum. However, the genetic-algorithms-based (GAB) approach proposed by Yin [14] can efficiently solve bilevel programming models.

Formulations (3) and (4) are basically p-median problems with UE formulations in the lower levels. The only decision variables in the upper levels are the shelter locations \(X_j\). Among previous works done on GAB for p-median problem, the approach of Alp, Drezner and Erkut [15] is suggested here due to its efficiency and simplicity. The Franke-Wolfe algorithm [13] is brought to bear on the UE component. Since the lower level formulations (3b) and (4b) are exactly the same as the conventional UE assignment problem given the shelter locations \(X_j\), the Franke-Wolfe algorithm is considered as a black box and not discussed here. Formulation (4) is basically a scaled-down version of formulation (3) so the solution algorithm described below can solve it with slight modifications.

Step 0:

Solution sets \(\{X_j\}\) in the upper level formulation are encoded as chromosomes; the genes are the indices of the \(p\) selected shelters. The fitness function is the objective function (3a). The regret for each scenario \(W_s^r\) can be calculated after solving the lower level UE formulation. \(\zeta\), in the fitness function, is the value satisfying \(f(\chi, \zeta^-) \leq \alpha \leq f(\chi, \zeta)\), which is the lowest possible regret level where the collective probability of all scenarios with regret not greater than this level is at least \(\alpha\). The second term in the fitness function is the expected value of the excess regret over \(\zeta\). Thus, the fitness function value can be evaluated once the UE formulation is solved and the regrets \(W_s^r\) are calculated for each scenario.

Step 1:

Calculate the population size and generate the initial population as described in Alp et al. [15].

Step 2:

Calculate the fitness function value for each chromosome of the whole population. Record the members with the highest and lowest fitness value.

Step 3:

Randomly select two members from the population and run the generation operator to obtain a candidate member as described in Alp et al. [15]. Mutation operation is omitted.

Step 4:

Calculate the fitness value of the candidate member. If the fitness value of the candidate member is lower than the highest fitness value in the current population, replace the one
with the highest fitness value with the candidate member. Update the members with the highest and lowest fitness value.

Step 5:

If the best solution found so far has not changed after \( \left\lfloor N\sqrt{p} \right\rfloor \) (the smallest integer greater than or equal to \( N\sqrt{p} \)) iterations, stop; otherwise return to Step 3.

**Model II**

*Description*

Now we address the dynamic routing strategy. The optimal location of safety shelters is taken from Model I and a routing strategy with minimum evacuation time is sought. Unlike conventional models, the travel time on any link is considered to be stochastic and varies between an upper and lower limit according to some probability distribution as well as time. The model here considers both the capacity of the links and the congestion on the links. It uses the shortest path algorithm to route the vehicles and recognizes delays caused by queuing in calculating the travel cost of any particular route. The model has the flexibility for a vehicle to change its route at any time depending on network conditions.

An SD is constructed linking the safety shelters so that the shortest path algorithm does not need to match nodes to safety shelters. The shortest path algorithm will be applied between the concerned node and this SD.

**Assumptions:**
- Queues are formed at nodes
- Queues are point queues and they do not spill over to other links of the network
- Vehicles join the end of a queue upon reaching a node
- Vehicles have perfect knowledge of the travel times
FIGURE 2  Illustration of Model II

**Notation**

- $i$ - Index of demand nodes subject to evacuation
- $j$ - Index of candidate safety shelters
- $k$ - Index of time interval $(k, k + 1]$ 
- $\Delta \tau$ - Time interval length
- $D_i$ - Total demand at $i$ to be evacuated
- $r_i$ - External rate at which vehicles enter node $i$
- $q_{ik}$ - Number of vehicles in queue at the start of time interval $(k, k + 1]$
- $c_{ijk}$ - Capacity in link $ij$ in time interval $(k, k + 1]$
- $t_{ijk}^o$ - Free flow travel time on link $ij$ in time interval $(k, k + 1]$, which varies according to some probability distribution between an upper and lower limit
- $t_{ijk}$ - Total travel cost on link $ij$ in time interval $(k, k + 1]$
\( P_{ijk} \) - Probability that a vehicle leaves node \( i \) in time interval \((k, k + 1]\) will arrive node \( j \) in time interval \((k', k' + 1]\), i.e. \( P[\Delta \tau(k' - k) \leq t_{ijk} \leq \Delta \tau(k' - k + 1)] \), and \( \sum_{k'} P_{ijk} = 1 \)

\( \delta_{ik} \) - Number of vehicles leaving node \( i \) in time interval \((k, k + 1]\)

\( z_{ijk} \) - Binary for a vehicle starting from node \( i \) encountering node \( j \) in time interval \((k, k + 1]\)

\( e_{ik} \) - External number of vehicles entering node \( i \) in time interval \((k, k + 1]\)

\( w_{ik} \) - Delay at node \( i \) in time interval \((k, k + 1]\)

Definitions:

\[
z_{ijk} = \begin{cases} 
1 & \text{if the first node encountered by node } i \text{ is node } j \\
0 & \text{otherwise}
\end{cases}
\]

Delay at node \( i \) is the amount of time spent by a vehicle in queue at the node, i.e. \( w_{ik} = \frac{q_{ik}}{c_{ij}} \)

Algorithm
Step 0:
Initially the queue at any node \( i \) is given by
\( q_{i0} = e_{i0} = \min \{ D_{i}, r_{i} \times \Delta \tau \} \quad \forall \quad i \)

Step 1:
The route with the least travel time is calculated using the shortest path algorithm.
Inputs are \( t_{ij}^{o}, w_{ik} \)

Where \( w_{ik} = \frac{q_{ik}}{c_{ij}} \)

The total travel time on any link is calculated as the sum of free flow travel time and the delay due to queuing.
\( t_{ij} = t_{ij}^{o} + \frac{q_{ik}}{c_{ij}} \)

Output is \( z_{ijk} \)

Step 2:
The number of vehicles that leave a particular node is the minimum of the number in queue and the capacity of the link.
\( \delta_{ik} = \min \{ q_{ik}, c_{ij} \times \Delta \tau \} \)

Step 3:
The matrix \( q \) is updated for the next time interval
Update matrix $q$ for $\{k + 1, k + 2\}$

$$q_{ik+1} = q_{ik} - \delta_{ik} + e_{ik}$$

Where $e_{ik} = \min \{D_i - k \times r_i \times \Delta \tau, r_i \times \Delta \tau\}$

Step 4:

The matrix $q$ is updated for the time interval when the vehicles reach their destination nodes.

Update matrix $q$ for $\{k', k'+1\}$

$$q_{jk'} \rightarrow q_{jk'} + \sum_i p_{ijk} \delta_{ik} z_{ijk} \forall j, k'$$

Step 5:

The routing process is stopped when the queues at all the nodes are dissipated.

Stop if $\sum_i q_{ik} = 0$

Otherwise go to Step 1 for $k + 1$

The procedure is represented in FIGURE 3.

In this algorithm, batches of vehicles are dispatched at regular intervals. For clarity, it is assumed that the departure time of a vehicle from a node is always at the start of an interval, regardless of when it departs during the interval. Higher accuracy can be achieved with shorter time intervals – the trade-off being higher computational requirements.

In an actual evacuation, people will not have perfect knowledge of the travel times and they will not be able to run shortest path algorithms to find the optimal route. However, information on the traffic volume passing into each node and the corresponding exit node is known at all times. Evacuation managers can use this information to divert traffic in an optimal manner.

**CONCLUSION**

The $\alpha$-reliable mean-excess regret model was developed to determine the locations of safety shelters in a transportation network so that evacuation time is minimized. Given possible future disruption scenarios and likelihoods of occurrence, the model minimizes the expected regret of worst-case scenarios whose total probability is less than $1 - \alpha$. The regret is calculated in terms of the maximum evacuation time to safety shelters. Travel times are calculated with the User Equilibrium assignment of vehicles in a transportation network. The model was reformulated as bilevel model with the upper level designed to locate safety shelters and the lower level designed to calculate the User Equilibrium travel times. The bilevel problem is solved in two tiers: the upper level is solved using a genetic algorithm and the lower level using the Franke-Wolfe Algorithm.

A second model was developed to provide dynamic routing control in a stochastic time varying transportation network during disruption. Locations of safety shelters obtained by the first model are the destinations in the second. It routes the vehicles using the shortest path algorithm accounting for the capacity of the links and delays due to congestion. The second model is useful in providing decision makers with a control strategy during emergency evacuations.
\[
q_{i0} = e_{i0} = \min \left\{ D_i, r_i \times \Delta \tau \right\} \quad \forall \ i
\]

Shortest path algorithm

Inputs are \( t^o, w_{ik} \); Output is \( z_{jk} \)

\[
\delta_{ik} = \min \left\{ q_{ik}, c_{ij} \times \Delta \tau \right\}
\]

\[
q_{ik+1} = q_{ik} - \delta_{ik} + e_{ik}
\]

where

\[
e_{ik} = \min \left\{ D_i - k \times r_i \times \Delta \tau, r_i \times \Delta \tau \right\}
\]

\[
q_{jk} \rightarrow q_{jk} + \sum_i p_{ijk} \delta_{ik} z_{jk} \quad \forall \ j, k'
\]

\[
\sum_i q_{ik} = 0
\]

STOP

FIGURE 3 Model II algorithm.

Testing the Model I algorithm and the Model II algorithm is currently underway. More work will be performed to test the feasibility and computational efficiency of these procedures. Particularly for Model II, short computational time is exceptionally important during the critical
moment of evacuation. The trade-off between time interval size, computational time and accuracy for this algorithm should be studied to determine the optimal time interval size with the highest accuracy given computational time constraints. In addition, queue spillover is a common evacuation phenomenon that is not addressed in Model II. It is believed that consideration of queue spillover will increase the realism of the model and achieve a stronger result.

REFERENCES


