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UNIVERSITY OF CALIFORNIA

Radiation Laboratory

ROTANGER: A PROGRAM FOR CALCULATING ORBITS IN A CYCLOTRON BY THE USE OF THE IBM-650 DATA-PROCESSING COMPUTER

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ABSTRACT

Three simultaneous polar-coordinate component equations in an azimuthally homogeneous magnetic field of a cyclotron are integrated by a fixed-point program on the IBM 650 computer to give the orbits of particles in the cyclotron. A regenerator action for extracting particles is computed as an inpulse once each turn as a subprogram when desired. The program is described and instructions for its use are included.

ROTANGER: A PROGRAM FOR CALCULATING ORBITS IN A CYCLOTRON BY THE USE OF THE IBM-650 DATA-PROCESSING COMPUTER

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INTRODUCTION

Particle orbits in a cyclotron have been studied to design and evaluate a regenerative extractor. This paper discusses the program for numerical integration of the equations of motion. The program was made for and used on the IBM 650 data-processing computer.

In general, the program consists of five subprograms: (a) the finite-difference process of numerical integration of three simultaneous equations; (b) the evaluation of the differential equations; (c) the computation of the third derivatives for the starting routine; (d) tests; and (e) the regenerator action. These subprograms are tied together by the process of entering and leaving them through tests and by the use of single locations in the memory for terms or parameters common to two or more subprograms.

The numerical integration of three simultaneous differential equations was programmed by Kent Curtis and Alper Garren[1] of this Laboratory and uses a Milne 5-point formula for the integration with a Milne 3-point formula and a special procedure to start the 3-point formula to initiate the orbit.[2] The memory locations of the numerical integration subprogram are listed in Appendix 1.

The three differential equations are the r-, θ-, z-component equations of the Lorentz equation for charged-particle motion in a static azimuthally homogeneous magnetic field. The calculation of the third derivatives from the second-order equations is required for the starting program and is included. The evaluation of the magnetic field components is performed by a Taylor-series expansion about the midplane and the synchronous orbit.

The regenerator action[3] is computed as an impulse occurring at the same azimuthal position for each turn. The magnitude of the impulse is computed by a power series and the velocity terms are changed by the addition of the impulse. The integration program then restarts with these new initial conditions. The tests include determining the regenerator position, the bounds on the axial amplitude z, the bounds on the radial amplitude r, the radius where

[1]A UCRL Report is in preparation by Curtis and Garren.

[2]William E. Milne, Numerical Calculus (Princeton Univ. Press, 1949).

[3]Warren F. Stubbins, Design of Regenerative Extractors for Synchrocyclotrons. I. Small-Amplitude Extraction, UCRL-3476 Rev., Nov. 22, 1957.

constants in the expansion of the magnetic field are changed, and others. An interpolation routine to fix the regenerator position is included.

The program is in fixed point, and values are given with eight decimals. This sets an upper bound on the magnitude of numbers, which can be only 99.99999999. An overflow occurs a little before this value, about 10% in the computation of v^2; however, a rescaling could prevent this. The interval of each advance in the numerical integration h is variable and has been used at 0.1 radian for most orbits. The error ϵ between successive values of the variables as computed in the iterative integration procedure is also adjustable and has been used at .00000020 for most cases. The similar terms ϵ_0 and h_0 are used in the starting program and are set at one-half the values for ϵ and h.

At the end of each integration step, the square of the total velocity is computed. The constancy of this term is used to judge the accumulation of errors in the integration of the three equations.

COMPUTATIONS

Equations of Motion

The three-component equations obtained for the Lorentz equation are:

$$\frac{d^2r}{dt^2} - r \left(\frac{d\theta}{dt}\right)^2 = - \frac{e}{m} r \frac{d\theta}{dt} B_z$$

$$r \frac{d^2\theta}{dt^2} + 2 \frac{dr}{dt} \frac{d\theta}{dt} = - \frac{e}{m} \left(\frac{dz}{dt} B_r - \frac{dr}{dt} B_z \right)$$

and

$$\frac{d^2z}{dt^2} = - \frac{e}{m} r \frac{d\theta}{dt} B_r ,$$

where we have assumed an azimuthally uniform field, i.e., $B_\theta = 0$, and note that $B_z = B_z(r, z)$ and $B_r = B_r(r, z)$.

The synchronous radius R corresponds to the beginning radius of the regenerator, and ρ is the orbit departure from this radius. The radius from the center of the cyclotron is $r = R + \rho$. The magnetic field values are normalized by dividing them by the field on the median plane at the synchronous orbit B_z (R, 0).

Our equations then become:

$$\frac{d^2\rho}{dt^2} - (R + \rho) \left(\frac{d\theta}{dt}\right)^2 = - \frac{e B_z (R0)}{m} (R + \rho) \frac{d\theta}{dt} \frac{B_z (r, z)}{B_z (R, 0)} ,$$

$$\frac{d^2\theta}{dt^2} + \frac{2}{(R+\rho)} \frac{d\rho}{dt}\frac{d\theta}{dt} = - \frac{e B_z(R,0)}{(R+\rho)\, m} \left[\frac{dz}{dt} \frac{B_r(r,z)}{B_z(R,0)} - \frac{d\rho}{dt} \frac{B_z(r,z)}{B_z(R,0)}\right],$$

and

$$\frac{d^2 z}{dt^2} = - \frac{e B_z(R,0)}{m} (R+\rho) \frac{d\theta}{dt} \frac{B_r(r,z)}{B_z(R,0)} .$$

We define

$$\omega = \frac{e B_z(R,0)}{mc} = \frac{e B_z(R,0)c}{E}$$

where E is the total energy, B is the magnetic flux density in gauss, e is the electronic charge in esu, and c is the velocity of light. Choosing a new time variable τ such that $\tau = \omega t$, we obtain

$$\frac{d^2\rho}{d\tau^2} - (R+\rho)\left(\frac{d\theta}{d\tau}\right)^2 = -(R+\rho)\frac{d\theta}{d\tau}\frac{B_z(r,z)}{B_z(R,0)},$$

$$\frac{d^2\theta}{d\tau^2} + \frac{2}{R+\rho}\frac{d\rho}{d\tau}\frac{d\theta}{d\tau} = -\frac{1}{R+\rho}\left[\frac{dz}{d\tau}\frac{B_r(r,z)}{B_z(R,0)} - \frac{d\rho}{d\tau}\frac{B_z(r,z)}{B_z(R,0)}\right],$$

and

$$\frac{d^2 z}{d\tau^2} = -(R+\rho)\frac{d\theta}{d\tau}\frac{B_r(r,z)}{B_z(R,0)} .$$

We denote derivatives with respect to τ with dots hereafter.

The magnetic-field components are expressed in a Taylor-series expansion about the synchronous orbit on the midplane as

$$\frac{B_z(r,z)}{B_z(R,0)} = 1 + \frac{1}{B_z(R,0)} \left\{ \frac{\partial B_z}{\partial r}\rho + \frac{1}{2}\frac{\partial^2 B_z}{\partial r^2}\rho^2 + \frac{1}{6}\frac{\partial^3 B_z}{\partial r^3}\rho^3 + \frac{1}{2}\left[-\frac{1}{(R+\rho)}\frac{\partial B_z}{\partial r} - \frac{\partial^2 B_z}{\partial r^2}\right] z^2 \right\}$$

$$= \alpha + \beta\rho + \gamma\rho^2 + \delta\rho^3 - \left[\frac{1}{2r}\beta + \gamma\right] z^2 ,$$

and

$$\frac{B_r(r,z)}{B_z(R,0)} = \frac{1}{B_z(R,0)} \left\{ \frac{\partial B_z}{\partial r} Z + \frac{\partial^2 B_z}{\partial r^2} \rho Z + \frac{1}{2} \frac{\partial^3 B_z}{\partial r^3} \rho^2 Z \right.$$

$$\left. + \frac{1}{6}\left[\frac{1}{r^2} \frac{\partial B_z}{\partial r} - \frac{1}{r} \frac{\partial^2 B_z}{\partial r^2} - \frac{\partial^3 B_z}{\partial r^3} \right] Z^3 \right\}$$

$$= \beta z + 2\gamma\rho Z + 3\delta Z\rho^2 + \left[\frac{1}{6r^2}\beta - \frac{1}{3r}\gamma - \delta \right] Z^3 ,$$

where

$$\alpha = 1.0,$$

$$\beta = \frac{1}{B_z(R,0)} \left. \frac{\partial B_z}{\partial r} \right|_{R,0} ,$$

$$\gamma = \frac{1}{2\,B_z(R,0)} \left. \frac{\partial^2 B_z}{\partial r^2} \right|_{R,0} ,$$

and

$$\delta = \frac{1}{6B_z(R,0)} \left. \frac{\partial^3 B_z}{\partial r^3} \right|_{R,0} .$$

The third-derivative equations required to start the numerical integration are obtained directly from the component equations of motion.

$$\dddot{\rho} = r\,\ddot{\theta} \left(2\dot{\theta} - \frac{B_z(r,z)}{B_z(R,0)} \right) + \dot{\rho}\,\dot{\theta} \left(\dot{\theta} - \frac{B_z(r,z)}{B_z(R,0)} \right) ,$$

$$\dddot{\theta} = -\frac{2}{r} (\dot{\theta}\,\ddot{\rho} + \dot{\rho}\,\ddot{\theta}) + \frac{1}{r} \left(\frac{B_z(r,z)}{B_z(R,0)} \ddot{\rho} - \frac{B_r(r,z)}{B_z(R,0)} \ddot{Z} \right)$$

$$+ \frac{\dot{\rho}}{r^2} \left(2\,\dot{\theta}\,\dot{\rho} + \frac{B_r(r,z)}{B_z(R,0)} \dot{Z} - \frac{B_z(r,z)}{B_z(R,0)} \dot{\rho} \right) ,$$

and

$$\dddot{Z} = (r\,\ddot{\theta} + \dot{\rho}\,\dot{\theta}) \frac{B_r(r,z)}{B_z(R,0)}$$

Computation of Free Orbits

The integration of these equations without any regenerator perturbation allows the calculation of the periods of free orbits, the radial velocity for escape, and the radial velocities at the regenerator position. These are used to obtain the radial-velocity changes required and accomplished by the regenerator to extract the particles. The effect of axial and radial oscillation amplitudes on the axial and radial periods and the coupling between them is obtainable from the free orbits with a variety of initial conditions.

A check of the computed features of the orbits against the features obtained by analysis confirms the use of proper values for the parameters of field expansion, etc. The integration time is approximately 20 min per full turn in the cyclotron when the independent variable is advanced 0.1 radian each step.

The computation is started by reading the initial conditions and a transfer card into the computer. The initial conditions are the three coordinates and the three first derivatives, i.e., slopes of the coordinates. Also the regenerator location is required to reset the computer. For free orbits the regenerator position, θ_1, is set at the largest value, 99.99999999 radians, so the orbit will not encounter this value and initiate the regenerator action. A change in the interval of integration, the bounds on the variables, the field parameters, or other constants can be included with the initial conditions. The transfer card starts the program by transferring control to location 0151. The memory locations for variables are shown in Appendix 2; location for magnetic-field computations are given in Appendix 3, and locations of constants used in regenerator calculations are shown in Appendix 4. Appendix 5 to 11 describe the program for the differential equations, the tests, and the regenerator action.

Computation of Regenerating Orbits

When the regenerator location, θ_1, is encountered, the program transfers from the point-by-point integration to the routine for determining the effect of the regenerator and initiating an orbit from this point on.

The program tests at each advance of the variable θ to see if the value θ_1 has been exceeded. If so, the previous values of the variables are used to interpolate to the actual regenerator azimuthal position. The interpolated values of the variables are used to compute the impulsive changes, which are added to the radial and axial velocities. The value of $\dot{\theta}$ is computed from the v^2 value from the last computed point. The new initial conditions are substituted in their memory locations, the value of θ_1 is advanced by 2π radians, and the orbit is restarted. (By including a test for radial position, the regenerator action is by passed for a particle inside the synchronous orbit at the regenerator angle. See Appendix 5)

The v^2 term, which is used to check the accumulation of errors, relates the three velocity terms as

$$v^2 = \dot{\rho}^2 + (r\,\dot{\theta})^2 + \dot{z}^2 \ .$$

A synchronous particle has $v^2 = R^2$. This is used to establish the initial conditions for a particle with synchronous energy.

The radial impulse $\delta(\dot{\rho})$ is expressed as a power series in ρ:

$$\delta\,(\dot{\rho}) = a_1\,\rho + a_2\rho^2 + a_3\rho^3 \ .$$

The change in $\delta(\dot{\rho})$ for off-midplane position is ignored in this program, but a subroutine for accounting for this change was used in a portion of the calculations. The axial impulse is related to the radial impulse through the slope of the regenerator field. The axial perturbation arises from the radial component of the regenerator field and is proportional to the amplitude of axial motion at the encounter

$$\delta\,(\dot{Z}) = (\alpha_0 + \alpha_1\rho + \alpha_2\rho^2 + \alpha_3\rho^3)\,Z \ ,$$

where

$\alpha_0 = -\,Ra_1$, $\alpha_1 = -\,2Ra_2$, $\alpha_2 = -\,3Ra_3 - a_2$, and $\alpha_3 = -\,2a_3$.

The regenerator constants are placed in memory locations shown in Appendix 4.

Output

The computation of each point is recorded on an output card. These cards have in the following order, radius in inches, θ in degrees (input card requires θ in radians), z in inches, $\dot{\rho}$, $\dot{\theta}$, $\dot{z}$, and v^2. The quantities are given in eight decimals except θ in degrees and v^2, which are given in six decimals.

Tests and Operation Hints

Because orbits that exceed the axial aperture of the dee cannot be successfully regenerated, the computation of these orbits is terminated when $z \geqslant \tilde{z}$. $\tilde{z}$ is in memory location 0111.

A similar test is applied at $\tilde{r}$ (memory location 1834) so that an escaping particle can be identified and the orbit computation stopped where the magnetic field is no longer adequately given by the Taylor series expansion. This test can be used to stop the program so that new constants may be inserted in the field or regenerator description.

The coefficients in the Taylor-series magnetic-field expansion were changed by the program when the orbit crossed a radius R_x because the coefficients that fit well outside R_x gave the improper field shape inside. The two sets of coefficients were stored (see Appendix 3) and substituted by the program depending upon the orbit's radial position. R_x is in 0035.

The test for the regenerator position, θ_1, has been mentioned. This test causes the program to branch for computing the regenerator impulse θ_1 is in 0101.

After the stop because of the $\tilde{r}$ or $\tilde{z}$ test, when a new value of the constant is read into the memory, the program is restarted manually from command 1828.

An overflow in θ occurs just short of 16 turns and requires a reset of θ or a new start of the program at a convenient point.

If the program is stopped temporarily, it should be stopped by address stop at 0326. A random stop may make the program inoperative because in the iterative sections the commands are altered within the program. A random stop, followed by a new set of initial conditions and a restart of the program, may prevent the restoration of the altered commands to their proper initial values.

The continuation of an orbit may be accomplished by using the output values for the initial conditions. The value of θ must be restated in radians, and the regenerator position θ_1 related to the restarting azimuth. A convenient way is to use the last regenerator values as the initial condition, set θ at zero radians at this point, and redefine θ_1 as 2π.

An alternate method is to obtain a memory dump from 1000 to 1070, which is read in as the initial conditions, and to transfer control to start at 1111.

A card deck of the program and typical initial conditions are available upon request. A program, Daida, for reading the output cards and evaluating the orbits is described in another report.

ACKNOWLEDGMENTS

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APPENDIXES

Appendix 1

Locations for the Numerical Integration Portion of the Program.

Iterative Program	1000 - 1405
	1416 - 1527
Starting Routine	0151 - 0251
	0263
	0266
	0269
	0271 - 0303
	0305 - 0310
	0379
	0390
	0404
	0412
	0434
	0444
	0448 - 0459
	0476
	0503 - 0898
	0911 - 0999
	1600 - 1747

Appendix 2

Locations of Variables.*

r	1061	z	1067
$\dot{r}$	1062	$\dot{z}$	1068
$\ddot{r}$	1063	$\ddot{z}$	1066
$\dddot{r}$	0263	$\dddot{z}$	0269
θ	1064	h_0	0412
$\dot{\theta}$	1065	h	1000
$\ddot{\theta}$	1066	ϵ_0	0690
$\dddot{\theta}$	0266	ϵ	1010

Synchronous radius	R	0100
Regenerator position	θ_1	0101
Max. allowable axial amplitude	$\tilde{z}$	0111
Max. allowable radius	$\tilde{r}$	1834
Velocity squared	v^2	0033
Radius for changing field coefficient	R_x	0036

*Values of r, $\dot{r}$, $\ddot{r}$, θ, $\dot{\theta}$, $\ddot{\theta}$, z, $\dot{z}$, and $\ddot{z}$ from the proceding point are stored in 1051, etc., those from the second preceding point in 1041 etc., and in turn to 1011 for the fifth preceding point.

Appendix 3

Locations for magnetic-field computation

For	$r \gg R_x$
	α	0903
	β	0910
	γ	1769
	δ	1823

For	$r < R_x$
	α'	1836
	β'	0097
	γ'	0034
	δ'	0036
	R_x	0035
	$\frac{B_z(r, z)}{B_z(R, 0)}$	0009
	$\frac{B_r(r, z)}{B_z(R, 0)}$	0012

Appendix 4

Memory Locations for Constants Used in Regenerator Calculation.

a_1	0112
a_2	0113
a_3	0114
α_0	0115
α_1	0116
α_2	0117
α_3	0118
θ_1	0101

Appendix 5

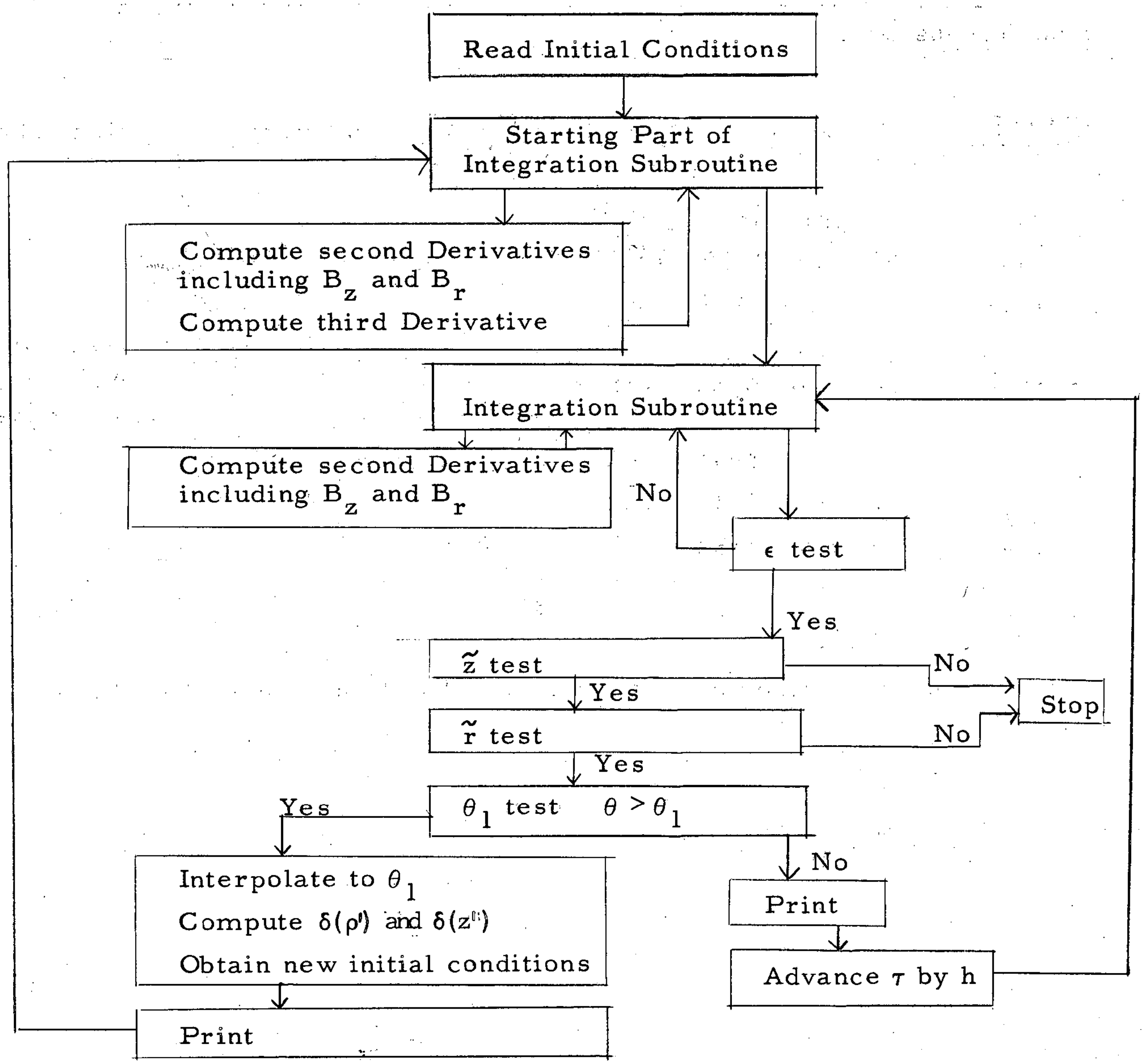
Test to Bypass Regenerator Action when Particle is Inside the Synchronous Orbit.*

Instruction location	Instruction			Operation	Storage location	Element	Remarks
1581	46	1937	1582	BRMIN			
1937	60	1061	1938	RAU		r	
1938	11	0100	1939	SU		r - R	
1939	46	1940	0340	BRMIN		r > R	Proceed with interpolation
						r < R	Avoid regenerator
1940	60	0101	1941	RAU		θ_1	
1941	10	0134	1942	AU		$\theta_1 + 2\pi$	
1942	21	0101	1582	STU	0101	$\theta_1 + 2\pi$	

*Cards for this subroutine are not supplied with the program deck.

Appendix 6.

Block Diagram of Rotanger



Appendix 7

Computer Control Setting for Program Operation.

Control	Position
Programmed	Stop
Half Cycle	Run
Control	Run
Display	Distributor
Overflow	Sense
Error	Stop

Appendix 8

Subprogram for evaluating the three equations of motion, including the computation of magnetic field components.

Instruction Location	Instruction	Operation	Storage Location	Element	Remarks
0313	2403121750	STD			
1750	6010611751	RAU		r_0	
1751	1101001752	SU		$r - R = \rho$	
1752	2100001753	STU	0000	ρ	
1753	6510610098	RAL		r	Starts test to choose proper field coefficient to give best fit in field expansion. R_x is radius at which coefficients are exchanged. Primed values used when $r < R_x$.
0098	1600350099	SL		$r - R_x$	
0099	4602670484	BRMIN			
0267	6918360268	LD		α'	
0268	2401050270	STD	0105	α'	
0270	6900970304	LD		β'	
0304	2401060311	STD	0106	β'	
0311	6900340324	LD		γ'	
0324	2401070402	STD	0107	γ'	
0402	6900360442	LD		δ'	
0442	2401080483	STD	0108	δ'	
0484	6909030485	LD		α	For coefficient in region $r > R_x$.
0485	2401050486	STD	105	α	
0486	6909100487	LD		β	
0487	2401060150	STD	106	β	
0150	6917690321	LD		γ	
0321	2401070322	STD	107	γ	
0322	6918230323	LD		δ	
0323	2401080483	STD	108	δ	
0483	6001081754	RAU		δ	Returns to program of calculation of field components.
1754	1900001755	MULT		$\delta\rho$	
1755	3100081756	SRD		$\delta\rho$	
1756	1501071757	AL		$\gamma + \delta\rho$	
1757	6080021758	RAU			
1758	1900001759	MULT		$\gamma\rho + \delta\rho^2$	
1759	3100081760	SRD			
1760	1501061761	AL		$\beta + \gamma\rho + \delta\rho^2$	
1761	6080021762	RAU			
1762	1900001763	MULT		$\beta\rho + \gamma\rho^2 + \delta\rho^3$	
1763	3100081764	SRD			
1764	1501051765	AL		$\alpha + \beta\rho + \gamma\rho^2 + \delta\rho^3$	
1765	2000011766	STL	0001	$\alpha + \beta\rho + \gamma\rho^2 + \delta\rho^3$	
1766	6010611767	RAU		r	
1767	1901041768	MULT		2r	
1768	3100021821	SRD			
1821	2000021822	STL	0002	2r	
1822	6001061770	RAU		β	
1770	3000021771	SRT			
1771	6400021826	DIVRU		$\frac{1}{2r}\beta$	
1826	3100021772	SRD			
1772	1501071773	AL		$\frac{1}{2r}\beta + \gamma$	
1773	6080021774	RAU			
1774	1910671775	MULT		$Z\left(\frac{\beta}{2r} + \gamma\right)$	

Instruction Location	Instruction	Operation	Storage Location	Element	Remarks
1775	3100081776	SRD			
1776	6080021777	RAU			
1777	1910671778	MULT		$Z^2 (\frac{\beta}{2r} + \gamma)$	
1778	3100081779	SRD			
1779	2000031780	STL	0003	$Z^2 (\frac{\beta}{2r} + \gamma)$	
1780	6000011781	RAU		$\alpha + \beta\rho + \gamma\rho^2 + \delta\rho^3$	
1781	1100031782	SU		B_Z/B_0	
1782	2100091783	STU	0009	$B_Z (\rho, Z)/B_0 (R, 0)$	Ends computation of axial component of magnetic field.
1783	6001081784	RAU		δ	
1784	1901091785	MULT		3δ	
1785	6080021805	RAU			
1805	1900001786	MULT		$3\delta\rho$	
1786	3100081787	SRD			
1787	2000041788	STL	0004	$3\delta\rho$	
1788	6001071789	RAU		γ	
1789	1901041790	MULT		2γ	
1790	1500041791	AL		$2\gamma + 3\ \delta\rho$	
1791	6080021792	RAU			
1792	1900001793	MULT		$2\ \gamma\rho + 3\ \delta\rho^2$	
1793	3100081794	SRD			
1794	1501061795	AL		$\beta + 2\ \gamma\rho + 3\ \delta\rho^2$	
1795	6080021796	RAU			
1796	1910671797	MULT		$Z (\beta + 2\gamma\rho + 3\ \delta\rho^2)$	
1797	3100081798	SRD			
1798	2000051799	STL	0005	$Z (\beta + 2\gamma\rho + 3\ \delta\rho^2)$	
1799	6010611800	RAU		r	
1800	1901091827	MULT		$3r$	
1827	3100021801	SRD			
1801	2000061802	STL	0006	$3r$	
1802	6001061803	RAU		β	
1803	6400021804	DIVRU		$\frac{\beta}{2r}$	
1804	3100021806	SRD			
1806	1601071807	SL		$\frac{\beta}{2r} - \gamma$	
1807	6080021808	RAU			
1808	3100041809	SRD			
1809	6400061810	DIVRU		$\frac{\beta}{6r^2} - \frac{\gamma}{3r}$	
1810	1601081811	SL		$\frac{\beta}{6r^2} - \frac{\gamma}{3r} - \delta$	
1811	2000071812	STL	0007	$\frac{\beta}{6r^2} - \frac{\gamma}{3r} - \delta$	
1812	6010671813	RAU		Z	
1813	1910671824	MULT		Z^2	
1824	3100081825	SRD			
1825	6080021814	RAU			
1814	1910671815	MULT		Z^3	
1815	3100081816	SRD			
1816	6080021817	RAU			
1817	1900071818	MULT		$Z^3 (\frac{\beta}{6r^2} - \frac{\gamma}{3r} - \delta)$	
1818	3100081819	SRD			
1819	1500051820	AL		$Z (\beta + 2\ \gamma\rho + 3\ \delta\rho^2) + Z^3 (\frac{\beta}{6r^2} - \frac{\gamma}{3r} - \delta)$	
1820	2000121840	STL	0012	$B_r (r, Z)/ B_0 (R, 0)$	Ends computation of radial component of magnetic field.

Instruction Location	Instruction	Operation	Storage Location	Element	Remarks
1840	6001051841	RAU			
1841	3000021842	SRT			
1842	6410611843	DIVRU		$\frac{1}{r}$	
1843	2000131844	STL	0013	$\frac{1}{r}$	
1844	6000121845	RAU		B_r/B_0	
1845	1910611846	MULT		$r\ B_r/B_0$	
1846	3100081848	SRD			
1848	6080021849	RAU			
1849	1910651850	MULT		$r\dot{\theta}\ B_r/B_0$	
1850	3100081851	SRD			
1851	2010691852	STL	1069	$d^2Z/d\tau^2 = \ddot{Z}$	Completes axial equation.
1852	6010651853	RAU		$\dot{\theta}$	
1853	1100091854	SU		$\dot{\theta} - B_Z/B_0$	
1854	2100141855	STU	0014	$\dot{\theta} - B_Z/B_0$	
1855	1910611856	MULT		$r\ (\dot{\theta} - B_Z/B_0)$	
1856	3100081857	SRD			
1857	6080021858	RAU			
1858	1910651859	MULT		$r\dot{\theta}\ (\dot{\theta} - B_Z/B_0)$	
1859	3100081860	SRD			
1860	2010631861	STL	1063	$\ddot{\rho} = \frac{d^2\rho}{d\tau^2}$	Completes radial equation.
1861	6000141865	RAU		$\dot{\theta} - B_Z/B_0$	
1865	1010651862	AU		$2\dot{\theta} - B_Z/B_0$	
1862	1910621863	MULT		$\dot{\rho}\ (2\dot{\theta} - B_Z/B_0)$	
1863	3100081866	SRD			
1866	2000151867	STL	0015	$\dot{\rho}\ (2\dot{\theta} - B_Z/B_0)$	
1867	6010681868	RAU		$\dot{Z}$	
1868	1900121869	MULT		$\dot{Z}\ B_r/B_0$	
1869	3100081870	SRD			
1870	1500151871	AL		$(2\dot{\rho}\dot{\theta} - \dot{\rho}\ (B_Z/B_0) + \dot{Z}\ (B_r/B_0) = -r\ddot{\theta}$	
1871	6080021872	RAU			
1872	1900131873	MULT		$-\ddot{\theta}$	
1873	3100081874	SRD			
1874	6680021875	RSL		$\ddot{\theta}$	
1875	2010660312	STL	1066	$\ddot{\theta}$	Complete azimuthal equation

Instruction location 0312 returns to integrating routine for the three simultaneous equation.

Appendix 9

Subprogram for evaluating third derivative equation used in starting the integration

Instruction location	Instruction	Operation	Storage location	Element	Remarks
0250	2402511876	STD			
1876	6010621877	RAU		$\dot{\rho}$	
1877	1910651878	MULT		$\dot{\rho}\dot{\theta}$	
1878	3100081879	SRD			
1879	2000161880	STL	0016	$\dot{\rho}\dot{\theta}$	
1880	6010611881	RAU		r	
1881	1910661882	MULT		$r\ddot{\theta}$	
1882	3100081883	SRD			
1883	2000171884	STL	0017	$r\ddot{\theta}$	
1884	1500161885	AL		$\dot{\rho}\dot{\theta} + r\ddot{\theta}$	
1885	2000181886	STL	0018	$\dot{\rho}\dot{\theta} + r\ddot{\theta}$	
1886	6080011887	RAU			
1887	1900121888	MULT		$B_r/B_0\ (\dot{\rho}\dot{\theta} + r\ddot{\theta})$	
1888	3100081889	SRD			
1889	2002691890	STL	0269	$\dddot{Z}$	Completes axial equation.
1890	6000181891	RAU		$\dot{\rho}\dot{\theta} + r\ddot{\theta}$	
1891	1900091892	MULT		$B_z/B_0\ (\dot{\rho}\dot{\theta} + r\ddot{\theta})$	
1892	3100081893	SRD			
1893	2000191894	STL	0019	$B_Z/B_0\ (\dot{\rho}\dot{\theta} + r\ddot{\theta})$	
1894	6000171895	RAU		$r\ddot{\theta}$	
1895	1910651896	MULT		$r\dot{\theta}\ddot{\theta}$	
1896	3100081897	SRD			
1897	6080021898	RAU			
1898	1901041899	MULT		$2r\dot{\theta}\ddot{\theta}$	
1899	2000201748	STL	0020	$2\ r\dot{\theta}\ddot{\theta}$	
1748	6000161749	RAU		$\dot{\rho}\dot{\theta}$	
1749	1910651528	MULT		$\dot{\rho}\dot{\theta}^2$	
1528	3100081529	SRD			
1529	1500201530	AL		$\dot{\rho}\dot{\theta}^2 + 2r\dot{\theta}\ddot{\theta}$	
1530	1600191531	SL		$\dot{\rho}\dot{\theta}^2 + 2r\dot{\theta}\ddot{\theta} - B_Z/B_0\ (\dot{\rho}\dot{\theta} + r\ddot{\theta})$	
1531	2002631532	STL	0263	$\dddot{\rho}$	Completes radial equation
1532	6010631533	RAU		$\ddot{\rho}$	
1533	1900091534	MULT		$\ddot{\rho}\ B_Z/B_0$	
1534	3100081535	SRD			
1535	2000211536	STL	0021	$\ddot{\rho}\ B_Z/B_0$	
1536	6010691537	RAU		$\ddot{Z}$	
1537	1900121538	MULT		$\ddot{Z}\ \ B_r/B_0$	
1538	3100081539	SRD			
1539	2000221540	STL	0022	$\ddot{Z}\ \ B_r/B_0$	
1540	6010631541	RAU		$\ddot{\rho}$	
1541	1910651542	MULT		$\ddot{\rho}\dot{\theta}$	
1542	3100081543	SRD			
1543	2000231544	STL	0023	$\ddot{\rho}\dot{\theta}$	
1544	6010621545	RAU		$\dot{\rho}$	
1545	1910661546	MULT		$\dot{\rho}\ddot{\theta}$	

Instruction location	Instruction	Operation	Storage location	Element	Remarks
1546	3100081547	SRD			
1547	1500231548	AL		$\ddot{\rho}\dot{\theta} + \dot{\rho}\ddot{\theta}$	
1548	6080021549	RAU			
1549	1901041550	MULT		$2(\ddot{\rho}\dot{\theta} + \dot{\rho}\ddot{\theta})$	
1550	1500221551	AL		$\ddot{Z}\ B_r/B_0 + 2(\ddot{\rho}\dot{\theta} + \dot{\rho}\ddot{\theta})$	
1551	1600211552	SL		$\ddot{Z}\ B_r/B_0 + 2(\ddot{\rho}\dot{\theta} + \dot{\rho}\ddot{\theta}) - \ddot{\rho}(B_Z/B_0) = P$	
1552	6080021553	RAU			
1553	1900131554	MULT		$\frac{1}{r}\ P$	
1554	3100081555	SRD			
1555	2000241556	STL	0024	$\frac{1}{r}\ P$	
1556	6010621557	RAU		$\dot{\rho}$	
1557	1900091558	MULT		$\dot{\rho}\ B_Z/B_0$	
1558	3100081559	SRD			
1559	2000251560	STL	0025	$\dot{\rho}\ B_Z/B_0$	
1560	6010681561	RAU		$\dot{Z}$	
1561	1900121562	MULT		$\dot{Z}\ B_r/B_0$	
1562	3100081563	SRD			
1563	2000261564	STL	0026	$\dot{Z}\ B_r/B_0$	
1564	6000161565	RAU		$\dot{\rho}\dot{\theta}$	
1565	1901041566	MULT		$2\dot{\rho}\dot{\theta}$	
1566	1500261567	AL		$2\dot{\rho}\dot{\theta} + \dot{Z}\ B_r/B_0$	
1567	1600251568	SL		$2\dot{\rho}\dot{\theta} + \dot{Z}\ B_r/B_0 - \dot{\rho}\ B_Z/B_0 = Q$	
1568	6080021569	RAU			
1569	1910621570	MULT		$\dot{\rho}Q$	
1570	3100081571	SRD			
1571	6080021572	RAU			
1572	1900131573	MULT		$\frac{1}{r}\ \dot{\rho}\ Q$	
1573	3100081574	SRD			
1574	6080021575	RAU			
1575	1900131576	MULT		$\frac{1}{r^2}\ \dot{\rho}Q$	
1576	3100081577	SRD			
1577	1600241578	SL		$\frac{1}{r^2}\dot{\rho}\theta - \frac{1}{r}\ P = \ddot{\theta}$	
1578	2002660251	STL	0266	$\ddot{\theta}$	Completes azimuthal equation.

Instruction 0251 returns to integrating routine.

Appendix 10

Subprogram for testing for maximum axial and radial amplitude and regenerator encounter, computing the velocity squared, and placing quantities in punch band for print out.

Instruction location	Instruction	Operation	Storage location	Element	Remarks
0902	6501011579	RAL		θ_1	Regenerator position.
1579	1601031580	SL		$\theta_1 - 10^{-8}$	
1580	1610641581	SL		$\theta_1 - 10^{-8} - \theta$	
1581	4603401582	BRMIN		$\theta \geqslant \theta_1$	Determines if integrated orbit has encountered regenerator.
1582	6010611583	RAU		r	
1583	2100271584	STU	0027	r	
1584	1910651585	MULT		$r\dot{\theta}$	
1585	2400311586	STD	0031	$\dot{\theta}$	
1586	3100081587	SRD		$r\dot{\theta}$	
1587	6080021588	RAU			
1588	1980011589	MULT		$(r\dot{\theta})^2$	
1589	2100371591	STU	0037	$(r\dot{\theta})^2$	
1591	6010640908	RAU		θ	
0908	1909070909	MULT		θ	Puts θ in degrees.
0909	6580031592	RAL			
1592	2400281593	STD	0028	θ in degrees	
1593	6510671594	RAL		Z	
1594	2400291595	STD	0029	Z	
1595	6010621596	RAU		$\dot{r}$	
1596	2400301597	STD	0030	$\dot{r}$	
1597	1980011598	MULT		$(\dot{r})^2$	
1598	3100081599	SRD			
1599	2000380314	STL	0038	$\dot{r}^2$	
0314	6010680315	RAU		$\dot{z}$	
0315	2100320316	STU	0032	$\dot{z}$	
0316	1980010317	MULT		$\dot{z}^2$	
0317	3100080318	SRD			
0318	1500381590	AL		$\dot{z}^2 + \dot{r}^2$	
1590	3000020319	SRT			
0319	1500370320	AL		$\dot{z}^2 + \dot{r}^2 + (r\dot{\theta})^2 = \nu^2$	To 8 decimals.
0320	2000331828	STL	0033	ν^2	
1828	6501111829	RAL		$\tilde{z}$	Maximum axial amplitude.
1829	1601031830	SL		$\tilde{z} - 10^{-8}$	
1830	1810671831	SABL		$\tilde{z} - 10^{-8} - \lvert Z \rvert$	
1831	4618321833	BRMIN			
1833	6518341835	RAL		$\tilde{r}$	
1835	1810611837	SABL		$\tilde{r} - \lvert r \rvert$	
1837	4618320326	BRMIN			
0326	7100270901	PCH			
1832	100001996	STOP			Program stops if r or Z are too large.

For θ having reached θ_1 (see instruction location 1581 above) the regenerator impulse is to be applied to the orbit. See Appendix 11.

Appendix 11

Subprogram for interpolating to regenerator position, computing ~~regenerator impulse, and establishing~~ new initial conditions.

Instruction location	Instruction	Operation	Storage Location	Element	Remark
0340	6510640341	RAL		θ_r	
0341	1610540342	SL		$\theta_n - \theta_{n-1}$	
0342	2000500343	STL	0050	$\theta_n - \theta_{n-1}$	
0343	6001050344	RAU			Interpolate to regenerator angle.
0344	3000020345	SRT			
0345	6400500346	DVRU		$\frac{1}{\theta_n - \theta_{n-1}}$	
0346	6080020347	RAU			
0347	1910000348	MULT		$\frac{h}{\theta_n - \theta_{n-1}} = \Delta_n t$	
0348	3100080349	SRD			
0349	2000510350	STL	0051	$\Delta_n t$	
0350	6510540351	RAL		θ_{n-1}	
0351	1610440352	SL		$\theta_{n-1} - \theta_{n-2}$	
0352	2000520353	STL	0052	$\theta_{n-1} - \theta_{n-2}$	
0353	6010000354	RAU			
0354	3000020355	SRT			
0355	6400520356	DIVRU		$\Delta_{n-1} t$	
0356	2000530357	STL	0053	$\Delta_{n-1} t$	
0357	6510640358	RAL		θ_n	
0358	1610440359	SL		$\theta_n - \theta_{n-2}$	
0359	2000540360	STL	0054	$\theta_n - \theta_{n-2}$	
0360	6000510361	RAU		$\Delta_n t$	
0361	1100530362	SU		$\Delta_n t - \Delta_{n-1} t$	
0362	3000020363	SRT			
0363	6400540364	DIVRU		$\Delta_n^2 t$	
0364	2000550365	STL	0055	$\Delta_n^2 t$	
0365	6001010366	RAU		θ_1	θ_1 is regenerator angle.
0366	1110440367	SU		$\theta_1 - \theta_{n-2}$	
0367	2100560368	STU	0056	$\theta_1 - \theta_{n-2}$	
0368	1900530369	MULT		$(\theta_1 - \theta_{n-2}) \Delta_{n-1} t$	
0369	3100080370	SRD			
0370	2000570371	STL	0057	$(\theta_1 - \theta_{n-2}) \Delta_{n-1} t = t_1$	
0371	6001010372	RAU		θ_1	
0372	1110540373	SU		$\theta_1 - \theta_{n-1}$	
0373	1900560374	MULT		$(\theta_1 - \theta_{n-1})(\theta_1 - \theta_{n-2})$	
0374	3100080375	SRD			
0375	6080020376	RAU			
0376	1900550377	MULT		$\theta_1 - \theta_{n-1})(\theta_1 - \theta_{n-2}) \Delta_n^2 t = -t_{n-2}$	
0377	3100080378	SRD			
0378	1500570380	AL		$t_1 - t_{n-2}$	
0380	2000580381	STL	0058	$t_1 - t_{n-2}$	
0381	1610000382	SL		$t_i - t_{n-1}$	
0382	2000590383	STL	0059	$t_1 - t_{n-1}$	
0383	6001050384	RAU		1	
0384	3000020385	SRT			
0385	6410000386	DIVRU		$\frac{1}{h}$	

Instruction location	Instruction	Operation	Storage location	Element	Remarks
0386	2000600387	STL	0060	$\frac{1}{h}$	
0387	6580010388	RAL			
0388	6401040389	DIVRU		$\frac{1}{2h}$	
0389	2000610391	STL	0061	$\frac{1}{2h}$	
0391	6010510392	RAU		r_{n-1}	
0392	1110410393	SU		$r_{n-1} - r_{n-2}$	Begin interpolating the radius.
0393	1900600394	MULT		$\Delta_{n-1} r$	
0394	3100080395	SRD			
0395	2000620396	STL	0062	$\Delta_{n-1} r$	
0396	6010610397	RAU		r_n	
0397	1110510398	SU		$r_n - r_{n-1}$	
0398	1900600399	MULT		$\Delta_n r$	
0399	3100080400	SRD			
0400	2000630401	STL	0063	$\Delta_n r$	
0401	6000630403	RAU			
0403	1100620405	SU		$\Delta_n r - \Delta_{n-1} r$	
0405	1900610406	MULT		$\Delta_n^2 r$	
0406	3100080407	SRD			
0407	2000640408	STL	0064	$\Delta_n^2 r$	
0408	6000590409	RAU		$(t_1 - t_{n-1})$	
0409	1900580410	MULT		$(t_1 - t_{n-1})(t_1 - t_{n-2}) = T^1_{n,2}$	
0410	3100080411	SRD			
0411	2000650413	STL	0065	$T^1_{n,2}$	
0413	6080010414	RAU			
0414	1900640415	MULT		$T^1_{n,2} \Delta_n^2 r$	
0415	3100080416	SRD			
0416	2000660417	STL	0066	$T^1_{n,2} \Delta_n^2 r$	
0417	6000580418	RAU		$t_1 - t_{n-2}$	
0418	1900620419	MULT		$(t_1 - t_{n-2}) \Delta_{n-1} r$	
0419	3100080420	SRD			
0420	1500660421	AL		$(t_1 - t_{n-2}) \Delta_{n-1} r + T^1_{n,2} \Delta_n^2 r = \Delta r$	
0421	1510410422	AL		$r_{n-2} + \Delta_r = r_1$	
0422	2010610423	STL	1061	r_1	Completes finding radius at regenerator.
0423	6010570424	RAU		z_{n-1}	
0424	1110470425	SU		$z_{n-1}\ z_{n-2}$	
0425	1900600426	MULT		$\Delta_{n-1} z$	
0426	3100080427	SRD			
0427	2000670428	STL	0067	$\Delta_{n-1} z$	
0428	6010670429	RAU		z_n	
0429	1110570430	SU		$z_n - z_{n-1}$	
0430	1900600431	MULT		$\Delta_n z$	
0431	3100080432	SRD			
0432	2000680433	STL	0068	$\Delta_n z$	
0433	6000680435	RAU		$\Delta_n z$	
0435	1100670436	SU		$\Delta_n z - \Delta_{n-1} z$	
0436	1900610437	MULT		$\Delta_n^2 z$	
0437	3100080438	SRD			
0438	6080020439	RAU			

Instruction location	Instruction	Operation	Storage location	Element	Remarks
0439	1900650440	MULT		$T^1_{n,2}\Delta^2_n z$	
0440	3100080441	SRD			
0441	2000690443	STL	0069	$T^1_{n,2}\Delta^2_n z$	
0443	6000670445	RAU		$\Delta_{n-1} z$	
0445	1900580446	MULT		$(t_1 - t_{n-2})\Delta_{n-1} z$	
0446	3100080447	SRD			
0447	1500690460	AL		$z_1 - z_{n-2}$	
0460	1510470461	AL		z_1	
0461	2010670488	STL	1067	z_1	Completes finding axial amplitude at regenerator.
0488	6010610489	RAU		r_1	
0489	1101000490	SU		ρ_1	
0490	2100740491	STU	0074	ρ_1	
0491	1980010492	MULT		ρ_1^2	
0492	3100080493	SRD			
0493	2000750494	STL	0075	ρ_1^2	
0494	6080010495	RAU			
0495	1900740496	MULT		ρ_1^3	
0496	3100080497	SRD			
0497	2000760498	STL	0076	ρ_1^3	
0498	6080010499	RAU		$a_3\rho_1^3$	
0499	1901140500	MULT			
0500	3100080501	SRD			
0501	2000770502	STL	0077	$a_3\rho_1^3$	
0502	6000751864	RAU		ρ_1^2	
1864	1901130252	MULT		$a_2\rho_1^2$	
0252	3100080253	SRD			
0253	2000780254	STL	0078	$a_2\rho_1^2$	
0254	6000740255	RAU		ρ_1	
0255	1901120256	MULT		$a_1\rho_1$	
0256	3100080257	SRD			
0257	1500780258	AL		$a_1\rho_1 + a_2\rho_1^2$	
0258	1500770259	AL		$a_1\rho_1 + a_2\rho_1^2 + a_3\rho_1^3 = \delta(\rho^1)$	
0259	2000790260	STL	0079	$\delta(\rho^1)$	Completes radial impulse
0260	6000760261	RAU		ρ_1^3	
0261	1901180262	MULT		$\alpha_3\rho_1^3$	
0262	3100080327	SRD			
0327	2000800328	STL	0080	$\alpha_3\rho_1^3$	
0328	6000750329	RAU		ρ_1^2	
0329	1901170330	MULT		$\alpha_2\rho_1^2$	
0330	3100080331	SRD			
0331	2000810332	STL	0081	$\alpha_2\rho_1^2$	
0332	6000740333	RAU		ρ_1	
0333	1901160334	MULT		$\alpha_1\rho_1$	
0334	3100080335	SRD			
0335	1501150336	AL		$\alpha_0 + \alpha_1\rho$	
0336	1500800337	AL		$\alpha_0 + \alpha_1\rho + \alpha_3\rho_1^3$	
0337	1500810091	AL		$\alpha_0 + \alpha_1\rho_1 + \alpha_2\rho_1^2 + \alpha_3\rho_1^3 = \frac{r}{z}\delta(\dot{z})$	
0091	6080020092	RAU			

Instruction location	Instruction	Operation	Storage location	Element	Remarks
0092	3000020093	SRT			
0093	6410610094	DIVRU		$\frac{1}{z}\ \delta(\dot z)$	
0094	6080020095	RAU			
0095	1910670096	MULT		$\delta(\dot z)$	
0096	3100080338	SRD			
0338	2000820339	STL	0082	$\delta(\dot z)$	Completes axial impulse.
0339	6010581406	RAU		$\dot z_{n-1}$	
1406	1110481407	SU		$\dot z_{n-1} - \dot z_{n-2}$	
1407	1900601408	MULT		$\Delta_{n-1} \dot z$	
1408	3100081409	SRD			
1409	2000831410	STL	0083	$\Delta_{n-1} \dot z$	
1410	6010681411	RAU		$\dot z_n$	
1411	1110581412	SU		$\dot z_n - \dot z_{n-1}$	
1412	1900601413	MULT		$\Delta_n \dot z$	
1413	3100081414	SRD			
1414	2000841415	STL	0084	$\Delta_n \dot z$	
1415	6000840039	RAU			
0039	1100830040	SU		$\Delta_n \dot z - \Delta_{n-1} \dot z$	
0040	1900610041	MULT		$\Delta_n^2 \dot z$	
0041	3100080042	SRD			
0042	6080020043	RAU			
0043	1900650044	MULT		$T^1_{n,2}\ \Delta_n^2 \dot z$	
0044	3100080045	SRD			
0045	2000850046	STL	0085	$T^1_{n,2}\ \Delta_n^2 \dot z$	
0046	6000830047	RAU		$\Delta_{n-1} \dot z$	
0047	1900580048	MULT		$(t_1 - t_{n-2})\ \Delta_{n-1} \dot z$	
0048	3100080049	SRD			
0049	1500850119	AL		$\dot z_1 - \dot z_{n-2}$	
0119	1510480120	AL		$\dot z_1$	
0120	1500820121	AL		$\dot z_1 + \delta(\dot z)$	
0121	2010680122	STL	1068	$z_1 + \delta(\dot z)$	Completes changed axial velocity for initial condition.
0122	6010520123	RAU		$\dot r_{n-1}$	
0123	1110420124	SU		$\dot r_{n-1} - \dot r_{n-2}$	
0124	1900600125	MULT		$\Delta_n \dot r_1$	
0125	3100080126	SRD			
0126	2000860127	STL	0086	$\Delta_{n-1} \dot r$	
0127	6010620128	RAU		$\dot r_n$	
0128	1110520129	SU		$\dot r_n - \dot r_{n-1}$	
0129	1900600130	MULT		$\Delta_n \dot r$	
0130	3100080131	SRD			
0131	2000870132	STL	0087	$\Delta_n \dot r$	
0132	6000870133	RAU			
0133	1100860135	SU		$\Delta_n \dot r - \Delta_{n-1} \dot r$	
0135	1900610136	MULT		$\Delta_n^2 \dot r$	
0136	3100080137	SRD			
0137	6080020138	RAU			
0138	1900650139	MULT		$T^1_{n,2}\ \Delta_n^2 \dot r$	
0139	3100080140	SRD			

Instruction location	Instruction	Operation	Storage location	Element	Remarks
0140	2000880141	STL	0088	$T^1_{n,2}\ \Delta^2_n\ \dot{r}$	
0141	6000860142	RAU		$\Delta_{n-1}\ \dot{r}$	
0142	1900580143	MULT		$(t_1 - t_{n-2})\ \Delta_{n-1}\ \dot{r}$	
0143	3100080144	SRD			
0144	1500880145	AL		$\dot{r}_1 - \dot{r}_{n-2}$	
0145	1510420146	AL		$\dot{r}_1$	
0146	1500790147	AL		$\dot{r}_1 + \delta\ (\dot{r})$	
0147	2010620462	STL	1062	$\dot{r}_1 + \delta\ (\dot{r})$	Completes changed radial velocity for initial condition.
0462	6010620463	RAU			
0463	1980010464	MULT		$[\dot{r}_1 + \delta\ (\dot{r})]^2$	
0464	3100080465	SRD			
0465	2009040466	STL	0904	$[\dot{r}_1 + \delta\ (\dot{r})]^2 = \dot{r}_1^2$	
0466	6010680467	RAU		$\dot{z}_1 + \delta\ (\dot{z})$	
0467	1980010468	MULT		$[\dot{z}_1 + \delta(\dot{z})]^2$	
0468	3100080469	SRD			
0469	2009050470	STL	0905	$[\dot{z}_1 + \delta\ (\dot{z})]^2 = \dot{z}_1^2$	
0470	6010610471	RAU		r_1	
0471	1980010472	MULT		r_1^2	
0472	6580030473	RAL			
0473	2009060474	STL	0906	r_1^2	
0474	6500330475	RAL		v^2	
0475	3500020899	SLT			
0899	1609040477	SL		$v^2 - \dot{r}_1^2$	
0477	1609050478	SL		$v^2 - \dot{r}_1^2 - \dot{z}_1^2$	
0478	3500060479	SLT			
0479	6409060480	DIVRU		$\frac{v^2 - \dot{r}_1{}^2 - \dot{z}_1{}^2}{r_1{}^2} = \dot{\theta}_1{}^2$	Goes to square-root routine.
0480	6904810160	LD			
0481	3000010482	SRT		$\dot{\theta}_1$	
0482	2010650148	STL	1065	$\dot{\theta}_1$	Consistent value of $\dot{\theta}$ for initial conditions.
0148	6501010149	RAL		θ_1	
0149	2010640264	STL	1064	θ_1	
0264	1501340265	AL		$\theta_1 + 2\pi$	
0265	2001010151	STL	0101	$\theta_1 + 2\pi$	

Final instruction transfers to integration routine, new initial conditions having been established following regenerator action. This routine may be skipped by placing a large value for θ_1, regenerator position, in memory location 0101; thus, many turns may be computed before the regenerator is reached.