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COMMON FEATURES AND DIFFERENCES BETWEEN FISSION AND HEAVY ION PHYSICS<sup>(\*)</sup>

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**Abstract** - The macroscopic approach to fission and fusion physics is formulated. A minimum set of three degrees of freedom is described qualitatively. The gross features of the potential energy in this configuration space are discussed. The problem of nuclear viscosity is mentioned and comparisons with liquid  $\text{He}^3$  are made. Some effects of large angular momenta are described.

## I - INTRODUCTION

We are entering a new stage in nuclear physics, characterized by the availability of very heavy nuclear projectiles. It is a good time to reflect on the place of the developing field of heavy ion physics in relation to nuclear fission and, more generally, in relation to nuclear physics as a whole.

During the 60 years of its history nuclear physics has had to contend with two limitations and the historic role of heavy ion physics will be to relax them. These limitations have been so pervasive that we have almost stopped being aware of them.

These limitations are:

1. The restriction of atomic numbers to less than about 100.
2. The almost exclusive restriction of nuclear shapes to those close to a sphere.

The introduction of accelerators for very heavy ions will relax both limitations. First, it will be possible to study at least transient nuclear systems with atomic numbers up to about

two hundred. Second, the enormous centrifugal forces created in off-center collisions of heavy nuclei will be sufficient to deform a nuclear system away from its customary near-spherical shape into more or less stretched-out configurations, sometimes resembling even a dumb-bell.

The extension of atomic numbers beyond a hundred may result in the discovery of superheavy elements in the vicinity of  $Z \approx 114$ ,  $Z \approx 154 - 164$ , and perhaps in some other regions.

The consequences of these anticipated discoveries are already beginning to be felt in theoretical chemistry and atomic physics. An even more fundamental consequence of the extension of the limit of nuclear systems from atomic numbers near 100 to atomic numbers near 200 has to do with the circumstance that the most intense electric fields occurring anywhere in the universe are to be found in the vicinity of heavy nuclei. The increase in atomic number from 100 to 200 increases this highest field only moderately, but it so happens that it is in this range of atomic numbers that an atomic electron becomes highly relativistic and the atomic properties of very heavy nuclei will test the limits of quantum electrodynamics under unusual conditions.

<sup>(\*)</sup>Supported by the U. S. Atomic Energy Commission.

The advent of very-heavy-ion accelerators will thus have an effect on chemistry, atomic physics and quantum electrodynamics, as well as on nuclear physics itself, to which I will now return.

## II - THE MACROSCOPIC APPROACH

To me the distinguishing feature of heavy ion physics is its macroscopic nature. For the first time we will have nuclear reactions where both the target and projectile satisfy well the criterion for a macroscopic approximation, namely  $A \gg 1$ . A kind of nuclear macro-physics, based on this approximation as a starting point, will come into its own.

This is to be contrasted with conventional nuclear reaction theory which, historically, has its roots in an idealization where the projectile is a structureless mass point. On the other hand it is the same nuclear macro-physics that is used in the theory of nuclear fission, and this is why nuclear fission may be used as a guide in formulating the framework of heavy ion physics.

What are the central features of the macroscopic approach and what are the principal unsolved problems?

In trying to answer these questions I would like to remind you that many problems in physics in general (not just nuclear physics) are solved according to a standard canonical scheme, which goes something like this:

First then, what are the relevant degrees of freedom in a macroscopic treatment of heavy ion and fission physics?

The characteristic feature of a macroscopic approach is that collective rather than single-particle degrees of freedom become convenient and relevant. This is saying no more than that if you have tens or hundreds of nucleons you will try to formulate your problem in terms of a few intelligently chosen groupings of the nucleon coordinates rather than of the whole set. This is a great simplification.

A further simplification immediately suggests itself in virtue of the relative thinness of the nuclear surface (the "leptodermous" character of most nuclei). Insofar as a nuclear system has a well-defined surface, that grouping of the nucleon coordinates which corresponds to the nuclear surface is the most relevant set of degrees of freedom. Thus one will try to describe the state of the system in terms of the shape of its surface and the development in time of this shape.

In general many degrees of freedom are needed to specify accurately the shape of a dividing or fusing nuclear system. There is a rule which suggests that for the fission of a nucleus into  $n$  parts, or in the simultaneous fusion of  $n$  nuclei, about  $9n - 4$  collective degrees of freedom should be adequate for many purposes. (If each fragment is thought of as an approximate ellipsoid, three axes describe its size and shape, three

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TABLE I

Guidelines for Solving Many Problems in Applied Physics:

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1. Isolate relevant degrees of freedom to be retained explicitly.
  2. Write down Equations of Motion (Schrödinger Equation) for these degrees of freedom.
    - (a) Potential Energy Terms.
    - (b) Dissipative Terms (representing the coupling to degrees of freedom not retained explicitly).
    - (c) Inertial Terms.
  3. Solve the Equations of Motion, using whatever techniques are applicable (e. g. statistical, semi-classical, or what have you).
  4. Compare with Experiment and re-cycle.
-

coordinates its location in space, and three Euler angles its orientation. This gives  $9n$  degrees of freedom for  $n$  fragments. The minus four constrains the total volume of the system to a standard value, and the center of mass of the system to a standard location.)

For a binary process of fission or fusion ( $n = 2$ ) this rule gives about 14 degrees of freedom. One may surely go below this number without making gross qualitative errors, but I believe three degrees of freedom is the barest minimum necessary to give the roughest qualitatively adequate description of nuclear shapes relevant for binary fission or two-ion fusion.

These degrees of freedom are something like this

1. A separation coordinate, say  $\alpha_2$ .
2. A necking or neck-healing coordinate, say  $\alpha_4$ .
3. A mass asymmetry coordinate, say  $\alpha_3$ .

(The  $\alpha$ 's may be related to but are not to be thought of as identical with the coefficients in a Legendre Polynomial expansion of the shape.)

The nuclear shapes described by these degrees of freedom can be displayed in a three-dimensional space, of which Fig. 1 attempts to represent a two-dimensional section at constant  $\alpha_3$ . The asymmetry coordinate  $\alpha_3$  would stick out of the plane of the paper and would correspond to changing the mass ratio of the left and right sides of the shapes.

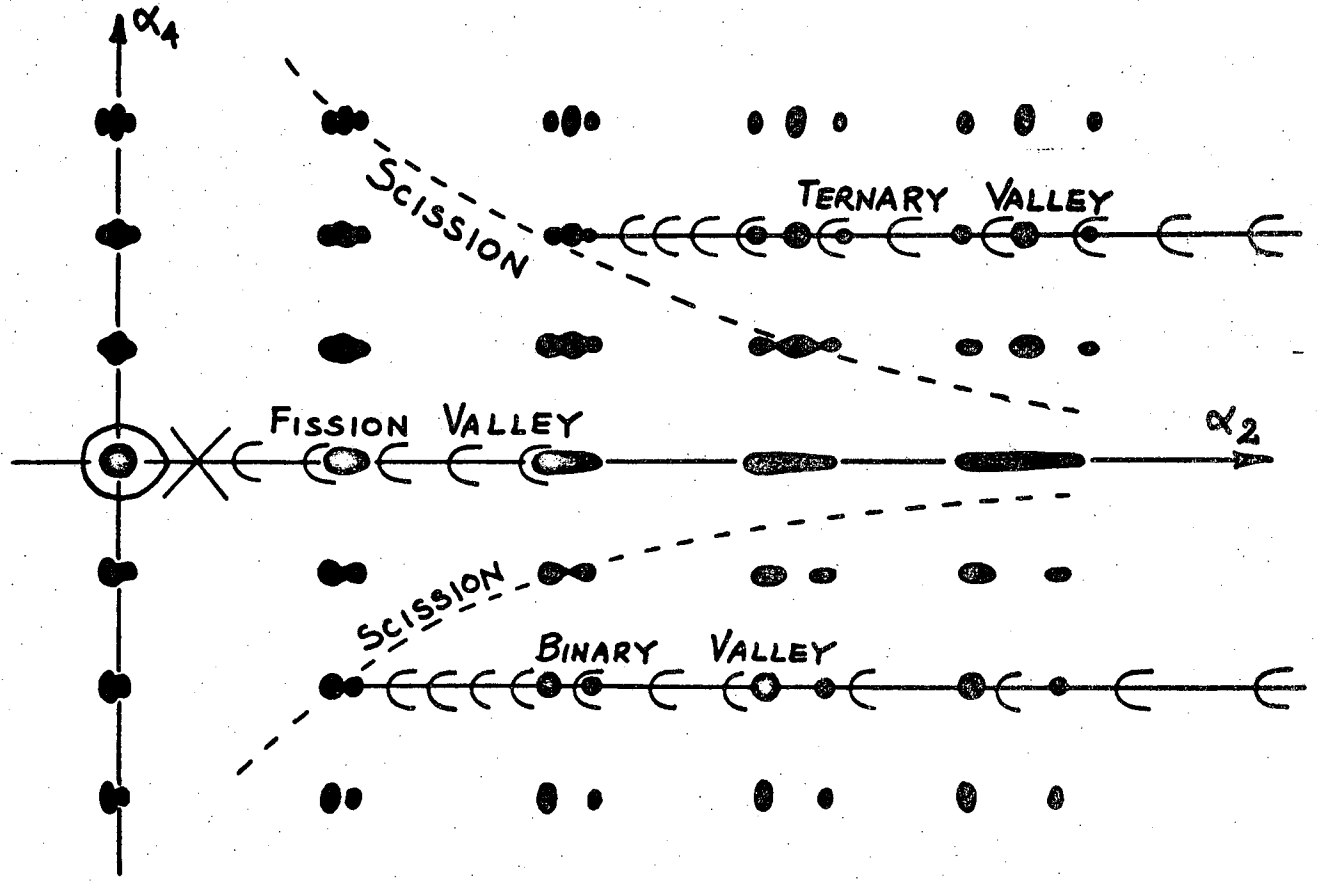


Fig. 1. An illustration of fission and fusion shapes described by an elongation coordinate  $\alpha_2$  and a necking or neck-healing coordinate  $\alpha_4$ . The asymmetry coordinate  $\alpha_3$  (which would point into the plane of the paper) is held fixed. The scission lines for binary and ternary divisions are indicated.

The time development of a collision between two heavy ions would be represented by a path starting somewhere on the right and proceeding to the contact (or scission) line, followed either by re-separation, fusion, or ternary division, depending on the conditions of the collision and other factors. Conversely, in nuclear fission one starts somewhere in the vicinity of the sphere and goes to the right, usually into the two-fragment valley, but sometimes, perhaps, into the three-fragment valley.

If the problem is treated quantum mechanically then instead of paths we shall be dis-

cussing the solution of a Schrödinger equation in the  $\alpha_2\alpha_3\alpha_4$  space, with  $|\psi(\alpha_2\alpha_3\alpha_4)|^2$  representing the probability of the system being in a configuration specified by  $\alpha_2\alpha_3\alpha_4$ , and  $\frac{\hbar}{i} \frac{d\psi}{dt} = H\psi$  giving the time development of the wave function  $\psi$ .

In order to construct a dynamical theory of the paths in  $\alpha_2\alpha_3\alpha_4$  space, or to solve the Schrödinger equation, we shall need information about certain basic properties of the nuclear systems considered. This brings us to the second item in the "Guidelines" in Table I: writing down the Equations of Motion. There will be three types of terms to consider:

Terms	Associated with time derivatives of the degrees of freedom	Relevant Quantities
1. Potential Energy Terms	ZEROTH	$V(\alpha_2\alpha_3\alpha_4 \dots)$
2. Friction, Damping or Dissipative Terms	FIRST	Rayleigh's Dissipation Function or $iW(\alpha_2\alpha_3\alpha_4 \dots)$
3. Inertial Terms	SECOND	Inertia Tensor $M_{\alpha_i\alpha_j}(\alpha_2\alpha_3\alpha_4 \dots)$

[ The number three of different terms is no accident: it is associated with the fact that (macroscopic) equations of motion contain zeroth, first and second time derivatives of the degrees of freedom, but no higher. ]

In classical mechanics the dissipative terms may be described by a quantity called the Rayleigh dissipation function (a generalized friction). In quantum mechanics the potential energy and the damping terms are sometimes combined in a complex potential  $V(\alpha_2\alpha_3\alpha_4) + iW(\alpha_2\alpha_3\alpha_4)$ . The inertial terms in classical as well as quantum mechanics give rise to a so-called inertia matrix or tensor  $M_{\alpha_i\alpha_j}(\alpha_2\alpha_3\alpha_4)$ , which describes the inertial response of the system to time variations of the

degrees of freedom. In any case there are three pieces of physics to consider in making a dynamical theory:

1. Potential
2. Damping
3. Inertia.

In the case of nuclear macro-physics the situation today is that we have a good understanding of 1, a little of 3, and very little of 2. I believe that in the future we will have to concentrate on pulling up the information in Inertia and Damping to a level that matches our understanding of the Potential Energy.

As regards our understanding of the nuclear potential energy, great progress has been

made in the last few years, principally as a result of the success of Strutinsky's prescription for combining macroscopic and microscopic theories. We are today in a position where we can calculate the potential energy of a nuclear system as a function of N, Z and the nuclear shape, with an accuracy of about ±1 MeV. This is 1 MeV out of a total binding energy of some 2000 MeV.

What have we learned? The potential energy as a function of  $\alpha_2\alpha_3\alpha_4$  is given by a pock-marked surface, consisting of a smooth part and shell effect pock-marks. The characteristic undulations of the smooth part are generally of the order of tens of MeV, the pock-marks are of the order of a few MeV. The theory underlying the smooth part is well understood. The pock-marks, though not so well understood, are also beginning to be related to simple features of the nuclear shape. In particular the last year or two have brought the realization that major shell effects in the nuclear potential energy are closely related to certain features of classical orbits in a potential well. For example if the well is such that a classical orbit closes up on itself, then major magic numbers are to be expected. In any case I feel that the first step in building up a theory of heavy-ion reactions is quite clear: to construct the potential-energy surface as a function of suitable deformation coordinates describing colliding and fusing nuclei, using the Strutinsky method (improved and refined where necessary).

There will, of course, be a wealth of structure in the resulting potential energy maps, especially in the pock-marks. Let me just point out some of the most primitive features to be expected in the gross structure of the maps.

There are four important features I wish to mention:

1. The equilibration of the neutron-proton ratio.
2. The existence of a critical mass asymmetry.
3. The existence of two misaligned valleys.
4. The effect of angular momentum.

#### Equilibration of N:Z ratio

The first point is rather trivial and I want to dispose of it quickly. It is that if two

nuclei with widely differing N:Z ratios are brought into contact, a re-distribution of neutron and proton densities will take place such that an approximately uniform value of N:Z will obtain throughout the system. There will be slight deviations from uniformity, and slight fluctuations around it, but it is a fair approximation to disregard these at first. For example, for tangent spheres of radii  $R_1, R_2$  one may work out in a closed form an expression for the  $Z_1:A_1$  ratio of one of the nuclei divided by the Z:A ratio of the whole system:

$$\frac{Z_1}{A_1} / \frac{Z}{A} = 1 + \frac{1}{8} \frac{(e^2/r_0)}{(\text{coeff})} A^{2/3} F\left(\frac{R_1}{R_2}\right),$$

where  $e$  is the proton charge,  $r_0$  is the nuclear radius constant (with  $e^2/r_0$  equal to about 1.2 MeV), "coeff" is the nuclear symmetry energy coefficient (about 30 MeV), and  $F$  is the following function

$$F(\lambda) = (1 - \lambda) [ (6/5)(1 + \lambda)^2 - 1 - \lambda - \lambda^2 ] \\ \times (1 + \lambda)^{-1} (1 + \lambda^3)^{-5/3}.$$

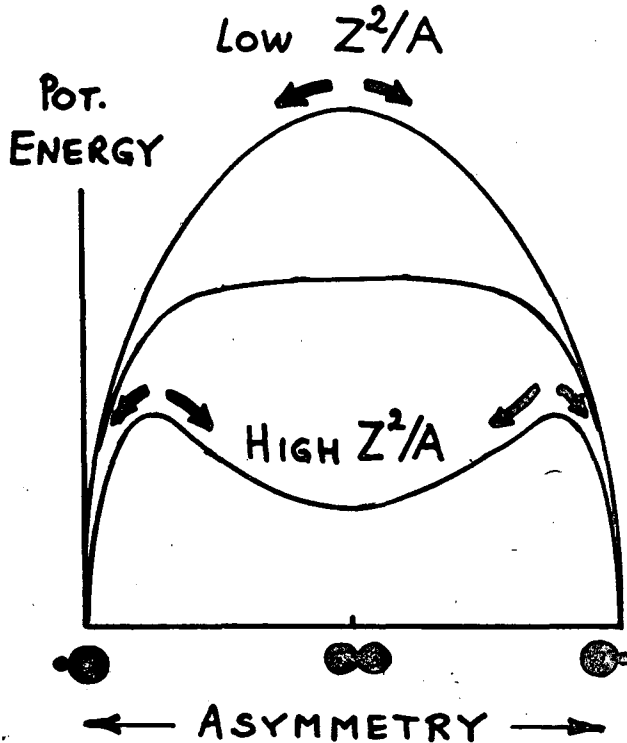
According to this formula the smaller of two tangent nuclei will have a slightly higher N:Z ratio, but only by at most a few percent. (The greatest deviation of  $\frac{Z_1}{A_1} / \frac{Z}{A}$  from unity occurs near  $R_1:R_2 = 0.3$  and is about 6% for  $A = 216$ .) For some purposes this charge re-distribution may be of importance, and there is experimental evidence for it both in fission and perhaps in heavy ion transfer reactions, but in my survey of gross properties I will not say more about it.

#### Existence of a critical mass asymmetry.

As regards the dependence of the gross potential energy on the asymmetry coordinate  $\alpha_3$ , the most important thing to bear in mind is that there exists a critical ratio of masses of target and projectile. For mass asymmetries more extreme than the critical (i. e. for a relatively light heavy ion and a heavy target) the target nucleus tends to suck up the projectile. For asymmetries less extreme than the critical (i. e. for heavy ions



more nearly comparable with the target) the projectile will tend to grow towards equality with the target (see Fig. 2). Most heavy ion experiments



**Fig. 2.** An illustration of the dependence of the relative deformation energy on asymmetry. For light systems (low  $Z^2/A$ ) asymmetric configurations tend to become even more asymmetric. For high  $Z^2/A$  this is still true for very asymmetric configurations, but moderately asymmetric configurations tend toward symmetry.

done to date lie on one side of the critical asymmetry. Most heavy ion experiments of the future, in particular those aiming at super-heavy nuclei, will lie on the other side of the critical asymmetry. The critical asymmetry is therefore an important feature to bear in mind when extrapolating from past experience to future experiments with heavy ions. The existence of a critical mass asymmetry is a result of the competition between the electric forces and the short-range nuclear forces (idealized as a surface energy), and may be understood with reference to configurations of tangent nuclei. For a sufficiently small nucleus in contact with a larger one the large pressure caused by the surface energy of the smaller nucleus tends to squirt

matter out of the small nucleus into the big one. If the electrostatic energy were negligible this would always be the case: the larger partner would tend to suck up the smaller one. For heavy systems, however, when the electrostatic energy is appreciable, the tendency is reversed, except for extreme asymmetries, when the pressure from the surface energy eventually begins to dominate.

There is more to this problem than I have indicated, in particular an inadequately understood qualitative change near  $Z^2/A \approx 40$ , but I will now go on to the third item.

#### Misaligned Valleys

The third important feature of the Nuclear Potential Energy maps in  $\alpha_2\alpha_3\alpha_4$  space is the existence of two (or more) valleys, similarly oriented but mis-aligned. Let me explain. Again think of a fixed mass-asymmetry, i. e., a section through the  $\alpha_2\alpha_3\alpha_4$  space along a fixed  $\alpha_3$ . The nuclear shapes as functions of  $\alpha_2$  and  $\alpha_4$  are shown in Fig. 1.

The simplest way to summarize the findings of many people who have investigated the potential energy in spaces like the  $\alpha_2\alpha_4$  space is to say that there are two principal valleys, as shown. One valley starts from the vicinity of the sphere. After a saddle, the energy goes down, but there is stability against changes of the necking coordinate for a fixed elongation coordinate. Below this valley is a roughly parallel two-fragment valley corresponding to approaching or separating fragments.

(Farther up there is a third valley, the Three-Fragment Valley, about which I will not say much more.)

How do the valleys fit together? I have shown an oversimplified sketch to give you a hint of what the situation looks like. A plan and an end view of the potential energy surface as functions of  $\alpha_2$  and  $\alpha_4$  are shown in Figs. 3a and 3b.

I hope you can see the fission valley with its saddle and stable spherical shape and the mis-aligned two-fragment valley. Between the two is a ridge running from A to C. Remember also that on top of what I described are shell-effect pock-marks. (One of these is shown: the magic hole H,

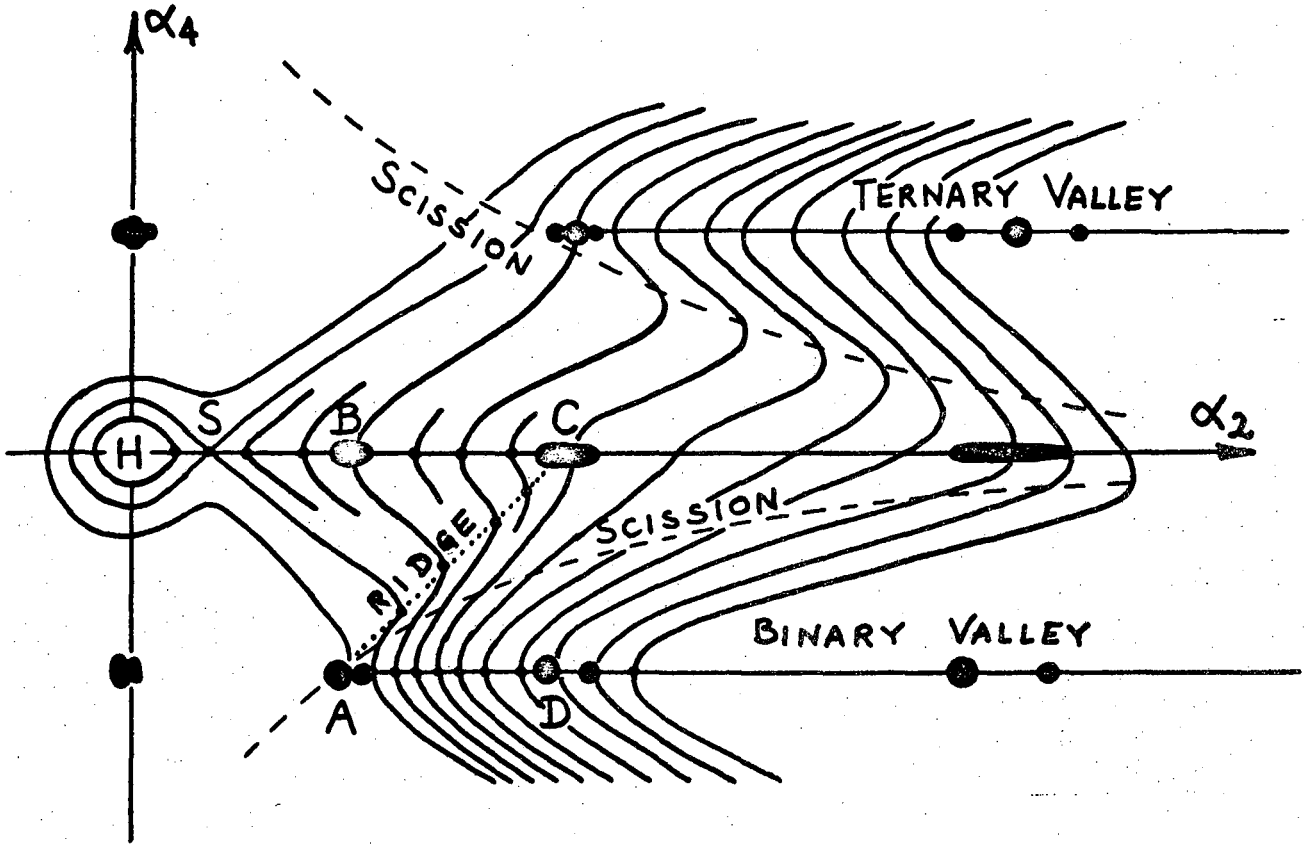


Fig. 3a. A sketch of the potential energy landscape in  $\alpha_2 - \alpha_4$  space, for a super-heavy nucleus. A few shapes are shown for orientation. The portion BC of the fission valley is separated from the portion AD of the binary valley by a ridge running from A to C.

responsible for the stability of a super-heavy nucleus.)

With this potential energy surface as background we can now sketch in a fission or a fusion path (corresponding to a dividing or fusing system. In fission the nucleus deforms, goes over the saddle and rolls down the fission valley. In the neighborhood of point C equilibrium against necking-in is lost and the system is injected into the two-fragment valley. Because of the misalignment of the valleys the injection is off-axis and the representative point will vibrate around the axis as it descends the two-fragment valley (or creep toward the bottom of the valley if there is appreci-

able damping). This vibration corresponds to changes in eccentricity of the fragments, i. e., to fragment excitation. The excitation energy is roughly the difference in energy between points C and D. Experimentally it is typically 20 - 40 MeV and is eventually dissipated in neutron evaporation from the fission fragments.

Now about fusion. The situation is analogous. We proceed up the Two Fragment Valley corresponding to approaching nuclei. At the point A, corresponding to tangency, equilibrium against an increasing eccentricity of the fragments is lost and the system is injected into the fission valley. Because of the off-center injection there is

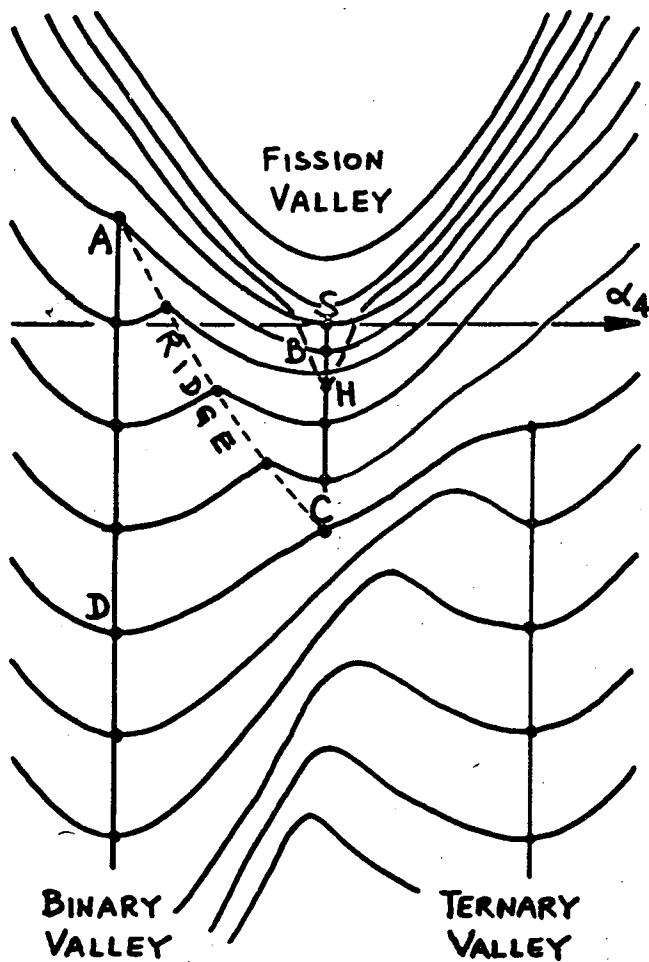


Fig. 3b. A view of the potential energy landscape in  $\alpha_2 - \alpha_4$  space looking up the three valleys in the direction of the spherical system in the hollow H. Coming out of H one goes over the saddle S and rolls down the fission valley along BC. At C injection into the binary valley takes place.

vibration about (or creep toward) the axis of the fission valley, which would eventually lead to excitation of the fused system. The amount of excitation is roughly the difference between the energy at A and at B.

An analogy to these fission and fusion paths may be constructed in terms of the path of a beam particle in a linear accelerator. Imagine a linear accelerator consisting of two misaligned segments. Each segment has radial focusing (e. g., quadrupole lenses). A short pre-accelerator (the

fission valley) injects a particle into the main accelerating tube, which, however, is misaligned. Conversely, in fusion, a particle is sent back up the main accelerator and is then injected up-hill into the pre-accelerator. Because of the misalignment, transverse oscillations are set up in the beam at injection. These oscillations correspond to fission fragment excitations in the case of fission, or to the excitation of the fused nucleus in the case of fusion.

The question of estimating the amount of excitation following a fusion reaction is one of the outstanding problems of heavy ion physics, especially when one is trying to make super-heavy nuclei. (If there is too much excitation one will not be able to make them.) Using the picture of misaligned valleys one estimates excitations ranging from perhaps 20 to 60 MeV for a typical case like that of  $^{232}\text{Th} + ^{76}\text{Ge}$  (which is one of the most promising candidates for making superheavies). These estimates are, however, extremely uncertain because of a crucial missing piece of information--namely, how large is nuclear viscosity. In other words, how strong is the coupling of the collective degrees of freedom to the single particle degrees of freedom that are not displayed explicitly in a macroscopic treatment. You can probably see at once that too much damping, too much viscosity, will make fusion difficult or impossible. This is because two nuclei like Th and Ge, when brought into contact, do not in general feel a driving force tending to fuse them into a spherical shape. On the contrary, even though the nuclear forces tend to fill in the neck region, the strong electric forces tend to push the bulks of the two nuclei apart and cause re-disintegration. In terms of the Potential energy map in Fig. 3 this means that in the vicinity of point B one is still some 10-15 MeV below the saddle at S and one is on a sloping part of the landscape, with a slope to the right, towards re-disintegration. In order to achieve fusion one would increase the bombarding energy above the contact energy (the coulomb barrier) hoping that this additional collective energy will carry the system over the saddle at S. If there were no viscosity--no conversion of collective into internal energy--an additional 15 MeV might be enough. But

if the viscosity is large--if say nine tenths of the extra energy goes into internal excitation, then one might have to go to bombarding energies  $\sim 150$  MeV above contact energy (coulomb barriers). It would be like trying to make two charged drops of honey coalesce by banging them together with high energy. Most of the collision velocity would go into heat, and after making partial contact the hot, charged honey drops would be torn apart again without ever reaching the spherical configuration.

So here we come across a central unanswered problem in heavy-ion physics. Are nuclei viscous like honey or mobile like water or mercury?

In hydrodynamics the distinction between extremely viscous and extremely nonviscous types of flow may be made quantitative in terms of the relative magnitude of the second and third terms in the equations of motion. For large viscosity the dissipative terms dominate over the inertial terms (which may then be neglected). For small viscosity the damping terms are neglected compared to the inertial terms. A textbook illustration is the case of small oscillations of a viscous liquid drop of radius  $R$ , density  $\rho$ , surface tension  $\gamma$  and coefficient of viscosity  $\eta$ . Such a drop, if distorted into a spherical shape, will either vibrate with a circular frequency  $\omega$  determined by  $\gamma$  and  $\rho$  (if viscosity is negligible), or creep back to the spherical shape with an e-folding time determined by  $\gamma$  and  $\eta$  (if inertia is negligible). The ratio of the e-folding time to the circular period  $1/\omega$  is given by

$$\frac{T_1}{T_2} = \frac{\text{e-folding time}}{1/\omega} = \frac{19}{5\sqrt{2}} \frac{\eta}{\sqrt{\gamma \rho R}}$$

This ratio (or the "creep parameter"  $\eta/\sqrt{\gamma\rho R}$  itself) may be used as a measure of the relative importance of viscosity. Here  $\eta, \gamma, \rho$  are given numbers, and the dependence on the size of the system enters through  $R$ . If for  $R$  we put  $R = r_0 A^{1/3}$  (with  $r_0 \approx 1.2 \times 10^{-13}$  cm for nuclei, or  $1.93 \times 10^{-8}$  cm for water) we find that  $T_1/T_2$  is

proportional to  $A^{-1/6}$ . For water at  $0^\circ\text{C}$  one finds  $\frac{T_1}{T_2} = \frac{40}{A^{1/6}}$ . If we put  $A = 10^{24}$  molecules ( $R = 1.93$  cm), we find  $\frac{T_1}{T_2} = 0.004$ , and for such large drops viscosity is negligible. But for  $A = 300$ ,  $\frac{T_1}{T_2} = 15$  and now we are in the extreme viscous or creep limit. This seems to be the case for all ordinary liquids. The ratio is less for ether, and is considerably less for water at  $100^\circ\text{C}$  ( $\frac{T_1}{T_2} = 2.5$ ), but we have here a somewhat ominous result: as is well known in hydrodynamics viscosity becomes dominant for sufficiently small systems, and for all ordinary liquids  $A = 300$  is indeed small in this sense. There would be great difficulties in the way of making a superheavy (!) drop of water with  $A \approx 300$  out of two droplets with  $A = 232$  and  $A = 76$  respectively (simulating the  $^{232}\text{Th} + ^{76}\text{Ge}$  reaction). Viscosity would in all likelihood prevent fusion of the drops before re-disintegration caused by the coulomb repulsion.

Now water may be an entirely misleading guide to the properties of a quantum fluid like nuclear matter. We should be able to get a better order of magnitude estimate by considering another quantum fluid, namely liquid  $\text{He}^3$ . Inserting the density, surface tension and radius constant of  $\text{He}^3$  at low temperatures we find, for  $A = 300$ ,  $\frac{T_1}{T_2} = 60,000 \eta$ , where  $\eta$  is in poises. The viscosity of  $\text{He}^3$  increases rapidly with decreasing temperature, and one finds that  $T_1/T_2$  is 120, 24, 1.4 at temperatures of  $0.04^\circ\text{K}$ ,  $0.1^\circ\text{K}$ ,  $1^\circ\text{K}$ , respectively. Nuclear temperatures of 1 or 2 MeV are expected to correspond to the lower range ( $0.04^\circ\text{K}$  to  $0.1^\circ\text{K}$ ) and we again come out with the indication that viscosity would be dominant for systems with  $A \approx 300$ . Table II summarizes these estimates.

These estimates suggest extremely high viscosity for nuclear matter (except at very high temperatures of many MeV), but they have to be taken with reservations. First of all it is possible that at a sufficiently low temperature a Fermi liquid like  $\text{He}^3$  would become superfluid, with very low viscosity, and the above estimates should at best be used as a guide at not too low temperatures (corresponding to nuclear excitations at which pairing effects are destroyed). Secondly the viscosities ordinarily calculated and measured for

TABLE II

Ratios of e-folding Time to Vibrational Times for Drops of Water and He<sup>3</sup>.

Water

$$\rho=1 \text{ g/cm}^3, \gamma=75 \text{ dynes/cm}, r_0=1.93 \times 10^{-8} \text{ cm}$$

at

$$\theta=0^\circ\text{C}: \eta=0.0179 \text{ poise}, \frac{T_1}{T_2} = 15$$

$$\theta=100^\circ\text{C}: \eta=0.00284 \text{ poise}, \frac{T_1}{T_2} = 2.5$$

He<sup>3</sup>

$$\rho=0.082 \text{ g/cm}^3, \gamma=0.15 \text{ dynes/cm}, r_0=2.43 \times 10^{-8} \text{ cm}$$

at

$$\theta=0.04^\circ\text{K (corresponding to 0.26 or 0.8 MeV)*:}$$

$$\eta=0.002 \text{ poise}, \frac{T_1}{T_2} = 120$$

$$\theta=0.1^\circ\text{K (corresponding to 0.65 or 2 MeV)*:}$$

$$\eta=0.0004 \text{ poise}, \frac{T_1}{T_2} = 24$$

$$\theta=1^\circ\text{K (corresponding to 6.5 or 20 MeV)*:}$$

$$\eta=0.000023 \text{ poise}, \frac{T_1}{T_2} = 1.4$$

\*The conversion of the Kelvin temperatures for He<sup>3</sup> to equivalent nuclear temperatures is not unambiguous. The first figure uses the mass of He<sup>3</sup> as mass of the Fermion in the Fermi fluid representing He<sup>3</sup>, the second uses the mass of a quasi-particle with an experimental effective mass equal to 3.08 times the mass of He<sup>3</sup>.

He<sup>3</sup> refer to very large systems, in which the mean free path of the particles (or quasi-particles) is small compared to the dimensions of the system. For small droplets, when this condition is not satisfied, a discussion of damping in terms of the usual viscosity coefficient may be grossly inadequate. An extreme case that illustrates this point is the damping of a first vibrational level of a nucleus. If this level happens to be the first excited state in the energy spectrum, then (in the absence

of electromagnetic decays) the system would go on vibrating indefinitely in that state, with no damping at all. Thus for small systems at low temperatures, where level spacings are large and individual levels stand out, the damping is no longer described by a viscosity coefficient and, in particular, may be very small. I wonder if one could throw some more light on the question of damping in small Fermi fluid systems by experiments on the properties of a mist of He<sup>3</sup>, consisting of droplets with A-values of tens or hundred of molecules.

What else can we do to fill in the gaps in our understanding of the viscosity problem? Bjørnholm has recently discussed the question of damping in relation to the presence or absence of vibrational levels in heavy nuclei (especially those with a spontaneously fissioning isomeric state). There will also soon be many heavy-ion experiments which in one way or another will depend on viscosity, and from these we shall gradually unravel the answer. However, there exists already a mass of relevant experimental data in the allied field of fission, which could be used to estimate the nuclear viscosity. These data are measurements of fission fragment kinetic energies, especially in their dependence on  $Z^2/A$ . For the heavier nuclei in particular (i. e., for high  $Z^2/A$ ) the saddle point shape for fission is cylinder-like, or even spheroidal, and this means that there is a considerable saddle-to-scission stage for which the dynamics will surely depend on the size of the viscosity. Thus if the descent from saddle to scission is creepy, little kinetic energy will be accumulated by the fragments during this stage, and the observed fragment kinetic energies ought to be less than if the descent were free and mobile. Why then hasn't the theory of fission already provided us with the answer as regards viscosity? Because the relevant calculations for the viscous descent from saddle to scission have not been made. Calculations for a non-viscous descent of an idealized drop are available, and the results do agree fairly well with experiment. One might take this as an indication that viscosity is small, but one cannot be sure, since one doesn't know how much viscosity the theory could stand before a discrepancy with experiment would begin to emerge.

The calculation of the viscous descent from saddle to scission remains thus an outstanding problem of direct relevance to heavy ion physics.

Before leaving the subject of viscosity let me mention a related problem on which progress appears to be possible. This is the question of the drag or friction experienced by two nuclei passing each other in a grazing collision. Imagine two Fermi gases passing each other with a relative velocity  $\Delta v$ . Imagine there is an area of contact  $\pi r^2$  (a neck or "window"), lasting for a time of the order of  $2r/\Delta v$ .

During the time this window is open particles will move to and fro between the two Fermi gases. Because there is a velocity mismatch there will be a transfer of linear momentum equal to  $M\Delta v$  every time a particle of mass  $M$  from one nucleus is captured by the other. The flux of particles across unit area in a Fermi gas with Fermi momentum  $P$  is  $\pi P^4/Mh^3$ , and this leads to a drag force of  $(\pi P^4/h^3) \Delta v$  per unit area of contact between the nuclei, or to  $\pi P^4/h^3$  as the "drag coefficient" per unit area, per unit velocity differential. The change in linear momentum of one of the nuclei (equal to the change in the other) is (force)  $\times$  (time) =  $\frac{\pi P^4}{h^3} \Delta v \cdot \pi r^2 \cdot \frac{2r}{\Delta v} = \frac{1}{4\pi} \left(\frac{r}{\lambda_F}\right)^3 P$ , where  $\lambda_F = \hbar/P$ . These simple predictions disregard the fact that the exclusion principle may inhibit the transfer of nucleons from one nucleus to the other. It would be interesting to compare the properly generalized formulae for the grazing drag with suitable experiments. The relation of this to the problem of viscosity is that, as in the calculation of viscosity, this is a problem in the transport of momentum across a plane in a (nuclear) fluid. The difference is that the velocity field, instead of being characterized by a uniform velocity gradient (as in standard viscosity problems) is characterized by a velocity discontinuity. But it is the latter velocity field which, though rare in ordinary hydrodynamics, is the standard initial condition for heavy ion collisions.

In the meantime the problem of nuclear viscosity remains very unclear. Can one do any

dynamical calculations at all before this question is cleared up? Not really--but there is a sort of half-dynamic, half-static stage which doesn't require a knowledge of damping terms. This is the problem of a steady rotation of a system without intrinsic change of shape. In this case the only dynamical term, the only kinetic energy, is the energy of rotation. This brings me to the fourth item I wanted to discuss, that of the effect of angular momentum on the potential energy surface.

#### Effect of Angular Momentum.

We all know that if you spin a deformable object, say a fluid mass, it tends to flatten, and if you spin it too hard it will fly apart (fission). Some of you may be aware of a finer point, namely that usually, as you increase the angular momentum, a fluid mass will go from a flat, axially symmetric equilibrium shape to a triaxial equilibrium shape--like an oval piece of soap--for a range of angular momenta before fission. What do we expect to happen to a nucleus as it is made to spin faster and faster?

To discuss the equilibrium shapes of a rotating nucleus in a macroscopic theory we set up an effective potential energy  $(PE)_{\text{eff}}(\alpha_2\alpha_3\alpha_4\cdots)$ , and look for configurations that make the effective potential energy stationary:

$$(PE)_{\text{eff}} = PE + E_{\text{rot}}$$

$$= E_{\text{Coulomb}} + E_{\text{surface}} + E_{\text{rotation}} + \text{shells.}$$

The rotational energy  $E_{\text{rot}}(\alpha_2\alpha_3\alpha_4\cdots)$  that is added to the usual PE may be written as

$$E_{\text{rot}}(\alpha_2\alpha_3\alpha_4\cdots) = \frac{\hbar^2 l(l+1)}{2J(\alpha_2\alpha_3\alpha_4\cdots)}$$

where  $J$  is some effective moment of inertia of the shape in question. As usual the problem of exploring  $(PE)_{\text{eff}}$  may be split into first looking at a smooth background and then adding shell structure wiggles. Little is known as yet about the full problem with shells. On the other hand the smooth background has been explored more thoroughly (although many of the results remain unpublished). I shall present a few of the results of making stationary the smooth part

$$(PE)_{\text{eff}} = E_{\text{coulomb}} + E_{\text{surface}} + E_{\text{rotation}}$$

where in  $E_{\text{rot}}$  the moment of inertia  $\mathcal{J}$  is taken as the rigid body moment.

I would like to give you a bird's eye view of what happens to this smooth part of the energy for a nucleus anywhere in the periodic table and for any amount of angular momentum. For this purpose it is convenient to introduce two dimensionless numbers specifying the relative sizes of the three energy components: coulomb, surface and rotational. We pick the surface energy of the spherical shape  $E_{\text{surface}}^{(0)}$  as a unit and specify the amount of charge on the nucleus by the usual fissility parameter  $x$

$$x = \frac{\frac{1}{2} E_c^{(0)}}{E_s^{(0)}} \approx \frac{1}{50} \frac{Z^2}{A}$$

where  $E_c^{(0)}$  is the coulomb energy of the spherical shape.

We specify the amount of angular momentum by  $y$

$$y = \frac{E_{\text{rot}}^{(0)}}{E_s^{(0)}} \approx 2.0 \frac{I^2}{A^{7/3}}$$

where  $E_{\text{rot}}^{(0)}$  is the rotational energy of a rigid sphere with the given angular momentum.

Now I shall discuss the results in an  $x$ - $y$  diagram, Fig. 4. This diagram says the following things: If you take any nucleus in the periodic table then, if there were no shell effects and if the moment of inertia were rigid, the nucleus would at first deform into a flat shape. The fission barrier decreases with increasing angular momentum and vanishes along the upper curve in Fig. 4. The middle curve shows the critical angular momentum at which the flat ground state shape goes over into a triaxial shape. (Some of these shapes are like slightly indented flattened dumbbells.) The dashed curve divides the  $x$ - $y$  plane into two regions. To the right the saddle point shape for the fission of the rotating drop is stable against asymmetry, to the left it is unstable. Note that beyond  $x = 0.81$  there are no triaxial shapes.

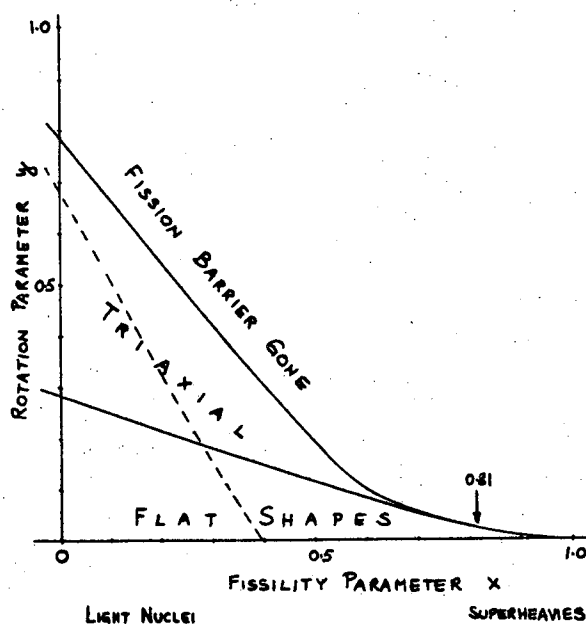


Fig. 4. A classification of rotating systems according to the fissility parameter  $x$  and the rotation parameter  $y$ . Triaxial equilibrium shapes disappear altogether at  $x = 0.81$ , but are almost gone at  $x = 0.6$ .

Let me illustrate the practical predictions that follow from this kind of calculation.

Consider a collision of a heavy ion of mass  $M_1$  and a nucleus of mass  $M_2$  at impact parameter  $b$  and center-of-mass energy  $E_{\text{cm}}$ . From conservation of energy and momentum it readily follows that the closest distance of approach of projectile and target is given by  $r_{\text{min}}$  where

$$\left( \frac{b}{r_{\text{min}}} \right)^2 = 1 - \frac{V(r_{\text{min}})}{E_{\text{cm}}}$$

Here  $V(r)$  is the interaction potential between the nuclei. For a given value of  $r_{\text{min}}$  this is a

hyperbola when  $b^2$  is plotted vs  $E_{\text{cm}}$ . (In such plots  $\pi b^2$  is proportional to a cross section.) If, for example,  $r_{\text{m}}$  is chosen to be the sum of the radii of the two nuclei,  $r_{\text{m}} = R_1 + R_2$ , the corresponding hyperbola divides the  $b^2$  vs  $E_{\text{cm}}$  plane into two regions: distant collisions where the nuclei pass each other without appreciable nuclear interactions, and close collisions where nuclear interactions take place (the corresponding  $\pi b^2$

gives then the reaction cross section). Because of the diffuseness of the nuclear surface and the finite range of nuclear forces, there is an intermediate diffuse transition region of grazing collisions. (See Fig. 5.)

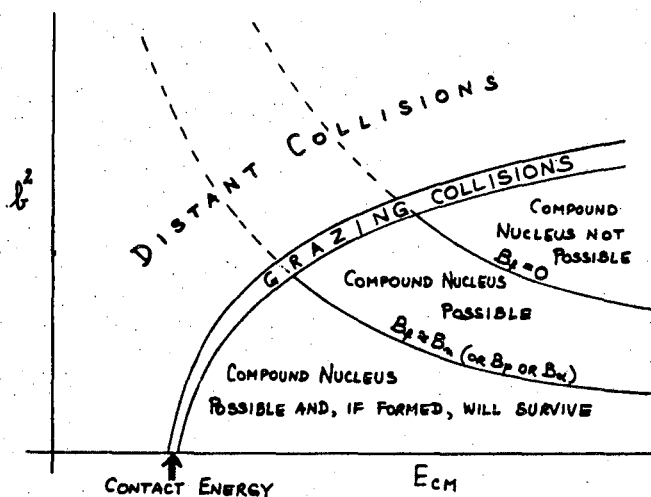


Fig. 5. A classification of nuclear collisions in the  $b^2$  vs  $E_{cm}$  plane. Distant collisions give place to close collisions along a diffuse region of grazing collisions. Close collisions are subdivided into three regions, as shown.

The  $b^2$  vs  $E_{cm}$  plane can be further subdivided by curves corresponding to loci of fixed angular momentum  $L$ . (Here  $L = \hbar l$ .) Since  $L$  is given by

$$L = b \sqrt{2E_{cm} M_{\text{reduced}}}, \quad M_{\text{reduced}} = \frac{M_1 M_2}{M_1 + M_2}$$

we have

$$b^2 = \frac{L^2}{2M_{\text{red}}} \cdot \frac{1}{E_{cm}}$$

For a given  $L$  this is again a hyperbola in a plot of  $b^2$  vs  $E_{cm}$ . If now from the upper curve in Fig. 4 we read off the value of  $y_{\text{crit}}$  (or  $L_{\text{crit}}$ ) at which the fission barrier has vanished, and insert this in the above equation for  $b^2$ , the resulting hyperbola will divide the  $b^2$  vs  $E_{cm}$  plane into two regions, as shown in Fig. 5. To the right

of the line marked  $B_f = 0$  (where the fission barrier  $B_f$  vanishes) the system has too much angular momentum to stay together and collisions corresponding to those values of  $b^2$  and  $E_{cm}$  would lead to redisintegration without the possibility of compound nucleus formation.

To the left of the hyperbola marked  $B_f = 0$  there exists a fission barrier and a compound nucleus is in principle possible. This is because a finite fission barrier ensures the existence in configuration space of a potential energy hollow, which can keep an excited system confined in its neighborhood (for times which decrease exponentially with decreasing height of the barrier). Thus if the system gets captured in the hollow, a compound nucleus will be formed. (Its lifetime is an exponentially decreasing function of the distance from the critical hyperbola marked  $B_f = 0$ .) But whether a more or less short-lived compound nucleus would, in fact, be formed for collisions to the left of  $B_f = 0$  is a different matter. It is a dynamical question of whether the colliding nuclei, starting off at the moment of tangency in some part of configuration space, would be captured in the potential energy hollow, or, on the contrary, whether they would miss it altogether or perhaps only pass through it without being captured.

Also shown in Fig. 5 is the hyperbola corresponding to the angular momentum at which the fission barrier has become equal to the binding energy of a neutron (or proton, whichever is lower). In this general neighborhood the de-excitation mode of a compound nucleus (if one were formed) would change from fission to particle emission and the compound nucleus, having survived the risk of fission, could be detected as such. There are indications that in some cases (e. g.,  $^{20}\text{Ne} + ^{27}\text{Al}$ ) the curve ABC does indeed predict the approximate energy-dependence of the cross section for the formation and survival of a compound nucleus. One should, however, remember that, as pointed out above, the prediction of the formation of a compound system is outside the scope of the considerations on which Fig. 5 is based. There is, in fact, a further curve, or family of curves, in the  $b^2$  vs  $E_{cm}$  plot, yet to be worked out on the



basis of the dynamics of fusion, which will describe the compound nucleus formation cross section. Only if this critical curve happens to be entirely above the curve ABC (i. e., if formation imposes no limitation) can the latter curve be expected to represent the cross section for the formation and survival of a compound nucleus. About the as yet undetermined critical curve (or curves) for compound nucleus formation we only know that it must lie below the  $B_f = 0$  hyperbola (and that in some cases, such as  $^{20}\text{Ne} + ^{27}\text{Al}$ , it seems to lie above the  $B_f \approx B_n$  hyperbola). But the calculation of the critical condition in a general case remains, as far as I know, an unsolved problem in fusion dynamics.

Let me summarize the main points of my talk.

1. Heavy Ion research is expected to have an impact on chemistry, atomic physics and quantum electrodynamics, as well as on nuclear physics.
2. Within nuclear physics heavy ion experiments will relax two age-old limitations: atomic numbers less than about 100, and near-spherical nuclear shapes.
3. Nuclear macro-physics based on exploiting  $A \gg 1$  should come into its own.
4. The first step in the new field is obvious: the working out of potential energy surfaces as functions of the degrees of freedom (three or more in number).
5. More difficult but essential steps are: understanding damping effects (viscosity) and effective inertias in dynamical problems.

I have been able to mention only a few of the outstanding problems in fission and fusion physics. I have biased my talk toward macroscopic aspects which I believe are the distinguishing features of heavy ion physics. I hope other speakers will complement the picture by stressing the microscopic approach which is, in principle at least, the more fundamental one.

References. An exhaustive list of references on recent calculations of Potential Energy surfaces, using the Strutinsky method, may be found in

J. R. Nix, *Ann. Rev. Nucl. Science* 22 (1972). The foundations of Strutinsky's method are described in M. Brack et al., *Rev. Mod. Phys.* 44, 320 (1972), where the formulation of the dynamics is also discussed. S. Björnholm's discussion of dynamics in fission and fusion, given at the Nordic-Dutch Accelerator Symposium at Ebeltoft, May 1971, is due to be published soon. Preliminary results on equilibrium shapes of rotating systems were given by S. Cohen, F. Plasil, and W. J. Swiatecki in Proceedings of the Third Conference on Reactions Between Complex Nuclei at Asilomar, edited by Ghirso, Diamond, and Conzett, University of California Press, 1963. A full account is in preparation. A recent paper which discusses grazing collisions of heavy ions is R. Basile et al., *Journal de Physique* 33, 9 (1972).

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