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PLACEMENT OF THE SYNCHROTRON BEAM ON AN INTERNAL TARGET. I

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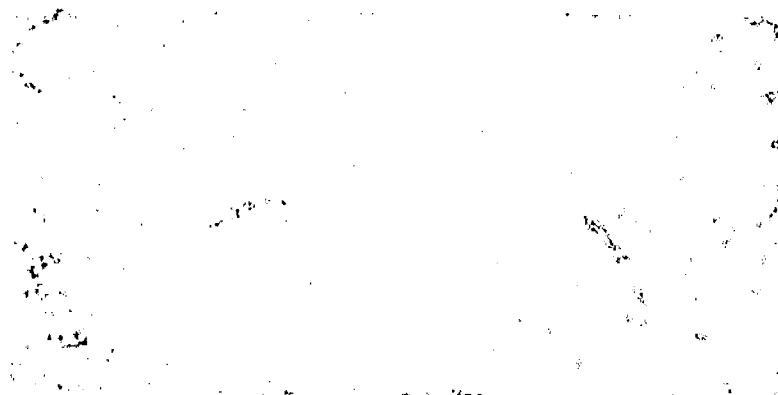
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Radiation Laboratory

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April 7, 1954

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Radiation Laboratory, Department of Physics,  
University of California, Berkeley, California

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ABSTRACT

High-energy electrons in a synchrotron may be placed on an internal target within one microsecond by using the forced radial oscillations that occur at  $n = 0.75$  in the presence of a first harmonic azimuthal variation of the magnetic field. The effect of the first harmonic in general and at  $n = 0.75$  is examined. The requirements for changing the radial field gradient from  $n = 0.67$  to  $n = 0.75$  are determined. An operating cycle is suggested.

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### INTRODUCTION

At the end of the acceleration cycle in synchrotrons, the radio-frequency voltage is removed and the magnetic field is static or changing very slowly. Under these conditions, the electrons spiral slowly inward because of their loss of energy by radiation<sup>1,2</sup>. A target placed at an inner radius intercepts the beam as it moves inward. In the Berkeley synchrotron the beam consumes about 15-25 microseconds in being decelerated by the target. The bremsstrahlung spectrum is present throughout.

In some experiments, an example of which is the measurement of the decay of a meson in flight, it is necessary to obtain a very short burst of radiation from the machine with no subsequent radiation. This paper tells how one-microsecond duration may be achieved by making use of a forced oscillation occurring at an  $n$  value of 0.75 with a first harmonic variation in the magnetic field asymmetry.

In the Berkeley synchrotron the variation of the field with radius must be modified to change the  $n$  value from 0.67 to 0.75 at the time the beam pulse is desired. The requirements for doing this are determined below.

A pulsed electrostatic deflection system has been tried and is unsuccessful for the following reasons: First, the voltage gradient required could not be maintained in the accelerating chamber, and second, the strong focusing action of the field reduces the initial deflection into an oscillatory path and further increases the voltage requirements.

Attempts to suddenly pulse the magnetic field to a higher value and stepwise reduce the radius of the orbit of the particles all involve a longer time in making the required change of field than is used by the beam in spiraling inward because of radiation loss.

# PARTICLE MOTION IN THE PRESENCE OF A FIRST HARMONIC OF AZIMUTHAL FIELD VARIATION

The derivation for the particle motion is made to show the effect of the first harmonic in general, and at the condition  $n = 0.75$  in particular.

For a circular path in a magnetic field the equilibrium orbit is obtained when the centrifugal forces equal the magnetic forces.

$$\frac{mv^2}{r_o} = \frac{eB_{r_o} v}{c} \quad \text{where } r_o \text{ is the radius of the equilibrium} \quad (1)$$

orbit and  $B_{r_o}$  is the magnetic field at the radius. For a particle not

on the equilibrium orbit a force arises given by  $F = \frac{mv^2}{r} - \frac{eBv}{c}$  (2)

Letting  $r = r_o + \rho$  and  $B = B_{r_o} + \frac{\partial B_{r_o}}{\partial r} \rho$ , and letting the azimuthal variation be such that  $B_{r_o} = B_{r_o} (1 + h \cos \theta)$ , we have a force

$$F = \frac{mv^2}{r_o + \rho} - \frac{ev}{c} \left[ B_{r_o} (1 + h \cos \theta) + \frac{\partial B_{r_o}}{\partial r} \rho (1 + h \cos \theta) \right],$$

noting that  $\frac{1}{r_o + \rho} = \frac{1}{r_o} - \frac{\rho}{r_o^2} + \frac{\rho^2}{r_o^3} \dots$

Since  $\rho \ll r_o$ , this series may be cut off after the second term. Then the force may be written

$$F = \left[ \frac{mv^2}{r_o} - \frac{ev}{c} B_{r_o} \right] - \frac{mv^2}{r_o^2} \rho - \frac{ev}{c} \frac{\partial B_{r_o}}{\partial r} \rho - \frac{evh}{c} \left[ B_{r_o} + \frac{\partial B_{r_o}}{\partial r} \rho \right] \cos \theta;$$

but--from Eq. 1--the first two terms on the right-hand side are equal, and thus the bracket is zero. The force may be replaced by the product of the mass of the particle and its acceleration (since the particle energy is assumed constant)

$$F = m \frac{d^2 \rho}{dt^2} = - \frac{mv^2}{r_o} \rho - \frac{e}{c} \frac{v}{r_o} r_o \frac{\partial B_{r_o}}{\partial r} \rho - \frac{eh}{c} \frac{v}{r_o} r_o \left[ B_{r_o} + \frac{\partial B_{r_o}}{\partial r} \rho \right] \cos \theta. \quad (3)$$

Letting  $\frac{eB}{mc} = \omega$  and  $\frac{v}{r_o} = \omega$ , and defining  $n = - \frac{r}{B_{r_o}} \frac{\partial B_{r_o}}{\partial r}$ ,

we may write the equation

$$\frac{d^2 \rho}{dt^2} + (1-n) \omega^2 \rho = \omega^2 h \cos \theta [n\rho - r_o]. \quad (4)$$

Since  $n < 1$  and  $\rho \ll r_o$ , this equation may be written as

$$\frac{d^2 \rho}{dt^2} + (1-n) \omega^2 \rho = -\omega^2 h r_o \cos \theta. \quad (5)$$

Letting  $\frac{d^2}{dt^2} = \omega^2 \frac{d^2}{d\theta^2}$ , one obtains  $\frac{d^2 \rho}{d\theta^2} + (1-n) \rho = -h r_o \cos \theta. \quad (6)$

The solution of equation (6) is of the form

$$\rho = A \sin \sqrt{1-n} \theta + B \cos \sqrt{1-n} \theta + \frac{h r_o}{n} \cos \theta. \quad (7)$$

Since for the synchrotron  $\sqrt{1-n}$  is not close to unity and since  $h$  is small, the third term is a forcing term far from resonance and it does not contribute a unidirectional increase in  $\rho$ . Thus, except when  $n \approx 0.75$ , the presence of a first harmonic of azimuthal field variation has no serious effect on the particle motion, as revealed by this linearized equation.

If the term  $n\rho$  is not dropped, the equation becomes

$$\frac{d^2 \rho}{d\theta^2} + [(1-n) - nh \cos \theta] \rho = -h r_o \cos \theta. \quad (8)$$

We may consider the auxiliary equation for Eq. (8) and substitute for the independent variable  $\theta = 2\phi$  to obtain

$$\frac{d^2 \rho}{d\phi^2} + 4 [(1-n) - nh \cos 2\phi] \rho = 0. \quad (9)$$

Eq. (9) is a Mathieu type equation whose canonical form<sup>3</sup> is

$$\frac{d^2 y}{dz^2} + (a - 2q \cos 2z) y = 0, \quad (10)$$

in which  $4(1-n) = a$

and  $4nh = 2q$ .

For small values of  $h$ , i. e., 5% - 10%, and values of  $n \neq 0.75$ , this equation is stable and its solution is oscillatory. However, at  $n = 0.75$ ,

$$\text{Eq. (8) becomes } \frac{d^2 \rho}{d\phi^2} + (1 - 3h \cos 2\phi) \rho = -hr_o \cos 2\phi, \quad (11)$$

$$\text{whose solution is } \rho = Ae^{\mu\phi} \phi(\phi) + Be^{-\mu\phi} \phi(-\phi) - \frac{hr_o}{3(1-h \cos 2\phi)} \cos 2\phi, \quad (12)$$

where  $\phi(\phi)$  is a periodic function of period  $\phi$  and  $\mu$  is an exponent which is real. The first term of the solution represents an exponential growth of the oscillatory motion.

For the conditions  $a = 1$ ,  $2q = 3h$ , the solution may be obtained from McLachlan<sup>3</sup> (Section 4.30) as  $\mu = \pm \frac{3}{4} h$ .

The amplitude of oscillation is increased by a factor of  $e$  each time  $\phi$  becomes  $\frac{1}{\mu}$ . Thus the number of turns required to give the growth of  $e$  is

$$N = \frac{4}{3\pi} \frac{1}{h}.$$

Thus as an example, a 1-percent first harmonic will require 43 turns to grow a factor of  $e$ . A 5-percent first harmonic will give an increase of  $e$  in about 9 turns. In 1 microsecond the electrons make 50 turns, so the increase in the latter case will be given by  $e^{50/9} \sim 260$ .

It is expected this increase will destroy the beam by bringing all particles onto the target.

## MODIFICATION OF THE SYNCHROTRON FIELD

The modification of the field pattern of the synchrotron may be made by adding current loops in the region of interest. The intention is not to change the mean field value, but merely to change the distribution within the gap.

Figure 1 shows field distribution under normal operating conditions. Figure 2 shows the physical arrangement of the gap, including compensating

coils. The compensating coils are closed through resistors whose value for each set was arrived at experimentally. Figure 3 shows the magnetic cycle and the cutoff for the radiofrequency signal.

From the definition of  $n = -\frac{r}{B} \frac{\partial B}{\partial r}$  at the peak field  $B_0 = 11,400$  gauss at  $r_0 = 39$  in., one may obtain the change in radial gradient required.

Table I

| r   | $\frac{\partial H}{\partial r}$<br>at $n=0.67$ | $\frac{\partial H}{\partial r}$<br>at $n=0.75$ | $\Delta \frac{\partial H}{\partial r}$ | $\Delta \frac{\partial H}{\partial r}$ |
|-----|------------------------------------------------|------------------------------------------------|----------------------------------------|----------------------------------------|
| 36" | -1.96%/inch                                    | -2.19%/inch                                    | -0.23%/inch                            | 26.2 gauss/in                          |
| 39" | -1.72                                          | -1.93                                          | -0.21                                  | 23.9                                   |
| 42" | -1.52                                          | -1.70                                          | -0.18                                  | 20.5                                   |

The change in the gradient due to one pair of coils is estimated as follows:

The field from a current-carrying conductor is given by  $B = \frac{0.2 I}{r}$

where  $B$  is in gauss,  $I$  in amperes, and  $r$  in cm. The presence of nonsaturated iron increases this by a factor of two, and the presence of the other member of the pair gives an additional factor of two. The vertical component at the median plane may be computed then from  $B_y = \frac{0.8 I x}{x^2 + y^2}$ .

Figure 4 shows the field from the No. 1 pair. The additional contribution from the first set of images is given for the first pair of coils. The sum of the field from the coil and its first reflection is shown. Finally, an approximation shown was taken as the contribution of the coil and all its reflections.

From the approximation, currents in the eight sets of coils were assumed to have various values to give the distribution desired. The best fit is obtained when coils 4 and 5 have about 192 amperes in each conductor and all the other coils have no current. The spacing between coils 4 and 5 is the optimum for this gap. Figure 5 shows the field from these two pairs.

The effect of changing gap width has not been considered in this approximation. The effect is to increase the gradient in the smaller gap

region and decrease it in the larger gap region. This is just the proper compensation to give the form implied in Table I. Adjustments in the currents in the pairs of coils permit additional trimming.

The force on the conductors is radial and is about one and one-quarter pounds per inch.

A sizable first harmonic in the full field of the synchrotron is believed to exist. A more pronounced harmonic may be induced by adjusting the quadrant-coil currents during the final portion of the cycle.

### OPERATION SEQUENCE

The change of the radial field gradient may take place slowly, with the rapid increase in radial oscillation amplitude occurring only when the  $n = 0.75$  condition is reached. Thus the beam may be destroyed in a very short time without the pulse difficulties of the other methods.

The rate of rise of current in the pairs of coils must meet two requirements. First, the rate must be small enough to avoid substantial cancellation of the field by the induced currents. Second, the duty cycle of the current must be short enough to avoid overheating the conductors.

A suitable cycling operation may be as follows. Near the end of the acceleration cycle current may be supplied to the pairs of coils, perhaps each turn being in series with the others. The rate of rise of this current may be not more than perhaps five times the maximum rate of rise of the main field. The calculated condition for radial blowup may be achieved after a sufficient period of inward spiraling to carry the beam near the target. The current may be stopped in any manner.

The slow rate at which the beam loses energy and decreases radius appears to preclude any extreme exactness in the cycling operation.

This work was done under the auspices of the Atomic Energy Commission.

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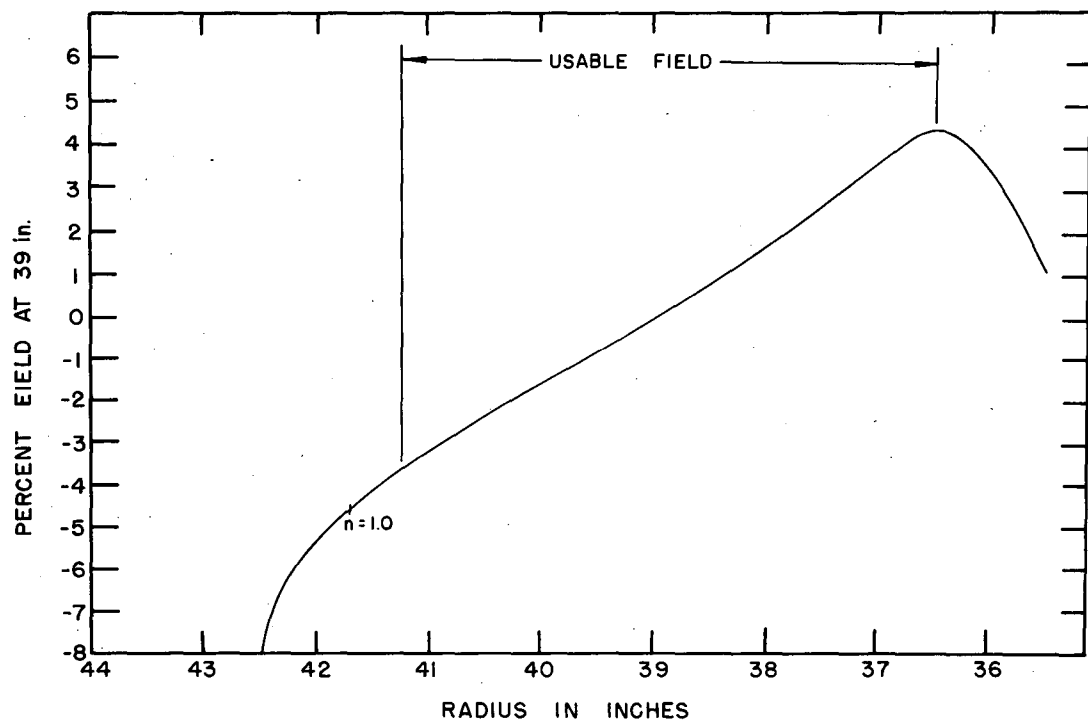


Fig. 1 --- Synchrotron Magnetic Field

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Fig. 1

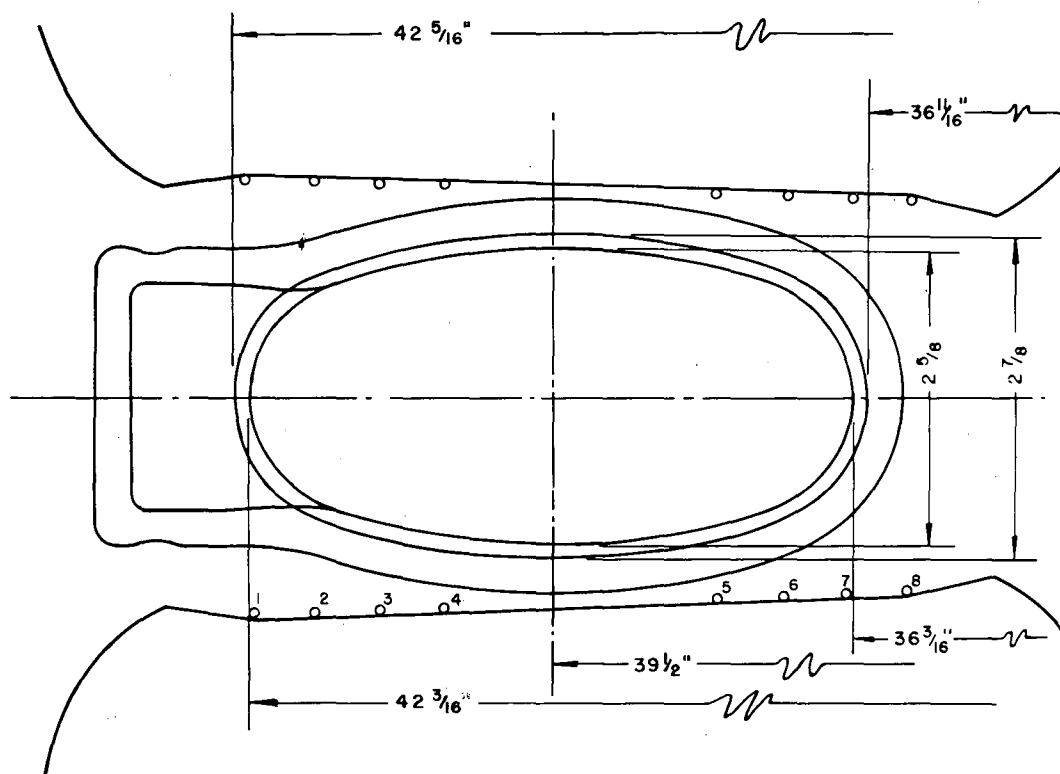


Fig. 2 --- Synchrotron Cross Section

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Fig. 2

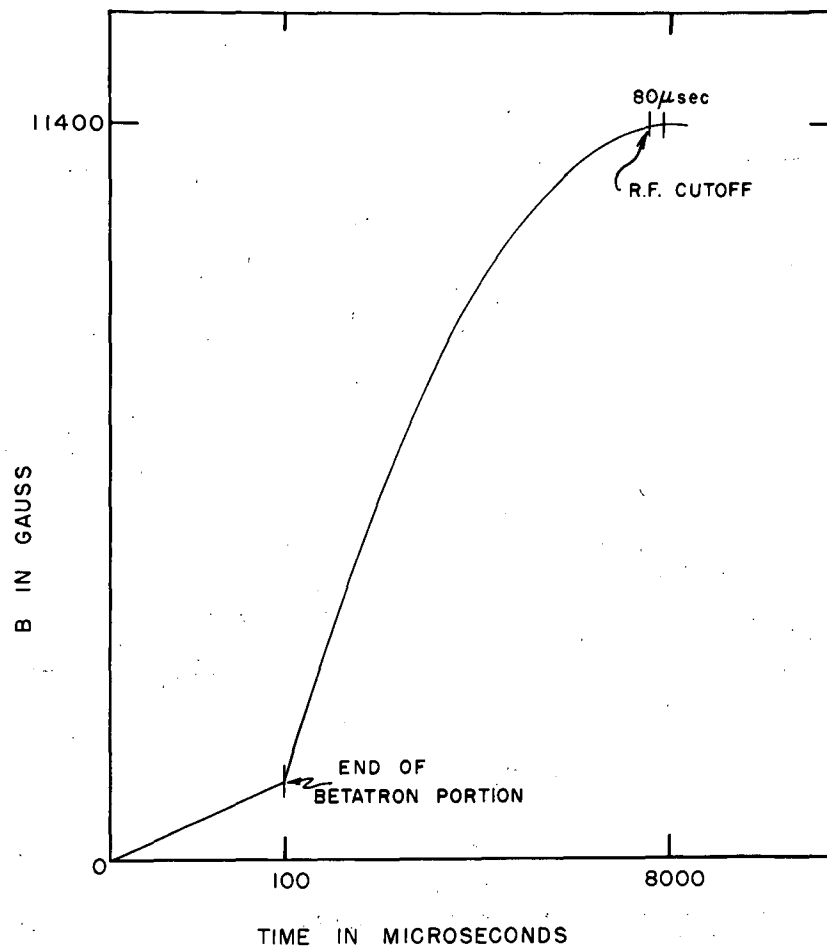


Fig. 3---Synchrotron Magnetic Field Variation  
(NOT TO SCALE)

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Fig. 3

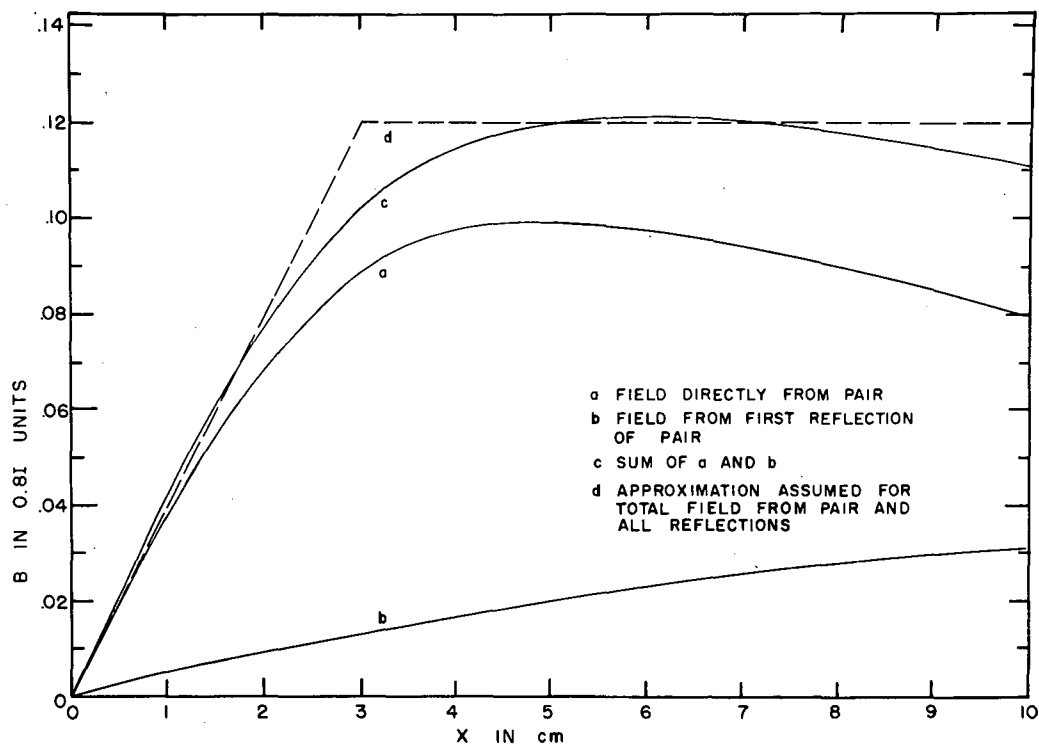


Fig. 4---The Vertical Component of the Magnetic Field at the Median Plane from No. 1 Pair of Wires.

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Fig. 4

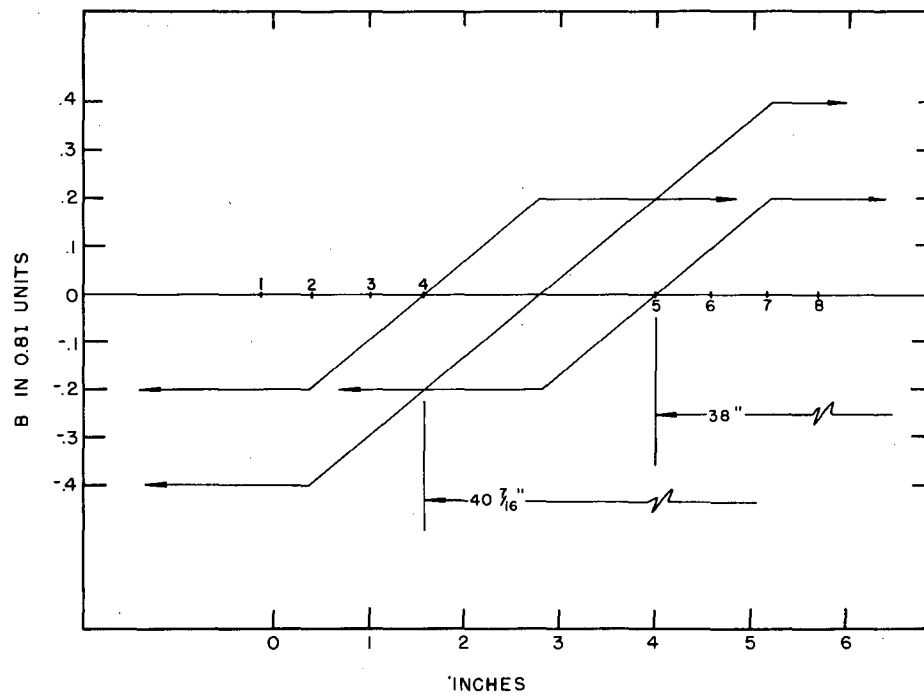


Fig. 5---Magnetic Field from Equal Currents in Pairs 4 & 5 Using the Approximation of Fig. 4

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Fig. 5