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ABSTRACT

A model is developed which allows one to calculate analytically the angular momentum removed, and the angular momentum misalignment created by the evaporation of light particles from an excited nucleus. The mass, temperature, and angular momentum of the emitting nucleus are explicitly considered. The formalism applies equally well to heavy ion and compound nuclear reactions.

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INTRODUCTION

When heavy ions undergo a deep inelastic collision, they form a short-lived intermediate complex or dinuclear system.[1] During the lifetime of this complex, angular momentum may be transferred from orbital motion into intrinsic rotation of the two fragments. The fragment spin generated in this way is essentially perpendicular to the reaction plane. Further, additional angular momentum can be generated in the two nuclei by the excitation of angular momentum-bearing modes of the intermediate complex.[2] In particular, angular momentum can be developed with nonvanishing components in the reaction plane leading to so-called misalignment. The magnitude of transferred angular momentum can be inferred, for instance, from gamma multiplicity measurements,[3] while angular momentum misalignment can be determined by measuring the angular distribution of particles or photons emitted by either or both of the fragments after the collision.[4] The angular distribution of sequential fission fragments also provides information from which angular momentum misalignment may be deduced.[4-7]

Collisions which are more head-on than those leading to deep inelastic reactions may result in the formation of a compound nucleus. Here too, orbital angular momentum is transferred into intrinsic spin. If the compound nucleus undergoes fission it assumes a shape reminiscent of that of the intermediate complex formed during a deep inelastic collision. During the fission phase angular momentum misalignment may be developed in fragments observed after the collision, just as in deep inelastic processes.

The ions which emerge from the heavy ion collision are excited and evaporate ejectiles, particularly neutrons. These ejectiles may remove substantial amounts of angular momentum from the ejector, be it a compound nucleus or an excited deep inelastic or fission fragment, and one must apply a particle evaporation correction in order to infer the initial fragment spin magnitude and alignment from measurements like gamma multiplicities and particle and/or photon angular distributions. Values for the first and second moments of the angular momentum removed by the evaporation of light particles from excited nuclei can be obtained from complicated particle evaporation codes. Unfortunately, the use of such codes is cumbersome and, more importantly, one tends to lose the feeling of the essential physics underlying the removal of angular momentum by the ejectiles. We endeavor to present simple models for light particle evaporation in order to obtain an accurate and simple description of the physics involved in the evaporation process. Our models, essentially based upon an earlier work by Moretto,[8] provide "pocket sized" formulae which show explicitely how the mass, temperature, and angular momentum of the ejector affect the value of the angular momentum removed by an ejectile.

Particle evaporation likewise causes misalignment of the angular momentum of the emitter; particle and photon angular distributions can not be properly used to obtain angular momentum misalignment without a particle evaporation correction. Our model calculates the dispersion of the emitter's angular momentum, both in the reaction plane and perpendicular to the reaction plane, arising from particle emission. For compound nuclei which do not fission we calculate dispersions parallel and perpendicular to the beam axis.

The model is applicable to ejectiles of any mass provided that the angular momentum removed by the ejectiles is not a large fraction of the angular momentum originally possessed by the emitting nucleus.

FIRST MOMENT OF ANGULAR MOMENTUM REMOVED BY EJECTILES

We assume that the evaporation process is controlled by a critical shape[8] (emitter/ejectile complex) consisting of the residual nucleus and emitted particle in contact with each other. The coordinate system we use is shown in Fig. 1. Its origin is at the center of mass of the emitter/ejectile complex. $\underset{\sim}{I}$ is the angular momentum of the complex before ejectiles are evaporated. The reaction plane is defined by the beam axis and the point at which one of the exit channel fragments is detected. The K axis (symmetry axis) is drawn through the centers of the emitting nucleus and the evaporated particle. The S axis is the axis along which the orbital angular momentum of the complex is directed. It is perpendicular to the K axis and is in the plane of the figure. The Y axis is perpendicular to both the S and K axes and points out of the figure. θ is the colatitude from which the ejectile is emitted.

We have specified three types of excited nuclei which evaporate light particles: Exit channel fragments from deep inelastic collisions, fission fragments from decaying compound nuclei, and compound nuclei which do not decay via fission. The average angular momentum carried away by any given evaporated ejectile depends, however, only on the mass, temperature and spin of the emitting nucleus; in no other way does the emitting nucleus remember the process through which it was created. We will consider exit channel fragments from deep inelastic collisions and fission fragments from decaying compound nuclei. These two types of excited fragments initially have their angular momentum predominantly aligned perpendicular to the reaction plane.

The angular momentum of the emitter/ejectile complex may be written:

$$\underset{\sim}{I} = I\cos\theta\,\hat{K} + I\sin\theta\,\hat{S} \quad ,$$

where $\hat{K}$ and $\hat{S}$ are the unit vectors along the K and S axes respectively.

With the origin at the center of mass of the emitter/ejectile complex, the moment of inertia tensor is written:

$$\underset{\approx}{\mathcal{J}} = \begin{array}{c|ccc} & K & S & Y \\ \hline K & J & 0 & 0 \\ S & 0 & J+\mu R^2 & 0 \\ Y & 0 & 0 & J+\mu R^2 \end{array}$$

where J is the moment of inertia of the emitting nucleus. Since

$$\underset{\approx}{\mathcal{J}} \cdot \underset{\sim}{\omega} = \underset{\sim}{I},$$

we obtain:

$$\underset{\sim}{\omega} = \frac{I\cos\theta\,\hat{K}}{J} + \frac{I\sin\theta\,\hat{S}}{J + \mu R^2} \; .$$

The orbital angular momentum associated with the emitter/ejectile complex is directed along the S axis. This is because both ejectile and emitter have their radius vector along the K axis and their center of mass momentum vector along the Y axis. Applying

$$\underset{\sim}{L} = \underset{\sim}{r} \times \underset{\sim}{p}_{cm},$$

where $\underset{\sim}{L}$ is the orbital angular momentum, $\underset{\sim}{r}$ is the radius vector, and $\underset{\sim}{p}_{cm}$ is the momentum of the center of mass, to both ejectile and emitter we find:

$$\underset{\sim}{L}_c = \frac{I\mu R^2 \sin\theta\, \hat{S}}{J + \mu R^2} = \frac{\mu R^2 \underset{\sim}{S}}{J + \mu R^2} \equiv (1 - F)\underset{\sim}{S} \quad , \tag{1}$$

where $\underset{\sim}{L}_c$ is the orbital angular momentum of the emitter/ejectile complex, and where $\underset{\sim}{S}$ is the component of $\underset{\sim}{I}$ along the S axis.

Initially we may assume that the ejectile is evaporated with no velocity in the rotating frame. The angular momentum removed by the ejectile is then the orbital angular momentum given by Eq. 1. Note that an ejectile emitted radially in the rotating frame likewise removes an amount of angular momentum given by Eq. 1. We thus call this model the radial emission model.[8] Before emission we have:

$$S^2 = I^2 - K^2. \tag{2}$$

If the ejectile has no intrinsic spin, we have, after emission:

$$(S')^2 = (FS)^2 = F^2(I^2 - K^2) \tag{3}$$

$$(I')^2 = (S')^2 + K^2 = F^2I^2 + (1 - F^2)\,K^2 \quad . \tag{4}$$

The primes designate the appropriate quantities for the residual nucleus after evaporation. The magnitude of I' is:

$$\begin{aligned}\sqrt{(I')^2} &= \sqrt{F^2I^2 - (1 - F^2)\,K^2} = FI\sqrt{1 + \left[\frac{(1 - F^2)\,K^2}{F^2I^2}\right]} \\ &\approx FI\left[1 + \frac{(1 - F^2)\,K^2}{2F^2I^2} - \frac{(1 - F^2)^2\,K^4}{8F^4I^4}\right] ,\end{aligned} \tag{5}$$

since $F \approx 1$.

We must now average over all possible values of K. For a given I the rotational energy of the complex is a function of K only[8]:

$$E_{rot} = \frac{I^2}{2J_s} + \frac{K^2}{2J_{eff}} = \text{const} + \frac{K^2}{2J_{eff}} \tag{6}$$

where $1/J_{eff} = 1/J_k - 1/J_s$, and where J_k and J_s are the moments of inertia about the K and S axes respectively. Thus with $K_o^2 \equiv J_{eff}T$, the average value of K^N is given by:

$$\left\langle K^N \right\rangle = \int_0^I K^N e^{-K^2/2K_o^2}\, dK \Big/ \int_0^I e^{-K^2/2K_o^2}\, dK \quad . \tag{7}$$

Upon taking the average value of Eq. 5 we find:

$$\langle I' \rangle = FI\left[1 + \frac{(1 - F^2)\langle K^2 \rangle}{2F^2 I^2} - \frac{(1 - F^2)^2 \langle K^4 \rangle}{8F^4 I^4}\right]$$

$$= I\left[F + \frac{1 - F^2}{2F}\left(\frac{\mathrm{erf}(Z)/2 - Ze^{-Z^2}/\sqrt{\pi}}{Z^2 \mathrm{erf}(Z)}\right)\right. \tag{8}$$

$$\left. - \frac{(1 - F^2)^2}{8F^3}\left(\frac{3\sqrt{\pi}\ \mathrm{erf}(Z)/4 - 3Ze^{-Z^2}/2 - Z^3 e^{-Z^2}}{\sqrt{\pi}\ \mathrm{erf}(Z)\ Z^4}\right)\right] ,$$

where $Z = I/(\sqrt{2}\, K_o)$.

It is convenient to define the functions:

$$g(Z) \equiv \frac{\mathrm{erf}(Z)/2 - Ze^{-Z^2}/\sqrt{\pi}}{Z^2 \mathrm{erf}(Z)}$$

$$h(Z) \equiv \frac{3\sqrt{\pi}\ \mathrm{erf}(Z)/4 - 3Ze^{-Z^2}/2 - Z^3 e^{-Z^2}}{\sqrt{\pi}\ \mathrm{erf}(Z)\ Z^4} \quad ; \tag{9}$$

with their help we may rewrite Eq. 8 as:

$$I' = I\left(F + \frac{1 - F^2}{2F}\, g(Z) - \frac{(1 - F^2)^2}{8F^3}\, h(Z)\right) \quad . \tag{10}$$

The functions g(Z) and h(Z) are plotted in Fig. 2. The limiting cases are:

$$\lim_{I\to 0} \left\langle K^2 \right\rangle = \frac{1}{3} I^2 , \quad \lim_{I\to 0} \left\langle K^4 \right\rangle = \frac{1}{5} I^4 ,$$

$$\lim_{I\to 0} \left\langle I' \right\rangle = I(F + (1 - F^2)/6F - (1 - F^2)^2/40\, F^3) ,$$

and

$$\lim_{I\to \infty} \left\langle K^2 \right\rangle = K_o^2 , \quad \lim_{I\to \infty} \left\langle K^4 \right\rangle = 3K_o^4 ,$$

$$\lim_{I\to \infty} \left\langle I' \right\rangle = I(F + (1 - F^2)K_o^2/2FI^2 - 3(1 - F^2)^2 K_o^4/8\, F^3 I^4) .$$

The first limit should be taken with a grain of salt since our model is clearly invalid when the angular momentum removed by the ejectile is calculated to be greater than the angular momentum initially present in the emitter/ejectile complex.

The functions g(Z) and h(Z) may be expanded in powers of Z. Their expansion to second order in Z yields:

$$\begin{aligned} g(Z) &\approx \frac{1}{3} - 4Z^2/45 \\ h(Z) &\approx \frac{1}{5} - 4Z^2/45 . \end{aligned} \tag{11}$$

Thus:

$$\left\langle I_{eject}\right\rangle = I - \langle I'\rangle \approx I\left[1 - F - \frac{1 - F^2}{2F}\left(\frac{1}{3} - \frac{4Z^2}{45}\right) + \frac{(1 - F^2)^2}{8F^3}\left(\frac{1}{5} - \frac{4Z^2}{45}\right)\right]. \quad (12)$$

As the nucleus evaporates ejectiles the parameter

$$F = \frac{J}{\mu R^2 + J}$$

changes. Similarly T and $Z = I/\sqrt{2}\,K_0$ change as particles are ejected. Thus, the angular momentum remaining in the residual nucleus after the emission of N ejectiles is:

$$\left\langle I'_N\right\rangle = I \prod_i \left(F_i + [1 - F_i^2]g(Z_i)/2F_i - [1 - F_i^2]^2 h(Z_i)/8F_i^3\right) , \quad (13a)$$

where F_i is the value of F before emission of the i^{th} ejectile and Z_i is similarly defined. In fact, F_i and $g(Z_i)$ change only slightly as particles are ejected, and one may reasonably write:

$$\left\langle I_{eject,N}\right\rangle = I - \left\langle I'_N\right\rangle = I\left[1 - \left(F_{ave} + \frac{(1 - F_{ave}^2)g(Z_{ave})}{2F_{ave}} - \frac{(1 - F_{ave}^2)^2 h(Z_{ave})}{8F_{ave}^3}\right)^N\right] \quad (13b)$$

if suitable values for F_{ave} and Z_{ave} are chosen, and provided $\left\langle I_{eject,N}\right\rangle$ is not nearly as large as is I.

DISPERSION OF ANGULAR MOMENTUM IN THE RESIDUAL NUCLEUS

One's intuitive feeling is that the angular momentum removed by ejectiles which are not ejected radially should more or less cancel. We shall later verify that this feeling is correct. On the other hand, ejectiles which are not emitted radially are expected to contribute substantially to the second moment of I'. These ejectiles are ignored by the radial emission model which is thus not adequate for computing the total dispersion of I'. Nevertheless, the radial emission model predicts a dispersion of I', both in the reaction plane and perpendicular to the reaction plane. This dispersion will prove to be useful when we consider a more reasonable model later on.

Consider the j component of I':

$$I'_j = I_j - I_{eject,j} \quad . \tag{14}$$

The variance is given by:

$$\sigma^2(I'_j) = \sigma^2(I_j - I_{eject,j}) =$$

$$= \langle I_j^2 \rangle - 2\langle I_j I_{eject,j} \rangle + \langle I^2_{eject,j} \rangle \tag{15}$$

$$- \langle I_j \rangle^2 + 2\langle I_j \rangle \langle I_{eject,j} \rangle - \langle I_{eject,j} \rangle^2 \quad .$$

Since $\underset{\sim}{I}$ is defined to be the angular momentum of the emitter/ejectile complex before the evaporation of an ejectile, it is a constant. Equation 15 may thus be written:

$$\sigma^2(I'_j) = \left\langle I^2_{eject,j} \right\rangle - \left\langle I_{eject,j} \right\rangle^2 = \sigma^2(I_{eject,j}) \ . \qquad (16)$$

We decompose $\underset{\sim}{I}_{eject}$ into two components:

$$\underset{\sim}{I}_{eject} = \underset{\sim}{I}_{eject,RP} + \underset{\sim}{I}_{eject,z} \ .$$

RP symbolizes "reaction plane" and the z axis is perpendicular to the reaction plane. We first consider the dispersion in the reaction plane. With

$$I_{eject} = (1 - F)\ S = (1 - F)\ I \sin\theta \quad , \qquad (17)$$

we find:

$$\left\langle I'^2 \right\rangle_{RP} = \left\langle I^2_{eject,RP} \right\rangle = (1 - F)^2\ I^2 \left\langle \sin^2\theta \cos^2\theta \right\rangle = 2\sigma_x^2 = 2\sigma_y^2 \ ; \qquad (18)$$

x and y are two equivalent orthogonal directions in the reaction plane. Remembering that $K = I \cos\theta$ we obtain:

$$\sigma_x^2 = \sigma_y^2 = \frac{I^2(1 - F)^2}{2} \left(\frac{\langle K^2 \rangle}{I^2} - \frac{\langle K^4 \rangle}{I^4} \right) = \frac{I^2(1-F)^2}{2}(g(Z) - h(Z)) \quad . \qquad (19)$$

The use of the approximations given in Eq. 11 will yield answers which differ from the exact solution we shall shortly obtain by $\lesssim$ 3%.

For the z direction we have:

$$\left\langle I^2_{eject,z} \right\rangle = (1 - F)^2 \; I^2 \left\langle \sin^2\theta \; \sin^2\theta \right\rangle \;, \tag{20}$$

or

$$\left\langle I^2_{eject,z} \right\rangle = I^2(1 - F)^2[1 - 2g(Z) + h(Z)]. \tag{21}$$

In order to estimate the dispersion in the z direction:

$$\sigma_z^2 = \left\langle I^2_{eject,z} \right\rangle - \left\langle I_{eject,z} \right\rangle^2 \;, \tag{22}$$

we need $\left\langle I_{eject,z} \right\rangle$. From Eq. 17,

$$I_{eject} = (1 - F)S = (1 - F) \; I \sin\theta \; .$$

The z component is:

$$(1 - F) \; I \sin^2\theta = (1 - F) \; I((I^2 - K^2)/I^2). \tag{23}$$

Thus

$$\left\langle I_{eject,z}\right\rangle = (1 - F)I\left\langle 1 - K^2/I^2\right\rangle = (1 - F)I(1 - g(Z)) \quad . \tag{24}$$

Combining Eqs. 20 through 24 yields:

$$\begin{aligned}\sigma_Z^2 &= I^2(1 - F)^2[1 - 2g(Z) + h(Z)] - [(1 - F)\ I(1 - g(Z))]^2 \\ &= I^2(1 - F)^2\ (h(Z) - g^2(Z)) \quad .\end{aligned} \tag{25}$$

ISOTROPIC EMISSION MODEL

Let us now consider a model which will allow us to explicitly calculate the contribution of non-radially emitted ejectiles to the second moment of I'. We will also find that the average angular momentum removed by ejectiles calculated with this isotropic emission model does not differ substantially from the first moment calculated earlier with the radial emission model. The coordinates used are defined in Fig. 3. We suppose that for a given colatitude, θ, ejectiles are emitted isotropically. The probability that an ejectile is emitted in a differential solid angle $d\Omega$ centered at (α,β) is:

$$P(\alpha,\beta)d\Omega \propto d\Omega = \sin\alpha d\alpha d\beta .$$

We may integrate this probability distribution over all β to obtain the probability, $P(\alpha)$, that the ejectile is evaporated at a polar angle α. The integration yields:

$$P(\alpha)d\alpha \propto \sin\alpha d\alpha .$$

The ejectile is emitted with momentum p in the rotating frame. That is:

$$\underset{\sim}{p}_{rot} = p \cos\alpha \hat{K} + p \sin\alpha \cos\beta \hat{S} + p \sin\alpha \sin\beta \hat{Y} \qquad (26)$$

where $\hat{K}$, $\hat{S}$ and $\hat{Y}$ are the unit vectors along the K, S and Y axes respectively. The vector $\underset{\sim}{p}_o = -p_o\hat{Y}$ is defined such that:

$$\underset{\sim}{p}_{lab} = \underset{\sim}{p}_{rot} + \underset{\sim}{p}_o \ . \qquad (27)$$

The radius vector is $\underset{\sim}{r} = r\hat{K}$. The angular momentum carried away by the evaporated particle is:

$$\underset{\sim}{r} \times \underset{\sim}{p}_{lab} = (rp_o - rp \sin\alpha \sin\beta) \hat{S} + rp \sin\alpha \cos\beta \hat{Y} \ . \qquad (28)$$

In the isotropic model:

$$\underset{\sim}{I}' = \underset{\sim}{I} - \underset{\sim}{I}_{eject}$$

$$= K\hat{K} + (S - (rp_o - rp \sin\alpha \sin\beta))\,\hat{S} + (- rp \sin\alpha \cos\beta)\,\hat{Y} \quad . \qquad (29)$$

Recalling that:

$$K = I \cos\theta \text{ and}$$

$$S = I \sin\theta$$

one obtains:

$$I' = \sqrt{\underset{\sim}{I}'\cdot\underset{\sim}{I}'} = (F^2I^2 + (1 - F^2)K^2 + 2FIrp \sin\alpha \sin\beta \sin\theta + r^2p^2 \sin^2\alpha)^{1/2} . \quad (30)$$

Keeping terms to 4th order in K and integrating over α, β, θ and p, we find that the radial emission model predicts greater average angular momentum removed per ejectile than does the isotropic model. The difference is:

$$\langle I_{eject}\rangle_R - \langle I_{eject}\rangle_I =$$

$$FI(r^2\langle p^2\rangle/6F^2I^2 + (2F^2 - 1)\langle K^2\rangle r^2\langle p^2\rangle/6F^4I^4 - r^4\langle p^4\rangle/15F^4I^4) \quad , \qquad (31)$$

where $\langle I_{eject}\rangle_R$ is the average angular momentum removed by ejectiles calculated in the radial emission model, and $\langle I_{eject}\rangle_I$ is the average angular momentum removed calculated via the isotropic model. With the estimates:

$$\langle p^2\rangle = 3mT \quad , \quad \langle p^4\rangle = 12\, m^2T^2 \quad , \quad \langle K^2\rangle \lesssim 1/3\; I^2, \qquad F = 1 \quad ,$$

we find the difference in $\langle I_{eject}\rangle$ predicted by the two models to be about $2mr^2T/3I$. The difference is proportional to the mass of the ejectile but so, roughly, is $\langle I_{eject} \rangle$. The two models thus agree to roughly 5% independent of the mass of the ejectile.

With Eq. 28 we may readily calculate the second moment of I'. We note that:

$$I'^2_{RP} = I'^2_s \cos^2\theta + I'^2_y = (rp_o \cos\theta)^2 - 2r^2 pp_o \sin\alpha \sin\beta \cos^2\theta$$
$$+ r^2p^2 \sin^2\alpha \sin^2\beta \cos^2\theta + r^2p^2 \sin^2\alpha \cos^2\beta \ . \tag{32}$$

Averaging over the independent variables α, β, θ, and p we find:

$$\langle I'^2 \rangle_{RP} = \langle r^2 p_o^2 \cos^2\theta \rangle + (1/3) r^2 \langle p^2 \rangle (1 + \langle \cos^2\theta \rangle) \ . \tag{33}$$

The first term in Eq. 33 is the second moment of I' in the reaction plane calculated via the radial emission model. We estimate $\langle p^2 \rangle =$ 3mT, where m is the mass of the ejectile and recall that $\langle \cos^2\theta \rangle =$ $\langle K^2 \rangle / I^2$. We find:

$$\sigma_x^2 = \sigma_y^2 = 1/2 \langle I'^2 \rangle_{RP} = 1/2 \langle I'^2_{RP,o} \rangle + (1/2)\ mr^2 T(1 + g(Z)) \tag{34}$$

where $1/2 \langle I'^2_{RP,o} \rangle$ is the variance in the x and y direction calculated with the radial emission model, and is given by Eq. 19. The second moment perpendicular to the reaction plane is calculated in an analogous way:

$$\langle I^2_{eject,z} \rangle = \langle I^2_{eject,z,o} \rangle + mr^2 T(1 - g(Z)) \ , \tag{35}$$

and $\left\langle I^2_{\mathrm{eject}\ z,o}\right\rangle$, the second moment of I_{eject} in the z direction calculated with the radial emission model, is given in Eq. 21.

$$\sigma_z^2 = \left\langle I^2_{\mathrm{eject},z,o}\right\rangle + mr^2T(1 - g(Z)) - \left\langle I_{\mathrm{eject},z}\right\rangle^2 \tag{36}$$

where $\left\langle I_{\mathrm{eject},z}\right\rangle$ is given in Eq. 24. One may find it convenient to avoid calculating the quantity $\left\langle I_{\mathrm{eject},z}\right\rangle$ and instead to use the value of $\left\langle I_{\mathrm{eject}}\right\rangle$ given by Eq. 12 for $\left\langle I_{\mathrm{eject},z}\right\rangle$ in Eq. 36. The error so introduced is $\lesssim$ 1% for both neutrons and alpha particles.

FLUCTUATIONS IN COMPOUND NUCLEI

It is not appropriate to use the coordinate system with which we have been working when discussing compound nuclei which do not fission, since for reactions leading to such nuclei, one does not usually define a reaction plane. A suitable coordinate system for non fissioning compound nuclei is shown in Fig. 4. Note that the x' axis is the beam axis. The distribution of $\underset{\sim}{I}$ for these compound nucleus reactions is confined to the y'z' plane and is isotropic therein. We use primed coordinates to distinguish the non fissioning compound nucleus from exit channel fragments from fissioning compound nuclei or heavy ion reactions.

While individual components of the variance depend on one's choice of coordinate axes, the total variance can not depend on the coordinate system chosen. That is:

$$\sigma_x^2 + \sigma_y^2 + \sigma_z^2 = \sigma_{x'}^2 + \sigma_{y'}^2 + \sigma_{z'}^2 \quad . \tag{37}$$

The x' axis is always perpendicular to the angular momentum $\underset{\sim}{I}$. The x' axis of the non fissioning compound nucleus corresponds to the x (or y) axis of the fissioning compound nucleus/heavy ion exit channel fragment. Here too the x and y axes are perpendicular to $\underset{\sim}{I}$. Thus

$$\sigma_x^2 = \sigma_y^2 = \sigma_{x'}^2 \quad . \tag{38}$$

The cylindrical symmetry of the non fissioning compound nucleus implies:

$$\sigma_{y'}^2 = \sigma_{z'}^2 \quad , \tag{39}$$

which, with Eqs. 37 and 38 yields:

$$\sigma_{y'}^2 = \sigma_{z'}^2 = \frac{1}{2}(\sigma_y^2 + \sigma_z^2) \quad . \tag{40}$$

A MORE FOCUSED MODEL

Eq. 30 is valid for any distribution of emitted ejectiles. For any distribution with cylindrical symmetry, that is with $P(\alpha,\beta)$ independent of β, we may integrate Eq. 30 over β, θ, and p and obtain:

$$\langle I_{eject}\rangle_R - \langle I_{eject}\rangle_F =$$

$$FI(r^2\langle\sin^2\alpha\rangle\langle p^2\rangle/4F^2I^2 + (2F^2-1)\langle K^2\rangle r^2\langle\sin^2\alpha\rangle\langle p^2\rangle/4F^4I^4 - r^4\langle\sin^4\alpha\rangle\langle p^4\rangle/8F^4I^4) \quad , \tag{41a}$$

where $\langle I_{eject}\rangle_F$ is the average angular momentum removed by ejectiles calculated via this more focused model. Since

$$r^4\langle\sin\alpha^4\rangle\langle p^4\rangle/8F^4I^4$$

is much less than the other terms in Eq. 41a (for neutron evaporation with $I \approx 25\hbar$ and $\langle K^2\rangle/I^2 \approx 0.2$, the ratio of the three terms in Eq. 41a is typically of order 1:0.2:0.003), we may approximate this equation by:

$$\langle I_{eject}\rangle_R - \langle I_{eject}\rangle_F =$$

$$FI(r^2\langle\sin^2\alpha\rangle\langle p^2\rangle/4F^2I^2 + (2F^2-1)\langle K^2\rangle r^2\langle\sin^2\alpha\rangle\langle p^2\rangle/4F^4I^4) \quad . \tag{41b}$$

We assumed an isotropic distribution in order to estimate values for $\langle p^2\rangle$ and $\langle\sin^2\alpha\rangle$ but we do not expect that these estimates are very accurate. In particular, the distribution of emitted ejectiles is skewed in the forward direction and favors ejectiles with relatively high momentum; $\langle \sin^2\alpha \rangle$ is in truth lower than the isotropic model predicts and $\langle p^2 \rangle$ is higher than is estimated by the isotropic model.

Referring to Fig. 3 we note that the probability of observing an emitted ejectile at any colatitude is proportional to v_k. The statistical distribution of velocities within the nucleus is:

$$P_{in}(v_k,v_s,v_y) \propto \exp(-m(v_k^2 + v_s^2 + v_y^2)/2T) \quad , \tag{42}$$

where m is the ejectile's mass and T is the nuclear temperature. Thus the distribution of velocities of evaporated ejectiles is:

$$P(v_k,v_s,v_y) \propto v_k \exp(-m(v_k^2 + v_s^2 + v_y^2)/2T) \quad . \tag{43}$$

In polar coordinates this becomes:

$$\begin{aligned} P(v,\alpha) &\propto v^3\exp(-mv^2/2T) \cos\alpha \sin\alpha \\ P(p,\alpha) &\propto p^3\exp(-p^2/2mT) \cos\alpha \sin\alpha \quad . \end{aligned} \tag{44}$$

With this probability distribution we obtain

$$\langle p^2 \rangle = 4mT \ , \ \langle \sin^2\alpha \rangle = \frac{\int_0^{\frac{\pi}{2}} \sin^2\alpha \ P(\alpha)d\alpha}{\int_0^{\frac{\pi}{2}} P(\alpha)d\alpha} = \frac{\int_0^{\frac{\pi}{2}} \sin^2\alpha \sin\alpha \cos\alpha \ d\alpha}{\int_0^{\frac{\pi}{2}} \sin\alpha \cos\alpha \ d\alpha} = 1/2 \ ,$$

where $P(\alpha)$ is proportional to the probability that the ejectile is emitted with polar angle α independent of the ejectile's momentum. With our isotropic model we obtained:

$$\langle p^2\rangle = 3mT \;,\; \langle \sin^2\alpha\rangle = \frac{\int_0^{\frac{\pi}{2}} \sin^2\alpha\, P(\alpha)d\alpha}{\int_0^{\frac{\pi}{2}} P(\alpha)d\alpha} = \frac{\int_0^{\frac{\pi}{2}} \sin^2\alpha\, \sin\alpha\, d\alpha}{\int_0^{\frac{\pi}{2}} \sin\alpha\, d\alpha} = 2/3.$$

Note that the form of $P(\alpha)$ is not identical in the isotropic and more focused models. In either case the product

$$\langle p^2\rangle\langle \sin^2\alpha\rangle = 2mT \quad .$$

Thus the first moment calculated by the isotropic model and this more focused model are essentially identical.

We now verify that the isotropic and more focused models give identical results for the second moment of I_{eject} and thus for the dispersion of I'. Before integration over α, the expressions for the second moment of I_{eject} are:

$$\langle I^2_{eject}\rangle_{RP} = \langle I'^2\rangle_{RP} = \langle I'^2{}_{RP,o}\rangle + (1/2)r^2 \langle \sin^2\alpha\rangle\langle p^2\rangle(1 + \langle \cos^2\theta\rangle) \tag{45}$$

$$\langle I^2_{eject}\rangle_z = \langle I^2_{eject,z,o}\rangle + (1/2)r^2 \langle \sin^2\alpha\rangle\langle p^2\rangle\langle \sin^2\theta\rangle \;.$$

Since the product $\langle \sin^2\alpha\rangle\langle p^2\rangle$ is identical in the isotropic and more focused models, so too must be the values of

$$\left\langle I^2_{eject} \right\rangle_{RP} \quad \text{and} \quad \left\langle I^2_{eject} \right\rangle_{Z} .$$

We note finally that if N particles are evaporated, the total variance is N times the variance for a single evaporated particle, in the limit that the parameters F, T, and Z do not change as particles are emitted.

CALCULATION

We consider a ^{108}Ag nucleus which has emerged from a heavy ion reaction. It has $T = 2.5$ MeV, $I = 35\hbar$ and it evaporates a single particle. In order to see the effects of some of our approximations, we include "crude" calculations in which the functions $g(Z)$ and $h(Z)$ have been expanded to second order in Z and in which we have taken $\langle I_{eject,z} \rangle = \langle I_{eject} \rangle$.

The parameters we use are:

$$F = \frac{J}{\mu R^2 + J} \qquad \text{(cf. Eq. 1)}$$

$$K_o^2 = \frac{T}{(1/J_k - 1/J_s)} = \frac{T J_k J_s}{J_s - J_k} \qquad \text{(Eq. 6)}$$

$$Z = I/\sqrt{2}\, K_o \qquad \text{(Eq. 8)} \quad .$$

The special functions are:

$$g(Z) = \frac{\mathrm{erf}(Z)/2 - Z\exp(-Z^2)/\sqrt{\pi}}{Z^2 \mathrm{erf}(Z)} \qquad \text{(Eq. 9)}$$

$$h(Z) = \frac{(3\sqrt{\pi}\,\mathrm{erf}(Z)/4 - 3Z\exp(-Z^2)/2 - Z^3\exp(-Z^2))}{\sqrt{\pi} Z^4 \mathrm{erf}(Z)} \qquad \text{(Eq. 9)} \quad .$$

In our crude calculations we approximate:

$$g(Z) \approx 0.333 - 4Z^2/45 \qquad \text{(Eq. 11)}$$

$$h(Z) \approx 0.200 - 4Z^2/45 \qquad \text{(Eq. 11)} \quad .$$

The relevant formulae are:

$$\langle I_{eject} \rangle = I - \langle I' \rangle = I\left[1 - F - \frac{(1 - F^2)}{2F} g(Z) + \frac{(1 - F^2)^2}{8F^3} h(Z)\right]$$

(cf. Eq. 10),

which is approximated in our crude calculation by:

$$\langle I_{eject} \rangle = I\left[1 - F - \frac{1 - F^2}{2F}\left(\frac{1}{3} - \frac{4Z^2}{45}\right) + \frac{(1 - F^2)^2}{8F^3}\left(\frac{1}{5} - \frac{4Z^2}{45}\right)\right]$$

(Eq. 12);

$$\sigma_x^2 = \sigma_y^2 = \frac{I^2(1 - F)^2}{2} (g(Z) - h(Z)) + \frac{mr^2T(1 + g(Z))}{2} \qquad \text{(Eqs. 19 and 34)} \quad ,$$

which, in our crude calculation, is approximated by:

$$\sigma_x^2 = \sigma_y^2 = \frac{I^2(1 - F)^2}{2} (0.133) + \frac{mr^2T(1.33 - 4Z^2/45)}{2} \quad ;$$

$$\sigma_z^2 = \langle I^2_{eject,z} \rangle - \langle I_{eject,z} \rangle^2 \quad ,$$

where:

$$\left\langle I^2_{eject,z} \right\rangle = I^2(1 - F)^2(1 - 2g(Z) + h(Z)) + mr^2T(1 - g(Z)) \quad \text{(Eqs. 21 and 35)}$$

$$\left\langle I_{eject,z} \right\rangle = I(1 - F)(1 - g(Z)) \quad . \quad \text{(Eq. 24)}$$

In our crude calculation we use the approximations:

$$\left\langle I^2_{eject,z} \right\rangle = I^2(1 - F)^2(0.533 + 4Z^2/45) + mr^2T(0.667 + 4Z^2/45)$$

and

$$\left\langle I_{eject,z} \right\rangle = \left\langle I_{eject} \right\rangle(\text{crude}) \quad . \quad \text{(Eq. 12)}$$

The results of our calculations are:

a) Neutron Evaporation

$$\left\langle I_{neut} \right\rangle = 0.738\hbar$$

$$\left\langle I_{neut} \right\rangle(\text{crude}) = 0.739\hbar$$

$$\sigma_x^2 = \sigma_y^2 = 1.889\hbar^2$$

$$\sigma_x^2 = \sigma_y^2(\text{crude}) = 1.891\hbar^2$$

$$\sigma_z^2 = 1.992\hbar^2$$

$$\sigma_z^2(\text{crude}) = 1.993\hbar^2$$

b) Alpha Evaporation

$\langle I_\alpha \rangle = 4.214\hbar$

$\langle I_\alpha \rangle(\text{crude}) = 4.277\hbar$

$\sigma_x^2 = \sigma_y^2 = 11.90\hbar^2$

$\sigma_x^2 = \sigma_y^2(\text{crude}) = 12.17\hbar^2$

$\sigma_z^2 = 14.41\hbar^2$

$\sigma_z^2(\text{crude}) = 14.41\hbar^2$.

CONCLUSION

We have developed simple models to calculate analytically the first moment and dispersion of the angular momentum removed from excited nuclei by the evaporation of light particles. At the same time, we tried to keep the physics of the problem as transparent as possible.

We equated the angular momentum removed by evaporated particles with the orbital angular momentum of the emitter/ejectile complex before evaporation, and took a suitable average over the colatitudes from which the ejectile may have been emitted. The fact that the ejectile may have been evaporated from any colatitude led to fluctuations in the angular momentum removed by the light particle even in the radial emission model. It was instructive, nonetheless, to consider two simple distributions for ejectiles in the rotating frame: an isotropic distribution, and a Maxwell velocity distribution weighted by the factor v_k, the velocity perpendicular to the surface at the point of evaporation. We were able to decompose the dispersions into their three orthogonal components. For exit channel fragments from deep inelastic collisions and for fission fragments from compound nucleus reactions, the coordinates were chosen to be perpendicular and parallel to the reaction plane. For non fissioning compound nuclei the coordinates were chosen to be perpendicular and parallel to the beam axis.

REFERENCES

1. L. G. Moretto and R. Schmitt, J. Phys. 37, (1976) C5-109 and references therein.
2. Luciano G. Moretto and Richard P. Schmitt, Phys. Rev. C. 21, 204 (1980).
3. M. Lefort and C. Ngô, Riv. Nuovo Cimento. 12, 1 (1979) and references therein.
4. L. G. Moretto, S. Blau, and A. Pacheco, LBL Report No. 10805 (to be published).
5. R. A. Broglia, G. Pollarolo, C. H. Dasso, and T. Døssing, Phys. Rev. Lett. 43, 1649 (1979).
6. D. v. Harrach, P. Glässel, Y. Civelekoglu, R. Männer, and H. J. Specht, Phys. Rev. Lett. 42, 1728 (1978).
7. R. J. Puigh, P. Dyer, R. Vandenbosch, T. D. Thomas, L. Nunnelley, and M. S. Zisman, Phys. Lett. 86B, 24 (1979).
8. L. G. Moretto, Nucl. Phys. A247, 211 (1975).

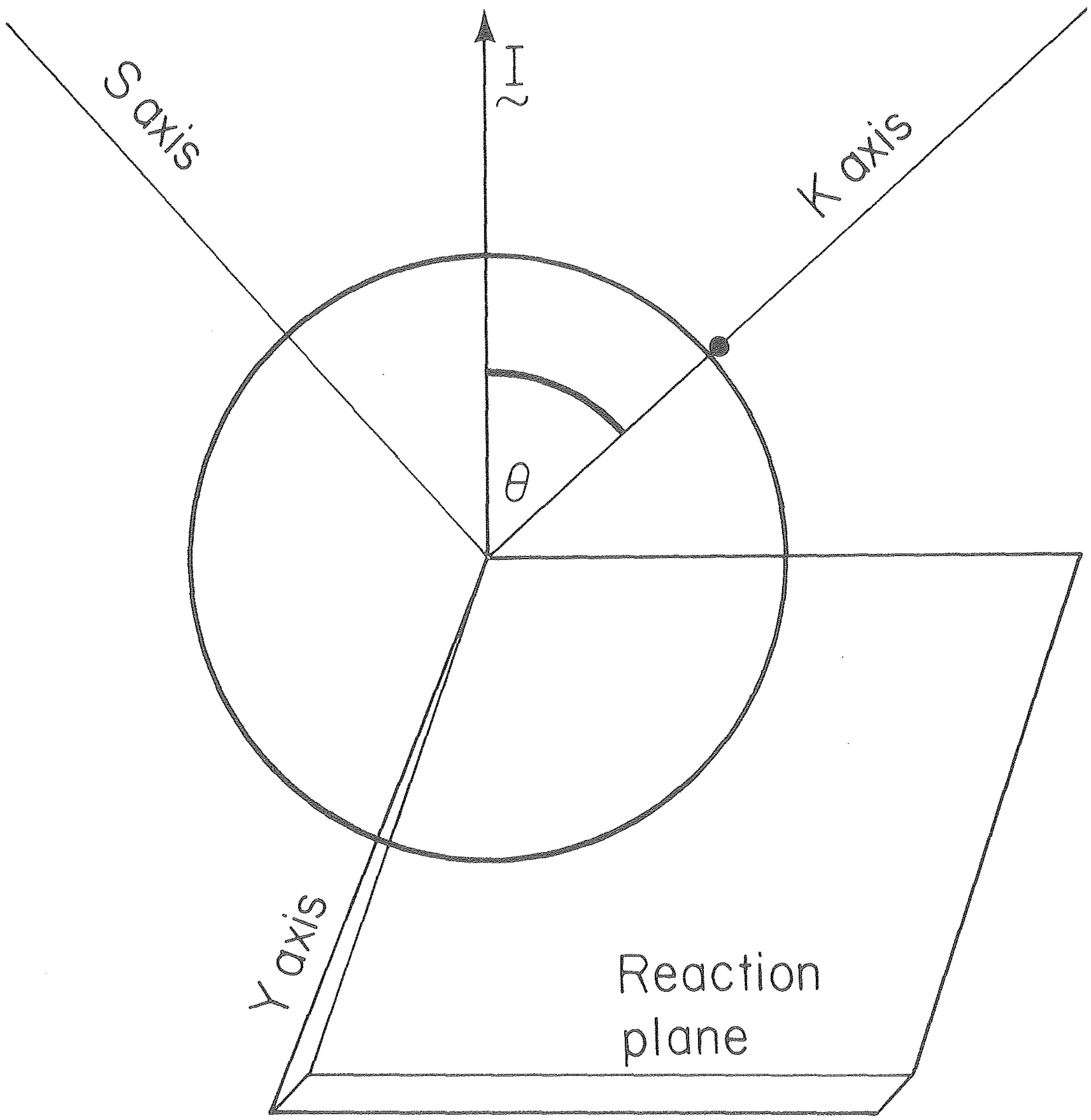
FIGURE CAPTIONS

Fig. 1. Coordinate system used to calculate the first moment of the angular momentum removed by an ejectile. The origin is at the center of mass of the emitter/ejectile complex.

Fig. 2. Functions g(Z) and h(Z) plotted against $Z = 1/\sqrt{2}\, K_0$.

Fig. 3. Coordinate system used to calculate the second moment of the angular momentum removed by an ejectile in the isotropic and focused models. The origin is at the center of the ejectile at the instant before it is evaporated. The ejectile is evaporated in the direction indicated by the arrow in the lower diagram.

Fig. 4. Coordinate system used in discussing non fissioning compound nuclei. The origin is at the center of the compound nucleus. The x' axis is the beam axis. The angular momenta of the non fissioning compound nuclei are distributed isotropically in the y'z' plane.



XBL-807-1536

Fig. 1

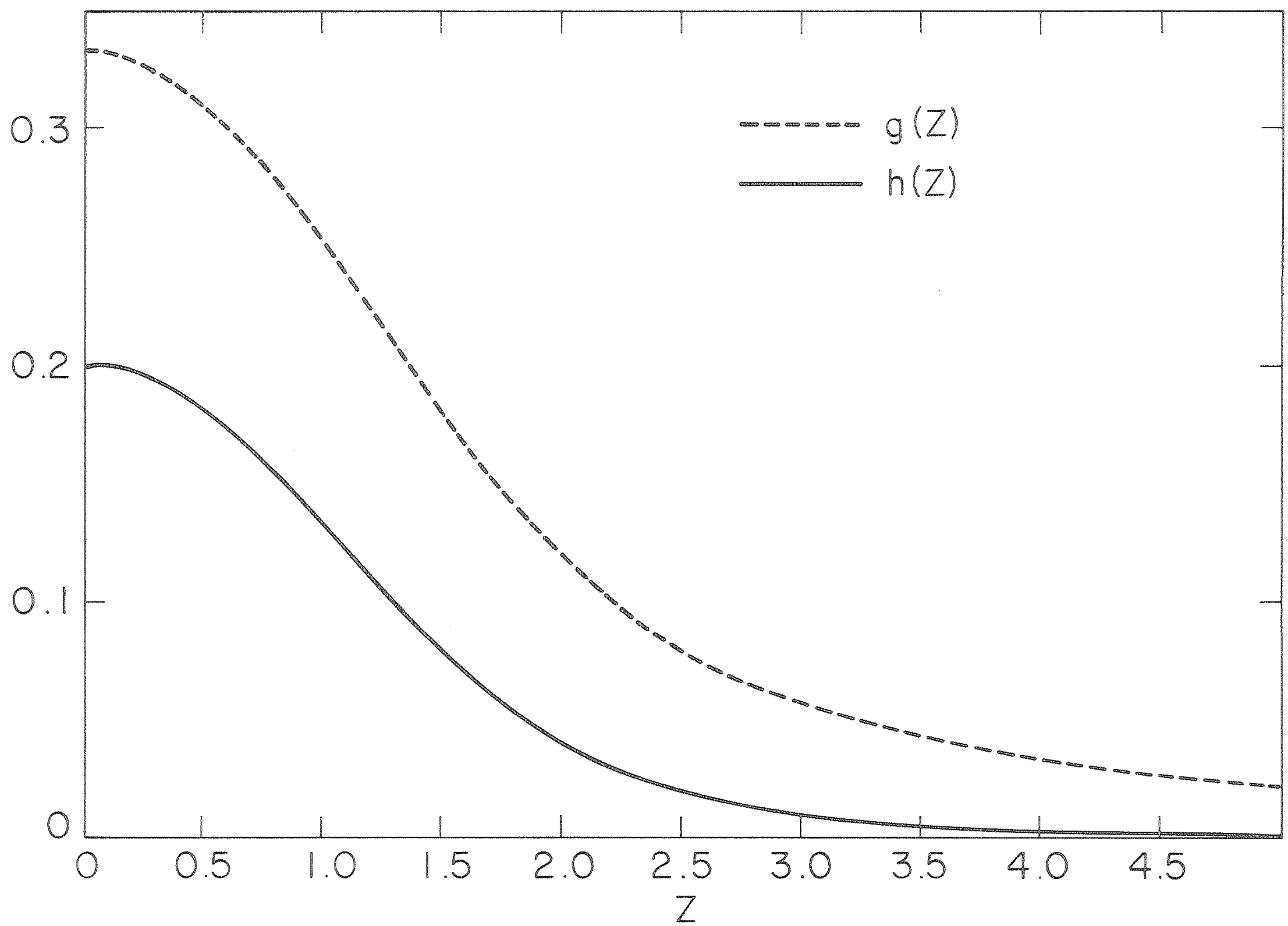


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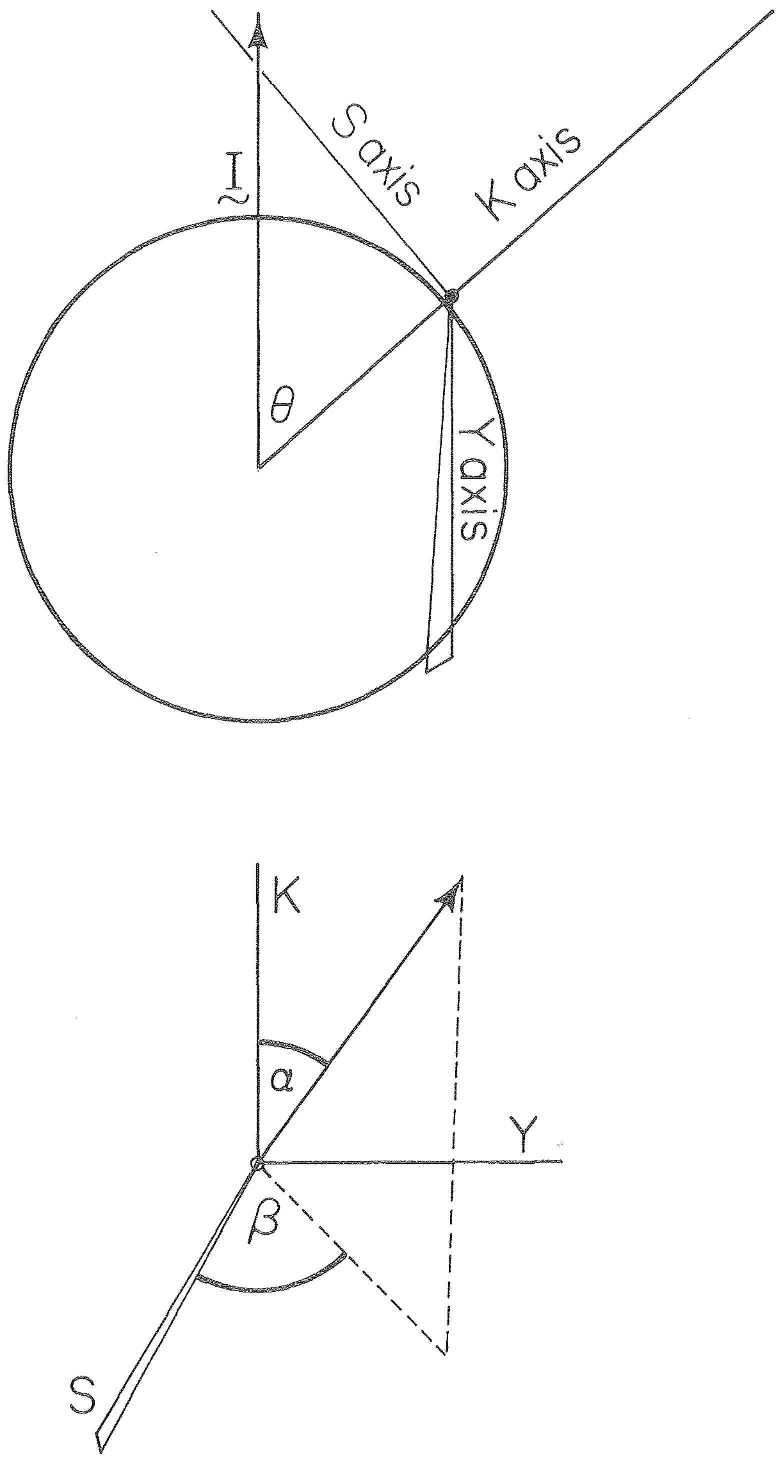
Fig. 2



XBL-807-1537

Fig. 3

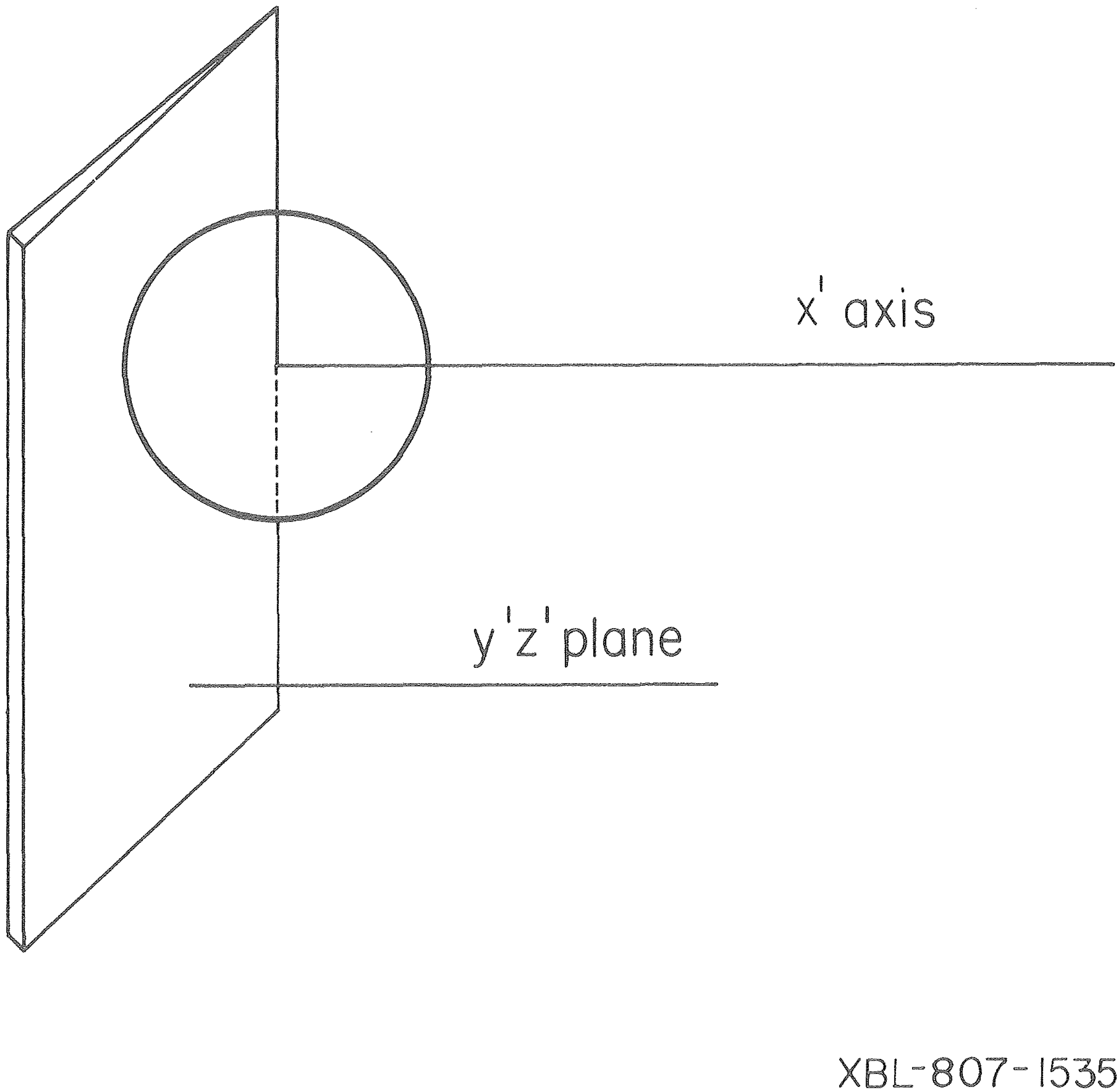


Fig. 4

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