Title

A Stratified Percolation Model for Saturated and Unsaturated Flow through Natural Fractures

Permalink

https://escholarship.org/uc/item/1381g65k

Authors

Pyrak-Nolte, L.J.
Cook, N.G.W.
Myer, L.R.

Publication Date

1990

LBL-28334

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

EARTH SCIENCES DIVISION

To be presented at the International High Level
Radioactive Waste Management Conference,
Las Vegas, NV, April 8–12, 1990, and
to be published in the Proceedings

A Stratified Percolation Model for Saturated and Unsaturated Flow through Natural Fractures

L.J. Pyrak-Nolte, N.G.W. Cook, and L.R. Myer

January 1990

Prepared for the U.S. Department of Energy under Contract Number DE-AC03-76SF00098.

DISCLAIMER

LBL-28334

A Stratified Percolation Model for Saturated and Unsaturated Flow through Natural Fractures

L. J. Pyrak-Nolte,[1] *N. G. W. Cook,*[2, 3] *and L. R. Myer*[2]

[1]Department of Earth and Atmospheric Sciences
Purdue University
West Lafayette, Indiana 47907

[2]Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

[3]Department of Materials Science
and Mineral Engineering
University of California

January 1990

This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, Engineering and Geosciences Division, and by the Director, Office of Civilian Radioactive Waste Management, Office of Facilities Siting and Development, Siting and Facilities Technology Division, of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

A STRATIFIED PERCOLATION MODEL FOR SATURATED AND UNSATURATED FLOW THROUGH NATURAL FRACTURES

L. J. PYRAK-NOLTE
Department of Earth
and Atmospheric Sciences
Purdue University
West Lafayette, IN 47907

N. G. W. COOK
Department of Materials Science
and Mineral Engineering and
Earth Sciences Division
Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

L. R. MYER
Earth Sciences Division
Lawrence Berkeley Laboratory
Berkeley, California 94720

ABSTRACT

The geometry of the asperities of contact between the two surfaces of a fracture and of the adjacent void spaces determines fluid flow through a fracture and the mechanical deformation across a fracture. Heuristically we have developed a stratified continuum percolation model to describe this geometry based on a fractal construction that includes scale invariance and correlation of void apertures. Deformation under stress is analyzed using conservation of rock volume to correct for asperity interpenetration. Single phase flow is analyzed using a critical path along which the principal resistance is a result of laminar flow across the critical neck in this path. Results show that flow decreases with apparent aperture raised to a variable power greater than cubic, as is observed in flow experiments on natural fractures. For two phases, flow of the non-wetting phase is likewise governed by the critical neck along the critical path of largest aperture but flow of the wetting phase is governed by tortuosity. Relative permeability curves show strong phase interference at all stresses and the crossover where the relative permeabilities of the phases are equal occurs at an invariant saturation of about 30 percent of the wetting phase.

INTRODUCTION

To isolate high-level radioactive waste in rocks where containment must last thousands of years, it is necessary first to understand the mechanical and hydraulic properties of a rock mass, both in the short term and the long term. Yucca Mountain is composed of tuffs some of which are porous and others heavily fractured. While saturated and unsaturated flow through porous media is relatively well understood, such flow through fractures is not. Fluid flow through fractures will be affected by changes in the state of the effective stress in a rock mass. Stresses at Yucca Mountain could change as a result of the excavation and emplacement of waste or as a result of tectonic activity, including faulting. It is important for performance assessment to be able to calculate or predict multiphase flow through fractures and the effects of changes in stress, pore pressure, or pore volume on such flow.

Fluid flow through a fracture is fundamentally different than fluid flow through a porous material. In a fracture, fluid flow occurs in a two-dimensional plane and the amount of flow is controlled by the aperture distribution. The complex flowpath geometry of a natural fracture arises because a natural fracture consists of two rough surfaces in partial contact. Between the areas of contact, there exist voids of various geometries and variable aperture. If a fracture is placed under stress, the fracture voids deform, which results in an increase in contact area, a reduction in void aperture, and a reduction in fracture void volume. Fracture geometry controls the flow of fluids through the fracture, the mechanical deformation of the fracture, and the propagation of seismic waves across the fracture.[1] The key to understanding the hydraulic, mechanical, and seismic properties of natural fractures is to quantify the fracture geometry and to understand how fluid flow and deformation properties are coupled through the fracture topology.

The void geometry of a fracture will be influenced by the roughness of the individual fracture surfaces and the correlations between the two surfaces.[2,3] Other investigators have made measurements of fracture contact area[4,5] fracture flowpath geometry,[6,7] and size and distribution of fracture apertures.[8] The basic result of all these studies is that void geometry is variable in size and shape, and the geometry of the voids determines fluid flow through the fracture and fracture deformation.

A standard approach has been to model fluid flow through a fracture as if it were between parallel plates. In this approach, fluid flow through the fracture is proportional to the cube of the aperture of the fracture ("cubic law"). Investigators have made measurements of fluid flow through both induced fractures and natural fractures and found relationships between fluid flow and fracture aperture involving an exponent much greater than cubic.[4,7,9,10] These discrepancies or deviations from cubic law behavior were attributed to variable void cross-section or to surface roughness and flow path tortuosity.[9,11]

MODEL DESCRIPTION

Examples of the flowpath geometry in a natural fracture at three different stress levels are shown in Figure 1. Flowpaths (filled with Wood's metal) are white while contact area is black. At the lowest stress (Fig. 1a) the contact area appears as isolated "islands" of contact. At higher stresses (Figs. 1b and c) these areas of contact become "continents" with "lakes" of metal connected by filamentary tortuous "streams" of metal. These images show that the distribution of voids and contact areas are heterogeneous, but correlated. Thus, a void site has a high probability of being surrounded by other void sites and, conversely, a point of contact has a high probability of being surrounded by other points of contact. The images also suggest that a fractal construction might be appropriate; the fractal dimension of the image in Figure 1c is 1.94.

We have constructed a stratified continuum percolation model to investigate flow in fractures and the changes in flow under varying stress. This approach incorporates the randomness of standard continuum percolation and and a fractal construction that reproduces the type of flow geometry observed in experiments.[12,13,14,15] A continuum model is used because the distribution of void apertures is continuous and there is no underlying lattice structure in a fracture. A fractal construction is used to incorporate the scale invariance and non-integer dimension of flow paths observed experimentally.

To construct the flowpath geometry a pattern is generated by placing N random sites within an area called a tier. Each one of these sites represents the center of a new tier which is smaller in size than the preceding tier by a scale factor b. In each of the new tiers, N sites that define the centers of yet another series of tiers which are smaller than the preceding tiers by the scale factor b are again randomly distributed. This process can continue for as many tiers as desired. The final result is a correlated pattern. The pattern in Figure 2a represents a fracture under low stress because of the small amount of contact

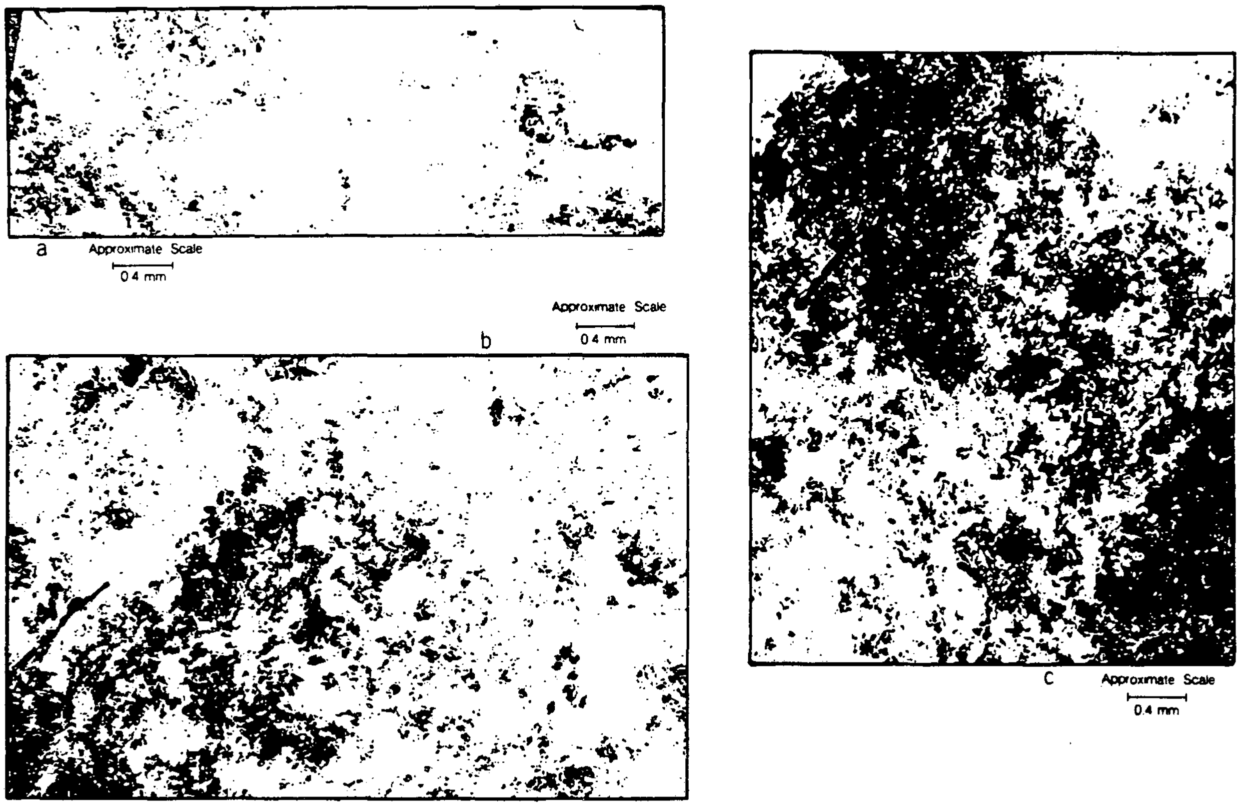

Figure 1. Composite from micrographs of a portion of a natural fracture at effective stresses of (a) 3 MPa; (b) 33 MPa; and (c) 85 MPa. Wood's metal injection technique was used.[6]

area. This pattern was generated using a five tier model with twelve points per tier and a scale factor of 2.37 between tiers.

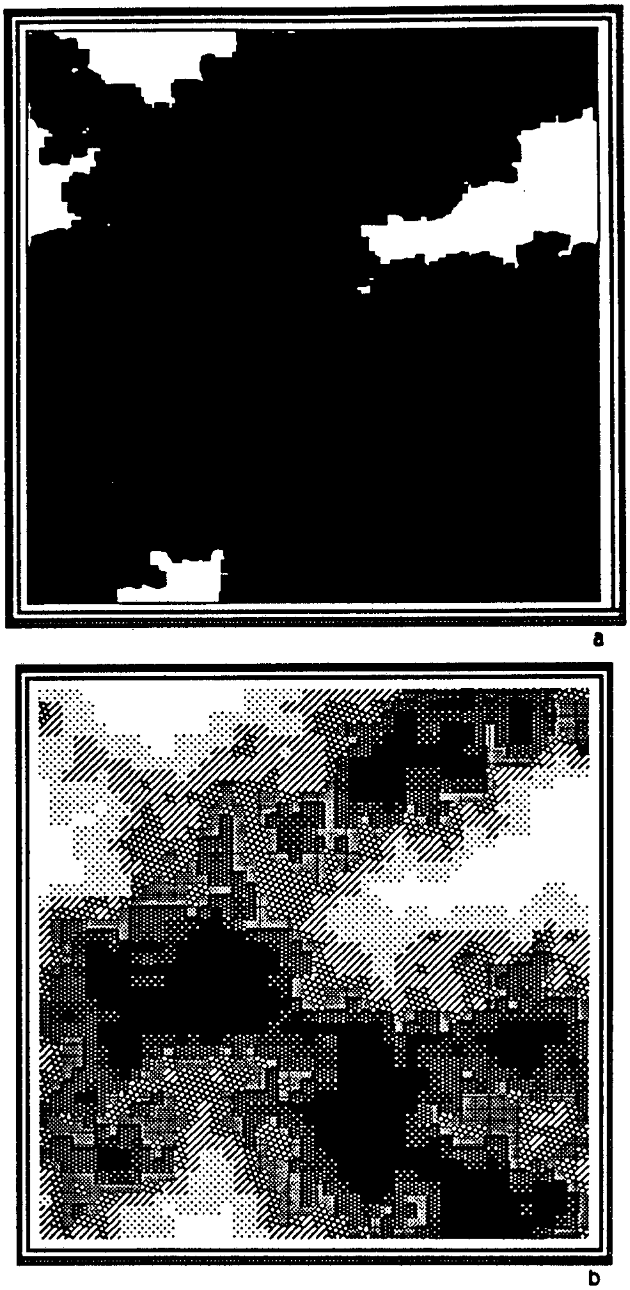

Figure 2. A stratified continuum percolation model of void spaces in a fracture under low stress; (a) Black represents the flow paths and the white regions represent contact area, D = 1.99; (b) Aperture contour plot of pattern shown in (a) White regions represent contact area, and increasing shades of gray to black represent increasing aperture. Scale of contour: 20 units of aperture.

On the final tier, as the points are being plotted to construct the pattern, points will overlap producing a stratified pattern. The amount of overlap that occurs at each pixel of the pattern is counted and equated to fracture aperture (arbitrary units). The aperture distribution of a generated pattern is related to the density of sites in the construction of the pattern. Figure 2b is a map of the aperture distribution of the pattern shown in Figure 2a. White areas in Figure 2b represent contact area and black areas represent sites of largest aperture. It is observed that the aperture distribution is also correlated; that is, sites of large apertures have a high probability of being surrounded by other sites of large apertures. In this paper we analyze fluid flow and mechanical deformation of the fracture flowpath geometry in Figure 2. This flow path geometry resembles that in Figure 1C and has a fractal dimension of 1.99 at zero stress.

SATURATED FLUID FLOW AND MECHANICAL DEFORMATION

Previously, measurements of mechanical displacement and fluid flow through three different natural fractures in samples of quartz monzonite were carried out.[6] The flow through these fractures was divided into two parts; a system-independent irreducible flow and the remaining aperture-dependent flow. Figure 3 shows the aperture-dependent portion of fluid flow and changes in apparent aperture as a result of increasing effective stress ($d_{max} - d$) on a log-log plot. If fluid flow through the fractures were behaving as fluid flow between parallel plates (cubic law), the data would lie along a line with a slope of one third. Instead, the fluid flow data deviate from cubic law behavior and show a dependence on apparent mechanical fracture aperture much greater than cubic, namely, 7.6, 8.3 and 9.8.

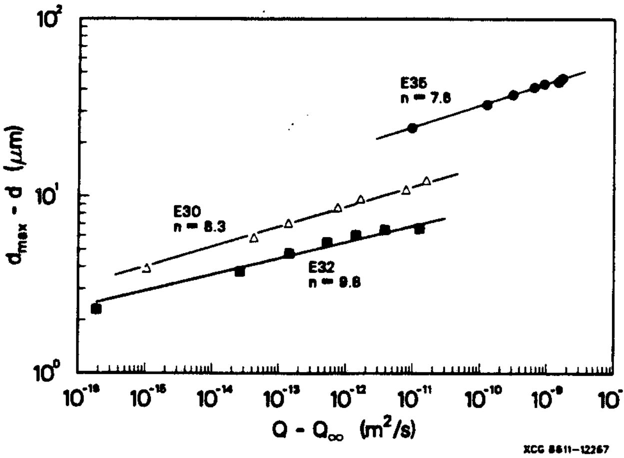

Figure 3. Fracture displacement versus flow per unit head drop after subtraction of irreducible flow for specimens E35, E30 and E32.[6]

To understand the deviations from cubic law behavior of the fluid flow, we analyzed both fluid flow and apparent aperture of the flow path geometry generated from the stratified continuum percolation model. Fluid flow through the generated patterns was analyzed using the results of percolation theory for standard random continuum percolation. Stratified continuum percolation and standard random continuum percolation are in the same universality class.[15] Thus, basic assumptions and equations from standard random continuum percolation can be applied to stratified continuum percolation. From percolation theory, flow through a pattern occurs mainly along a critical path and it is the critical neck along this critical path which dominates the flow. The critical path through the pattern is the connected path of highest apertures that extends from one side of the pattern to the other. The critical neck is the smallest aperture along the critical path. To calculate fluid flow through the simulated fracture flowpath, we assume laminar flow in the critical neck; that is, flow is proportional to the cube of the aperture.

In addition to determining the aperture of the critical neck, the apparent mechanical aperture of the pattern must be determined. In the laboratory, the apparent mechanical aperture of a fracture is determined from measurements of far-field displacement across the fracture. The far-field fracture displacement includes both the reduction in void aperture and the deformation of the asperities (points of contacts) that surround the voids. To account for the mechanical deformation of the points of contacts, we used a zeroth order approach by assuming conservation of rock volume during fracture deformation.[12] If the void space is reduced one unit of aperture, the material in the asperities does not simply vanish or interpenetrate into the opposite fracture surface. To account for this material, rock volume is conserved by redistributing the deformed asperity material across the whole cross-section of the fracture. Thus, the far-field displacement is not equal to aperture closure but instead to some fraction of the aperture reduction, depending upon the relative area of asperities.[12] The difference between the initial aperture and mechanical displacement is often taken to represent the apparent mechanical fracture aperture in laboratory experiments.

Using the fracture geometry represented by Figures 2a and b as a basis, the far-field mechanical displacement was calculated assuming conservation of rock volume during fracture deformation. Figure 4 is a graph of the apparent aperture and the void aperture closure. If conservation of volume were not applied, the apparent aperture and the void aperture closure would be equal yielding the linear relationship shown in the figure. However, by assuming conservation of volume, a nonlinear relationship exists between apparent aperture and void aperture closure. The amount of void closure is greater than the amount of mechanical displacement. This results in the critical neck, being closed faster than the apparent aperture.

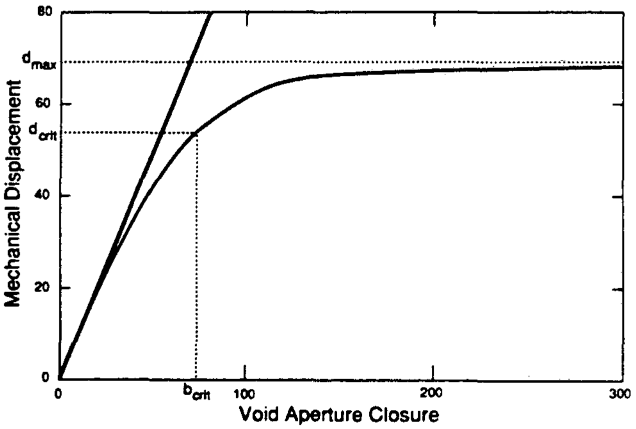

Figure 4. Apparent fracture aperture versus closure of void space for pattern shown in Figure 2.

The non-linear relationship between the mechanical far-field displacement and the closure of void aperture is important to understanding the relationship between fluid flow through a fracture and changes in apparent fracture aperture (Fig. 5). If fluid flow through a fracture depended on the cube of the difference between the initial aperture, (d_{max}) and mechanical displacement (i.e., apparent fracture aperture), then the data would plot along the line labeled cubic in Figure 5. However, if flow per unit head is calculated based on the cube of the aperture of the critical neck but still as a function of apparent aperture, deviations from cubic law will arise. The deviation from cubic behavior for fluid flow through a fracture is a result of the nonlinear relationship between the apparent mechanical aperture and the void aperture. A comparison between Figures 3 and 5 shows that the nearly linear part of Figure 5 at small flows has a high negative exponent as does the curve for the experimental data in Figure 3.

UNSATURATED FLUID FLOW

We have also analyzed unsaturated flow in a fracture, again using stratified continuum percolation. Unsaturated flow occurs when there are two or more phases present in the fracture. Relative permeability of fluids in a fracture differs from relative permeability of fluids in a porous medium in that the flow paths through a fracture lie in two dimensions. We investigated the relative permeabilities of immiscible fluids in a simulated fracture geometry and considered the effect of stress on relative permeabilities as do Pruess and Tsang.[16] This investigation does not deal with invasion percolation or trapping

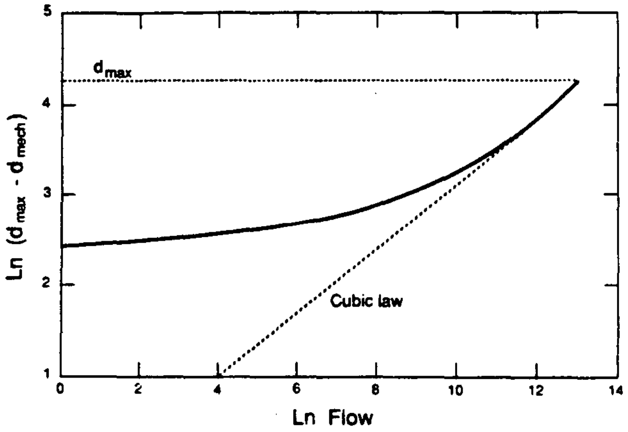

Figure 5. Apparent mechanical aperture $d_{max} - d_{mech}$ versus flow for pattern shown in Figure 2.

percolation. We allow the non-wetting phase to occupy the large voids and assume steady-state conditions when we calculate relative permeabilities. The results from this unsaturated flow investigation are based on ten simulations of fracture flowpath geometry, all of which use a five tier model with twelve points per tier and a scale factor of 2.37. A representative pattern is shown in Figure 2.

Analysis of multiphase flow through a simulated fracture begins by saturating the void space with the wetting phase. The non-wetting phase is introduced into the largest apertures and then allowed to occupy progressively smaller apertures. The phases are assumed to have the same density and viscosity, but different surface tensions. As the non-wetting phase is allowed into smaller apertures, it eventually forms a connected path when it occupies the critical neck along the connected path of highest apertures, and begins to flow (Fig. 6a). Both the wetting phase and the non-wetting phase then flow through the fracture. If the non-wetting phase is allowed into even smaller apertures it will eventually cut off the percolating path of the wetting phase which then ceases to flow (Fig. 6b) and only the non-wetting phase percolates through the fracture.

The first critical neck for the wetting phase, which is also the minimum aperture along the connected path of highest apertures, is operable until it is occupied by this phase which then begins to flow. When the non-wetting phase begins to flow, this phase will always flow along the critical path because the non-wetting phase has been introduced into the largest apertures. The final critical neck for the wetting phase is the critical connection that maintains a percolating path for the wetting phase. It operates until filled with non-wetting phase, causing the wetting phase to cease to flow.

In order to determine the relative permeabilities of the two phases in a fracture, relative flow of each phase was evaluated. To calculate fluid flow through the model, a zeroth order approach is taken which includes only the simplest dependences, which are: (1) the cubic law describes the local dependence of flow on aperture; and (2) the two-dimensional critical behavior is included by a

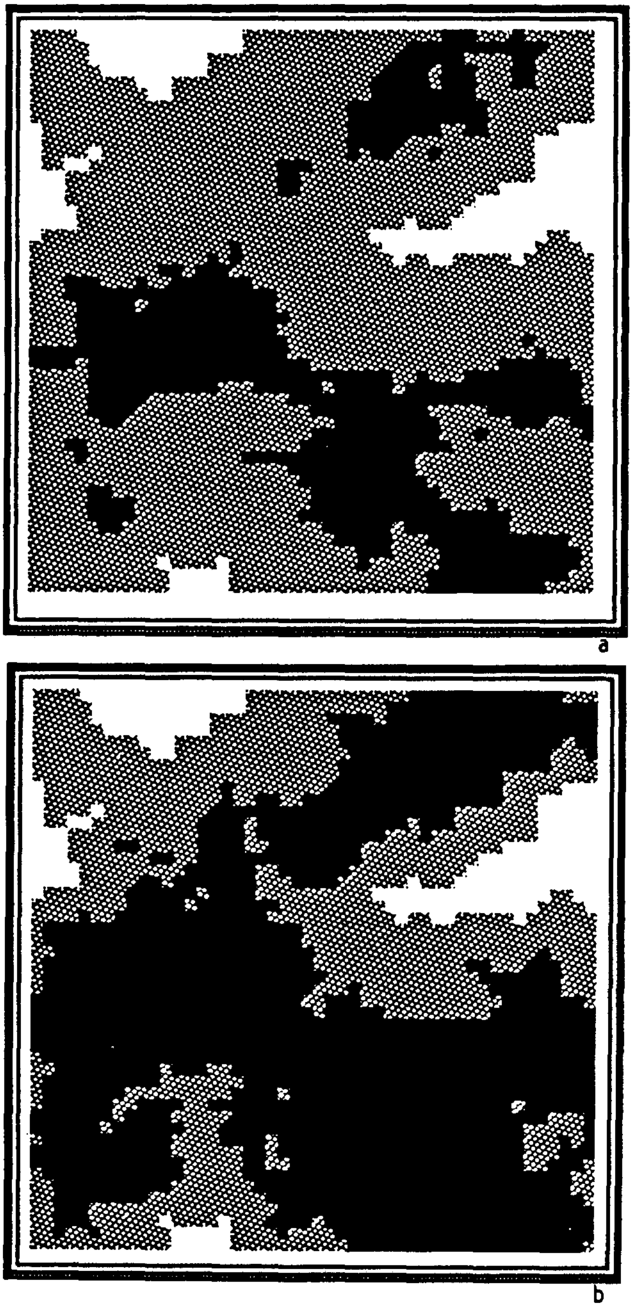

Figure 6a,b. Depiction of area of void spaces occupied by wetting phase (gray), non-wetting phase (black), and contact area (white) for non-wetting phase in apertures larger than (a) 102 units of aperture, and (b) 56 units of aperture.

scaling law that describes changing tortuosity. Laminar, parallel plates (cubic law) flow is assumed for fluid flow of both phases through the critical neck. Tortuosity is important only for calculating the wetting phase permeability because, as the non-wetting phase is introduced, the wetting phase is robbed of its high aperture path or critical path and its path becomes more tortuous as it retreats.

The expression for wetting phase flow through the pattern is

$$Q_w \propto \{b_{wc1}^3 \, [a_w - a_{wc1}]^{1.9}\} + \{b_{wc2}^3 \, [a_w - a_{wc2}]^{1.9}\} \, , \quad (1)$$

where

w - wetting phase
c - critical
Q - flow
a - area normalized with respect to the whole area of the fracture
b - aperture of critical neck
1 or 2 - first or final critical neck

Tortuosity is incorporated into the expression for relative permeability of the wetting phase (Eq. (1)) through a scaling term, $(a_w - a_{wc1})^t$, where a is the normalized area occupied by the wetting fluid, a_c is the normalized critical area of the wetting phase at the percolation threshold, and the exponent t is a critical exponent.[17] The critical exponent, t, can range between 1.7 and 2.7 for the standard random continuum percolation. We have assumed a value of 1.9 for the critical exponent. This equation applies mainly in the vicinity of the percolation threshold.

The non-wetting phase flow, in contrast, is always dominated by the main critical path because it occupies the largest apertures of the pattern. Therefore, the tortuosity of the non-wetting phase does not change with increasing non-wetting phase saturation. However, the width of the non-wetting phase flow path does change with increased non-wetting phase saturation and must be accounted for in the relative flow expression. The expression for non-wetting phase flow (nw) is

$$Q_{nw} \propto b_{nwc1}^3 \, [a_{nw} - a_{nwc1}] \, . \quad (2)$$

The expression for the non-wetting phase is simply a cubic dependence on aperture and a linear dependence on change in area occupied by the non-wetting phase.

Assuming that the densities and the viscosities of the two phases are the same, relative permeabilities for the two phases were calculated and plotted as a function of wetting phase saturation is shown in Figure 7. A rapid decrease in non-wetting phase permeability is observed with an increase in wetting phase saturation. For low saturations of wetting phase (around forty-five percent), the non-wetting phase ceases to flow. The cross-over in permeabilities, where the permeability of both phases is equal, occurs at a wetting phase saturation of around 32 percent.

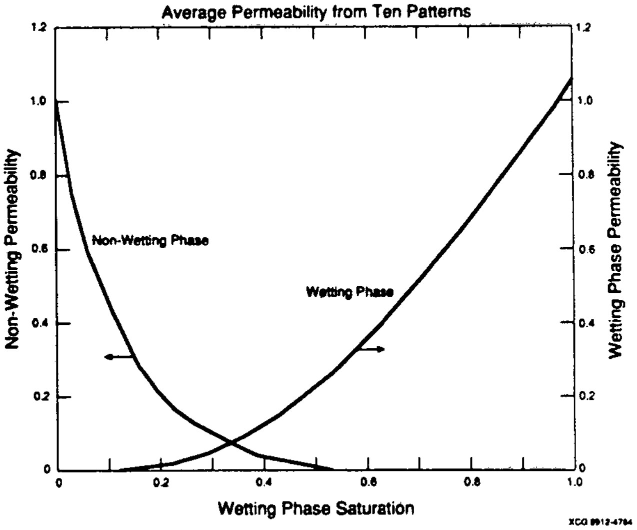

Figure 7. Relative permeabilities of the non-wetting phase and wetting phase as a function of wetting phase saturation.

The effects of stress on relative permeabilities of fluids in a fracture was evaluated by applying stress to the generated patterns of fracture geometry and analyzing the relative flows. Application of stress is viewed as a reduction of all the apertures, an increase in the contact area, and a reduction in the void volume. We investigated the effect of stress on multiphase flow for three stresses which are referred to by the amount of aperture closure (5, 20, 50). A closure of 50 units of aperture is about a third of the largest apertures in the pattern. The overall effect of stress is to reduce the relative permeabilities (Fig. 8a). To illustrate the effects of stress, the relative permeabilities under stress have not been normalized to unity but are shown relative to unit normalized permeability at zero stress. The rapid decrease in non-wetting phase flow with wetting phase saturation is still observed. In addition it is seen in Figure 8b that the cross-over point is essentially invariant with respect to stress. For all stresses the cross-over occurs between 26-32 percent wetting phase saturation. For this model, this leads to the conclusion that if the percentage saturation of one of the phases is known, one can determine which phase dominates the flow at any stress.

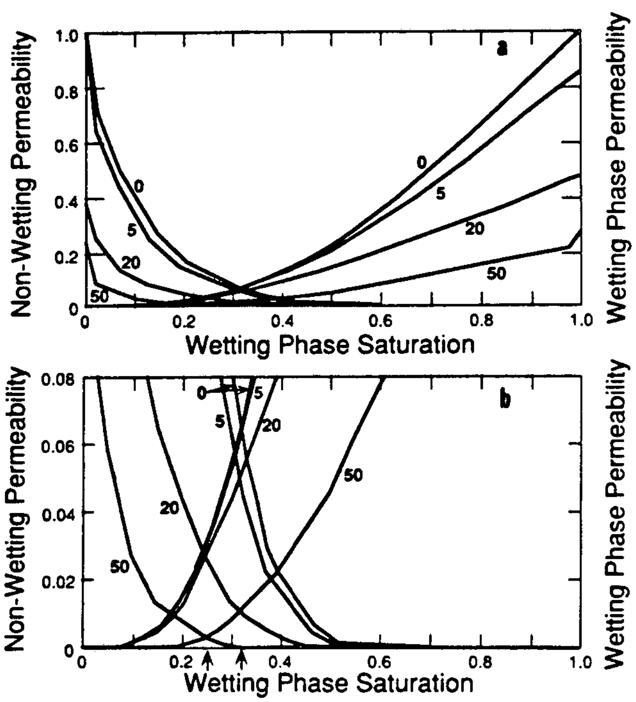

Figure 8a,b. Effect of stress (a) on relative permeabilities for a reduction in aperture of 5, 20 and 50 units of aperture. (b) An enlargement of the cross-over region of (a). The arrows indicate the minimum and maximum value of wetting phase saturation for the cross-overs in relative permeabilities as a function of stress. The cross-overs in relative permeability are essentially invariant of stress.

CONCLUSIONS

Using a model based on a fractal construction, we were able to simulate the experimentally-observed flow path geometry of a fracture. The stratified continuum percolation model was used because the percolation properties are in the same universality class as those of standard random continuum percolation and, hence, this body of knowledge can be used to analyze fluid flow through a fracture. The stratified percolation model yields correlated aperture distributions. Employing percolation results and conservation of volume, we were able to understand deviations from cubic law behavior that have been observed for saturated fluid flow through single natural fractures. Also, we were able to analyze unsaturated fluid flow through fractures using the stratified percolation model.

The relationship between behavior measured on small laboratory samples (less than a meter in dimension) and fluid flow through fractures in the field (with dimensions of perhaps kilometers) is not known. Clearly, this relationship needs to be established if the results of research are to have useful applications. Many phenomena in geology have been shown to possess a large degree of scale invariance over a range of dimensions such as those of concern here. Fractals have been used to describe these invariant properties. The scale invariance of flow paths and contact areas in laboratory samples of rock fractures over a scale of tens of millimeters to hundredths of millimeters led us heuristically to propose a stratified percolation model based on a fractal construction to simulate the geometry of the void spaces between fracture surfaces. This construction has been shown to replicate many of the features of real fractures. The attraction of the model is two fold. First, it may capture effectively those essential properties of the void space and contact area that result from the genesis and subsequent metamorphosis of fractures in rock. Second, it offers the prospect, through scale invariance, of transposing the results of a study on a laboratory scale to the field scale.

Changes in fracture aperture as a result of changes in effective stress have been shown experimentally to have a profound effect on fluid flow. Clearly, this derives from changes in void space geometry brought about by deformation of the rock in the vicinity of the voids. In principle, for given mechanical properties of the rock, it is possible to calculate the changes in void geometry as a function of effective stress. In practice, such a calculation would be computationally intensive and would require an unreasonably detailed knowledge of the void geometry. Again, the question arises whether or not a more simple model might capture the essential features of the phenomena involved. The essential process involved in the deformation is that the complex stress redistribution adjacent to the voids as a result of this deformation does not involve any changes in the net volume of the rock in this region. Therefore, we chose to model changes in void aperture so that the volume of rock would be conserved. The volumetric change in void space is distributed across the whole projected area of the fracture to relate it to changes in measured fracture aperture. The final abstraction involves the use of a critical path and a critical neck to analyze the resistance to single phase flow or the resistance to the flow of a non-wetting phase, and scaling to estimate the effects of tortuosity on the flow of the wetting phase for two phase flow.

Numerical simulations based on these essential abstractions that is, the fractal construction of the void spaces, conservation of volume of rock and the dominance of the critical neck yield results very similar to those observed in laboratory experiments. In particular, the relationship between changes in measured (mechanical aperture) and void closure are shown to be non-linear, leading to much higher exponents between measured aperture and flow than given by the cubic law. We have used a similar approach to derive the relative permeabilities of fractures for two-phase flow but as yet have no laboratory data against which to compare these results.

Though the simple approach described above has been effective in simulating the behavior of some natural fractures in laboratory experiments, it remains to be established whether or not it is sufficiently effective and robust to describe a wider range of laboratory experiments and to form the basis for extending this work to the field. Clearly, the latter requires some measurements on a field scale of the geometry of the void spaces and contact areas between fracture surfaces, fluid flow through such fractures, and the effects of stress or pore pressure on flow. Hopefully, it will ultimately be possible (if not practical) to characterize fractures in the field in such a way as to enable the properties of these fractures to be analyzed with as simple a model as described.

ACKNOWLEDGMENTS

This work was supported by the DOE Assistant Secretary for Energy Research, Office of Basic Energy Sciences and by the Director, Office of Civilian Radioactive Waste Management, Office of Facilities Siting and Development, Siting and Facilities Technology Division of the U.S. Department of Energy Contract No. DE-AC03-76SF00098.

REFERENCES

1. L. R. MYER, L. J. PYRAK-NOLTE, D. L. HOPKINS and N. G. W. COOK, "Seismic Characterization of Fracture Properties," First Annual International High-Level Radioactive Waste Management Conference, American Nuclear Society (this volume) (1990).

2. G. SWAN, "Determination of Stiffness and Other Joint Properties from Roughness Measurements," *Rock Mechanics and Rock Engineering,* 16, p. 19-38 (1983).

3. S. R. BROWN, R. L. KRANZ and B. P. BONNER, "Correlation between the Surfaces of Natural Rock Joints," *Geophysical Research Letters,* 13, p. 1430-1434 (1986).

4. K. IWAI, "Fundamentals of Fluid Flow through a Single Fracture," Ph.D. thesis, University of California, Berkeley, California, 280 p. (1976).

5. S. C. BANDIS, A. C. LUMSDEN, and N. R. BARTON, "Fundamentals of Rock Joint Deformation," *International Journal of Rock Mechanics and Mining Science and Geomechanical Abstracts,* 20(6), p. 249-268 (1983).

6. L. J. PYRAK-NOLTE, L. R. MYER, N. G. W. COOK and P. A. WITHERSPOON, "Hydraulic and Mechanical Properties of Natural Fractures in Low Permeability Rock," in *Proceedings, International Society for Rock Mechanics, 1,* The International Congress on Rock Mechanics, Montreal, Canada, p. 225-231, August 1987.

7. S. GENTIER, D. BILLAUX AND L. VAN VLIET, "Laboratory Testing of the Voids of a Fracture," to appear in International Journal of Rock Mechanics and Mining Science and Geomechanical Abstracts, (1989).

8. E. HAKAMI, "Water Flow in Single Rock Joints," Ph.D. thesis, Lulea University of Technology, Sweden, p. 99 (1988).

9. T. ENGELDER and C. H. SCHOLZ, "Fluid Flow Along Very Smooth Joints at Effective Pressure up to 200 Megapascals," in Mechanical Behavior of Crustal Rocks, The Handin Volume, American Geophysical Union Monograph 24, p. 147-152 (1981).

10. K. G. Raven and J. E. Gale, "Water flow in a Natural Rock Fracture as a Function of Stress and Sample Size," Abstract, *International J. Rock Mech. Min. Sci. Geomech.,* 22(4), pp. 251-261 (1985).

11. Y. W. Tsang, "The Effect of Tortuosity on Fluid Flow through a Single Fracture," *Water Resources Research,* 20(9), p. 1209-1215 (1984).

12. L. J. Pyrak-Nolte, N. G. W. Cook and D. D. Nolte, "Fluid Percolation through Single Fractures," *Geophysical Research Letters,* 15(11), p. 1247-1250. (1988).

13. D. D. NOLTE, L. J. PYRAK-NOLTE and N. G. W. COOK, "Fractal Flow Paths in Rock and the Approach to Percolation," Abstract, *EOS, American Geophysical Union Transactions,* 67(44), p. 871. (1986).

14. D. D. NOLTE, L. J. PYRAK-NOLTE and N. G. W. COOK, "The Fractal Geometry of Flow Paths in Natural Fractures in Rock and the Approach to Percolation," *Pure and Applied Geophysics,* 131(1/2), p. 111 (1989).

15. D. D. NOLTE, "Invariant Fixed Point in Stratified Continuum Percolation," *Rapid Communications in Physical Review A,* 40, p. 4817 (1989).

16. K. PRUESS and Y. W. TSANG, "On Two-Phase Relative Permeability and Capillary Pressure of Rough-Walled Rock Fractures," accepted for publication, *Water Resources Research,* (1990).

17. B. I. HALPERIN , S. FENG and P. N. SEN, "Differences between Lattice and Continuum Percolation Transport Exponents," *Phys. Rev. Lett.* 54, pp. 2391-2394 (1985).