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CALCULATION OF PARITY NONCONSERVING EFFECTS
IN FORBIDDEN M1 TRANSITIONS IN CESIUM*

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ABSTRACT

Calculations are presented of the E1 amplitude expected in forbidden M1 transitions of Cs if parity conservation is violated in the neutral weak e-N interaction, as proposed in a number of gauge models, including that of Weinberg and Salam. Valence electron wave functions are generated as numerical solutions of the Dirac equation in a Tietz central potential, and are used to calculate excited state lifetimes, hfs splittings and Stark E1 transition amplitudes. These are compared with experiment and are in good agreement. Contributions to the $6^2S_{1/2}$ g-factor anomaly and to the forbidden $6^2S_{1/2} - 7^2S_{1/2}$ and $6^2S_{1/2} - 8^2S_{1/2}$ transitions from relativistic effects, Breit interaction, interconfiguration interaction and hfs mixing are calculated, and it is found that this theoretical description is not entirely adequate. The parity-nonconserving E1 amplitude $\mathcal{E}_{PN}$ for the $6^2S_{1/2} - 7^2S_{1/2}$ and $6^2S_{1/2} - 8^2S_{1/2}$ transitions is evaluated. The results

$$\mathcal{E}_{PN}(6S - 7S) = i \, 3.50 \times 10^{-11} \, Q_W |\mu_B|$$

$$\mathcal{E}_{PN}(6S - 8S) = i \, 1.48 \times 10^{-11} \, Q_W |\mu_B|$$

are obtained. With a measured value of the M1 amplitude $\mathcal{M}_{\text{expt}}$ and the Weinberg value $Q_W = -99.$, we find a circular dichroism

$$\delta = 1.64 \times 10^{-4}$$

for the $6^2S_{1/2} - 7^2S_{1/2}$ transition.

1. INTRODUCTION

Existence of a neutral, weak, parity-nonconserving electron-nucleon interaction implies that forbidden M1 transitions in heavy atoms, e.g., $6^2P_{1/2} - 7^2P_{1/2}$ in thallium (Tl) and $6^2S_{1/2} - 7^2S_{1/2}$, $6^2S_{1/2} - 8^2S_{1/2}$ in cesium (Cs) should exhibit circular dichroism. In a previous paper¹ (hereafter referred to as I) we presented calculations of the atomic properties of Tl relevant to the interpretation of observations of circular dichroism in the thallium transition in terms of the Weinberg-Salam gauge field model. Here we present analogous calculations for the Cs transitions. In both cases experiments are currently underway to detect the parity nonconserving effect.

Our approach is the one electron central field (OECF) approximation. We find numerical solutions to the Dirac equation for the valence electron in a "Tietz" central potential:²

$$V(r) = -\frac{e^2(Z-1)}{r(1+\eta r)^2} - \frac{e^2}{r} \quad (1)$$

where parameter η is chosen to give agreement between the observed and calculated $6^2S_{1/2}$ energies. The wave-functions obtained are used to calculate fine and hyperfine structure splittings, and allowed (E1) transition rates and excited state lifetimes. These are compared with experimental results (see Section 2). The $6^2S_{1/2} - 7^2S_{1/2}$, $6^2S_{1/2} - 8^2S_{1/2}$ M1 amplitudes and corrections to $g_J(6^2S_{1/2})$ are calculated in Section 3 and compared with experiment. Relativistic contributions to the matrix elements, as well as the "Lamb" correction and corrections due to interconfiguration interaction and hyperfine mixing are included. We find that a theoretical formulation limited to these small effects is not entirely adequate. In Section 4 we present calculations of the parity-nonconserving E1 amplitudes

$\mathcal{E}_{\text{PN}}(6^2S_{1/2}-7^2S_{1/2})$, $\mathcal{E}_{\text{PN}}(6^2S_{1/2}-8^2S_{1/2})$ based on the Weinberg-Salam model.³

We find

$$\mathcal{E}_{\text{PN}}(6S-7S) = 3.50i \times 10^{-11} Q_W |\mu_B| \quad (2)$$

and

$$\mathcal{E}_{\text{PN}}(6S-8S) = 1.48i \times 10^{-11} Q_W |\mu_B| \quad (3)$$

Here $|\mu_B| = |e\hbar/2m_e c|$ and $Q_W = (1 - 4\sin^2\theta_W)Z-N$, where θ_W is the "Weinberg" angle. Results (2) and (3) are somewhat smaller than earlier estimates by Bouchiat and Bouchiat⁴ (see Section 4). Finally, in Section 5 we calculate Stark matrix elements for the transitions $6^2S_{1/2}-7^2S_{1/2}$ in an external electric field, and compare our results to earlier calculations by Bouchiat and Bouchiat,⁴ and to the experimental results of Bouchiat and Pottier.⁵

2. CESTUM WAVE FUNCTIONS IN THE ONE-ELECTRON CENTRAL FIELD APPROXIMATION

2.1. Construction of Electronic Wave-Functions

As in I, we solve the Dirac equation for the valence electron in a centrally symmetric potential $V(r)$. The latter approximates the nucleus and 54 core electrons as a fixed charge distribution. With

$$\psi = \begin{pmatrix} \frac{f(r)}{r} & \chi_{\kappa}^{\mu}(\theta, \phi) \\ i \frac{g(r)}{r} & \chi_{-\kappa}^{\mu}(\theta, \phi) \end{pmatrix}, \text{ the Dirac equation}$$

reduces to the coupled radial equations:

$$\left. \begin{aligned} \frac{df}{dr} &= -\frac{\kappa}{r} f + [2 - E - V(r)]g \\ \frac{dg}{dr} &= [E + V(r)]f + \frac{\kappa}{r} g \end{aligned} \right\} \quad (4)$$

Our units are $\hbar = m_e = c = 1$, E is the ionization energy, and other notation is defined in I. The parameter η of the potential of Eq. (1) is found to be

$$\eta = 355.12 \lambda^{-1} = 2.5914 a_0^{-1} \quad (5)$$

by requiring agreement between observed and calculated $6^2S_{1/2}$ energies. The wave-functions are calculated by integrating Eqs. (4) stepwise from the nuclear radius $R_0 = .016\lambda$ as described in detail in I. Table 1 presents calculated $S_{1/2}$, $P_{1/2}$, $P_{3/2}$ energies along with the observed values (obtained from the tables of C. E. Moore⁶).

2.2. Hyperfine Splittings

In first order perturbation theory the hyperfine energy is given by:⁷

$$W_F = \frac{8\kappa}{4\kappa^2 - 1} e g_N \mu_N [F(F+1) - I(I+1) - J(J+1)] \int_0^\infty \frac{f(r)g(r)dr}{r^2} \quad (6)$$

For Cs^{133} (the only stable isotope), $I = 7/2$, $g_N = 5.16$,⁸ leading to $F=4,3$ for $J=1/2$ states and $F=5,4,3,2$ for $J=3/2$ states. Hyperfine splittings ΔE are calculated between the highest and lowest F levels. These are related to the usual hfs interaction constants A by $\Delta E_{J=1/2} = 4A_{1/2}$ and $\Delta E_{J=3/2} = 12A_{3/2}$. The results are presented in Table 1, and compared with experimental values. Agreement is reasonably good.

2.3. Allowed E1 Transition Rates

For $P_{1/2} - S_{1/2}$ and $P_{3/2} - S_{1/2}$ E1 transitions the Einstein A coefficient is

$$A = \frac{4}{9} e^2 \omega^3 |\langle P_J | r | S_{1/2} \rangle|^2 \quad (7)$$

In Table 2 we present radial integrals and transition rates for $P_{1/2} - S_{1/2}$,

TABLE 1

State	Ionization energy (calculated) ($m_e c^2 = 1$)	Ionization energy (measured) ⁶	Hyperfine energy splitting (calculated) GHz	Hyperfine energy splitting (observed) GHz
6S _{1/2}	7.62024×10^{-6}	7.62024×10^{-6}	9.212	$9.193 \pm .001^c$
7S _{1/2}	3.1232×10^{-6}	3.1229×10^{-6}	2.346	$2.185 \pm .012^e$
8S _{1/2}	1.7201×10^{-6}	1.7117×10^{-6}	0.935	$0.876 \pm .006^e$
9S _{1/2}	1.0839×10^{-6}	1.0909×10^{-6}	0.468	$0.438 \pm .008^e$
6P _{1/2}	4.9081×10^{-6}	4.9622×10^{-6}	1.642	$1.168 \pm .001^c$
6P _{3/2}	4.7732×10^{-6}	4.7713×10^{-6}	0.723	$0.611 \pm .006^b$
7P _{1/2}	2.3392×10^{-6}	2.3301×10^{-6}	0.498	$0.377 \pm .001^d$
7P _{3/2}	2.2953×10^{-6}	2.2715×10^{-6}	0.224	$0.199 \pm .001^b$
8P _{1/2}	1.3824×10^{-6}	1.3711×10^{-6}	0.220	
8P _{3/2}	1.3624×10^{-6}	1.3450×10^{-6}	0.100	$0.0916 \pm .0002^a$
9P _{1/2}	0.9146×10^{-6}	0.9064×10^{-6}	0.117	0.093^d
9P _{3/2}	0.9037×10^{-6}	0.8924×10^{-6}	0.054	

^aH. Bucka and G. von Oppen, Ann. Phys. 10, 119 (1962).

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^cJ. Abele, M. Baumann, W. Hartmann, Phys. Lett. A 49A, 205 (1974).

^dP. Tsekarris, J. Farley, R. Gupta, Fifth International Conf. on Atomic Physics, Abstract J13, 250 (1976).

^eR. Gupta, W. Happer, L. K. Lam, and S. Svanberg, Phys. Rev. A8, 2792 (1973).

^fD. Feiertag, A. Sahm, and G. Zu Putlitz, Z. Physik 255, 93 (1972).

TABLE 2. A-coefficients in Cs

Transition	$\langle r \rangle_{fi}$ radial integral ($\text{\AA}$)	A-coefficient (10^6 sec^{-1})
$6P_{1/2} - 6S_{1/2}$	-861.4	37.3
$7P_{1/2} - 6S_{1/2}$	- 80.4	2.40
$8P_{1/2} - 6S_{1/2}$	- 30.8	0.582
$9P_{1/2} - 6S_{1/2}$	- 18.0	0.245
$6P_{3/2} - 6S_{1/2}$	-846.8	41.82
$7P_{3/2} - 6S_{1/2}$	-104.0	4.11
$8P_{3/2} - 6S_{1/2}$	- 46.6	1.34
$9P_{3/2} - 6S_{1/2}$	- 28.6	0.623
$6P_{1/2} - 7S_{1/2}$	747.3	8.00
$7P_{1/2} - 7S_{1/2}$	-1777.3	3.83
$8P_{1/2} - 7S_{1/2}$	-181.8	4.39
$9P_{1/2} - 7S_{1/2}$	- 73.9	0.148
$6P_{3/2} - 7S_{1/2}$	830.3	7.80
$7P_{3/2} - 7S_{1/2}$	-1730.0	4.27
$8P_{3/2} - 7S_{1/2}$	-230.3	0.729
$9P_{3/2} - 7S_{1/2}$	-101.9	0.286
$6P_{1/2} - 8S_{1/2}$	184.8	2.79
$6P_{1/2} - 8S_{1/2}$	1605.4	1.54
$8P_{1/2} - 8S_{1/2}$	-3016.4	0.883
$9P_{1/2} - 8S_{1/2}$	-322.8	0.137
$6P_{3/2} - 8S_{1/2}$	186.8	2.50
$7P_{3/2} - 8S_{1/2}$	1750.9	1.47
$8P_{3/2} - 8S_{1/2}$	-2919.4	0.983
$9P_{3/2} - 8S_{1/2}$	-396.6	0.217

$P_{3/2} - S_{1/2}$ transitions. These numerical values are required for computation of $\mathcal{E}_{PN}$ and Stark amplitudes (Sections 4 and 5).

To judge the accuracy of these transition rates, we calculate values of Cs excited state lifetimes. The lifetime of a state $|L_J\rangle$ is given by

$$T_{L_J} = \left(\sum_{L'_J} A_{|L_J\rangle \rightarrow |L'_J\rangle} \right)^{-1}$$

where the sum is over all states $|L'_J\rangle$ with energy less than that of $|L_J\rangle$. Table 3 compares available measurements of Cs lifetimes with our calculated values; agreement is, again, reasonably good. These lifetime calculations include calculated values of $A_{|P_J\rangle \rightarrow |D_{J'}\rangle}$.

3. MAGNETIC DIPOLE TRANSITION RATES

The relativistic contribution to the 6S-7S or 6S-8S M1 transition amplitude is

$$\mathcal{M}_{REL} = e \int g_1 \frac{(kr)}{\omega} (f_i g_f + g_i f_f) dr \quad (8)$$

where $g_1(kr) = \sqrt{\pi/2kr} J_{3/2}(kr)$ is a spherical Bessel function, and k and ω are the wave-number and angular frequency of the absorbed photon, respectively. The formula for $nP_{1/2} - n'P_{1/2}$ M1 transitions (as in thallium) was derived in I and is identical to Eq. (8) except for sign. We use our OECF radial wave-functions to compute the numerical results

$$\mathcal{M}_{REL}(6S-7S) = 9.05 \times 10^{-6} |\mu_B| \quad (9)$$

$$\mathcal{M}_{REL}(6S-8S) = 5.68 \times 10^{-6} |\mu_B| \quad (10)$$

These results and additional corrections are summarized in Table 4. The "Lamb correction," discussed in I, arises from the interaction between

TABLE 3. Lifetimes of Cs states

State	Measured lifetime (nsec)	Calculated lifetime (nsec)
$6P_{1/2}$	34.0 ± 0.6^a	26.8
$6P_{3/2}$	29.7 ± 0.2^b	23.9
$7P_{1/2}$	158 ± 5^c	149.0
$7P_{3/2}$	135 ± 1^b	113.0
$8P_{1/2}$	307 ± 14^e	351.0
$8P_{3/2}$	274 ± 12^e	270.0
$8S_{1/2}$	87 ± 9^d	82.0

^aJ. K. Link, J. Opt. Soc. Am. 56, 1195 (1966).
^bS. Svanberg and S. Rydberg, Z. Phys. 227, 216 (1969).
^cD. W. Pace and J. B. Atkinson, Can. J. Phys. 53, 937 (1975).
^dJ. Marek, Physics Lett. A, 60A, 190 (1977).
^eJ. Marek and K. Niemax, J. Phys. B: Atom. Molec. Phys. 9, L483 (1976).

valence electron spin and core electron orbits. For $S_{1/2}$ - $S_{1/2}$ transitions this is given by:

$$\mathcal{M}_L = -\frac{1}{3} e^2 \langle W \rangle \mu_B . \quad (11)$$

Here $\langle W \rangle = \int_0^\infty F(r_1) [\int_{r_1}^\infty \rho(r_2) r_2 dr_2] F'(r_1) r_1^2 dr_1$ where F, F' are the non-relativistic 6S, 7S(8S) radial wave-functions respectively, and $\rho(r_2)$ is a spherically symmetric core electron density, as in I. The "orbit-orbit" correction vanishes for $S_{1/2}$ - $S_{1/2}$ transitions.

The relativistic and "Lamb" contributions to the g-factor anomaly for the $6^2S_{1/2}$ state may be computed in the same way. As previously noted by Perl⁹ and by Phillips,¹⁰ the calculation of relativistic effects leads to a g-factor anomaly which is too small and of the wrong sign when compared with experimental results. It has been suggested by a number of authors that interconfiguration interaction^{10,4,11} might be responsible for the discrepancy. As discussed in I, electrostatic interaction of the outer electron with excited core states does not by itself affect M1 transition amplitudes or the g-factor anomaly since it mixes only those configurations which have the same total angular momentum and spin ($^2S_{1/2}$). However, in second order, spin orbit coupling allows an admixture of different L-S states (such as $^2P_{1/2}, ^4P_{1/2}$ in Cs) which can give rise to finite contributions to M1 transitions or g-anomalies. Our detailed nonrelativistic calculation of this effect is similar to that presented for thallium in I, and differs only slightly from the work of Phillips.¹⁰ The ground configuration of Cs is $1s^2 \dots 5p^6 6s$. For first-order excited configurations, we take $1s^2 \dots 5p^5 6s 6p$ or $1s^2 \dots 5p^5 7s 6p$. The outer s and excited p electrons can form 1P or 3P states which we label by ψ_1^n, ψ_2^n

respectively (where n corresponds to the nS valence electron). Thus the perturbed $6S$, $7S$ states are written

$$|\bar{6S}\rangle = |6S\rangle + \alpha_1 \psi_1^6 + \alpha_2 \psi_2^6 + \beta_1 \psi_1^7 + \beta_2 \psi_2^7 \quad (12)$$

$$|\bar{7S}\rangle = |7S\rangle + \gamma_1 \psi_1^6 + \gamma_2 \psi_2^6 + \delta_1 \psi_1^7 + \delta_2 \psi_2^7 \quad (13)$$

L.S coupling mixes the $^2S(^1P)$ states with $^2P(^1P)$ states, and also mixes $^2S(^3P)$ states with $^2P(^3P)$ and $^4P(^3P)$ states. Thus we obtain in second order:

$$\begin{aligned} |\bar{6S}\rangle = & |6S\rangle + \dots + \alpha_1 A_1(^2\phi_1^6) + \alpha_2 A_2(^2\phi_3^6) \\ & + \alpha_2 A_2'(^4\phi_3^6) + \beta_1 B_1(^2\phi_1^7) + \beta_2 B_2(^2\phi_3^7) + \beta_2 B_2'(^4\phi_3^7) \end{aligned} \quad (14)$$

$$\begin{aligned} |\bar{7S}\rangle = & |7S\rangle + \dots + \gamma_1 C_1(^2\phi_1^6) + \gamma_2 C_2(^2\phi_3^6) \\ & + \gamma_2 C_2'(^4\phi_3^6) + \delta_1 D_1(^2\phi_1^7) + \delta_2 D_2(^2\phi_3^7) + \delta_2 D_2'(^4\phi_3^7) \end{aligned} \quad (15)$$

The m_{ϕ}^n are $m_P(^lP)$ states with s -electron radial quantum number n . The A_i , B_i , C_i , D_i , are determined by the electrostatic interaction between outer electrons. The expressions for this interaction are as presented by Phillips except that we find a result $\sqrt{6}$ times larger from anti-symmetrizing initial and final states. For example,

$$A_1 = \frac{\sqrt{6}}{2} \frac{F_0 + G_1}{\Delta E} \quad (16)$$

where F_0 and G_1 are the direct and exchange electrostatic integrals and ΔE is the perturbation energy denominator.

The second order coefficients α_i , β_i , γ_i , δ_i are determined by fine structure matrix elements of the $5p$ electron state, as computed by Phillips. For example, $\alpha_1 = \xi/\sqrt{2} \Delta E$, where ξ is the spin-orbit parameter of the

5p-hole. Our value of $\xi/\Delta E = 0.07$ calculated with OECF wave functions differs slightly from Phillips' estimate $\xi/\Delta E \cong 0.10$. The coefficients are evaluated numerically using OECF wave-functions and contribute as follows to the 6S-7S M1 amplitude:

$$\begin{aligned} \mathcal{M}_{II}(6S-7S) = & (\alpha_1 A_1 \gamma_1 C_1 + \beta_1 B_1 \delta_1 D_1) \left(\frac{g(^2P) - g(^2S)}{2} \right) \\ & + (\alpha_2 A_2 \gamma_2 C_2 + \alpha_2 A_2' \gamma_2 C_2' + \beta_2 B_2 \delta_2 D_2 + \beta_2 B_2' \delta_2 D_2') \cdot \left(\frac{g(^4P) - g(^2S)}{2} \right) \end{aligned} \quad (17)$$

The results for $\mathcal{M}_{II}(6S-7S)$ and similar corrections for the 6S-8S M1 amplitude and the 6S g-factor anomaly are presented in Table 4. Similar corrections due to the $(5p^5 6p5d)$ configuration have been calculated; however, these are smaller ($\sim 25\%$ of that obtained from Eq. 17)). The overall uncertainty in the interconfiguration interaction correction could be as much as a factor of two or three. However, as can be seen from Table 4, this calculated correction is too small to account for the observed 6S g-factor and 6S-7S M1 amplitude by an order of magnitude. This discrepancy is not reduced much by including contributions of $5p^5 n' pns$ ($n' > 6$) or $5p^5 n' pnd$ ($n' > 6, n > s$) configurations since their contributions diminish rapidly as n, n' increase. Our conclusion, consistent with that of Phillips, is that the observed anomalies are not due to interconfiguration interaction of this type.

An appreciable correction to the M1 amplitude arises from hyperfine mixing. The size of this effect can be derived from I-59, as modified for Cs $^2S_{1/2}$ states. We find

$$\left(\langle n'S, F | H_{HFS} | 6S, F \rangle - \langle 6S, F' | H_{HFS} | nS, F' \rangle \right) \cdot \frac{\langle nS_{1/2}, F' | M1 | nS_{1/2}, F \rangle}{E_{6S} - E_{nS}} \quad (18)$$

The amplitude vanishes for $F = F'$; thus unlike the other amplitudes it only

affects $F=3 \rightarrow F'=4$ or $F=4 \rightarrow F'=3$ transitions. The hyperfine integrals are evaluated numerically, and we employ:

$$\langle nS_{\frac{1}{2}}F' | M1 | nS_{\frac{1}{2}}F \rangle = -\frac{e}{2} \langle F'm'_F | g | F_1 m_F \rangle \quad (19)$$

The numerical results are summarized in Table 4. An observation of the $3 \rightarrow 4$ and $4 \rightarrow 3$ transitions with the same accuracy that Bouchiat and Pottier¹² reported for the $4 \rightarrow 4$ and $3 \rightarrow 3$ components of the $6S$ - $7S$ transitions would clearly reveal the hyperfine correction.

TABLE 4. Summary of contributions to the M1 transition rates.

	$6S_{\frac{1}{2}}$ g-factor anomaly ($g = g_e + \delta_g$)	$6S_{\frac{1}{2}}-7S_{\frac{1}{2}}$	$6S_{\frac{1}{2}}-8S_{\frac{1}{2}}$
Relativistic	$+1.75 \times 10^{-5}$	$+9.05 \times 10^{-6}$	$+5.68 \times 10^{-6}$
Lamb	$+6.2 \times 10^{-6}$	$+2.87 \times 10^{-6}$	$+1.78 \times 10^{-6}$
Interconfigu- ration Inter- action	-8.3×10^{-6}	-7.0×10^{-6}	-5.9×10^{-6}
Hyperfine mixing	--	$8.36 \times 10^{-6} \times$ ($F - F'$)	$4.02 \times 10^{-6} \times$ ($F - F'$)
Observed value	$-1.181 \pm 0.002 \times 10^{-4}{}^a$	$-4.24 \pm 0.34 \times 10^{-5}{}^b$	--

^aP. A. Vanden Bout, et al., Phys. Rev. 165, 88 (1968).

^bM. A. Bouchiat and L. Pottier, Jour. de Phys. Lettres 37, L-79 (1976).

The poor agreement indicates that we do not fully understand the small (10^{-4} to 10^{-5} , up to fourth order) effects contributing to the M1 amplitudes. These do not affect the calculation of $\mathcal{E}_{PN}$ since that calculation depends on large, first-order, amplitudes such as $\langle E1 \rangle_{SP}$ and $\psi_{s,p}(\vec{r} \rightarrow 0)$. The small size of $\mathcal{E}_{PN}$ is determined only by the small size of the Fermi coupling constant G .

4. CALCULATION OF PARITY VIOLATING E1 AMPLITUDE

According to the Weinberg-Salam model, the parity nonconserving electron-nucleus interaction provides the following interaction matrix element (I-64):

$$\langle \psi_1 | H_{PN} | \psi_2 \rangle = - \frac{GQ_W}{2\sqrt{2}} \psi_1^*(\vec{x}) \gamma_5 \psi_2(\vec{x}) \Big|_{(\vec{x}=0)} \quad (20)$$

This mixes S-states with opposite parity p-states, as follows:

$$|\overline{nS_{1/2}}\rangle = |nS_{1/2}\rangle + \sum_{n'} \frac{\langle n'P_{1/2} | H_{PN} | nS_{1/2} \rangle}{E_{nS} - E_{n'P}} |n'P_{1/2}\rangle \quad (21)$$

Thus Eq. (20) can be reduced to:

$$\langle n'P_{1/2} | H_{PN} | nS_{1/2} \rangle = i \frac{GQ_W}{8\sqrt{2}\pi} (f_p g_s - f_s g_p) \Big|_{r=0} \cdot \delta_{m_S m_P} \quad (22)$$

The "r=0" symbol indicates that the expression is averaged over the nuclear volume, and we assumed a constant nucleon density for $r < 0.016\lambda$.

An alternative procedure would be to assume a point-like nucleus and evaluate $\langle H_{PN} \rangle$ at the nuclear radius; this produces a value 2% larger.

An E1 transition amplitude is now possible between the perturbed S-states. Its value is given by

$$\mathcal{E}_{PN} = \langle \overline{nS_{1/2}} | E1 | \overline{6S_{1/2}} \rangle = \sum_{n'} \left\{ \langle nS_{1/2} | E1 | n'P_{1/2} \rangle \frac{\langle n'P_{1/2} | H_{PN} | 6S_{1/2} \rangle}{E_{6S} - E_{n'}} + \frac{\langle nS_{1/2} | H_{PN} | n'P_{1/2} \rangle}{E_{nS} - E_{n'}} \langle n'P_{1/2} | E1 | 6S_{1/2} \rangle \right\} \quad (23)$$

where

$$\langle nS_{1/2} | E1 | n'P_{1/2} \rangle = e \langle nS_{1/2} | \vec{\epsilon} \cdot \vec{r} | n'P_{1/2} \rangle \cong \frac{e}{3} \int f_s r f_p dr \quad (24)$$

and the last expression is derived for the particular case $m_S = m_P = -\frac{1}{2}$, $\vec{\epsilon} = \hat{e}_z$. The numerical results are summarized in Table 5, where Eq. (23)

has been evaluated by two methods:

1. A finite sum over the nearest four intermediate p-states
2. The use of the Dirac Green's function.

The Green's function automatically includes all intermediate states, including continuum and auto-ionizing states as shown in I. The two methods give similar results, as shown in Table 5. The Green's function method is considered more accurate, since it is more complete.

In the Weinberg model, with $\sin^2 \theta_W = 0.30$ as suggested by the experiment of Reines et al.,¹²

$$Q_W = -((4\sin^2 \theta_W - 1)Z + N) = -99 \quad (25)$$

for Cs^{133} . This leads to a value of $\mathcal{E}_{PN} = -i3.47 \times 10^{-9} |\mu_B|$ for the $6S_{1/2} - 7S_{1/2}$ transition. This corresponds to a circular polarization (circular dichroism) of:

$$P = \frac{2\text{Im}(\mathcal{E}_{PN})}{\mathcal{M}_{\text{exp}}} = 1.64 \times 10^{-4} \quad (26)$$

Bouchiat and Bouchiat,⁴ using non-relativistic wave functions with a relativistic correction factor for $\langle H_{PN} \rangle$, a modified Bates-Damgaard method for $e\langle \vec{\epsilon} \cdot \vec{r} \rangle$, and a finite sum over the nearest four P states, obtained a somewhat higher estimate of $-i4.7 \times 10^{-9} |\mu_B|$ for $\mathcal{E}_{PN}$ in this transition, and a similarly higher result for the 6S-8S transition.

Using our analysis of hyperfine structure and excited states decay rates, we can form an estimate of errors. Our hyperfine structure and fine structure calculations indicate that the magnitudes of the P-state wave functions as $r \rightarrow 0$ are ~10% lower than physically accurate. However, decay rate comparisons indicate that our $\langle E1 \rangle$ matrix

TABLE 5A. Calculation of ϵ_{PN} for the $6S_{1/2}-7S_{1/2}$ transition.

Method 1:

Intermediate P-state	$\frac{e}{3} \frac{\langle 7S r nP_{1/2} \rangle \langle nP_{1/2} H_{PN} 6S \rangle}{E_{6S} - E_n}$	$\frac{e}{3} \frac{\langle 7S H_{PN} nP_{1/2} \rangle \langle nP_{1/2} r 6S \rangle}{E_{7S} - E_n}$
$ 6P\rangle$	$-i 7.823 \times 10^{-11} Q_W \mu_B $	$+i 6.912$
$ 7P\rangle$	$+i 5.259$	$-i 0.809$
$ 8P\rangle$	$+i 0.303$	$-i 0.093$
$ 9P\rangle$	$+i 0.084$	$-i 0.031$
Total	$-i 2.18$	$+i 5.98$
	$= i 3.80 \times 10^{-11} Q_W \mu_B $	

Method 2:

$$\begin{aligned}
 & -i 1.75 \qquad \qquad \qquad +i 5.24 \\
 & = i 3.50 \times 10^{-11} Q_W |\mu_B|
 \end{aligned}$$

TABLE 5B. Calculation of ϵ_{PN} for the $6S_{1/2} \rightarrow 8S_{1/2}$ transition.Method 1:

Intermediate P-state	$\frac{e}{3} \frac{\langle 8S r nP_{1/2} \rangle \langle nP_{1/2} H_{PN} 6S \rangle}{E_{6s} - E_{np}}$	$\frac{e}{3} \frac{\langle 8S H_{PN} nP_{1/2} \rangle \langle nP_{1/2} r 6S \rangle}{E_{8s} - E_{np}}$
$ 6P\rangle$	$-i 1.935 \times 10^{-11} Q_W \mu_B $	$+i 2.445$
$ 7P\rangle$	$-i 4.751$	$+i 0.647$
$ 8P\rangle$	$+i 5.027$	$-i 0.303$
$ 9P\rangle$	$+i 0.366$	$-i 0.054$
Total	$-i 1.29$	$+i 2.74$
$= i 1.44 \times 10^{-11} Q_W \mu_B $		

Method 2:

$$\begin{aligned}
 & -i 0.81 \qquad \qquad \qquad +i 2.29 \\
 & = i 1.48 \times 10^{-11} Q_W |\mu_B|
 \end{aligned}$$

elements are too large by $\sim 10\%$. These errors cancel in the evaluation of $\mathcal{E}_{\text{PN}}$ and our $\mathcal{E}_{\text{PN}}$ error should not be greater than $\sim 10\%$.

5. CALCULATION OF THE STARK EFFECT E1 TRANSITIONS

In actual experimental practice (see Bouchiat and Pottier) $\mathcal{M}$ and $\mathcal{E}_{\text{PN}}$ are measured in interference with the E1 transition induced by an external electric field. $\mathcal{M}$ and $\mathcal{E}_{\text{PN}}$ are not directly measured but are compared to the induced Stark effect amplitude, $\mathcal{E}_S$, which is calculated. Therefore, it is important to calculate a reliable value of $\mathcal{E}_S$.

The coordinate system used in the calculation is illustrated in Fig. 2, and is the same used in I. An electric field $E_0 \hat{e}_y$ is perpendicular to the photon propagation vector $\hat{e}_x$. The photon has polarization $\vec{\epsilon} = \cos\theta \hat{e}_y + \sin\theta \hat{e}_z$, and the $^2S_{1/2}$ states are mixed with $P_{1/2}$, $P_{3/2}$ states by Stark effect.

$$|nS_{1/2}\rangle = |nS_{1/2}\rangle + \sum_{n'P_{1/2}} \langle n'P_{1/2} | e\vec{E}_0 \cdot \vec{r} | nS_{1/2} \rangle \frac{|n'P_{1/2}\rangle}{\Delta E_{1/2}} + \sum_{n'P_{3/2}} \langle n'P_{3/2} | e\vec{E}_0 \cdot \vec{r} | nS_{1/2} \rangle \frac{|n'P_{3/2}\rangle}{\Delta E_{3/2}} \quad (27)$$

There is an E1 transition amplitude $\mathcal{E}_S$ between the perturbed states, which we represent as a 2×2 matrix whose rows and columns are labeled by $m_J(6S_{1/2})$ and $m_J(nS_{1/2})$ respectively

$$\begin{aligned} \mathcal{E}_S &= \langle nS_{1/2} | e\vec{\epsilon} \cdot \vec{r} | 6S_{1/2} \rangle \\ &= e^2 E_0 \cdot \begin{array}{c|cc} & \frac{1}{2} & -\frac{1}{2} \\ \hline m_J(7S_{1/2}) = \frac{1}{2} & \alpha \cos\theta & -i\beta \sin\theta \\ -\frac{1}{2} & -i\beta \sin\theta & \alpha \cos\theta \end{array} = m_J(6S_{1/2}) \end{aligned} \quad (28)$$

$$\alpha = \frac{1}{9} \sum_{n'P_{1/2}} R_{nS, n'P_{1/2}} R_{6S, n'P_{1/2}} \left(\frac{1}{E_n - E_{n'P_{1/2}}} + \frac{1}{E_6 - E_{n'P_{1/2}}} \right) \\ + \frac{2}{9} \sum_{n'P_{3/2}} R_{nS, n'P_{3/2}} R_{6S, n'P_{3/2}} \left(\frac{1}{E_6 - E_{n'P_{3/2}}} + \frac{1}{E_n - E_{n'P_{3/2}}} \right) \quad (29)$$

$$\beta = \frac{1}{9} \sum_{n'P_{1/2}} R_{nS, n'P_{1/2}} R_{6S, n'P_{1/2}} \left(\frac{1}{E_6 - E_{n'P_{1/2}}} + \frac{1}{E_n - E_{n'P_{1/2}}} \right) \\ + \frac{1}{9} \sum_{n'P_{3/2}} R_{nS, n'P_{3/2}} R_{6S, n'P_{3/2}} \left(\frac{1}{E_n - E_{n'P_{3/2}}} + \frac{1}{E_6 - E_{n'P_{3/2}}} \right) \quad (30)$$

where

$$R_{6S, n'P_{1/2}} = \langle 6S_{1/2} | r | n'P_{1/2} \rangle, \quad E_6 = E(6S_{1/2}), \quad (31)$$

etc. The quantities α and β have been evaluated by summation over the nearest $P_{1/2}$, $P_{3/2}$ states, and also by use of the Green's function. The results are summarized in Table 6.

Our results can be compared with the calculation of Bouchiat and Bouchiat, which was used in the experimental determination of $\mathcal{M}(6S \rightarrow 7S)$.⁵ Their calculation used the E1 oscillator strengths calculated by Stone and they determined signs by the Bates-Damgaard method and summed over the four lowest energy levels. Their value is $e^2 \alpha = -1.62 \times 10^{-5} \quad |\mu_B| / \frac{V}{\text{cm}}$ and $|\alpha/\beta| \cong 7.0$ for $\mathcal{S}(7S-6S)$. Our value of $|\alpha/\beta|$ is 10.1 and agrees more closely with the experimental result 8.8 ± 0.4 . However, our analysis of excited state lifetimes leads us to suspect that our value $e^2 \alpha = -1.97 \times 10^{-5} \quad |\mu_B| / \frac{V}{\text{cm}}$ is ~10% to 20% too large, so the true

value of e^2_α is probably somewhere between our result and that of Bouchiat and Bouchiat.

In Table 7 we combine the calculations of $\mathfrak{E}_S$, $\mathcal{M}$, and $\mathfrak{E}_{PN}$ in a single 2×2 matrix so that the interference among these amplitudes can be readily extracted. Table 7 gives the $(6S \rightarrow nS)$ transition amplitudes with the photon directed along $\hat{e}_x$ with polarization $\vec{\epsilon} = \vec{e}_y \cos\theta + \vec{e}_z \sin\theta$.

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TABLE 6. Stark effect E1 amplitudes.

A. $\langle \overline{7S} E1 \overline{6S} \rangle$	$e^2 \alpha$ $\left(\frac{ \mu_B }{\text{volts/cm}} \right)$	$e^2 \beta$ $\left(\frac{ \mu_B }{\text{volts - cm}} \right)$	$\left \frac{\alpha}{\beta} \right $
Finite sum method:	-2.043×10^{-5}	-1.78×10^{-6}	11.5
Green's function:	-1.972×10^{-5}	-1.96×10^{-6}	10.06
Experimental value:	--	--	8.8 ± 0.4
B. $\langle \overline{8S} E1 \overline{6S} \rangle$			
Finite sum method:	-3.132×10^{-5}	-3.71×10^{-6}	8.45
Green's function:	-3.166×10^{-5}	-3.97×10^{-6}	7.86

TABLE 7
 $n^2S_{1/2} - 6^2S_{1/2}$ Transition amplitudes.

$$\langle \overline{nS} | E1 + M1 + \epsilon_S | \overline{6S} \rangle =$$

		$+ \frac{1}{2}$	$- \frac{1}{2} = m_z(6^2S_{1/2})$
m_z $(nS_{1/2})$	$= +\frac{1}{2}$	$\alpha' \cos\theta$ $+ \mathcal{M} \cos\theta$ $- \epsilon_{PN} \sin\theta$	$-i \beta' \sin\theta$ $+i \mathcal{M} \sin\theta$ $+i \epsilon_{PN} \cos\theta$
	$= -\frac{1}{2}$	$-i \beta' \sin\theta$ $-i \mathcal{M} \sin\theta$ $-i \epsilon_{PN} \cos\theta$	$\alpha' \cos\theta$ $- \mathcal{M} \cos\theta$ $+ \epsilon_{PN} \sin\theta$

$$\vec{\epsilon} = \hat{e}_y \cos\theta + \hat{e}_z \sin\theta$$

$$\alpha' = e^2 \alpha E_y$$

$$\beta' = z e^2 \beta E_z$$

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FIGURE CAPTIONS

Fig. 1. Low-lying S and P levels of the Cs atom. Fine structure is enlarged and hyperfine structure is not resolved. The forbidden M1 transitions at 5395Å and 4112Å are shown.

Fig. 2. Coordinate system and orientations of electric field $\vec{E}$, photon beam and polarization. Detectors are placed as in the experiment on T1 of Chu et al.¹³

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Figure 1.

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Figure 2.

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