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ABSTRACT

A traffic signal is a substantially different traffic sign compared with othe traffic devices. The uniqueness of traffic signals is manifested in their displaying an alternate message and not a constant one. The transition period from one message to another creates a decision problem for drivers. An inappropriate decision might create a risk of a rear-end collision. This article presents a disaggregate behavioral model for drivers' decision when the green light ends. It is demonstrated, and supported by field data, that a large option zone increases the indecision of drivers. The increase in indecision creates a greater risk of rear-end collisions, as experienced at many intersections. The influence of distance from the intersection and of approach speed on drivers' decision is examined through the model.

1. INTRODUCTION

The traffic signal is a unique type of sign; whereas all other traffic signs display the same directive constantly, a traffic signal changes its instruction periodically. By its nature, the traffic signal should display whether or not one direction of traffic has the right of way at an intersection. Thus, ideally the signal should be a binary device indicating whether a driver approaching the intersection may continue or has to stop. Since drivers cannot instantaneously change their action when the directive of the traffic signal changes, a typical traffic signal has three directives, the third indicating a transition state from go to stop.

There are two reasons that a driver cannot effect instantaneous change. One is the simple physical truth that it takes time to bring a car to a full stop once it is in motion. The second reason is based on human behavior: It takes time for a driver first to perceive the instruction of the traffic signal and then to reach a decision whether to stop or to continue and cross the intersection. This perception and reaction process is quite complex; furthermore, the difficulty it presents is compounded by the dynamic changes occurring at the traffic signal. These dynamic and behavioral elements have serious implications for the proper design of the transition period.

The term "transition period" (or "change interval") means the interval sequence at a signalized intersection that occurs from the moment that the continuous green light ends for one direction and a green light starts for a conflicting approach. This period might consist of various combinations and propertions of yellow and red for the conflicting approaches.

The existing design process for this transition period uses a normative deterministic approach. The approach is nominally based on actual behavior, but it lacks the capability of accounting for the complex behavioral process that actually occurs when a driver approaches an intersection. On a macroscopic level, this behavior is manifested in the conflicts that take place at the intersection approach, resulting in rear-end collisions.

In order to better understand the behavioral process that occurs when a driver approaches an intersection, an appropriate model, capable of representing the phenomena at hand, may be constructed. Such a model has to be stochastic in nature, since it does not seem reasonable that all the complex relationships involved in this situation can be represented with deterministic certainty. The number of variables influencing a driver approaching an intersection might be very large. Traditionally, we can divide those factors into three groups: The first group has to do with the driver, the person's idiosyncratic characteristics, driving ability, and driving history; the second group relates to traffic conditions and traffic mix at the approach to the intersection; the third consists of the geometric characteristics of the intersection as well as the design of the length of the signal phases. It is from this last group that there emerges the intriguing question of how does the length of the warning phase influence driver's behavior. The warning period is a subset of the transition period which lasts from the moment the continuous green stops to the time the red goes on at an intersection approach.

Two main reasons explain why the warning period is of significant importance. First, from a practical point of view, the length of the warning period can be fully determined by the traffic engineer; thus, if length influences behavior, it can easily be set properly. Second, from a theoretical point of view, this period is in a sense superfluous. As mentioned before, the purpose of the traffic signal is to display the right of way for traffic movements. The transition period is, in a sense, forced on the planners because of physical and behavioral reasons. During the transition period, however, drivers are forced to execute a decision that might cause traffic conflicts. Hence, a proper understanding of the behavioral process during this interval may reduce conflicts and thus lessen rear-end accidents at the approach to signalized intersections.

The literature seems to contain a general consensus on the effect of traffic signals on rear-end collisions. Most researchers have concluded that signalizing an intersection significantly increases the number of these collisions. For example, Hakkert and Mahalet (1978) found in a sample of 34 urban intersections that after the introduction of a signal control, the number of rear-end collisions increased from 33 to 77 in three years. Short, et al. (1982), in a similar study of 31 intersections in Milwaukee, found an increase of 37 percent in the number of such accidents. King and Goldblatt (1975) observed the same phenomenon of increased rear-end collisions in their statistical analysis of nationwide accident data from the United States.

In spite of the fact that rear-end collisions at signalized intersections are significantly more frequent than right-angle collisions, the former have not received much attention either in the literature or in practice. The purpose of this paper is to demonstrate through a behavioral model, how the risk of rear-end collisions at signalized intersections can be evaluated theoretically. Moreover, this model was applied under controlled conditions to analyze the influence of the length of the warning interval on the probability of rear-end collisions.

2. RISK-GENERATING PROCESS OF A REAR-END COLLISION

Most rear-end collisions at a signalized intersection occur when two successive drivers approaching the intersection make conflicting decisions at the end of the green light phase. A high risk of a rear-end collision will exist if the first driver decides to stop while the second one wishes to cross the intersection. When the collision actually occurs, it is reasonable to assume that the following driver did not anticipate the stopping decision of the driver in front, and thus could not react in time to prevent the accident. The highest probability of a rear-end collision exists when the probability of two successive drivers' reaching conflicting decisions about whether to cross the intersection or stop it the highest. The probability of a conflicting decision is a function of the distance of the two drivers from the intersection at the appearance of the yellow light. Thus, the probability of conflicting decisions can be derived from a stopping probability function taking into account this distance. Such a function may be constructed from macroscopic observations, without any behavioral basis. It will be shown later that the same probability function can also be derived from a behavioral model of a driver's decision process.

The macroscopic probability function can be defined as follows: Let $P(x)$ be the probability of stopping, and x the distance from the intersection when the green phase ends. The probability of deciding to cross the intersection will then be $1 - P(x)$. Note that this function (Figure 1a) represents a realization of independent Bernulli trials carried out at various distances from the stop line the moment the green light ends. The probability of stopping is high when the distance is relatively far from the intersection and low when the driver is close. Two Bernulli trials are shown schematically in Figure 1a. The limiting envelope of these trials can be represented as a cumulative normal distribution.

The probability of two successive drivers' reaching a conflicting decision about whether to cross or to stop will be highest when the expression $P(x) * [1 - P(x)]$ obtains a maximum value. This happens when $P(x) = 0.5$. Figure 1b illustrates this probability. The zone around the point at which the stopping probability has a value of 0.5 is most difficult for a driver to reach a decision on the proper action to take when the green light ends. The probability becomes lower when this value is greater or smaller as the distance to the stop line increases or decreases.

In practice (see Zegeer, 1977; Parsonson, 1978; Sheffi & Mahmassani, 1981) it is customary to describe the area between the 10^{th} and 90^{th} percentiles of the stopping probability function as an indecision zone. An example of the implementation of the concept of an indecision zone is found in Parsonson (1978), who suggested placing a detector loop in this zone, the purpose of which would be to prevent, in unsaturated cycles, a situation in which a driver is caught in the indecision zone at the beginning of the yellow light.

A necessary condition for the occurrence of a rear-end collision is, of course, the presence of vehicles in the intersection approach when the yellow light appears. The probability of a rear-end collision increases when the number of vehicles in the indecision zone increases. The actual number of rear-end accidents is thus a function of the two following factors:

1. Traffic volume: The larger the volume of vehicles in the approach to the intersection, the higher the probability that vehicles will be located in the indecision zone when the green light ends. There probably exists a critical volume of traffic beyond which drivers do not

operate their vehicles independently from others and driving speeds are extremely low. When traffic reaches this level, the number of rear-end collisions might not increase with volume, or even decrease; however, the number of conflicting decisions might still rise.

2. The range of the indecision zone: The larger the indecision zone, the higher is the probability that vehicles will be located in the zone when the green light ends. The range of the indecision zone depends on the value of the variance of the random process that generates the stopping probability function.

The second point deserves additional clarification. Imagine, for a moment, a situation in which a driver's decisions and actions are instantaneous, as are changes in vehicular motion. In such a world, a traffic signal could be a binary device, as discussed in the introduction. Under these conditions, the stopping probability function would degenerate to a deterministic step function, as shown in Figure 2. The size of the indecision zone would then be equal to zero, and no conflicting decisions would be generated by drivers approaching the intersection. All drivers located to the right of the vertical line of the step function would stop when the green light ended, and all drivers to the left of this line would continue and cross the intersection. In the real world, the size of the indecision zone is a function of the variance of the cumulative probability function, which approximates the set of Bernulli trials discussed above. Figure 2 presents two stopping probability functions that differ in their underlying variances. It is easy to see that the larger this variance, the larger is the indecision zone.

The next section focuses on the deterministic normative method of analyzing the intersection-approach problem and its relation to the probabilistic macroscopic approach presented above.

3. DILEMMA & OPTION ZONES

Many studies (Gaziz et al., 1960; May, 1968; Bissell & Warren, 1981), as well as Transportation & Traffic England (1976), concerned with events occurring on the approach to signalized intersections are based on the normative deterministic behavior pattern of a reasonable driver. This method is thus different from the stochastic macroscopic model presented above. The phenomena occurring on the intersection approach will be analyzed through the use of dilemma and option zones, which are mathematically defined and based on the normative deterministic behavior of drivers and on simple motion equations.

Drivers who are located in the dilemma zone at the end of the green light can neither stop their vehicles before the stop line nor cross the line on the other side before the light turns red. Drivers who are in the option zone when the signal turns yellow can either stop their vehicles at the stop line or cross it before the light turns red. The ability of a driver to cross the stop line or to stop is based on deterministic normative values. It is usually assumed that deceleration takes place at a rate of about 10 feet/second^2 and that when there is an attempt to cross the intersection, the driver will continue at a constant speed or will accelerate at a rate of 5 feet/second^2 (see May, 1968). Figure 3 presents the shape of the dilemma and option zones as a function of approach speed.

The importance of the definition of a dilemma and an option zone lies in a normative ability to analyze and judge various actions taken by drivers at an intersection approach. For example, May (1968) defined a risk-measurement factor based on the events occurring in such zones. It is important to realize, however, that these zones describe, under normative deterministic assumptions, what a driver can and may do in each zone. They do not describe what a driver will actually do, not even in the stochastic sense. Thus, it can be concluded that dilemma and option zones are tools of diagnostics or analysis; they are not, and cannot describe, the actual behavior of drivers.

In many of the studies that were carried out following the work by Gazis, et al. (1960), special emphasis was placed on reducing the size of the dilemma zone. The motivation behind this objective was to lessen the risk of right-angle collisions. A manifestation of this school of thought is the "Proposed Recommended Practice for Determining Vehicle Change Interval" (1985). In these guidelines, the proposed speed approach for determining the length of the yellow light is the 85^{th} percentile of the actual speed distribution or of the posted speed limit. This recommendation indicates that the tendency is to use a relatively high approach speed to reduce the size of the dilemma zone. The rationale is that the chance will then be smaller that a driver who is not able to stop before the stop line when the red signal lights up will cross during the red light. This is, of course, in line with legal attitudes as expressed in traffic laws.

The direct implication of determining the length of the yellow light according to the relatively fast drivers is to create a large option zone for the slower drivers. Such an option zone provides these drivers with a relaxed decision situation, since whether they decide to stop or cross the intersection, they can do so within the legal time frame. Although this situation may be desired by the individual driver, it has serious implications at the system level.

The option zone, by definition, is an area in which either decision, to stop or to cross, is legitimate; thus, one may expect a high proportion of conflicting decisions by the various drivers located in this zone. The high proportion of possible conflicting decisions by itself creates a high potential for rear-end collisions. To demonstrate this contention, imagine that a stop sign is considered by some drivers to be a recommendation to stop and by others to be a recommendation to cross the intersection. This situation, by its very nature, will create conflicts and, thus, rear-end collisions. This hypothetical situation is, of course, analogous to the interpretation of the option zone advanced here.

4. BEHAVIORAL INTERPRETATION OF THE RELATION BETWEEN OPTION & INDECISION ZONES

An increases in the length of the warning period has direct implication on the size of the option zone, as shown in Figure 3. In the previous section, it was stipulated that increasing the option zone would increase the number of conflicting decisions by drivers. The aim of the present analysis is to use field experiments to evaluate the influence of the duration of the warning period on drivers' behavior. Toward this end, a disaggregate (microscopic) behavioral model of drivers behavior approaching a signalized intersection when the green light ends was developed. The model was estimated base on data from experiments carried out at four urban intersections in Tel Aviv.

4.1 The Model

The microscopic disaggregate behavioral model chosen is derived from behavioral theories long used in consumer choice theory by McFadden (1973), and is similar to the model used by Sheffi and Mahamassani (1981). For the sake of clarity, we derive the model based on fundamental behavioral assumption. The model is based on the following two basic assumptions:

1) The driver faced with decisions regarding future actions is a rational human being. This driver will not stop if the intersection line can be crossed before the red appears, on the one hand, and will stop if the line cannot be crossed in time.

2) The driver makes driving decisions based on his perception of the surrounding traffic situation and the directives of the traffic signals. The driver's perception might or might not be subject to error; in any case, we, as observers of drives behavior, always have incomplete information about all the variables influecing these decisions.

These assumptions can be operationalized in a mathematical model: Let T_i be the time perceived by driver i chosen at random to reach the intersection line from the moment the green light ends:

$$T_i = t_i + \xi_i$$

where t_i is an observed or calculated value, and ξ_i is a random disturbance term.

In a real-life experiment, T_i cannot be observed. This relation can be expanded to include many variables that influence the perceived value, T_i. The simplest relation is probably the one stated above, in which t_i is calculated on the basis of constant traveling speed, as we assumed in the present work. Let $T_{cr,i}$ be the time left before the light turns red as perceived by driver i, but the value of which is not known with certainly either to the driver or to us as observers of his behavior, $T_{cr,i}$ can be defined by the following relation:

$$T_{cr,i} = t_{cr,i} + \varepsilon_i$$

where $t_{cr,i}$ is the objectively measured time interval from the moment that the green ends till the instance when the red appears; in other words, $t_{cr,i}$ is the duration of the warning period for driver i, and ε_i is defined to be a random disturbance term.

Had all the information been known with certainty, based on the first behavioral assumption, the driver would have stopped if $T > T_{cr}$ and would have crossed the intersection if $T_{cr} > T$. This deterministic relation defines the step function presented in Figure 2.

Since the driver does not have full deterministic information and since we as observers have even more stochastic noise, the above relation has to be stated in probabilistic terms, as follows:

$$\Pr[\text{stop}] = \Pr[T_i > T_{cr,i}] = \Pr[t_i + \xi_i > t_{cr,i} + \varepsilon_i]$$

$$\Pr[\text{stop}] = \Pr[t_i - t_{cr,i} > \varepsilon_i - \xi_i] \qquad (1)$$

To implement the model, the stochastic properties of the disturbance terms ε_i and ξ_i have to be defined. Both terms are influenced by many intervening variables; for mathematical convenience, we assume that each disturbance term is normally distributed with a zero mean and a specific variance. Thus, their difference can be defined as follows: $\eta = \varepsilon_i - \xi_i$; and is distributed as $\eta \sim N(0, \sigma^2)$. Hence,

$$\Pr[\text{stop}] = \int_{-\infty}^{t_i - t_{cr,i}} \frac{1}{\sqrt{2\Pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{\eta}{\sigma}\right)^2\right] d\eta \; ; \; \sigma > 0 \qquad (2)$$

$$Pr[stop] = \Phi \left(\frac{t_i - t_{cr,i}}{\sigma} \right) \quad (3)$$

It may be observed that Equation (3), which is the final form of the disaggregate stochastic model, defines a normal cumulative curve similar to the curves presented in Figure 2. This curve was previously defined from a macroscopic point of view; here it is derived with behavioral disaggregate assumptions.

Equation (3) defines a simple binary probit model, and its estimation process is a straightforward maximum likelihood procedure. In the present work, a computer program termed CHOMP, developed by Daganzo and Schoenfeld (1978), was used in the estimation process.

4.2 Description of the Experiment

The sample consisted of events recorded at four signalized intersections in Tel Aviv, two sets of observations for each intersection. The basic traffic information for each sample is presented in Table 1. The size of the sample in each set is the number of vehicles that were actually exposed to a stopping or crossing decision at the end of the green light. The samples do not include vehicles forced to stop by vehicles in front. The events were recorded on film with a cine camera twice, once with a short option zone and once with a larger zone. A detailed description of the data is given by Becker (1971). The increase in the option zone was achieved, not by lengthening the yellow light, but by substituting the last 3 seconds of the green light with a flashing green. The flashing green was not new to the Israeli driver, since it has been in use for years at most interurban signalized intersections in Israel and at those urban intersections that have high approach speeds. Thus, it can be assumed, as it was in this study, that the flashing green is perceived by the Israeli driver mostly as an extension of the warning period, indicating the close appearance of the red light.

In spite of the familiarity of the Israeli driver with the flashing green, special attention was given to the fact that changes in the length of the warning interval were made at those intersections. First, the data were recorded at the intersections in their normal operational mode (with or without a flashing green). The signal was then changed to the other mode. These changes were announced on radio, and the second set of measurements at those intersections were made a month or two after the change. It was thus assumed that most drivers passing the intersection during the period of the experiment had already become used to the new mode of operation.

The first three intersections are characterized by a low approach speed (20-40 km/hr), and the fourth by a higher approach speed (about 60 km/hr). The duration of each of the two warning periods at each intersection was as follows:

1. 3-second period: A yellow light appeared for 3 seconds after the continuous green light and was followed by the red light;
2. 6-second period: A flashing green of 3 seconds duration appeared after the continuous green, followed by a yellow light for 3 seconds, and then the red light. Altogether, the warning period lasted for 6 seconds.

For each vehicle, the data included its position at the end of the continuous green light, its speed, and its deceleration rate if it stopped. Sample sizes for each intersection were as follows: 52, 256, 255, and 60, respectively, for the 3-second period; and 47, 341, 239, and 131, respectively, for the 6-second period.

4.3 Model Validation and Estimation Results

Data from the four sampled intersections was analyzed separately throughout the analysis. It was not combined so that differences which might exist at those intersections regarding geometry, traffic mix and volumes, sight distance, approach speed, etc., will not introduce additional noise into the estimation process. Estimation results of the probit model stated in Equation 3 for the two warning periods at the four intersections are presented in Table 2. The validity of the stated behavioral assumptions can be tested using the estimation results. A basic assumption underlying the proposed model was that T_{cr} is the perceived duration of the warning period and t_{cr} is the objectively measured duration of the same period. This behavioral assumption can be tested by a formal statistical test since the exact length of the warning periods was 3 seconds and 6 seconds at all intersections. The t values of the test performed to examine the hypothesis: H_0: t_{cr} = Length of the warning period; versus H_1: Not H_0; are presented in Table 3. In four of the eight tests H_1 can not be rejected. This is especially true for three experiments: the two warning periods at intersection 2 and the 6-second warning period at intersection 4. The discrepancy between the estimation results and the behavioral assumptions might have been caused by the way our data was recorded and coded. It was mentioned in Section 4.2 that the speed of each vehicle was recorded at the moment the solid green ended. To calculate the value of t_1--the amount of time it takes the vehicle to reach the intersection line--we assumed constant traveling speed. At some intersections, where the approach speeds are low and thus the stopping distances short, many drivers accelerate their vehicle's speed when the solid green ends. In such a cases it will take them less time to reach the intersection than is calculated by us. The effect of this behavior will be equivalent to reduction in the length of the warning period under the assumption of the stated model. Observe that the largest discrepancies between the actual and estimated length of the warning period occurred at intersection 2 were the average approach speed was the lowest--about 22 km/hr.

Accepting the finding that drivers behave differently on the approaches to various intersections we should expect, based on our behavioral assumptions, that their behavior won't change when the warning period length changes. Thus the difference between the values of t_{cr} for the same intersection for the two warning periods should be 3 seconds in length. The appropriate t test can be stated as follows:

$$H_0: \ t_{cr}[6] - t_{cr}[3] = 3 \qquad\qquad H_1: \ \text{Not } H_0 \qquad (4)$$

The results of this tests for the four intersections are presented in Table 4a. The hypothesis that the difference is equal to 3 seconds cannot be rejected for three intersections at 5 percent significance level and for all four at the 1 percent level. For reasons which will be explained later, the samples at the three last intersections were divided into two speed groups as shown in Table 4b. The first intersection was excluded because of its small size. For the different speed groups at each intersection a t-test was performed to check whether the difference in the values of t_{cr} for the two warning periods is equal to 3 seconds. The results of this tests are presented in Table 4b. Here again all tests indicate that the hypothesis that the difference in t_{cr} = 3.0 cannot be rejected at the 1 percent level of significance for all cases. For four of the six tests it cannot be rejected even at the 5 percent level. Based on this analysis we can conclude that the simple probit behavioral model is adequate to analyze drivers behavior on intersection approach when the solid green ends. This is especially true if one is interested in the differences in drivers behavior on the same intersection approach as a function of the length of the warning period. It can be further concluded that based on the revealed differences in behavior samples from different intersections should not be combined for analysis. The present simple probit model can probably be improved if the acceleration of vehicles speed on the intersection approach is taken into account.

The main goal of the present analysis was to estimate a behavioral models which will provide the tool to investigate the changes in the shape of the decision function when the length of the warning period changes. As was shown earlier, the larger the variance of the stopping probability function, the higher is the number of conflicting decisions. An increase in the amount of conflicting decisions increases the probability of rear-end collisions on the intersection approach. It may be observed in Table 2 that the variance of the decision function (stopping probability function) increased significantly at the first three intersections. For example, the variance at intersection 1 increased from 0.31 to 6.14. The direct implication of the increased variances is an increase in the range of the indecision zone. As can be seen, this increase is in the order of 90 to 350 percent. To continue with the example of intersection 1, the range of the indecision zone increased from 1.42 seconds to 6.34 seconds. The stopping probability curves of Olson and Rothery (1961), which were also estimated for the two warning periods, show a tendency of longer range indecision zones during the longer warning period.

The range of the indecision zone did not change at only one intersection; it seems, however, that this result may not be random. As previously mentioned, intersection 4 is characterized by a relatively high approach speed. Stopping distances, therefore, are longer here than those at the other intersections. This fact means that drivers who can stop are situated farther from the intersection. It is reasonable to assume that the temptation to cross when one is at a short distance from the stop line is higher than when at a long distance; thus, a driver is more likely to stop at option zones with high speed approaches than at option zones with low approach speeds. This conclusion relates to the hypothesis advanced by Mahalel and Zaidel (1985) that drivers' stopping decisions are more strongly influenced by their distance from the stopping line than by their approach speed. At low approach speeds drivers are likely not to stop even if they can they can easily increase their speed and cross they intersection in time. The chance the slow moving driver takes upon himself by doing so is very small. This is due to the short stopping distance he has even after acceleration and because of the good information which he possesses on the conditions at the intersection due to his closeness to it. This type of behavior can explain the short values of tcr observed in this study at intersection 2 which had an average approach speed of 22 km/hr. At high approach speeds, whenever drivers can stop, they do so with high probability. Evidence that drivers' decisions reflect a higher sensitivity to distance than to speed may also be found in Chang, et al. (1985).

The foregoing analysis suggests that travel speed influences the magnitude of the value of t_{cr} and σ^2. The value of t_{cr} should increase with the speed of travel while σ^2 should decrease. Evidence that the phenomenon actually occurs can be found in Table 2. Observe the values of t_{cr} and σ^2 for intersections 2 and 4. Intersection 2, which has the slowest approach speed (about 22 km/hr), has the shortest t_{cr} for both warning periods and the highest, with one exception, value of σ^2. Intersection 4 which has the fastest approach speed (about 60 km/hr) has, with one exception, the longest

t_{cr} and in most cases the shortest σ^2. To test whether this phenomenon happens, the data for each intersection and each warning period was divided into two speed categories. One speed group included the lower speed vehicles and the other the higher speeds. A separated decision function was estimated for each speed group. The results are presented in Table 5. Since the sample size for intersection 1 was too small, it was dropped from the present analysis. Because of the difference in character between intersections 2 and 3, on the one hand, and intersection 4, on the other, the speed groups were defined differently. For intersections 2 and 3, the first speed group included all vehicles traveling below or at 30 km/hr, and the second all other vehicles. For intersection 4, the dividing speed value was 55 km/hr.

The results presented in Table 5 indicate a trend that is in line with the hypothesis advanced in this study; i.e., in most cases when the speed increases, the value of t_{cr} increases and σ^2 decreases. This trend, however, is not supported by a formal statistical test. The likelihood ratio test, based on the likelihood of each of the speed groups and the likelihood of the total sample at each intersection indicates that in 5 of 6 tests performed, the hypothesis that the parameter of the two speed groups are equal could not be rejected. This statistical result indicates that, in spite of the fact that the trend of the results is in line with expectations, it cannot be supported statistically. The reason for this may be that the samples were too small or the speed distributions at the different intersections were too narrow. Another explanation for this finding may be that the shape of the decision function is determined by the speed of travel of the traffic stream rather than by the speed of individual cars. That this might be true receives support from the fact that drivers' decisions are not based solely on the speed and distance of their car, but also on the vehicle's relative speed and location compared to other vehicles.

5. DISCUSSION

This article has pointed out the relationship between the option zone and the range of the indecision zone. It has demonstrated with empirical evidence that at low approach speeds, an increased option zone causes a significant increase in the indecision zone. The latter result probably holds for all approach speed and traffic-flow conditions on the intersection approach, as expressed by the total average speed. When the approach speeds of individual drivers are considered, the same trend may exist although it was not found to be statistically significant. Nevertheless, from a practical point of view, a characteristic decision function can apparently be estimated for a specific intersection approach under given traffic-flow conditions. This function can be used by a smart traffic signal controller, with the help of detectors, to minimize the chances of rear-end collisions at the intersection approach.

Analysis of the risk of rear-end collisions indicates that risk of accidents increases as the range of the indecision zone increases. In fact, various studies (Mahaled & Zeidel, 1985; Hakkert & Mahalel, 1978; Hocherman & Prashker; 1983) have reported that the number of rear-end collisions at urban intersections with a flashing green signal is significantly higher than at other signalized intersections. This important finding corroborates the hypothesis regarding the relationship between the range of the indecision zone and the risk of rear-end collisions.

The operational implication of this relationship calls for additional research effort to be invested in order to find the pattern of warning intervals that will minimize the range of the indecision zone. In other words, ways should be found to shape the stopping probability function to be as close as possible to a step function.

Table 1: Basic Data of the Four Intersections Included in the Study

Intersection No.	Characteristic	Warning Period [1]	
		3 sec.	6 sec.
1	No. of cycles	75	67
	Sample size (veh)[2]	52	47
	Volume (VPH)	320	330
	Average speed (km/h)[3]	31.7	37.8
	Speed variance	76	147
2	No. of cycles	80	83
	Sample size (veh)	256	341
	Volume (VPH)	950	865
	Average speed (km/h)	22.1	21.4
	Speed variance	95	72
3	No. of cycles	73	68
	Sample size (veh)	255	239
	Volume (VPH)	1317	1404
	Average speed (km/h)	37.1	34.7
	Speed variance	98	111
4	No. of cycles	42	47
	Sample size (veh)	60	131
	Volume (VPH)	Missing Data	1065
	Average speed (km/h)	59.4	63.0
	Speed variance	156	85

[1] The 6 sec. warning period is composed of 3 sec. flashing green and and 3 sec. yellow.

[2] The sample size refers to the number of vehicles at the end of the green. The vehicles preceded by stopped vehicles are not included.

[3] Speed of vehicles at the end of the continuous green.

TABLE 2 : SUMMARY OF ESTIMATION RESULTS FOR A SINGLE INTERSECTION EQUATION

Intersection	Warning Interval(1) (sec)	$\hat{t}_{cr}$ (2) (sec)	$\hat{\sigma}^2$	$\mathcal{L}^*$(3)	Indecision Zone Boundaries		
					Inner boundary (sec)	Outer boundary (sec)	Total length (sec)
1	3	2.42 (0.309)	0.31 (0.014)	- 8.64	1.71	3.13	1.42
	6	5.27 (1.202)	6.14 (0.931)	- 58.03	2.10	8.44	6.34
2	3	1.45 (0.020)	0.87 (0.072)	- 46.52	0.26	2.64	2.38
	6	4.64 (0.037)	3.16 (0.318)	-122.17	2.36	6.91	4.55
3	3	3.12 (0.014)	0.53 (0.018)	- 53.60	2.19	4.05	1.86
	6	7.08 (0.135)	4.98 (0.147)	- 88.77	4.22	9.94	5.72
4	3	4.46 (0.121)	1.39 (0.431)	- 16.59	2.95	5.97	3.02
	6	6.64 (0.117)	1.42 (0.348)	- 28.04	5.11	8.16	3.05

Note
(1) A warning interval of 3 sec. consists of 3 sec. amber. A warning interval of 6 sec. consists of 3 sec. flashing green and 3 sec. amber.

(2) In parenthesis is the variance of the estimate.

(3) Likelihood at convergence.

TABLE 3: COMPARISON BETWEEN THE ACTUAL AND ESTIMATED LENGTH OF THE WARNING PERIOD

INTERSECTION	t_{cr} (sec)	WARNING PERIOD (sec)	t TEST
1	5.27	6.0	0.67
1	2.42	3.0	1.04
2	4.64	6.0	7.12
2	1.45	3.0	11.02
3	7.08	6.0	2.94
3	3.12	3.0	1.06
4	6.64	6.0	1.89
4	4.46	3.0	4.19

TABLE 4a: DIFFERENCES IN THE LENGTH OF t_{cr} FOR THE TWO WARNING PERIODS

INTERSECTION	$t_{cr}[6]-t_{cr}[3]$ (sec)	t TEST (sec)
1	2.85	0.12
2	3.19	0.80
3	3.95	2.48
4	2.19	1.66

TABLE 4b: DIFFERENCES IN THE LENGTH OF t_{cr} FOR THE TWO WARNING PERIODS STRATIFIED BY SPEED

INTERSECTION	SPEED (km/hr)	$t_{cr}[6]-t_{cr}[3]$ (sec)	t TEST (sec)
2	≤ 30	2.78	0.88
2	> 30	4.35	2.15
3	≤ 30	3.80	1.42
3	> 30	4.06	1.44
4	≤ 55	3.05	0.07
4	> 55	1.47	2.45

TABLE 5 : SUMMARY OF ESTIMATION RESULTS STRATIFIED BY SPEED

Intersection	speed group limit (Km/h)	3 Sec. warning period				6 Sec. warning period			
		Ave.speed (Km/h)	$\hat{t}_{cr}$ (sec)	$\hat{\sigma}^2$	$\mathcal{L}^*$	Ave. Speed (Km/h)	$\hat{t}_{cr}$ (sec)	$\hat{\sigma}^2$	$^*\mathcal{L}$
2	$\leq$ 30	18.3	1.26 (0.031)	0.98 (0.120)	-33.83	18.5	4.05 (0.031)	1.85 (0.156)	-82.18
	> 30	36.5	1.42 (0.056)	0.33 (0.059)	- 9.39	35.5	6.04 (0.350)	5.95 (7.070)	-30.10
3	$\leq$ 30	25.6	3.14 (0.077)	1.01 (0.267)	-14.61	24.5	6.94 (0.241)	6.23 (4.94)	-40.00
	> 30	41.2	3.09 (0.164)	0.44 (0.016)	-31.92	40.8	7.15 (0.379)	4.54 (3.137)	-47.94
4	$\leq$ 55	45.7	3.41 (0.306)	1.24 (1.120)	- 6.55	50.6	6.76 (0.204)	1.90 (1.534)	-13.95
	> 55	67.4	4.66 (0.132)	0.43 (0.328)	- 7.93	66.9	6.13 (0.260)	0.69 (0.330)	-13.65

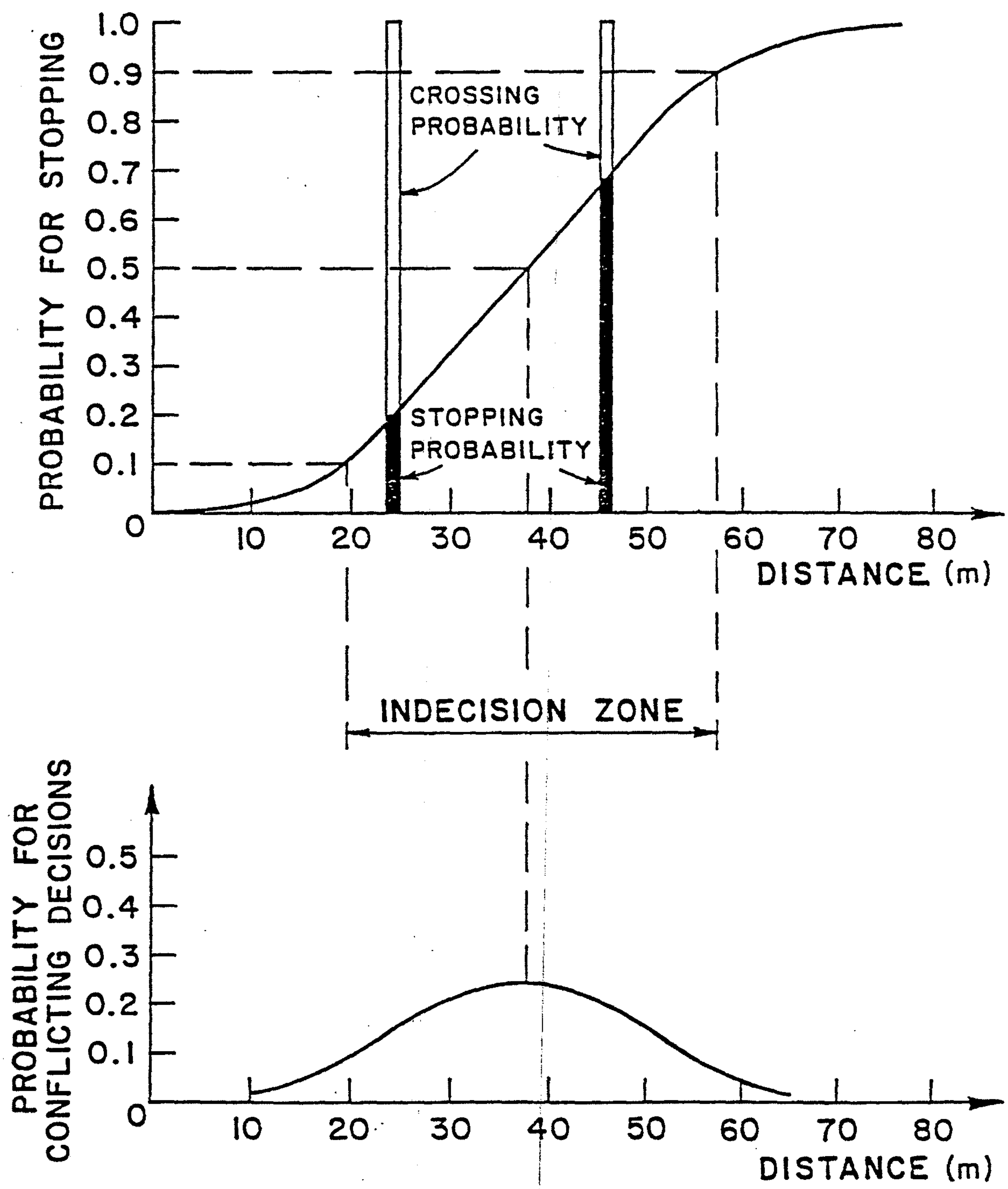

Fig. 1: Hypothetical stopping probability function and the probability for conflicting decisions.

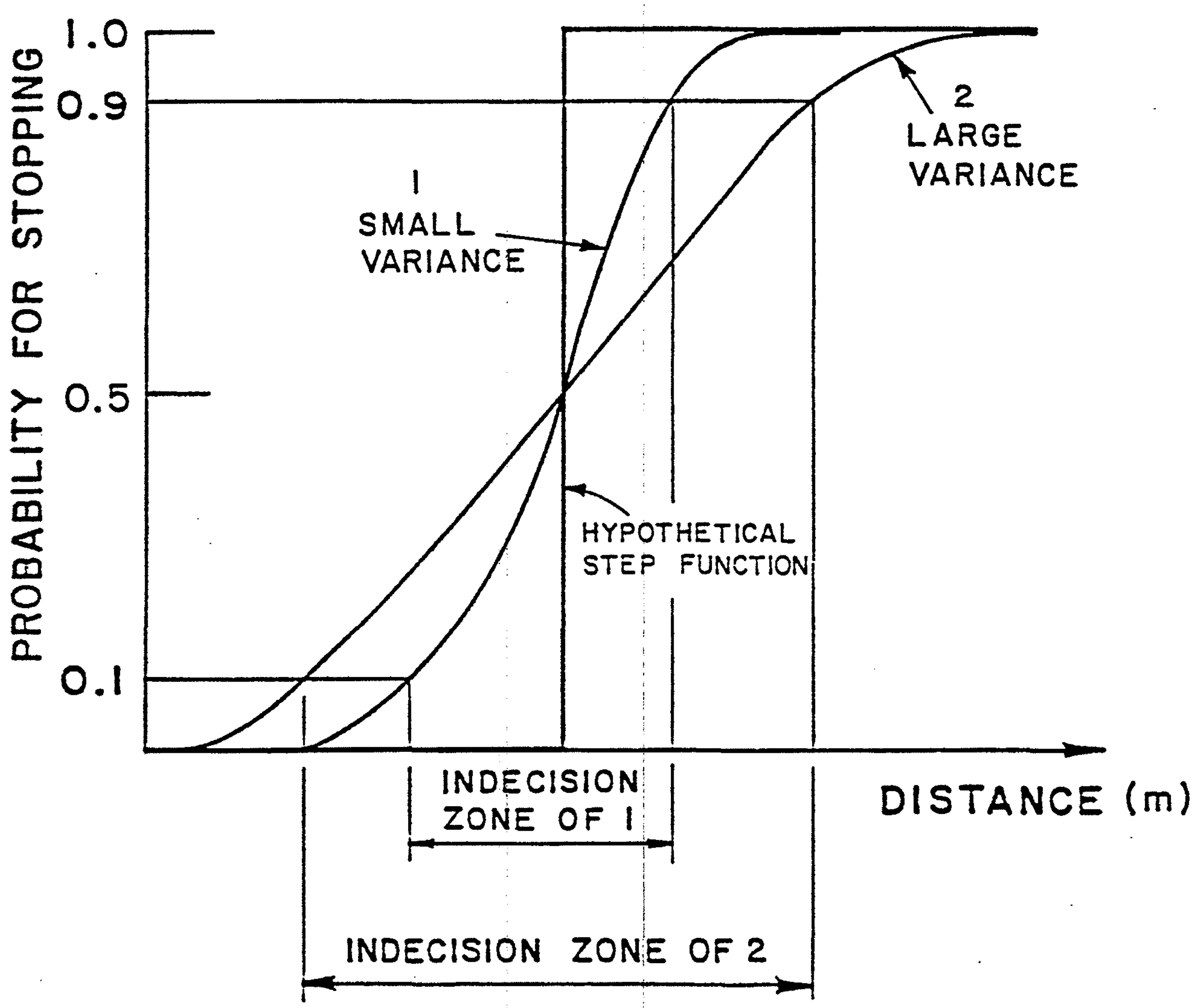

Fig. 2: Three hypothetical stopping probability functions in which P(x)=0.5 occours at the same x and which have different underlying variance.

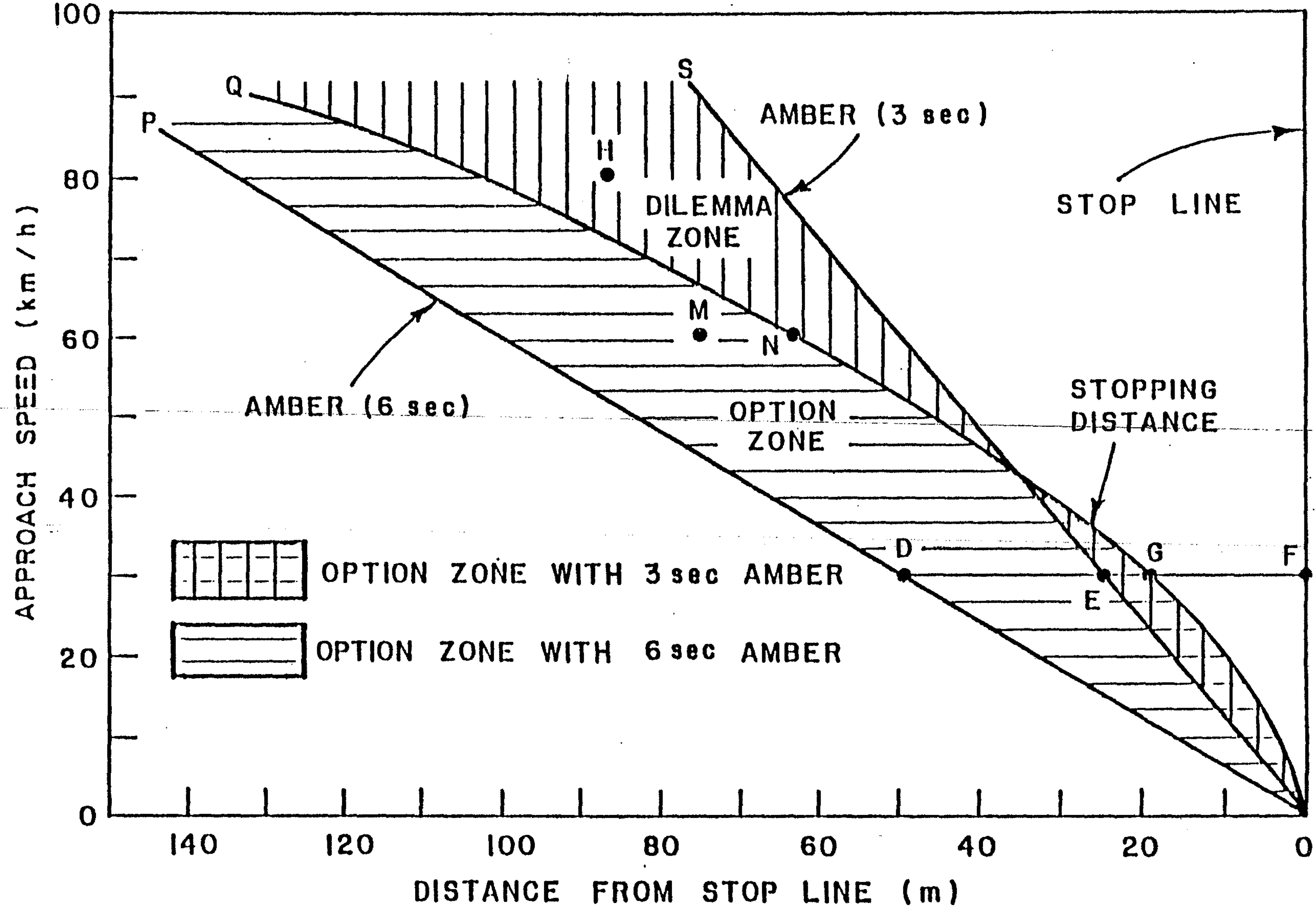

Fig. 3: Options and dilemma zones for two warning periods.

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