

Resource Allocation for Multicast Services in Multicarrier Wireless Communications

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Abstract— We consider a multicast resource allocation problem for the downlink in OFDM-based wireless cellular network systems. In a conventional multicast system, to accommodate users with bad channel conditions, the transmission is based on the worst case user. We show that such a multicast system saturates the capacity when the number of users increases in fading environments. We exploit the multicarrier nature of OFDM and advances in coding techniques such as MDC (multiple description coding), in which arbitrary combinations of layers can be decoded at the receiver. Different MDC layers are carried over different subcarriers and users with good channels receive data from more subcarriers than users with poor channel conditions. We present an optimal subcarrier/bit allocation method requiring full search of possible candidates. To reduce the complexity, we propose a two-step suboptimum algorithm by separating subcarrier allocation and bit loading. Numerical results show that the proposed heuristics significantly outperform the conventional multicast transmission scheme. The difference between optimum and heuristic solutions is less than 5%.

Index Terms— Multicast, multicarrier, multiple description coding (MDC), resource allocation.

I. INTRODUCTION

MULTICAST radio resource allocation is a challenging problem in wireless networks due to different channel conditions of users. One solution is to transmit data based on the worst case user, which causes inefficient use of radio spectrums. The problem is how to satisfy different users without sacrificing efficiency. One approach to the problem is to use multiple layer coding [1]. Controlling the number of layers provides the degree of freedom to satisfy different users. Another approach is to utilize multiple subcarriers in wireless systems such as OFDM (orthogonal frequency division multiplexing). By allocating different number of subcarriers intelligently, the inefficiency issue can be handled. The authors in [2] followed a similar approach.

Even though the aforementioned works address inefficiency issues in one way or another, little research has been done on multicast over multicarrier systems with multiple layer coding. Moreover, typical fairness and efficiency issues have barely been addressed. In this paper, we visit a multicast

resource allocation problem over multicarrier wireless systems to address fairness and efficiency issues. Unlike single-carrier systems, user difference can be better accommodated in multicarrier systems by assigning subcarriers intelligently. Similarly, multiple layer coding provides another degree of freedom to exploit different user conditions. Our work is different in that we use both degrees of freedom (multiple subcarrier and multiple layers) to formulate the resource allocation problem for multicast data. As far as we know, this is the first work that addresses the problem.

In our systems, we assume multiple description coding (MDC) [3], in which an arbitrary combination of layers can be decoded regardless of layer hierarchies. In MDC, a single media source is fragmented into n (≥ 2) independent substreams called ‘descriptions’ of equal importance. One important property of MDC is that any description can be used to decode the media stream and that the quality improves with more descriptions. For example, if there are four descriptions, $2^4 - 1 = 15$ possible combinations of different qualities exist for the original media stream. In our work, the decoding flexibility of MDC is well exploited to use radio resources more efficiently.

II. CAPACITY LIMITATIONS OF CONVENTIONAL MULTICAST DATA

In multicast systems, the system capacity increases linearly proportional to the number of users. If there are K active users, then the system capacity is $K \cdot r$, where r is the data rate of a broadcasting channel. In this section, we address the same question for cellular-based multicast systems and conclude that in fading environments, the system capacity becomes saturated. We consider Rayleigh and Rician fading channels.

Let $T(K)$ be the multicast system capacity when there are K users: $T(K) = K \cdot r(K)$, where $r(K)$ is the multicast transmission rate when there are K users in the system. As K increases, $r(K)$ typically decreases because a typical multicast transmission rate is adjusted to the worst case user. In other words, the first term is increasing while the second term is decreasing on K .

Rayleigh Fading Channel: In a Rayleigh fading channel, we prove that if $r(K)$ is based on the worst user, then the system capacity saturates under the mild assumption of I.I.D channel gains for all users (see Theorem 1). We skip the detailed proof due to the page limit. For details, see the full version of the paper [4].

Let X_k designate the channel gain of user k and suppose that X_k 's are independently distributed with a parameter of α_k as follows: $Pr(X_k \leq x; \alpha_k) = 1 - \exp\left(-\frac{x}{\alpha_k^2}\right)$,

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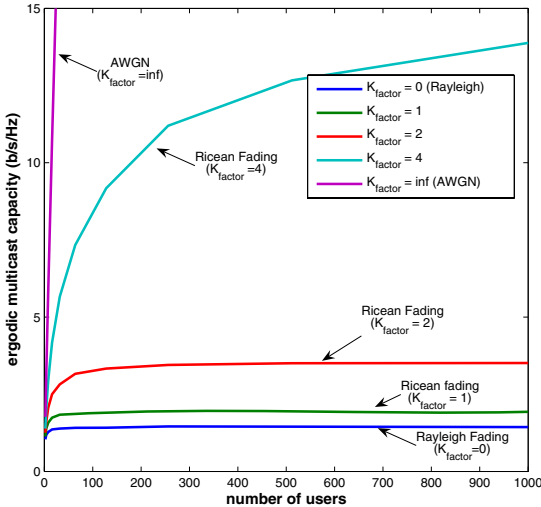


Fig. 1. Simulation results of multicast system capacity in fading channels ($P = 1$, $\sigma^2 = 1$, and K_{factor} is identical for all the users).

for all k . In conventional multicast transmission, the capacity is determined by the smallest channel gain: $Y_1 = \min\{X_1, X_2, \dots, X_K\}$. Therefore, an ergodic capacity for multicast services is defined by

$$E[T(K)] \triangleq K \cdot E \left[\log_2 \left(1 + \frac{P}{\sigma^2} Y_1 \right) \right], \quad (1)$$

where K is the number of users, P is transmission power, and σ^2 is noise variance.

Theorem 1. Assume that channel status r.v.'s X_k are independent with different α_k 's for all k . We further assume that there exists $\bar{\alpha}$ such that $\alpha_k \leq \bar{\alpha}$ for all k . Then, as $K \rightarrow \infty$, the multicast system capacity is bounded, i.e., $\lim_{K \rightarrow \infty} E[T(K)] < \infty$.

Sketch of Proof: The proof is done in two steps. In the first step, we show that $E[T(K)]$ is bounded under the assumption of the same α_k for all k . In the second, we use the stochastic relationship to show that $E[T(K)]$ with $\bar{\alpha}$ is larger than that of the original system. By applying the first result, we can obtain the Theorem. \square

Ricean Fading Channel: The Ricean fading channel is used to model a multipath channel with a line-of-sight content. Due to the complicated form of Ricean distribution including a modified Bessel function, we relied on simulation instead of analysis. In Fig. 1, $K_{factor} \triangleq \frac{m^2}{2\eta^2}$, where m denotes the peak amplitude of the dominant signal and η^2 is the variance of multipath. $K_{factor} = 0$ and $K_{factor} = \infty$ indicate Rayleigh fading and AWGN channels, respectively. Here we can see that when the line-of-sight component is weak (i.e., K_{factor} is small), the similar tendency of capacity saturation is more noticeable. However, when the line-of-sight component is very strong (i.e., K_{factor} is very large), the saturation is less obvious.

Using analysis and simulation (Theorem 1 and Fig. 1), we observed that the transmission scheme based on the worst case user can be limiting in multicast systems. In the following

section, we formulate a multicast resource allocation problem and suggest a subcarrier/bit allocation scheme to overcome capacity limitations of conventional multicast data.

III. PROBLEM FORMULATION

We consider a multicast transmission problem over a multi-carrier system with N subcarriers and K users using multiple description coding (MDC). For each subcarrier n , the BS needs to decide the power level P_n and the number c_n of bits to carry over subcarrier n .

Each user k experiences a different channel condition. Let $\alpha_{k,n}$ denote the channel gain of user k over subcarrier n . For user k to decode transmitted data c_n over subcarrier n , the received power $\alpha_{k,n}^2 P_n$ should be greater than the required power $f(c_n)$. In other words, the required power level P_n should satisfy $P_n \geq \frac{f(c_n)}{\alpha_{k,n}^2}$.

By introducing an allocation vector $\rho_{k,n}$ of which the value is 1 or 0, inequality $P_n \geq \frac{f(c_n)}{\alpha_{k,n}^2}$ can be rewritten as (3). With these notations, we formulate the problem as follows:

$$\text{(MRA)} \quad \max_{(\rho_{k,n}, P_n, c_n)} \sum_k w_k R_k = \sum_k w_k \left[\sum_n c_n \cdot \rho_{k,n} \right] \quad (2)$$

$$\text{subject to } \frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \leq P_n, \quad \forall k, \forall n, \quad (3)$$

$$\sum_n P_n \leq P_T, \quad P_n \geq 0, \quad (4)$$

$$\rho_{k,n} \in \{0, 1\}, \quad c_n \in \{0, 1, \dots, M\} \quad \forall k, \forall n. \quad (5)$$

The objective function is a weighted sum of data rate (R_k) for all the users. Note that R_k is expressed as an arbitrary sum of data rates over subcarrier n i.e. $\sum_n c_n \rho_{k,n}$. In the MDC, an arbitrary combination of layers assigned to different carriers is decodable. The weight vector $\{w_k\}$ is introduced to generalize the problem. By changing the value of w_k , (MRA) can cover throughput maximization and also proportionally fair (PF) allocation. If $w_k = 1$, the objective is throughput maximization. If $w_k = \frac{1}{E[R_k]}$, then it is PF allocation. If we choose $w_k = \frac{1}{E[R_k]^\infty}$, it becomes *max-min* fair allocation. Therefore, the formulation can even cover the fairness issue.

Note also that $f(c_n)$ is a nonlinear function of c_n . For example, in the case of M -ary quadrature amplitude modulation (M-QAM), $f(c)$ can be represented as $f(c) = \frac{N_o}{3} [Q^{-1}(p_e/4)]^2 (2^c - 1)$, where p_e is the required bit error rate (BER), $N_o/2$ denotes the variance of the additive white Gaussian noise (AWGN), and $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt$.

IV. OPTIMUM ALGORITHM

The problem belongs to the category of nonlinear integer programming, which is known to be NP-hard, another way of saying that it is difficult. The multiplication of integer variables, $c_n \cdot \rho_{k,n}$ and $f(c_n) \cdot \rho_{k,n}$, make it more difficult. Our approach is a brute force algorithm that considers all possible combinations of allocation matrix $[\rho_{k,n}]$. For a given instance

of $[\rho_{k,n}]$, the nonlinear terms disappear and (MRA) is reduced to (MRA $_{\rho}$).

$$\text{(MRA}_{\rho}) \quad \max_{(P_n, c_n)} \sum_k w_k R_k \quad (6)$$

$$\text{subject to } \max_{k \in I_n} \left[\frac{f(c_n)}{\alpha_{k,n}^2} \right] \leq P_n, \forall n, \quad (7)$$

$$\sum_n P_n \leq P_T, \quad (8)$$

$$P_n \geq 0, c_n \in \{0, 1, \dots, M\} \quad \forall n, \forall k. \quad (9)$$

Here, $I_n = \{k | \rho_{k,n} = 1\}$ is an index set for the users allocated in n . To see the equivalence of (3) and (7), remember that we can remove constraints in (3) for $\rho_{k,n} = 0$ and that the power P_n is determined by the worst user in I_n . Using the fact that the allocation can be optimal when P_n is fully utilized, this problem can further reduced into the following (MRA' $_{\rho}$) having only one control variable (c_n). Here, a constant value W_n is defined as $\sum_k w_k \rho_{k,n}$ and $\alpha_n^* = \min_{k \in I_n} [\alpha_{k,n}]$. Note that Eqs. (7) and (8) are expressed into one equation (11).

$$\text{(MRA}'_{\rho}) \quad \max_{(c_n)} \sum_n W_n c_n \quad (10)$$

$$\text{subject to } \sum_n \frac{f(c_n)}{\alpha_n^{*2}} \leq P_T, \quad (11)$$

$$c_n \in \{0, 1, \dots, M\}, \quad \forall n \quad (12)$$

We can solve (MRA' $_{\rho}$) by comparing the objective function values of $(M+1)^N$ possible bit-loading choices, ranging from $(0, 0, \dots, 0)$ to (M, M, \dots, M) .

Complexity: The complexity of the proposed optimal algorithm is $(K+1)^N (M+1)^N$. The first term $(K+1)^N$ indicates the number of all possible subproblems and the second term $(M+1)^N$ denotes the complexity of each subproblem (MRA' $_{\rho}$). Note that there are $(K+1)$ possible allocations of $(\rho_{k,n}, k = 1, 2, \dots, K)$ for each subcarrier n . To understand this, consider the fact that $\alpha_{k,n} > \alpha_{k',n}$ implies $\rho_{k,n} \geq \rho_{k',n}$ in the optimal allocation. Otherwise, a contradiction occurs. In other words, if user k' with bad channel is allocated for subcarrier n , then user k having better channel status should be allocated for subcarrier n as well. Since there are N subcarriers, the total number of possible $(\rho_{k,n})$ is equal to $(K+1)^N$. Therefore, to find the optimal solution of (MRA), it is required to evaluate the objective value $\sum W_n c_n$ for $[(K+1)(M+1)]^N$ cases. Note that W_n is a function of c_n and also $[\rho_{k,n}]$. To reduce the complexity, we consider a simpler suboptimum algorithm in the next section.

V. SUBOPTIMUM ALGORITHM-TWO STEP APPROACH

In an attempt to avoid the full search algorithm in the preceding section, we devise a suboptimum two-step approach. In the first step, the subcarriers are assigned assuming the constant transmit power of each subcarrier. This assumption is used only for subcarrier allocation. Next, bits are loaded to the subcarriers assigned in the first step. Although the separation causes suboptimality of the algorithm, it makes the complexity significantly low. In fact, the separation concept has been already employed in OFDMA systems and also its efficacy

has been verified in terms of both performance and complexity [5]. However, the specific algorithm proposed in this paper is unique in dealing with multicast resource allocation.

A. Subcarrier Allocation

For a given power allocation vector $\mathbf{P} = (P_1, P_2, \dots, P_N)$ for each subcarrier, (MRA) is separable with respect to each subcarrier. The subcarrier problem with respect to subcarrier n is:

$$\text{(MRA-n)} \quad \max_{c_n, \rho_{k,n}} c_n \sum_{k=1}^K w_k \rho_{k,n} \quad (13)$$

$$\text{subject to } \max_k \left(\frac{f(c_n) \rho_{k,n}}{\alpha_{k,n}^2} \right) \leq P_n, \quad \forall n.$$

If we fix c_n to be c , then the solution $\rho_{k,n}^c$ of (MRA-n) is determined by the following allocation:

$$\rho_{k,n}^c = \begin{cases} 1, & \text{if } \alpha_{k,n}^2 \geq \frac{f(c)}{P_n} \\ 0, & \text{otherwise.} \end{cases} \quad (14)$$

The optimality of $\rho_{k,n}^c$ for given $c_n = c$ can be argued by contradiction that the objective function would not be maximized otherwise. Since the optimal $\rho_{k,n}^c$ for a given c is easily determined, Theorem 2 implies a simple algorithm to solve (MRA-n).

Theorem 2 For a given P_n , (MRA-n) is equivalent to

$$c_n^* = \arg \max_{c=1,2,\dots,M} c \cdot \sum_k w_k \cdot \rho_{k,n}^c, \quad (15)$$

where w_k is the weight of user k and $\rho_{k,n}^c$ is defined in equation (14).

With the help of Theorem 2, we can provide the following polynomial time algorithm that requires only M evaluations of $c \sum_k w_k \rho_{k,n}^c$. Note that computation of $\sum_k w_k \rho_{k,n}^c$ can be done in linear time. Therefore, the following algorithm is a linear algorithm: **Algorithm SUB-ALLOC**

- 1) Compute $\rho_{k,n}^c, \forall c = 1, 2, \dots, M$.
- 2) Determine c_n^* using equation (15), $\forall n = 1, 2, \dots, N$.
- 3) Store $W_n = \sum_k w_k \rho_{k,n}^{c_n^*}$ corresponding to $c_n^*, \forall n$.

B. Bit Loading

The subcarrier allocation $\rho_{k,n}$ is given by SUB-ALLOC algorithm. In the bit loading step, we determine subcarrier power P_n and corresponding c_n based on their channel quality $\alpha_{k,n}$. For a given $\rho_{k,n}$, (MRA) becomes

$$\max_{c_n} \sum_{n=1}^N c_n \cdot W_n, \quad \text{subject to } \sum_{n=1}^N \frac{f(c_n)}{\alpha_{k_n^*,n}^2} \leq P_T, \quad (16)$$

where $W_n = \sum_k w_k \rho_{k,n}$ and $\alpha_{k_n^*,n}$ is the lowest $\alpha_{k,n}$ such that $\rho_{k,n} = 1$. We modified the Levin-Campello algorithm in [6] to determine the number c_n of loaded bits. Let $\Delta P_n(c)$ be the additional power needed for transmitting one additional bit:

$$\Delta P_n(c) = \frac{f(c+1) - f(c)}{\alpha_{k_n^*,n}^2 \cdot W_n}. \quad (17)$$

The factor W_n is required because the incremental power is shared by the group of users. This is a main difference from

the well-known Levin-Campello algorithm. Using Eq. (17), the bit loading algorithm is summarized below.

Algorithm MOD-LEV-CAMP

1) Initialization

0. Let $c_n = 0$ and compute $\Delta P_n(0), \forall n$.
1. Let P_T^* be the tentative transmit power and set $P_T^* = 0$.

2) Bit Loading Iteration

2. $n^* = \arg \min_n \Delta P_n(c_n)$.
3. Update $P_T^* = P_T^* + \Delta P_{n^*}(c_{n^*}) \cdot W_{n^*}$ and $c_{n^*} = c_{n^*} + 1$.
4. If $c_{n^*} = M$, set $\Delta P_{n^*}(c_{n^*}) = \infty$ else evaluate $\Delta P_{n^*}(c_{n^*})$.
5. If $P_T^* \geq P_T$ stop; Otherwise, go to 2.

In the above procedure, if c_{n^*} reaches M , ΔP_{n^*} should be set to the infinite value to prevent more bit loading. Even though the above algorithm is not necessarily optimal, it plays as a good heuristic. The optimality is guaranteed when $W_n = 1, \forall n$ [6].

Complexity: The algorithm **SUB-ALLOC** has the complexity of $O(MN)$ since it needs M evaluation of $\sum w_k \rho_{k,n}^c$ to perform (15) for all N subproblems. Similarly, the algorithm **MOD-LEV-CAMP** has the complexity of $O(MN)$ since the maximum number of iterations is MN and each step from 2 to 5 takes constant time. Note that there exists a big difference between the complexity of $[(K+1)(M+1)]^N$ and $O(MN)$ complexity of the heuristic.

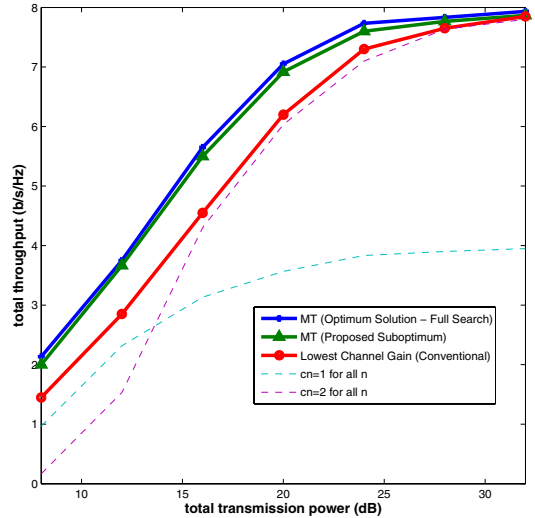
VI. NUMERICAL RESULTS

In this section, the proposed suboptimum algorithms for MT and PF are compared with the full-search optimum solution and the conventional scheme (the lowest channel gain (LCG) method). In the LCG method, all the subcarriers are shared by all the users but bits are loaded using the modified Levin-Campello algorithm. In simulations, we assume a frequency-selective Rayleigh fading channel with an exponential gain profile. The users are uniformly distributed in a cell and the large-scale path loss exponent is two. The average channel gain indicating long-term fading is set to constant, but the short-term fading channel is independently generated in every scheduling slot. In fact, 4000 independent short-term fading channels are generated and the results in figures denote the average values. In the PF case, the following weighting factor is used:

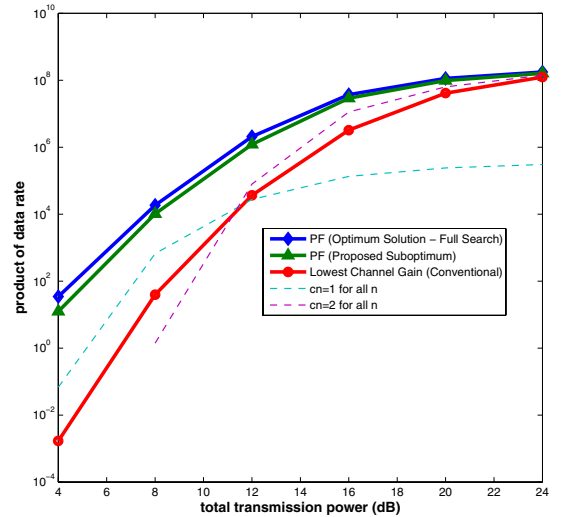
$$\frac{1}{w_k(t+1)} = \frac{1-\gamma}{w_k(t)} + \gamma \sum_{n=1}^N c_n \rho_{k,n}, \quad (18)$$

where γ is set to $1/1000$ suggested for CDMA-HDR systems. The required BER is $p_e = 10^{-4}$ and the noise variance $N_o/2 = 1$. The number K of users is between four to 128.

Fig. 2 shows a comparison of an optimum full search algorithm and suboptimum two-step algorithm when the number N of subcarriers is eight, the number K of users is four, maximum loaded bits M is two, and the number of channel taps is four. Fig. 2(a) shows the MT case, where performance difference between the optimum and suboptimum ones is within about 5% for a wide range of transmission power. It



(a)



(b)

Fig. 2. Comparison of IP optimum algorithm and suboptimum two-step algorithm for $N = 8$, $M = 2$, and $K = 4$; (a) MT; (b) PF.

implies that the equal power assumption for all the subcarriers is reasonable during subcarrier allocation. Compared to the LCG scheme and fixed modulation, e.g., $c_n = 1$ or $c_n = 2$, the performance gap is not significant. For large transmission power, observe that throughput is saturated regardless of any type of algorithms. It is because the maximum loaded bits are limited by two.

Fig. 2(b) shows the comparison for the PF case. As a measurement of proportional fairness, we used the product of throughput (equivalent to the sum of logarithmic throughput). When compared to the LCG method, the performance gap is not significant, i.e., falls within 5%. It is shown that the proposed suboptimum algorithm also has performance gain in the PF case.

Since the optimum solution is based on a full search

TABLE I
THROUGHPUT(B/S/HZ)-FAIRNESS COMPARISON OF DIFFERENT
SCHEDULING ALGORITHMS

Number of Users	4	6	8	16	32	64
Throughput-MT	5.74	8.08	10.46	18.51	34.94	66.83
Throughput-PF	5.58	7.76	9.96	17.90	33.58	64.24
Throughput-LCG	3.68	4.18	4.61	5.95	7.23	8.53
Fairness-MT	0.86	0.81	0.80	0.80	0.79	0.79
Fairness-PF	0.93	0.90	0.90	0.90	0.88	0.88
Fairness-LCG	1.00	1.00	1.00	1.00	1.00	1.00

algorithm, we could not find the results for large parameter sets. Instead, by comparing the conventional LCG method, we showed the efficacy of the proposed suboptimum algorithm in practical OFDM systems such as 802.11 adopting $N = 64$ and $M = 5$ [7]. Table I shows the throughput-fairness comparison of three scheduling algorithms, i.e., MT, PF, and LCG.

As we expected, the throughput of the LCG method becomes saturated as the number of users increases, while the throughput of the proposed one increases with K . Note that the performance gap between MT and PF schedulers is small when compared to the LCG method. Although the throughput difference between MT and PF becomes large with the increasing K , it is still within about 5% for a wide range of number of users. As a measurement of fairness, we adopted the following fairness index (FI) defined in [8]: $FI = \frac{(\sum_k R_k(t))^2}{K(\sum_k R_k^2(t))}$. The LCG method has the best performance among three scheduling schemes and the PF scheduling has compromised performance between the MT and LCG methods.

VII. CONCLUSION

In this paper, we considered the resource allocation problem for multicast services over multicarrier systems using the

assumption of multiple description coding (MDC). We first showed that conventional scheduling based on the worst case user may not be a good technique under fading channels. Under Ricean or Rayleigh fading channels, the result implies that the worst-user-based scheduling shows the saturation of the system throughput with an increasing number of users. Secondly, we formulated the power control/bit loading algorithm for maximum throughput and proportional fairness. In an attempt to reduce high computational complexity of the optimum solution based on a full search algorithm, we proposed suboptimum algorithms by separating subcarrier allocation and bit loading. The proposed two-step heuristic is validated against the optimal solution and the performance difference between the two algorithms was less than 5%.

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