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November 17, 1966

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ABSTRACT

The suggestion is made that the particle-pole correspondence be more freely employed in the interpretation of experimental data. The resulting removal of ambiguities in the characterization of unstable particles is stressed. Also emphasized is the variety of ways in which high precision data can be used to discover new particles.

* This work was done under the auspices of the U. S. Atomic Energy Commission.

I. INTRODUCTION

Frequently one hears raised the question, "What precisely is a resonance?" Is it the same as a "particle"? Is a threshold enhancement different from a resonance? Is a single peak associated with several different sets of quantum numbers a resonance? Can the location and width of a resonance depend on the production mechanism? And so on. Within the framework of the analytic S matrix there has long existed a natural and unambiguous meaning for the concept of "particle" or "resonance" that resolves all such questions, but there continues to be some hesitance in accepting this meaning. The purpose of the present review is to urge uninhibited adoption of a uniform and natural experimental definition of the term "particle" or "resonance", at least with respect to hadrons.* Semantic precision in physics in itself often promotes progress in real understanding of nature. One object here is thus to illustrate how insight into strong-interaction dynamics may be enhanced by a precise definition of the particle-resonance concept. A more important object, however, is to increase experimenter's awareness of the variety of possibilities for establishing new particles through high precision measurements.

Why is careful handling of the resonance concept more important in high-energy physics than in atomic physics or even classical (low energy) nuclear physics, where particles and resonances are equally

* Zero-mass particles present special problems that we wish to ignore here.

prominent. The difference is that these latter domains are non-relativistic and the concept of wave function is correspondingly well-defined. The results of experiment are usually translated into statements about wave functions, and it is unnecessary to be precise about the definition of a resonance so long as the wave function is unambiguously specified. In the high-energy domain, however, it has so far proved impossible to define a wave function that can be determined from experiment. The combination with quantum mechanics of relativistic requirements seems to preclude a meaning for the concept of particle "position," when the required precision is $\sim \hbar/Mc$, M being the particle mass. "High-energy physics" by definition concerns itself with wavelengths that fall into this difficult range.

Even though localized wave functions lack meaning at high energy, scattering and reaction amplitudes continue to be well-defined --the complete collection of all reaction amplitudes constituting the elements of what is called the "S Matrix."* Basic to the discussion here is the fact that S-matrix elements are analytic functions of the initial and final energies and angles of particles appearing in the various reactions. The ground rules employed in this paper are to state without proof certain properties of the analytic S matrix that are noncontroversial among theorists. Often there is controversy about where one should begin in deriving these principles, but the validity of the principles to be employed here is not seriously challenged.¹

* In the nonrelativistic limit, the information content of the analytic S matrix has been shown to be equivalent to that of the wave function. In other words, the nonrelativistic wave function can be unambiguously constructed, given a knowledge of the S matrix.

II. CERTAIN BASIC S-MATRIX CONCEPTS

There is little doubt that the physical phenomena which physicists have felt comfortable in describing as "resonances" or "particles" are associated with simple poles of the S matrix. A straightforward and obvious approach, then, is simply to decree a one-to-one correspondence between "particles" and poles. This is what we are advocating. (It might be felt that the term "resonance" should be used in a more narrow sense than "particle," being reserved for poles that occur in the special regions of the complex energy-space discussed below in Sec. III. However, when we come to consider these regions, it will be evident that they cannot be precisely delineated. With semantic precision as a goal, therefore, it seems best not to attempt any distinction between the terms "particle" and "resonance". Here these nouns will be employed interchangeably.) A possible objection to a universal pole-particle identification is that the S matrix might have several different kinds of poles, only one category being appropriately associated with particles. Such a possibility was, in fact, intensively considered during the early days of S-matrix theory, but subsequent work has shown that all simple poles in individual "channel invariants" are of a single basic type.¹

What is meant by the term "channel invariant?" A channel is any collection of more than one particle and the "channel invariant" is the square of the total energy in the barycentric system of the channel. (For the sake of clarity we adopt the convention here that each channel has a well-defined set of all conserved quantum numbers,

not only charge and baryon number but angular momentum and, for hadrons, hypercharge, parity, and isotopic spin.) A pole that appears in a particular channel invariant at a fixed real value $s_c = m_j^2$ on the physical sheet, independently of other variables (such as momentum transfer), has long been interpreted as a stable particle of mass m_j that "communicates" with the channel c . Our major task here is to discuss poles that do not appear on the physical sheet and do not correspond to stable particles.

By the term "communicating" channel we mean that if sufficient energy were available the particle could decay into this channel. Channels that communicate with the same particle also communicate with each other. Generally speaking, channel communication is blocked only by selection rules, or in other words, only by a failure of the channels in question to share the same set of quantum numbers. The invariants of all the different members of a set of communicating channels share the same set of stable particle-poles, and conversely, any individual pole appears in every S-matrix element connecting channels that communicate with the corresponding particle. This important property can be proved to hold true for poles both off and on the physical sheet.

The term "physical sheet" has been introduced in the foregoing. How is this defined? So far as the variable s_c is concerned, the conventional physical sheet is obtained by drawing cuts from each "normal threshold" branch point in s_c along the positive real axis to $+\infty$. The physical threshold s_c^t of channel c occurs at the square of the sum of the masses of constituent particles. If we consider the set of

all S-matrix elements $S_{c',c}(s)$ representing transitions between the members of a set of communicating channels, realizing that $s_{c'} = s_c = s$ by energy-momentum conservation, it turns out that each element $S_{c',c}(s)$ has branch points in s at every communicating channel threshold (not just at s_c^t and $s_{c'}^t$). These branch points mean that we must think of a Riemann surface in s , with a particular sheet specified by the manner in which cuts are drawn. The surface topology is common to all communicating S-matrix elements.*

Normal threshold branch points divide the physical region into sectors, and it is necessary to prescribe how two adjacent physical sectors in s are connected through analytic continuation. The rule is that, if one is moving along the real s axis in a physical region and encounters a threshold branch point, one goes infinitesimally above the branch point in the s -complex plane and on returning to the real axis one will be in the adjacent physical region. (Going below the branch point or more than halfway round leads generally to a unphysical region.) Reviewing the content of the last two paragraphs, one sees that in general the physical region of s is reached from the physical sheet by approaching the positive real axis from above.

It turns out that poles on the physical s sheet can occur only on the positive real s axis, below the lowest normal threshold

* In partial-wave S-matrix elements (of definite angular momentum) there are additional branch points not associated with normal thresholds in s . One may usually ignore these so-called "left-hand" branch points, however, in specifying the position of a pole.

branch point, as shown in Fig. 1. Other poles must lie on unphysical sheets, but our basic contention will be that any pole in an individual channel invariant is equally entitled to be interpreted as a particle communicating with the set of channels in question.

At this point the experimenter usually objects that poles in unphysical regions cannot be experimentally established. In fact they can be, subject to the uncertainties attendant on any physical measurement. Given that the S matrix is an analytic function which may be measured with arbitrary precision in physical regions, one can extrapolate unambiguously to unphysical regions. The practical difficulty is that extrapolation with accuracy over large intervals requires a correspondingly high precision in the physical region. Inevitably "distant" poles are more difficult to establish than are "nearby" poles, but this kind of situation is not new in physics. To illustrate how certain unphysical sheet poles have been quite reliably determined in the past we review in the following sections some well-known special techniques of extrapolation.

III. THE BREIT-WIGNER EXTRAPOLATION

The most familiar type of unphysical-sheet pole lies at a complex point in the s plane slightly below the physical region, let us say at a position

$$s = s_p = s_{pr} - i\Delta_p, \quad (\text{III.1})$$

where Δ_p is a small positive real quantity. The meaning of the adjective "small" as applied to Δ_p is developed in the following paragraph. If one wishes to display the pole position and the physical region simultaneously, cuts must be drawn so as to expose part of an unphysical sheet, at the same time necessarily obscuring part of the physical sheet. A possible way to draw the cuts is shown in Fig. 2.

If Δ_p is small compared to the distance from s_p to the next nearest singularity of the S matrix (which in Fig. 2 would be a threshold branch point), then one may expand the product

$$f(s) = (s - s_p) S_{c,c}(s) \quad (\text{III.2})$$

in a power series about the point $s = s_p$,

$$f(s) = f(s_p) + (s - s_p) f'(s_p) + \dots, \quad (\text{III.3})$$

with the series converging in a circle that includes part of the physical region, as shown in Fig. 3. If the physical point $s = s_{pr}$ is not far from the center of the circle, the leading term of the series will be a good approximation for s near s_{pr} . Then we are led to the familiar Breit-Wigner formula,

$$S_{c'c}(s) \underset{s \text{ near } s_{pr}}{\approx} \frac{-f(s_p)}{s_{pr} - s - i \Delta_r} + f'(s_p) \quad (\text{III.4})$$

from which one easily identifies the conventional width parameter Γ as

$$\Gamma = \Delta_p / \sqrt{s_{pr}} \quad (\text{III.5})$$

Unitarity and time reversal give constraining relations between $f(s_p)$ and $f'(s_p)$ which we shall not consider here. The point is that the fitting of data with a simple Breit-Wigner formula corresponds to the most elementary type of extrapolation from physical region to unphysical-sheet pole. The extrapolation can be and sometimes is improved by keeping more terms in the expansion (III.3). These correspond to what is usually called the "background" of the resonance. When the background is substantial the actual observed width of a cross section peak need not be Γ . Nevertheless this parameter has a unique value by extrapolation.

It should also be remarked here that no simple and completely general statement is possible concerning the behavior of the phase shift in the physical region near a resonance. The "position" of the resonance ought to be considered synonymous with the complex position of the pole, not with a point where the phase shift assumes any particular value. In Sec. VII the definition of the mass of an unstable particle is dealt with more generally. It should already be clear from the foregoing, however, that the Breit-Wigner location parameters, s_{pr} and Δ_p , do

not depend on the particular reaction in which the resonance is observed. It is the residue of the pole and the background that differ, not the position.

If the next nearest singularity happens to be a two-particle channel threshold and the S-matrix element in question connects two-particle channels, then the threshold branch point may be removed by employing the K matrix rather than the S matrix.² (An example of the K matrix is given in the following section.) In fact, any finite number of two-particle channel thresholds may be removed through the K matrix. This technique enlarges the circle of convergence of the power series and correspondingly improves the extrapolation accuracy. It also may allow several different S-matrix elements that share the same pole and threshold branch points to be employed collaboratively in finding the common pole.

IV. EXTRAPOLATION TO POLES ON THE REAL AXIS

As a second example let us consider a pole on the real s axis at $s = s_p$, slightly below the lowest threshold. Figure 1 shows such a pole on the physical sheet, where we have already stated the physical interpretation as that of a stable particle of mass $\sqrt{s_p}$. Figure 4 shows a pole equally close to the beginning of the physical region but lying on an unphysical sheet. Either type of pole may be reliably deduced by extrapolation, if the data near the first threshold is sufficiently accurate and if other singularities lie sufficiently far away.

Suppose, for example, that we consider $S_{11}(s)$, where c_1 is the channel with the lowest threshold. It can be shown² that the branch point at this threshold is absent in the function

$$K_1(s) = i \rho_1(s) \frac{S_{11}(s) + 1}{S_{11}(s) - 1}, \quad (\text{IV.1})$$

where

$$\rho_1(s) = q_1(s)^{2\ell+1} \quad (\text{IV.2})$$

is the phase-space factor for channel 1, if q_1 is the barycentric-system momentum for channel 1 and if ℓ is the orbital angular momentum. It follows, furthermore, from the form of (IV.1), that $K_1(s)$ does not contain any of the poles of $S_{11}(s)$, so if we expand $K_1(s)$ about the threshold s_1 for channel 1,

$$K_1(s) = K_1(s_1) + (s - s_1) K'_1(s_1) + \dots, \quad (\text{IV.3})$$

the expansion converges in a circle passing either through the next nearest branch point of $S_{11}(s)$ or the nearest zero of $S_{11}(s) - 1$, whichever is closer. It is easily verified that in the physical region

$$K_1(s) = \rho_1(s) \cot \delta_1(s), \quad (\text{IV.4})$$

so we are here talking about what is usually called an effective-range expansion. If the pole under study is sufficiently close to s_1 , it is a reasonable approximation to keep only the first term of (IV.3), leading to the function

$$S_{11}(s) = \frac{K_1(s) + i \rho_1(s)}{K_1(s) - i \rho_1(s)} \quad (\text{IV.5})$$

having its pole roughly at

$$\rho_1(s) \approx -i K_1(s_1). \quad (\text{IV.6})$$

For $\ell = 0$, we usually write $K_1(s_1)$ as a_1^{-1} , where a_1 is called the scattering length. The position of the pole is then at

$$q_1(s) \approx -i a_1^{-1}, \quad (\text{IV.7})$$

on the physical sheet if a_1 is negative but on an unphysical sheet if a_1 is positive.* The best known examples of such poles are in the two

* The function $q_1(s)$ behaves like $+\sqrt{s - s_1}$ just above s_1 in the physical region. Therefore $q_1(s)$ behaves like $+i\sqrt{s - s_1}$ on the real axis of the physical sheet just below s_1 and like $-i\sqrt{s - s_1}$ on the unphysical sheet.

$l = 0$ states of the two-nucleon system, where the isotopic-spin-0 pole lies on the physical sheet (the deuteron), while the isotopic-spin-1 pole is on an unphysical sheet. In both these cases the second and third terms of the expansion (IV.3) have been accurately determined, and the poles turn out to be close, so the positions and residues are known with great precision.

A reminding word of caution is required about the above extrapolation method, and in fact about all extrapolations. The form of the expression (IV.5) suggests that necessarily there are poles of S_{11} somewhere, but for a pole excessively distant from the physical region, the position and residue will be difficult to establish. In particular the expansion (IV.3) is unreliable if the pole falls into a region populated by the "left-hand" branch points mentioned in the following section. Fortunately the location of all such branch points can be determined in advance from general principles.¹

V. GENERAL PROBLEM OF EXTRAPOLATION, PARTICLE MULTIPLICITY

No attempt is made here to survey all the extrapolation methods that over the years have been devised. Some, for example, have managed to include the nearby "left-hand" cuts of partial-wave amplitudes,³ that arise from singularities originally present in momentum-transfer variables but transmitted to the energy variable in the process of projecting out a definite J value.* Some techniques have eliminated or at least suppressed the effect of multiparticle channel thresholds, as well as the two-particle thresholds.⁴ We also do not propose here to evaluate the reliability of the specific pole determinations that have actually been attempted. In many cases the experimental data currently available is insufficient to reach firm conclusions about whether a pole is or is not present in a particular region. The essential point however is that, in principle, data of sufficient accuracy always will answer this question. Thus, to discover a new particle the task of the experimenter is to establish by extrapolation of his data the existence of a simple pole in some channel invariant of the S matrix. To assist the extrapolation he should employ all available information about other singularities.

The quantum numbers of the particle are the quantum numbers of the communicating channels, and here we should mention the matter of multiplicity--a semantic question that sometimes causes confusion. If the S matrix possesses an exact symmetry, such as rotational invariance, then there exist multiplets of equivalent channels and,

* Although left-hand branch points may be ignored in specifying pole location, they do affect the extrapolation.

correspondingly, multiplets of equivalent poles. Rotational invariance, for example, leads to multiplicities $2J + 1$ and one might say that there always exist $2J + 1$ different particles of exactly the same mass and other quantum numbers. The usual convention, of course, refers to each rotational multiplet as one particle.

Such a convention causes no confusion for an exact and completely understood symmetry, but with approximate or accidental degeneracies it seems better to say that each different simple pole corresponds to a different particle. Most of the time this latter practice is followed, but occasionally one hears the term "particle" or "resonance" employed in referring to a "bump" which detailed analysis has shown to be associated with two or more different poles of the S matrix that happen to lie near each other. Such terminology is misleading and should be avoided. Unless an exact symmetry is involved, each simple pole of the S matrix is in principle separately identifiable and may be assigned a definite set of conserved quantum numbers.

Theorists are not quite unanimous in their attitude toward poles of higher order than first in the S matrix. No experimental indication of such poles has ever been found, and theoretical models of the S matrix have not produced multiple poles--except accidentally. It would seem sensible to reserve the term "particle" for simple poles. When and if experiments show the existence of multiple poles, we can face the problem of finding a good name for them. It goes without saying that the term particle or resonance should not be employed to describe phenomena associated with branch points. Sometimes branch points of the S matrix, acting alone, are of such character as to produce energy

peaks in cross sections. In defining a particle, however, it is the pole and not the peak that is of paramount significance. Careful experimental analysis can reveal the difference between poles and branch points.

Having made these last remarks, we must hasten to add that a pole hiding behind a threshold branch point, as in Fig. 4, should be considered just as good a particle as any other. Often one hears used the term "threshold enhancement" to describe the kind of situation shown in Fig. 4, with the implication that one is not dealing here with a "true" particle. If, however, extrapolation around the branch point clearly indicates the existence of a pole, the experimenter has discovered a particle in as valid a sense as when no branch point intervenes. He should not be diffident in reporting his discovery.

We add parenthetically the remark that if, near a threshold, any individual partial-wave cross section varies rapidly by an amount comparable to the unitarity limit, careful extrapolation has usually revealed a nearby pole as the culprit. This remark is no substitute for detailed analysis, which must always be carried out, but it is a fact that most so-called "threshold enhancements" have turned out to be reflections of nearby poles. When poles are absent, threshold effects are usually too weak to be observable.

The type of branch point that can most easily produce confusing energy peaks is not associated with simple energy thresholds but instead is a so-called "Landau singularity" whose position depends on both energy and momentum transfer.⁵ The position in momentum transfer may come close

to the physical region for a narrow range of energies, so that a peaking effect results.* The clue to identifying such a phenomenon is the simultaneous dependence on momentum transfer. The peaking occurs not only in energy but also in angle--favoring either forward or backward directions. A true particle-pole appears in only one channel invariant.

* A good example is pion-deuteron elastic scattering where the strong forward angular peak exhibits a peaking in energy around points corresponding to pion-nucleon resonances. These peaks are not pion-deuteron resonances. The dominant Landau singularity in this example may be identified with the so-called "impulse approximation."

VI. POLE LOCATIONS AND PARTICLE MASSES

In the illustrations of Sections III and IV we discussed two quite different types of pole locations on unphysical sheets, and theoretical models of the S matrix suggest that an almost unlimited variety of regions can be populated by poles. No doubt one reason that experimenters draw back from an uninhibited pole-particle association is that for most pole locations they find the concept of particle-mass confusing. On the physical sheet, of course, one makes the association

$$m_p^2 = s_p, \quad (\text{VI.1})$$

and it is natural to attempt an extension of this formula to define the "mass" of any particle corresponding to an unphysical-sheet pole. If the pole is far from the physical region, however, the usual intuitive meaning for "mass" becomes blurred. In fact the "mass" is not simply a complex number, since a specification is required of the sheet on which the pole lies. Thus, so long as the experimenter insists that a particle be completely characterized by a set of numbers for spin, mass, etc., he is prevented from considering the great bulk of particle-poles.

Take the example of Fig. 4. To say that the "mass" of this particle is $\sqrt{s_p}$ would ignore the distinction with Fig. 1. The pole of Fig. 4 is not a stable particle. Often a special term, "virtual state", or "antibound state" is used to describe this particular pole location, but such terms tend to perpetuate the myth that this not a true particle. The only viable attitude is to accept

pole "position" in the Riemann surface as one of the properties to be specified for each particle. The mass concept must be generalized in this fashion.

The majority of particle poles discovered to date have locations corresponding to Fig. 2, the position being specified by two real numbers s_{pr} and Δ_p if it is understood that the sheet involved is that immediately adjacent to the physical region. Such poles are discovered with relative ease for the obvious reason that they are the closest to the physical region. As experimental techniques develop, however, poles will be identified in wider and wider regions. For example, a possible new type of pole location that may soon be clarified is shown in Fig. 5. The gross manifestation of such a pole has been called the "woolly cusp".⁶ The pole hides behind the threshold branch point for an unstable-particle channel (such as $N\Delta$) in much the same sense as in Fig. 4. The difference is that the shielding branch point is now itself displaced from the physical region, so the observed effect in the physical region is relatively smooth.

VII. DYNAMICAL EQUIVALENCE OF DIFFERENT POLE POSITIONS

An important general characteristic of a particle-pole is that the dependence of the residue on initial and final channels may be factored. Thus, if we have

$$S_{c'c}(s) \underset{s \rightarrow s_p}{\sim} \frac{\gamma_{c'c}}{s - s_p}, \quad (\text{VII.1})$$

then

$$\gamma_{c'c} = g_{c'} g_c, \quad (\text{VII.2})$$

where g_c is often referred to as the "coupling constant" of the particle p for the channel c . This factorizability of the residue is important to the interpretation of the pole as a particle. It means that the state of the system represented by the pole has no "memory" of its origin. The state decays or interacts with other particles in a manner independent of the mechanism that originally produced the state. A crucial issue for our point of view is that not only stable-particle poles but also all unphysical-sheet simple poles in individual channel invariants should have factorizable residues. This property has in fact been established by a variety of theoretical arguments. (If K-matrix methods are employed in extrapolating physical data, pole residues emerge automatically in a factored form.)

The demonstration of factorizability does not depend on the location of the pole, which may be on the physical sheet or many sheets removed from the physical sheet. It may be near or far from other singularities. There is no hint in this property of a distinction

between one class of poles and another.

Potential models of the S matrix, as well as certain well-established experimental facts, give additional powerful reasons for believing that all positions for hadron poles stand on a fundamentally equivalent basis. Consider, for example, the $\frac{3}{2}^+$ decuplet of baryons which have been so beautifully correlated with SU_3 rules. One member of the decuplet, the Ω^- , is stable with respect to strong interactions, while the other nine members are unstable. Thus one pole is on the physical sheet, while nine are in the position category discussed in Sec. III. What better evidence could exist of the basic equivalence of these two regions of the Riemann surface? Furthermore, from the point of view of a potential model, a pole that is primarily a bound state of channels with orbital angular momentum greater than zero can be moved continuously from the unstable region of Sec. III onto the physical sheet by smoothly increasing the attractive strength of the potential. Whether a pole appears on the physical sheet or off is an "accident" depending on the precise potential strength.

Potential models for $l = 0$ similarly show how a pole on the physical sheet can move up to the lowest threshold as the attraction is decreased and then continuously back on the real axis of the adjacent unphysical sheet into the location discussed in Sec. IV. Again, it is a dynamical accident of no great significance that determines precisely where the pole is located.

A related and striking feature of potential models is the inevitable occurrence of an infinite number of poles, any one of which

can be brought onto the physical sheet by making the attraction sufficiently strong. At interaction strengths of the magnitude observed in nature for hadrons, relatively few poles are close enough to the physical region to be easily observable. But as experimental ingenuity and precision increase we must expect an ever-increasing number to reveal themselves--in all sorts of locations. There is no hint either in potential models or in relativistic S-matrix theory of dividing lines that can be used to sharply classify different regions of the Riemann surface.

The most awkward obstacle to a one-to-one pole-particle correspondence, so far recognized by theorists, is that associated with "weak" threshold branch points that happen to occur near a pole. Dynamically speaking, a weak threshold branch point arises from a channel relatively weakly coupled (usually because of phase-space considerations) to the pole. A good example is the $\pi\pi N$ threshold, which is close to the communicating $\Delta(\frac{3}{2}^+, 1238)$ pole while at the same time the partial width is small for the Δ to decay into the $\pi\pi N$ channel. Because the change in the S matrix is small in making one circuit around the weak branch point (that's what "weak" means), there must be a pole on the corresponding adjacent sheet whose location and residue are close to those of the original pole. (See Fig. 6.) The existence of the second pole is so closely connected to that of the first that physicists usually find it unnatural here to speak of two different particles.

Again, however, there will inevitably be cases of intermediate-strength threshold branch points for which the "shadow" pole is sufficiently displaced in position and residue so that it becomes difficult to identify uniquely with a "primary" pole. If a completely general and precise definition is sought for the term "particle", therefore, it does not seem possible to group poles into unique families--an entire family corresponding to one particle--unless recourse is made to S-matrix principles that are still controversial.*

A similar-looking but less awkward complication for a one-to-one pole-particle correspondence arises from the S-matrix property called "Hermitian analyticity",¹ which decrees that poles in complex locations must occur in complex-conjugate pairs. Here there is no ambiguity about the pairing, however, and extrapolation techniques conforming to Hermitian analyticity (such as the K-matrix method) automatically include both members of the pole-pair. It may be remarked that one member of the pair is always closer to the physical region than the other and correspondingly makes its presence more obvious to the experimenter.

* Jones has studied the problem of pole families for poles lying on Regge trajectories.⁷ It is well known that each trajectory encompasses an infinite set of poles of different angular momentum. Jones emphasized that in addition each trajectory possesses a unique continuation onto any sheet of the Riemann surface in s and may cross the same physical J value on two or more different sheets. It is unambiguous and perhaps natural to group together all poles of the same J that belong to the same Regge trajectory and to call this collection one particle.

VIII. CONCLUSION

The motivation for this paper arose from the impression, acquired from many conversations, that experimenters by and large regard the notion of S-matrix poles as a controversial theoretical conjecture that has a dubious role in the interpretation of data. In fact, many properties of poles are solidly established and non-controversial. We have here emphasized the following:

- (a) the uniqueness of pole-position (in the Riemann surface determined by threshold branch points) independent of the particular reaction considered. This position is thus the natural and unambiguous generalization of the particle-mass concept,
- (b) the factorizability of all pole residues,
- (c) the dynamical equivalence of a variety of different regions on the unphysical sheets of the Riemann surface,
- (d) the possibility of unique extrapolation from the physical region in order to determine pole positions and residues.

There seems no reason why experimenters should hesitate to use pole properties in data interpretation, any more than they hesitate to use S-matrix properties such as Lorentz invariance and unitarity. Poles have implicitly been recognized for many years through the use of the Breit-Wigner formula, but their relevance to other extrapolation procedures, such as the K matrix (effective range expansion), has not always been appreciated. The possibility of disentangling poles by careful measurement and analysis of such phenomena as "woolly cusps" remains totally unexplored.

The reader may wonder why no mention has been made here of poles in momentum transfer variables, whose existence follows from the S-matrix principle of "crossing". The reason is purely practical: momentum-transfer poles generally lie so far from the physical region that their determination by extrapolation is orders of magnitude more difficult than for the corresponding energy poles. The single exception is the pion-pole, which happens to lie close to the edge of the physical momentum-transfer region in a number of reactions. Experimenters are well aware of this exceptional case and familiar with methods for exploiting it--which are commonly described by the term "peripheralism".

The reader also may wonder why Regge poles have been ignored (except for the footnote in Sec. VI). The reason is that at present this remains an area of serious controversy among theorists. The spirit of this review has been to emphasize only principles that have received overwhelming theoretical acceptance.

REFERENCES

1. Fairly complete treatments of noncontroversial S-matrix properties may be found in papers by D. Olive, Phys. Rev. 135, B745 (1964) and H. P. Stapp, Phys. Rev. 139, B257 (1965). See also R. J. Eden, P. Landshoff, D. Olive, and J. C. Polkinghorne, The Analytic S Matrix (Cambridge University Press, New York, 1966).
2. See, for example, R. H. Dalitz and S. F. Tuan, Ann. Phys. (N. Y.) 10, 307 (1960). Also F. Uchiyama-Campbell and R. K. Logan, Phys. Rev. 149, 1220 (1966).
3. H. P. Noyes and D. Wong, Phys. Rev. Letters 3, 191 (1959).
4. J. Ball and W. Frazer, Phys. Rev. Letters 7, 204 (1961).
5. L. D. Landau, Nuc. Phys. 13, 181 (1959).
6. M. Nauenberg and A. Pais, Phys. Rev. 126, 360 (1962).
7. C. E. Jones, Ann. Phys. 31, 481 (1965).

FIGURE LEGENDS

Fig. 1. A typical physical sheet, crosses denoting normal threshold branch points and the dot a possible stable particle pole. The physical region is denoted by a dashed line, while the heavy lines are branch cuts, drawn at a slight angle so as not to conceal one another.

Fig. 2. The s complex plane with cuts drawn so as to expose a pole lying just below that physical sector which is bounded by the second and third normal thresholds.

Fig. 3. The circle of convergence of the Breit-Wigner expansion when the first neglected singularity is a threshold branch point.

Fig. 4. Unphysical-sheet pole lying on the real s axis below the lowest threshold.

Fig. 5. A conjectured pole location for channels with zero strangeness and baryon number two. Experiments suggest such a pole for $J = 2^+$, $I = 1$.

Fig. 6. The Δ pole and its "shadow" reached by a single circuit around the $\pi\pi N$ normal threshold branch point.

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Fig. 1

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Fig. 2

S

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Fig. 3

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Fig. 4

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Fig. 5

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Fig. 6

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