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IN NUCLEAR EMULSIONS

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January 12, 1954

Berkeley, California

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ABSTRACT

Stacks of nuclear emulsion varying in thickness from 18.35 g/cm^2 to 56.83 g/cm^2 were exposed in the external proton beam of the Berkeley cyclotron, and the average specific ionization dT/dx (in $\text{Mev}(\text{g/cm}^2)^{-1}$) was measured by determining the equivalent thickness of Cu needed to remove the same amount of energy from the beam as a given thickness of emulsion. The energies range from 230 to 320 Mev. The emulsion stacks consisted of layers of 200- μ Ilford C-2 stripped emulsions of average density $3.81 \pm 0.01 \text{ g/cm}^3$. The specific ionization was found to be 1.5 percent higher for emulsion than for Cu measured over the same energy interval. From these data the ratio of the mean ionization potentials for emulsion and Cu was determined as $I_E/I_{Cu} = 0.89 \pm 0.09$. The range of 342.5-Mev protons in emulsion was found to be $92.68 \pm 0.25 \text{ g/cm}^2$, which is 1.1 percent lower than the measured range of the same beam in Cu ($93.65 \pm 0.20 \text{ g/cm}^2$). The measured range in emulsion is about 1.5 percent shorter than the value calculated by Vigneron (and extended to higher energies by Barkas). They assumed 332 ev as the mean ionization potential of emulsion.

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I INTRODUCTION

The recent introduction of nuclear emulsion stacks into work with cosmic rays and with high-energy accelerators has led to the systematic investigation of very long tracks. The range-energy relation for such tracks becomes of considerable importance in many problems such as the decay schemes of heavy mesons and hyperons or the energy balance in nuclear stars; it was thus decided to extend the experimental measurements of the range-energy relation to higher energies.

Considerable experimental data exist for proton energies up to about 40 Mev,¹ including an accurate measurement recently performed at this laboratory at 33.64 Mev proton energy.² These energies correspond to track lengths of the order of 0.5 cm, whereas the tracks studied in emulsion stacks may be 10 cm and more in length. A calculation of the proton ranges up to 230 Mev has been carried out by Vigneron¹ and extended by Barkas³ to 30 Bev.

The purpose of this experiment is thus (1) to measure a point on the range-energy curve at a proton energy corresponding to a track length of about 25 cm which would serve as an experimental check on the calculated value of the range, and (2) to measure the specific ionization of emulsion relative to that of copper. We shall define specific ionization to mean the quantity $\frac{dT}{dx}$ expressed in units of $\text{Mev}(\text{g}/\text{cm}^2)^{-1}$. The latter measurement allows one to calculate the mean ionization potential of emulsion, assuming some previously determined value for the mean ionization potential of copper.

II EXPERIMENTAL PROCEDURE

Let us first consider the experimental procedure for the measurement of the specific ionization. The experimental arrangement is shown in Fig. 1. The external proton beam of the 184-inch cyclotron passes through a collimator, 40 in. long and of adjustable opening, and emerges from the concrete shielding surrounding the machine at the snout S. The beam next passes through a thin ion chamber (I. C. 1), through a known thickness of emulsion E, and then through a variable thickness of copper consisting of a thick copper absorber Cu and thin copper absorbers mounted on the carriage C. This carriage is remote-controlled, and the amount of copper in the beam at any one time can be changed from 0.56 g/cm^2 to 3.36 g/cm^2 in six equal steps of 0.56 g/cm^2 . Finally the beam passes through a second ion chamber (I. C. 2) identical with the first one.

If we make a plot of the ratio of the ion chamber outputs I_2/I_1 as a function of absorber thickness we obtain the well known Bragg curve (see Fig. 2). We first determine the amount of energy ΔT removed from the beam by a thickness of emulsion ΔR_E . This is done by simply measuring the range of the beam (in copper) with and without the emulsion in the position shown in Fig. 1. The difference between the two range values for copper gives the thickness of copper ΔR_{Cu} needed to remove the same increment of energy ΔT from the beam as the emulsion. The quantities $\Delta T/\Delta R_E$ and $\Delta T/\Delta R_{Cu}$ give us the average specific ionization in emulsion and in copper measured over the same energy interval.

The apparatus for the measurement of the range in emulsion was the same as described above except that all the copper, including the absorbers on the carriage, was replaced by emulsion. Using this arrangement another Bragg curve was run. Here again the energy of the beam was determined by its range in copper.

The emulsion absorber consisted of layers of 200- μ C-2 Ilford stripped emulsions held in a clamping device. Their size varied from 2 in. x 2 in. at the front to 4 in. x 5 in. at the back, to allow for the broadening of the beam due to multiple Coulomb scattering as the protons slow down. The entire emulsion stack was machined and weighed after it was clamped together to determine the thickness in the direction of the beam. The density was determined from the known volume and weight of the emulsion stack, and in addition the water content was determined by a chemical analysis before and after each run.

The ion chambers were filled with argon at slightly higher than atmospheric pressure. The collecting foils are about 2 in. apart and 4 in. in diameter, and are operated at 1,000 V positive with respect to a grounded foil in the center of the chamber.

III DISCUSSION

The experimental results consist of curves giving the ratio of the ion-chamber outputs as a function of absorber thickness, as illustrated in Fig. 2. From a curve of this type we now wish to obtain the "range" of the particles in the absorber and--for copper--determine the energy of the particles by use of the known range-energy relations. We shall here outline only very briefly the method employed for obtaining the range, further details of which may be found in references 4 and 5.

The response of the ion chamber placed at a distance R from the end of the range in the absorber material is of the form

$$I(R) = N_0 \int_{-\infty}^{\infty} P(r, R) i(r, R) dr \quad (1)$$

where $P(r, R)$ is the probability that a particle in the interval between r and $r + dr$ has an energy corresponding to a residual range R , (see Fig. 3) and $i(r, R)$ is the response of the ion chamber to a particle which has the residual range R when it passes the position r . Both r and R are measured from the end of the range of the particle, being positive towards the source of the beam. The total number of particles is N_0 , which we assume to be constant. We thus neglect the attenuation of the beam due to large-angle scattering and other nuclear interactions in going from one experimental point to the next. It can be shown that this introduces only a very minor change in the shape of the curve.

We now substitute the usual Gaussian range-straggling distribution (with a standard deviation α) for $P(r, R)$ and an empirical relation⁵ of the form

$$i(r, R) = i(r') = \text{constant } (r')^{-0.46} \quad (2)$$

as the ion chamber response for a single particle. The variable $r' = 2R - r$ represents the total residual range of the particle (in the absorber material) as it passes through the ion chamber. It is easy to see that $i(r') = 0$ for negative values of r' .

Making another substitution of variables in order to obtain the curve in units of a , we get as the final response of the ion chamber

$$I(x) = (\text{const.}) \int_0^{\infty} e^{-\frac{(x+t)^2}{2}} t^{-0.46} dt \quad (3)$$

where $x = -\frac{R}{a}$ and $t = \frac{r'}{a}$. Thus, if we assume only statistical straggling to be present the curve of $I(x)$ vs x should be identical with the experimentally observed Bragg curve. Note that the curve $I(x)$ vs x is independent of a .

The point on the experimental curve which corresponds to the value $x = 0$ on the theoretical curve gives us the experimental range R^* , since we have chosen the end of the range as our origin for x . (See curve a, Fig. 2.) To locate this point we first evaluate the ratio $I(0)/I(x)_{\max}$ by numerical integration of equation 3. This ratio turns out to be 0.82. We thus choose a point on the experimental curve whose ordinate is 0.82 times the peak ordinate, and the abscissa of this point represents the experimental range R^* .

From the experimental range R^* determined above we obtain the rectified experimental range R by correcting for the shortening of the tracks by multiple small-angle scattering in the absorber. It can be shown⁵ that this correction is fairly small, and for protons it is approximately given by

$$\frac{R - R^*}{R} = \frac{Z}{6400}$$

or about 1/2 percent for copper.

A comparison of the shapes of the theoretical and experimental response curves of the ion chamber yields an estimate of the energy spread of the proton beam. If we assume that the energy spread of the proton beam is gaussian we may fold this gaussian into the statistical straggling gaussian, and we again obtain a gaussian but with a larger standard deviation. We thus ascribe any broadening of the experimental curve with respect to the theoretical curve to the energy spread of the beam and hence

$$\sigma_{\text{Beam}} = \sqrt{\sigma_{\text{exp}}^2 - \sigma_{\text{th}}^2} = \frac{\Delta T}{\left(\frac{dT}{dx}\right)} \quad (4)$$

In the above expression σ_{th} is obtained by a numerical integration of the relativistic straggling formula,⁶

$$\sigma_{th}^2 = 4\pi z^2 e^4 N_a Z \int_0^T \left(\frac{dT}{dR}\right)^{-3} \left(\frac{1 - \beta^2/2}{1 - \beta^2}\right) dT. \quad (5)$$

The experimental standard deviation σ_{exp} is obtained by noting that the horizontal distance between $I(x)_{max}$ and $I(o)$ is $0.85 \sigma_{th}$ on the theoretical curve (curve a, Fig. 2). The same relation holds to a good approximation for the experimental curve, and we thus take the difference between the abscissa of the peak and the range as $0.85 \sigma_{exp}$. Substituting the values of σ_{exp} , σ_{th} , and the value of $\frac{dT}{dx}$ in Cu for 340-Mev protons, we obtain $\Delta T = \pm 1.5$ Mev.

Since the experiment indicated that both the ranges and the specific ionization are very closely the same for nuclear emulsions and for copper, the average ionization potential I_E for emulsion was evaluated relative to that of copper using the method outlined below.

At the energies we are considering, the rate of energy loss of a proton in matter can be written in the form⁷

$$\frac{dT}{dx} = \left(\frac{4\pi}{\beta^2}\right) \left(\frac{e^2}{mc^2}\right)^2 N \ln \frac{2 mc^2 \beta^2 e^{-\beta^2}}{\left(1 + \frac{m}{m_p}\right)^2 (1 - \beta^2) I} (mc^2/cm) \quad (6)$$

where $\beta = \frac{v}{c}$ for the incident particle

$\frac{e^2}{mc^2}$ = classical electron radius

N = number of electrons/cm³ of absorber

m = electron mass

m_p = proton mass

I = mean ionization potential

T = kinetic energy of incident particle

In this formulation the units of $\frac{dT}{dx}$ are mc^2/cm rather than the usual Mev/cm.

From the kinematical relationship between T and β we obtain an expression for dT as a function of β and $d\beta$

$$dT = \frac{(m_p c^2) \beta d\beta}{(1 - \beta^2)^{3/2}} \quad (7)$$

We now substitute the expressions for $\frac{dT}{dx}$ and dT from formulas 6 and 7 respectively into

$$\Delta R = \int_{T_1}^{T_2} \frac{dT}{\left(\frac{dT}{dx}\right)} \quad (8)$$

which thus becomes entirely a function of β and I . It will have the form

$$\Delta R = \frac{m_p c^2}{4\pi \left(\frac{e}{mc^2}\right)^2 N} \int_{\beta_1}^{\beta_2} \frac{\beta^3 d\beta}{(1 - \beta^2)^{3/2} \ln \frac{2 mc^2 \beta^2 e^{-\beta^2}}{\left(1 + \frac{m}{m_p}\right)^2 (1 - \beta^2) I}} \quad (9)$$

We now make the assumption that $I_E = \lambda I_{Cu}$ where λ is a constant whose numerical value is close to 1. Let us now introduce the following abbreviations:

$$A(\beta) = \frac{\beta^3}{(1 - \beta^2)^{3/2}}; \quad B(\beta) = \frac{2 mc^2 \beta^2 e^{-\beta^2}}{\left(1 + \frac{m}{m_p}\right)^2 (1 - \beta^2)} \quad \text{and } I = I_{Cu}.$$

We can then write

$$\frac{N_E \Delta R_E}{N_{Cu} \Delta R_{Cu}} = \frac{\int_{\beta_1}^{\beta_2} \frac{A(\beta) d\beta}{\ln \frac{B(\beta)}{I} - \ln \lambda}}{\int_{\beta_1}^{\beta_2} \frac{A(\beta) d\beta}{\ln \frac{B(\beta)}{I}}} \quad (10)$$

But since $\ln \frac{B(\beta)}{I} \gg \ln \lambda$ we can expand the integrand in the numerator of (10) in terms of the small quantity $\frac{\ln \lambda}{\ln \frac{B(\beta)}{I}}$. This enables us to solve for $\ln \lambda$ which is given by

$$\left\langle \ln \frac{2 mc^2 \beta^2 e^{-\beta^2}}{\left(1 + \frac{m}{m_p}\right)^2 (1 - \beta^2) I_{Cu}} \right\rangle^{-1} \ln \lambda = \left(\frac{N_E \Delta R_E}{N_{Cu} \Delta R_{Cu}} - 1 \right) \quad (11)$$

where the $\ln$ term has been averaged over the range of β appropriate to the ΔR under consideration. The $\ln$ term in equation 11 is a slowly varying function of both β and I , but λ is a fairly sensitive function of the electron densities N_E and N_{Cu} and the experimentally measured ratio $\frac{\Delta R_E}{\Delta R_{Cu}}$.

The density and composition of a nuclear emulsion is a function of its water content and we thus have to find a relation that gives us the electron density N_E for a given value of the physical density ρ . We assume that for small variations in ρ this relation may be written

$$N_E = a \rho + b \quad (12)$$

where the constants a and b are evaluated by substituting the known values of N_E and ρ at two points. The manufacturer⁸ furnishes the chemical composition of emulsions at two different values of the density: for dry emulsion, ($\rho = 4.18 \text{ g/cm}^3$), and for $\rho = 3.92 \text{ g/cm}^3$. The expression becomes

$$N_E = (0.253 \rho + 0.080) \times 10^{24} \text{ electrons/cm}^3. \quad (13)$$

From this formula we obtain an electron density of 1.044×10^{24} for the emulsion used in this experiment ($\rho = 3.81 \text{ g/cm}^2$).

IV RESULTS

In Table I are listed the experimental results for the measurement of the average specific ionization in emulsion over the energy intervals indicated. The beam energies were determined from the proton ranges in Cu and the range-energy curves of Aron.⁴ The energies listed in col. 2 are the midpoint energies of the intervals listed in col. 1. The values of $\frac{dT}{dx}$ shown in col. 5 and 6 were obtained by dividing ΔT (col. 1) by ΔR (col. 3 and 4 resp.). The values of λ listed were calculated from equation 11.

Table I
Experimental Results for Measurement of
Specific Ionization

column 1	col. 2	col. 3	col. 4	col. 5	col. 6	col. 7
$\Delta T(\text{Mev})$	Midpoint Energy T_M	$(\Delta R)_{EM}$ g/cm^2	$(\Delta R)_{Cu}$ g/cm^2	$\left(\frac{dT}{dx}\right)_{EM}$ $\text{Mev}(\text{g/cm}^2)^{-1}$	$\left(\frac{dT}{dx}\right)_{Cu}$ $\text{Mev}(\text{g/cm}^2)^{-1}$	λ
345 - 302	323.5	18.35	18.53	2.34	2.32	0.91
298 - 272	285.0	10.45	10.65	2.53	2.48	0.86
342 - 272	307.0	28.80	29.28	2.43	2.39	0.87
272 - 193	232.5	28.03	28.30	2.82	2.79	0.92
342 - 193	267.5	56.83	57.58	2.62	2.59	0.89

The average value of λ is 0.89 ± 0.09 and the average percent difference between $\left(\frac{dT}{dx}\right)_{EM}$ and $\left(\frac{dT}{dx}\right)_{Cu}$ = 1.36 percent. The estimated error of 10

percent in the value of λ is based on the estimated accuracy in the measurements of ΔR_E , ΔR_{Cu} and the values of N_E and N_{Cu} .

The second measurement was of the range of high-energy protons in emulsion. The experimental results and the calculated range (rectified experimental ranges R) are:

Range of beam in Cu	$93.65 \pm 0.20 \text{ g/cm}^2$
Energy of beam	$342.5 \pm 1.5 \text{ Mev}$
Range in emulsion	$92.68 \pm 0.25 \text{ g/cm}^2$
Calculated range	94.22 g/cm^2

The energy of the beam is again obtained from the curves of Aron,⁴ and the calculated value of the range in emulsion is from the work of Vigneron¹ and Barkas.³

The experiment thus indicates that the specific ionization (in $\text{Mev (g/cm}^2)^{-1}$) for protons in the energy region of 200 to 300 Mev is about 1 percent higher than the same quantity measured in Cu. It was also found that the range of 342-Mev protons in emulsion is about 1 percent lower than their range in Cu, which is in agreement with the $\frac{dT}{dx}$ measurements.

This fact is of some practical utility, since it permits one to substitute the well-known range-energy and $\frac{dT}{dx}$ curves of copper for the corresponding emulsion curves whenever calculations do not call for a high degree of accuracy.

The value of the mean ionization potential of emulsion upon which Vigneron based his range-energy calculations is $I_E = 332 \pm 25 \text{ ev}$. We found that $I_E = (0.89 \pm 0.09) I_{Cu}$, and if we take Aron's value⁴ for I_{Cu} (309.9 ev) which is based on the experiment of Mather and Segrè, we obtain for the mean ionization potential of emulsion $I_E = 276 \pm 28 \text{ ev}$.

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EXPERIMENTAL ARRANGEMENT.

MU-7027

Fig. 1

ION CHAMBER RESPONSE vs RANGE.

MU-7026

Fig. 2

RESIDUAL RANGE R AND POSITION VARIABLE r IN ABSORBER BLOCK.

MU-7025

Fig. 3