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UNIVERSITY OF CALIFORNIA  
SANTA CRUZ

**OCEAN TIDAL DISSIPATION AND ITS ROLE IN SOLAR SYSTEM SATELLITE  
EVOLUTION**

A dissertation submitted in partial satisfaction  
of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

EARTH SCIENCES (PLANETARY SCIENCES)

by

**Erinna M. Chen**

September 2013

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# Table of Contents

|                                                                                     |             |
|-------------------------------------------------------------------------------------|-------------|
| <b>List of Figures</b>                                                              | <b>vi</b>   |
| <b>List of Tables</b>                                                               | <b>ix</b>   |
| <b>Abstract</b>                                                                     | <b>x</b>    |
| <b>Dedication</b>                                                                   | <b>xii</b>  |
| <b>Acknowledgments</b>                                                              | <b>xiii</b> |
| <b>1 Introduction</b>                                                               | <b>1</b>    |
| 1.1 Tides and their role in satellite evolution . . . . .                           | 1           |
| 1.2 Tidal heating in global scale oceans: an overview . . . . .                     | 2           |
| 1.3 Portions of this thesis for publication . . . . .                               | 4           |
| <b>2 Tidally-driven flow in global satellite oceans</b>                             | <b>5</b>    |
| 2.1 Introduction . . . . .                                                          | 5           |
| 2.2 Shallow-water description of a global satellite ocean . . . . .                 | 6           |
| 2.2.1 Tidal potentials . . . . .                                                    | 7           |
| 2.2.2 Semi-analytic method of solution . . . . .                                    | 9           |
| 2.2.3 Derived quantities . . . . .                                                  | 13          |
| 2.2.4 General features of the shallow-water tidally-forced system . . . . .         | 19          |
| 2.3 Scaling laws for tidally-driven ocean flows . . . . .                           | 20          |
| 2.3.1 Scalings for the response to the westward obliquity tidal potential . . . . . | 22          |
| 2.3.2 Effective tidal quality factor $Q$ in a satellite ocean . . . . .             | 26          |
| 2.3.3 Scalings for the response to the eastward obliquity tide . . . . .            | 26          |
| 2.3.4 Scalings for the response to the eccentricity tide . . . . .                  | 29          |
| 2.4 Effective viscosity in a global ocean . . . . .                                 | 34          |
| 2.4.1 Numerical method . . . . .                                                    | 35          |
| 2.4.2 Scaling arguments . . . . .                                                   | 37          |
| 2.4.3 Summary . . . . .                                                             | 38          |
| 2.5 Implications for the outer Solar System satellites . . . . .                    | 39          |

|          |                                                                                                                  |           |
|----------|------------------------------------------------------------------------------------------------------------------|-----------|
| 2.5.1    | Calculating the available thermal energy . . . . .                                                               | 39        |
| 2.5.2    | Relative contributions from eccentricity and obliquity . . . . .                                                 | 42        |
| 2.5.3    | Global tidal heating pattern . . . . .                                                                           | 42        |
| 2.5.4    | Triton . . . . .                                                                                                 | 43        |
| 2.5.5    | Caveats . . . . .                                                                                                | 44        |
| 2.6      | Conclusion . . . . .                                                                                             | 45        |
| <b>3</b> | <b>Tidal dissipation in the lunar magma ocean and its effect on the early evolution of the Earth-Moon system</b> | <b>57</b> |
| 3.1      | Introduction . . . . .                                                                                           | 57        |
| 3.2      | Methods . . . . .                                                                                                | 58        |
| 3.2.1    | Coupled thermal-orbital evolution . . . . .                                                                      | 59        |
| 3.2.2    | Magma ocean dissipation . . . . .                                                                                | 62        |
| 3.2.3    | Cassini state obliquity . . . . .                                                                                | 64        |
| 3.2.4    | Solidification of the ocean . . . . .                                                                            | 65        |
| 3.3      | Results . . . . .                                                                                                | 66        |
| 3.4      | Discussion . . . . .                                                                                             | 67        |
| 3.4.1    | Inclination damping . . . . .                                                                                    | 69        |
| 3.4.2    | Dissipation in the early Earth . . . . .                                                                         | 70        |
| 3.4.3    | Caveats . . . . .                                                                                                | 71        |
| 3.5      | Conclusion . . . . .                                                                                             | 71        |
| <b>4</b> | <b>Summary</b>                                                                                                   | <b>79</b> |
| 4.1      | Constraining dissipation in a global scale ocean . . . . .                                                       | 79        |
| 4.2      | Applications of ocean tidal heating . . . . .                                                                    | 80        |
| 4.2.1    | Thermal budget of satellites in the outer Solar System . . . . .                                                 | 80        |
| 4.2.2    | The evolution of the early Earth-Moon system . . . . .                                                           | 80        |
| <b>A</b> | <b>Useful spherical harmonic relations</b>                                                                       | <b>82</b> |
| <b>B</b> | <b>Reconciliation of various forms of the Cassini state constraint equation</b>                                  | <b>83</b> |

|                                                                                               |           |
|-----------------------------------------------------------------------------------------------|-----------|
| <b>C A discrepancy in the Mignard equations for the evolution of the orbital eccentricity</b> | <b>84</b> |
| <b>References</b>                                                                             | <b>88</b> |

## List of Figures

- 1 Power spectrum with varying spherical harmonic degree  $l$  of the full semi-analytic solution described in §2.2.2 using parameters relevant to an ocean on Europa (Tables 1 and 2), here with a viscosity of  $\nu = 1.0 \times 10^2$  m<sup>2</sup>/s for both the obliquity and eccentricity tides. The response of each tidal component is separated. Note the significant separation between degrees 1 and 2 in the case of the westward obliquity response and degrees 3 and 4 for all other tidal components, justifying the scaling arguments presented in §2.3, §2.3.3 and §2.3.4. . . . . 50
- 2 Average kinetic energy associated with each of the tidal responses for an ocean on Europa with parameters from (Tables 1 and 2), where symbols indicate the values calculated from scaling arguments (§2.3, §2.3.3 and §2.3.4) and the solid lines indicate values calculated through the semi-analytic solution (§2.2.2). The shaded region indicates where the Reynolds number is less than 50 for this model ocean and the scaling argument predictions are no longer within 10% of the semi-analytic solution value (see (2.54)). The dashed line represents the effective viscosity for the response to the eccentricity tide and the dot-dashed line represents that for the obliquity tide, both of which are numerically-derived (see §2.4.2) using the same coefficient of drag ( $c_D = 0.002$ ). Note that these effective viscosities are not the same, making the obliquity tide the likely dominant contributor to the ocean energy dissipation (see §2.5.2 for further details). . . . . 51
- 3 Similar to Figure 2 but for average energy dissipation. . . . . 52
- 4 Results (diamonds) and linear fits (solid lines) from numerical runs presented in §2.4.2. The slopes are  $k_{obl} = 0.40$  and  $k_{ecc} = 0.13$  for (a) obliquity and (b) eccentricity, respectively. . . . . 53

|   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     |    |
|---|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 5 | Energy dissipated in various satellite oceans due to the obliquity and eccentricity tides calculated from the scalings summarized in Table 4. The parameters used are $h = 30$ km, $c_D = 0.002$ , and satellite parameters from Table 2. For satellites lying to the right of the dotted line, the obliquity response is the dominant contribution to dissipation. Here for plotting purposes, Triton has energy dissipation associated with the eccentricity, but in reality, this value should be zero. . . . .                                                                                                                                                                                                                                  | 54 |
| 6 | Thermal heat budget for various outer Solar System satellites assuming solid-body $Q = 100$ , a bottom drag of $c_D = 0.002$ , and ocean depth of $h = 30$ km; solid body tidal heating components are calculated assuming a (a) viscoelastic homogeneous internal structure and (b) completely fluid response. Details of the calculations are presented in §2.5.1, while the numerical quantities are presented in Table 3. Pie charts indicate relative contributions from radiogenic heating (blue), solid body eccentricity tidal heating (green), solid body obliquity tidal heating (yellow), ocean obliquity tidal heating (red), and ocean eccentricity tidal heating (grey). The size of the pie charts are scaled by $\log(R)$ . . . . . | 55 |
| 7 | Time-averaged energy dissipation pattern associated with (a) obliquity tidal response and (b) eccentricity tidal response for a model ocean on Europa. These are calculated from the numerical model presented in §2.4 with $c_D = 0.002$ and $h = 30$ km. . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | 56 |
| 8 | Geometry of the orbit described by the model in §3.2.1. The relevant angles are the inclination $i$ , the angle between the orbit normal ( $\hat{n}$ ) and the invariable pole ( $\hat{k}$ ), and the Moon's obliquity $\theta_0$ , the angle between the spin axis ( $\hat{s}$ ) and the orbit normal ( $\hat{n}$ ). The Earth's obliquity is denoted by $I$ . We assume that the Moon's obliquity occupies a Cassini state where $\hat{n}$ , $\hat{k}$ , and $\hat{s}$ all remain coplanar while $\hat{n}$ and $\hat{s}$ precess. . . . .                                                                                                                                                                                                         | 74 |
| 9 | Schematic for the coupled model described in §3.2. . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 75 |

|    |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |    |
|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 10 | Orbital and thermal evolution for a model without magma ocean dissipation for various tidal time lags of the Earth. Changing the time lag does not change the trajectory of the evolution, but controls the rate of orbital evolution. The dashed line in the top two panels indicates the current value of the orbital inclination. Even for large time lags, the inclination is not damped significantly. . . . .                                                                                                                                                                                                                                                                                                  | 76 |
| 11 | Orbital and thermal evolution for a model without magma ocean dissipation with $\Delta t_E = 10$ (black line as in Figure 10) and with the fully coupled model including ocean dissipation (red curves). Dissipation in the magma ocean can result in significant inclination damping. The Moon's obliquity controls this ocean dissipation. The Moon's obliquity is unlikely to have reached greater than $\sim 1^\circ$ prior to magma ocean dissipation; otherwise, the inclination would have been damped to a value lower than the presently observed value. . . . .                                                                                                                                            | 77 |
| 12 | Semi-major axis evolution for an analytic orbital evolution model with only dissipation in the Earth with constant tidal quality factor $Q$ . The grey shaded region is the timescale for magma ocean solidification. The red dotted line indicates the semi-major axis where the obliquity of the Moon is greater than $\sim 1^\circ$ ; if the Moon is still molten at this point, dissipation in the ocean is significant enough to damp the inclination below the present value. Thus, $Q$ of the early Earth is unlikely to be as low as it is at present ( $Q \sim 12$ ). We present estimates for equivalent $\Delta t_E$ for comparison to the fully coupled thermal-orbital model presented in §3.2. . . . . | 78 |

## List of Tables

|   |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |    |
|---|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| 1 | Constant parameters utilized in thermal energy calculations . . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 47 |
| 2 | Satellite specific parameters utilized in thermal energy calculations. The physical parameters on the left half of the table are taken from Schubert et al. (2004) for the Galilean satellites, Thomas (2010), Zebker et al. (2009) and Jacobson et al. (2006) for the Saturnian satellites, Thomas (1988) and Jacobson et al. (1992) for the Uranian satellites, and Thomas (2000) and Jacobson (2009) for Triton. The orbital parameters are taken from JPL satellite ephemerides ( <a href="http://ssd.jpl.nasa.gov/?sat_elem">http://ssd.jpl.nasa.gov/?sat_elem</a> ). Calculations for the quantities on the right side of the table are described in §2.5.1. The gravity coefficients for the Galiean satellites are from <i>Galileo</i> measurements (Schubert et al., 2004); others are calculated from a hydrostatic assumption. The obliquity for Titan is taken to be the observed value (Stiles et al., 2008) and is not a theoretical Cassini state value. . . . . | 47 |
| 3 | Energy contributions calculated from parameters presented in Tables 1 and 2 as described in §2.5.1. . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     | 48 |
| 4 | Summary of scalings presented in §2.3, §2.3.3 and §2.3.4. These scalings are accurate to 10% of the full semi-analytic solution of §2.2.2 when $Re = \frac{\Omega R^2}{\nu} > 50$ and $\epsilon = \frac{4\Omega^2 R^2}{gh} \lesssim \frac{4}{15}$ . The values presented for $\nu$ are numerically obtained (see §2.4.2). . . . .                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               | 49 |
| 5 | Physical parameters utilized in the model for Earth-Moon system evolution                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | 73 |

## Abstract

Ocean tidal dissipation and its role in satellite evolution

by

Erinna M. Chen

The history of satellites in the Solar System is quite diverse. For example, satellites like Io and Enceladus exhibit active volcanism currently, while satellites like Ganymede and Tethys show signs of geologic activity in the deep past, but not at present. The energy dissipated by tides has been identified as a major heat source for satellites, but calculations for satellite tidal dissipation primarily focus on dissipation in a solid layer, such as the ice shell. An exciting discovery of the NASA spacecraft missions *Galileo* and *Cassini* is that global-scale, deep, liquid water oceans are present on many of the outer Solar System satellites. Tyler (2008) suggested that tidal dissipation due to flow in these oceans could potentially be a significant and previously neglected source of heat. However, a critical free parameter in Tyler's model is the effective turbulent viscosity in the ocean. The value of the effective viscosity is unconstrained and because the amount of tidal dissipation scales with this parameter, the amount of ocean tidal dissipation is also unconstrained.

In order to address this uncertainty, we developed a numerical model that solves the shallow-water equations on a spherical shell and includes a nonlinear bottom friction parameterization for viscous dissipation. The bottom friction coefficient has a well-established value in the terrestrial literature; however, the nonlinearity of this term in the equations of motion make the model far more computationally expensive than a model that includes turbulent viscosity. Thus, we provide numerically-derived scalings that map the bottom friction coefficient and satellite parameters to an equivalent effective turbulent viscosity. Because tides depend on both the thermal structure of a satellite as well as characteristics of the satellite's orbit, models that couple thermal and orbital evolution are required to understand the history of a satellite. We use our numerically-derived scalings to adapt a coupled thermal-orbital model to include the effects of ocean tides on satellite evolution.

We applied these methods to explore whether or not ocean tidal dissipation was significant in the evolution of satellites of the Solar System. In the case of the outer Solar System

satellites, we find that the dominant contributors to the present thermal budgets are radiogenic heating and solid-body eccentricity tidal heating. The one notable exception, where ocean tidal heating may be important, is Neptune's satellite Triton. Because Triton's orbit is evolving inwards toward Neptune, ocean tides are growing over time. The dissipation of these tides could have led to the recent geologic activity on Triton.

A global magma ocean was present on the early Moon. By including the effects of tidal dissipation in this ocean in the orbital evolution of the early Earth-Moon system, we find that for consistency with the present day Earth-Moon system configuration, our models require that the global lunar magma ocean solidify prior to an orbital semi-major axis of  $\sim 30$  Earth radii. The timing of this orbital configuration is controlled by dissipation in the Earth and suggests that the Hadean Earth was significantly less dissipative than the present Earth.

To my family and friends.

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While I consistently complain about the city of Santa Cruz, I have no doubt that it was the perfect incubator for me. I was provided with the true necessities for growth: an incredibly beautiful natural environment and a safe supportive community. I will nostalgi-

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# 1 Introduction

One of the most exciting discoveries of the NASA spacecraft missions *Galileo* and *Cassini* is that global-scale, liquid water oceans are present and may be common on outer Solar System satellites. Based on spacecraft data, subsurface water oceans have been inferred to exist presently on at least Europa, Ganymede, Callisto, Titan and perhaps Enceladus (Khurana et al., 1998; Kivelson et al., 2002; Bills and Nimmo, 2011; Iess et al., 2012; Postberg et al., 2011). From observations of surface geology, it is likely that other satellites have had oceans play a role in their history (Smith et al., 1989; Pappalardo et al., 1997; Giese et al., 2008).

Large liquid water reservoirs are exciting from an astrobiological standpoint; on Earth, liquid water is a prerequisite for life. Even in the most extreme environments, e.g. within and underneath glaciers or on the deep ocean floor, where liquid water is present, some form of life has been found (Yayanos, 1995; Priscu et al., 1998; Abyzov et al., 1998; Kargel et al., 2000). From a geophysical standpoint, the long-term presence of subsurface oceans may help explain the geologic diversity that we observe amongst the satellites of the outer Solar System. In the case of the icy satellites, the presence of a global-scale ocean decouples the ice shell from a, presumably, more rigid, silicate-dominated mantle. A decoupled ice shell tends to have a significantly larger response to tides than a crust that is coupled to the silicate mantle. On satellites such as Europa, Ganymede, and Enceladus, widespread geologic resurfacing has been attributed to such an enhanced tidal response (Squyres et al., 1983; Ross and Schubert, 1989; Tobie et al., 2005; Bland et al., 2009).

## 1.1 Tides and their role in satellite evolution

Tides are perhaps the most important reason why satellite evolution can differ from the evolution of the terrestrial planets. Through tides, a satellite can tap into the potential energy associated with its orbit, which is a far larger reservoir of energy than any energy source internal to the satellite. Therefore, the thermal history of a satellite can be drastically altered by tides and their ability to transfer orbital energy into heat. The most dramatic example of this process is Io and the presence of active volcanism on this body (Peale et al., 1979). Understanding the evolution of a satellite is a coupled problem requiring understanding

of both the thermal and orbital history of the satellite. The magnitude of satellite tides is related to parameters of the orbit, such as eccentricity and the semi-major axis, as well as the internal structure of the satellite; as tides dissipate energy in the form of heat, both the orbit and the thermal structure and dissipative properties of the satellite are altered. The coupled behavior can be complicated and may involve thermal runaway and non-monotonic behavior, as is the case for Io (Peale et al., 1979; Ojakangas and Stevenson, 1986).

The details of tidal dissipation are poorly understood. Tidal dissipation is specific to the properties of the body; for primarily fluid bodies, such as gas giants, the breaking of internal gravity waves likely account for most of the dissipation (Ogilvie and Lin, 2004). For primarily solid bodies, such as satellites, tides and their associated dissipation are typically calculated assuming a viscoelastic rheology, such as Maxwell, Andrade or Burgers (e.g. Ross and Schubert (1989); Tobie et al. (2005); Efroimsky and Williams (2009); Castillo-Rogez et al. (2011); Nimmo et al. (2012)). While viscoelastic models are widely employed, they may not capture the full dissipative behavior of a body. In the case of Enceladus, it is difficult to explain the observed high heat flow with a standard Maxwell viscoelastic model (Porco et al., 2006; Meyer and Wisdom, 2007; Roberts and Nimmo, 2008; Howett et al., 2011).

## **1.2 Tidal heating in global scale oceans: an overview**

Even though it has been widely recognized that tides can dramatically alter the evolution of a satellite and that the presence of a subsurface global ocean likely enhances tidal responses, tidal dissipation within oceans themselves has been widely ignored. This dismissal may be attributed to two factors. First, the molecular viscosity of liquid water is quite small ( $\sim 10^{-6} \text{ m}^2/\text{s}$ ) and thus viscous dissipation should be negligible. However on Earth, a significant amount of energy is dissipated by tides in the ocean through turbulence and bottom friction (Munk and MacDonald, 1960); molecular viscosity does not necessarily determine the dissipative properties of the fluid. Second, for satellite solid-body tides, the dominant contributor to tidal dissipation is eccentricity tides. Assuming an equilibrium response to the eccentricity tide, the flow velocity in the ocean is quite small, less than 1 mm/s (Moore and Schubert, 2000; Tyler, 2008) and tidal dissipation is correspondingly

small. However, as was pointed out in Tyler (2008), the dominant response in a satellite ocean should be a dynamic/resonant response to the obliquity tide and not an equilibrium response due to the eccentricity tide. The resonant response is significantly larger than what would be predicted for an equilibrium response, i.e. flow velocities of  $\sim 10$  cm/s for a dynamic response as compared to less than 1 mm/s for an equilibrium response. This enhanced flow may dissipate significant energy even at small obliquities ( $\sim 0.1^\circ$ ) (Tyler, 2008).

While Tyler (2008, 2009, 2011) highlighted a previously unrecognized source of thermal energy, many questions stem from his work. First, for ocean tidal dissipation to be important in satellite evolution, the effective viscosity in the ocean cannot be the molecular viscosity of water; it must be significantly enhanced from the molecular value. Tyler (2011) leaves this effective viscosity as a free parameter. For satellite oceans, there is little indication of what the viscosity should be, and by analogy to liquid iron, the uncertainty in effective viscosity could be as large as ten orders of magnitude (Lumb and Aldridge, 1991). Because dissipation scales with this value, the amount of dissipation and, therefore, its relative importance is essentially unknown. Thus, a major open question is what the effective viscosity in a global satellite ocean is. In Chapter 2, we describe a model that employs a bottom friction formulation for dissipation to address this question. The benefit of our model is that the value for the bottom friction coefficient is well-established to within an order of magnitude (Jeffreys, 1921; Jayne and St. Laurent, 2001; Sohl et al., 1995). The challenge in using such a formulation is that the amount of computation required to solve such a model is orders of magnitude larger than a model that employs a linear viscous term such as that of Tyler (2011). We have therefore also developed scaling relationships to allow for easy implementation of ocean dissipation in thermal evolution codes; Chapter 3 describes such an implementation.

With the ability to constrain the amount of tidal dissipation occurring in a satellite ocean, we also present in Chapter 2 a calculation of the present thermal budget of the major outer Solar System satellites. We find that for many of these satellites, the present thermal budget is dominated by radiogenic heating and solid-body tidal dissipation due to eccentricity. The one notable exception is Triton, the presumably captured satellite of Neptune (Agnor and

Hamilton, 2006). Since surface imagery of Triton was returned by *Voyager I*, an open question has been why Triton looks to be recently resurfaced (Schenk and Zahnle, 2007). Eccentricity tides are unlikely to have caused the recent surface changes because the orbit is currently circular; however, obliquity tides have likely grown over time as Triton's orbit evolves inwards and can plausibly explain the young surface age.

For many of the outer Solar System satellites, tidal dissipation in a global ocean is likely limited because of small obliquity values at present. But it has long been recognized that the Moon likely had a period in its past where its obliquity was quite high, even as large as  $77^\circ$  (Ward, 1975). In Chapter 3, we present a coupled thermal-orbital model for the evolution of the early Earth-Moon system where we account for dissipation in a lunar magma ocean using the methods developed in Chapter 2. From this model, we have determined new constraints for the early Earth and Moon: first, the lunar magma ocean likely solidified prior to the large obliquity excursion described in Ward (1975); and second, the early Earth was much less dissipative than it is at present and this requirement likely precludes the presence of large scale water oceans on the Earth's surface during the Hadean epoch.

### **1.3 Portions of this thesis for publication**

The chapters constituting the body of this dissertation have been submitted for publication as follows:

- Chapter 2: Chen, E.M.A, F. Nimmo and G.A. Glatzmaier (2013), Tidal heating in icy satellite oceans, *Icarus*, *in revision*.

## 2 Tidally-driven flow in global satellite oceans

### 2.1 Introduction

Tidal heating clearly influences the present-day behavior of some planetary bodies, such as Io (Peale et al., 1979) and Enceladus (Spencer et al., 2006; Howett et al., 2011). It probably also played a role at earlier times elsewhere, including Europa (Husmann and Spohn, 2004), Ganymede (Showman et al., 1997), Triton (Jankowski et al., 1989), and the Moon (Garrick-Bethell et al., 2010), and may be important in some super-Earth exoplanets (Henning et al., 2009).

For solid bodies, the effects of tides and their associated dissipation are typically calculated assuming a viscoelastic rheology, such as Maxwell, Andrade or Burgers (e.g. Ross and Schubert (1989); Tobie et al. (2005); Efroimsky and Williams (2009); Castillo-Rogez et al. (2011); Nimmo et al. (2012)), though other processes (such as frictional heating, e.g. Nimmo and Gaidos (2002)) may also play a role. For primarily fluid bodies, such as giant planets, a significant component of dissipation is likely to be due to the breaking of internal gravity waves (Ogilvie and Lin, 2004). Lastly, fluid layers on or within solid bodies may also be a source of dissipation. On the Earth it is well-known that tidal dissipation occurs mainly in the oceans (Munk and MacDonald, 1960; Egbert and Ray, 2000; Ray et al., 2001). Global subsurface oceans are thought to occur on at least Europa, Ganymede, Callisto, Titan and perhaps Enceladus (Khurana et al., 1998; Kivelson et al., 2002; Bills and Nimmo, 2011; Iess et al., 2012; Postberg et al., 2011); the focus of this chapter is to examine tidal dissipation within such oceans.

In a prescient paper, Ross and Schubert (1989) discussed the possibility of tidal heating on Enceladus arising from turbulent dissipation in a subsurface ocean. More recently, Tyler (2011) expanded an analysis initially developed by Longuet-Higgins (1968) to investigate energy dissipation in tidally-driven satellite oceans. In Tyler (2011), a key free parameter is the linear drag constant  $\alpha$ , which can be related to a tidal quality factor  $Q$ . It is worth noting that the value of  $\alpha$  or  $Q$  is *a priori* very poorly known and the total energy dissipation scales linearly with the model's prescribed value.

In this chapter, we follow an analysis similar to that of Tyler (2011). However, we

depart from his approach in two important respects. First, we provide an estimate for  $Q$  using an approach and parameter values developed in studies of the Earth's oceans. Second, we present approximate scaling relationships which allow fluid dissipation to be calculated in a manner analogous to the well-known equations for solid body dissipation (e.g. Segatz et al. (1988); Ross and Schubert (1989); Wisdom (2008)). These scalings will facilitate investigation of long-term satellite evolution, in which the thermal and orbital histories are coupled (e.g. Ojakangas and Stevenson (1989); Hussmann and Spohn (2004); Bland et al. (2009); Meyer et al. (2010); Zhang and Nimmo (2012)).

The rest of this chapter is organized as follows. §2.2 reviews the shallow water equations appropriate for flow in global fluid layers on a rotating spherical shell. For clarity, we summarize a semi-analytic solution to these equations, similar to that adopted by Longuet-Higgins (1968) and Tyler (2011), and in addition, explicitly present the method and equations utilized to calculate quantities such as the average kinetic energy and energy dissipation. §2.3 simplifies this system and carries out an analytical study of the response of a shallow global ocean to tidal forcing, building on the method presented in §2.2. This novel analysis derives approximate scaling relationships for ocean tidal flow and the resulting dissipation under typical icy satellite parameters. The algebra involved can be tedious; Table 4 summarizes the key results. The advantage of these relationships is that they retain the fundamental physical effects while being somewhat simpler to implement than the method presented in §2.2. The results of §2.3 are expressed in terms of an unknown effective (presumably turbulent) viscosity. In §2.4, we present an estimate for this viscosity using a numerical technique based on analogy to frictional ocean dissipation on Earth. We discuss the applications and implications of these results in §2.5. In particular, ocean dissipation is unlikely to be a significant heat source unless the orbital eccentricity is very small; Triton is thus the most likely candidate for a satellite in which ocean tidal dissipation is significant.

## **2.2 Shallow-water description of a global satellite ocean**

Here we briefly review the equations of motion for a shallow global satellite ocean. These equations are equivalent to equations 3 and 4 presented in Tyler (2011); we present fully dimensional equations and explicitly expand these equations for the solutions to unknown

spherical harmonic coefficients.

The forced, dissipative, linear shallow water equations on a rotating sphere are (cf. Longuet-Higgins (1968) equations 13.1-13.3 or Tyler (2011) equations 3-4, noting the sign difference in the tidal potential term)

$$\frac{\partial \vec{u}}{\partial t} + 2\Omega \cos \theta \hat{r} \times \vec{u} = -g\vec{\nabla}\eta - \vec{\nabla}U - \alpha\vec{u} + \nu\nabla^2\vec{u} \quad (2.1)$$

and

$$\frac{\partial \eta}{\partial t} + h\vec{\nabla} \cdot \vec{u} = 0. \quad (2.2)$$

where  $\vec{u}$  is the radially-averaged, horizontal velocity vector,  $\Omega$  is the constant rotation rate,  $\hat{r}$  is the unit vector in the radial direction,  $g$  is the surface gravity,  $\eta$  is the vertical displacement of the surface, and  $h$  is the constant ocean depth. To maintain the linearity of the system, the dissipative term can be represented as either a linear process with a linear coefficient  $\alpha$  or a Navier-Stokes type viscosity with a viscous diffusivity of  $\nu$ . We assume no radial gradients in the unknown quantities  $\vec{u}$  and  $\eta$  such that the Laplacian operator,  $\nabla^2$ , has no radial components and thus,  $\nu$  represents an effective horizontal diffusivity.  $U$  represents the forcing potential due to tides, either eccentricity- or obliquity-related, also radially-averaged. Equations (2.1) and (2.2) are valid for incompressible flow under the assumptions that the thickness of the fluid layer is much smaller than the radius of the body ( $h \ll R$ ), the vertical displacement is much smaller than the layer thickness ( $\eta \ll h$ ) and fluid properties are constant (e.g.  $\alpha$  and  $\nu$ ). These equations ignore ocean stratification, and thus do not include the effects of internal tides. In addition, they do not include overlying ice shell rigidity, though this effect should be small (Matsuyama, 2012).

### 2.2.1 Tidal potentials

For synchronously rotating satellites, such as the regular satellites of Jupiter and Saturn, we are concerned with the ocean flow driven by the eccentricity,  $e$  of the orbit and the obliquity,  $\theta_0$ , the tilt of the rotational axis relative to the orbital axis. The forcing tidal potentials can be derived by assuming the planet is a point mass and calculating the gravitational potential due to this at every point on the satellite (cf. Kaula (1964); Murray and Dermott (1999)). The

tidal potentials are expressed to  $O(e^2)$  for the eccentricity tide and  $O(\theta_0)$  for the obliquity tide.

The obliquity tidal potential at a point of colatitude  $\theta$  and longitude  $\phi$  on a synchronously rotating satellite with small obliquity  $\theta_0$  (in radians) is a standing wave and can be written as the sum of an eastward and a westward propagating potential (cf. Tyler (2011) eq. (34))

$$U_{obl} = \frac{-3}{2} \Omega^2 R^2 \theta_0 \sin \theta \cos \theta (\cos(\phi - \Omega t) + \cos(\phi + \Omega t)). \quad (2.3)$$

We define Laplace spherical harmonics of degree  $l$  and order  $m$ ,  $Y_l^m$ , as

$$Y_l^m(\theta, \phi) \equiv \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos \theta) e^{im\phi} \quad (2.4)$$

employing a Condon-Shortley phase factor of  $(-1)^m$  for  $m > 0$ . These spherical harmonics are orthogonal under

$$\int_0^{2\pi} \int_0^\pi Y_l^m Y_{l'}^{m'*} \sin \theta d\theta d\phi = \delta_{l,l'} \delta_{m,m'} \quad (2.5)$$

where  $*$  denotes the complex conjugate and  $\delta$  is the Kronecker delta.

The obliquity tidal potential thus can be expressed in spherical harmonics as a westward propagating potential  $U_{obl,W}$ ,

$$\begin{aligned} U_{obl,W} &= \frac{3}{2} \sqrt{\frac{2\pi}{15}} \Omega^2 R^2 \theta_0 (e^{i\Omega t} Y_2^1 - e^{-i\Omega t} Y_2^{-1}) \\ &= 2 \left( \frac{3}{2} \sqrt{\frac{2\pi}{15}} \Omega^2 R^2 \theta_0 \right) \Re(e^{i\Omega t} Y_2^1) \equiv 2U_2^1 \Re(e^{i\Omega t} Y_2^1), \end{aligned} \quad (2.6)$$

and a symmetric eastward propagating potential  $U_{obl,E}$  for which the  $e^{i\Omega t}$  term is replaced by  $e^{-i\Omega t}$ .

The eccentricity tidal potential can be expressed as (Kaula, 1964) (cf. Tyler (2011) eq. (35))

$$U_{ecc} = \frac{-3}{4} \Omega^2 R^2 e \left[ -(3 \cos^2 \theta - 1) \cos \Omega t + \sin^2 \theta (3 \cos 2\phi \cos \Omega t + 4 \sin 2\phi \sin \Omega t) \right]. \quad (2.7)$$

For the subsequent analysis, the eccentricity tidal potential can be split into three separate components. There is an axisymmetric component,  $U_{ecc,rad}$ , (Tyler (2011) calls this the “radial” component)

$$\begin{aligned} U_{ecc,rad} &= \left( 3\sqrt{\frac{\pi}{5}} \Omega^2 R^2 e \cos \Omega t \right) Y_2^0 = \frac{1}{2} \left( 3\sqrt{\frac{\pi}{5}} \Omega^2 R^2 e \right) (e^{i\Omega t} + e^{-i\Omega t}) Y_2^0 \\ &\equiv U_2^0 (e^{i\Omega t} + e^{-i\Omega t}) Y_2^0 \end{aligned} \quad (2.8)$$

There is also an asymmetric librational component,  $U_{ecc,lib}$ , that can be split into a westward propagating potential

$$\begin{aligned} U_{ecc,lib,W} &= \left( \frac{3}{4} \sqrt{\frac{2\pi}{15}} \Omega^2 R^2 e \right) (e^{i\Omega t} Y_2^2 + e^{-i\Omega t} Y_2^{-2}) \\ &= 2 \left( \frac{3}{4} \sqrt{\frac{2\pi}{15}} \Omega^2 R^2 e \right) \Re(e^{i\Omega t} Y_2^2) \equiv 2U_{2,W}^2 \Re(e^{i\Omega t} Y_2^2) \end{aligned} \quad (2.9)$$

and an eastward propagating potential

$$\begin{aligned} U_{ecc,lib,E} &= \left( \frac{-21}{4} \sqrt{\frac{2\pi}{15}} \Omega^2 R^2 e \right) (e^{-i\Omega t} Y_2^2 + e^{i\Omega t} Y_2^{-2}) \\ &= 2 \left( \frac{-21}{4} \sqrt{\frac{2\pi}{15}} \Omega^2 R^2 e \right) \Re(e^{-i\Omega t} Y_2^2) \equiv 2U_{2,E}^2 \Re(e^{-i\Omega t} Y_2^2). \end{aligned} \quad (2.10)$$

## 2.2.2 Semi-analytic method of solution

The equations of motion (2.1) and (2.2) subject to the forcing tidal potentials described in §2.2.1 can be solved using a semi-analytic method set forth in Tyler (2011) and summarized here. In §2.3 following, we describe a similar method of solution which makes additional simplifying assumptions and in doing so, eliminates the need for numerical computation.

The horizontal velocity can be expressed using a Helmholtz decomposition as the sum of the gradient of a scalar potential  $\Phi$  and the curl of a streamfunction  $\Psi \hat{r}$  where (cf. Tyler (2011) equation 5)

$$\vec{u} = \vec{\nabla} \Phi + \vec{\nabla} \times \Psi \hat{r} = u_\theta \hat{\theta} + u_\phi \hat{\phi}. \quad (2.11)$$

with the components of velocity given by

$$u_\theta = \frac{1}{R} \frac{\partial \Phi}{\partial \theta} + \frac{1}{R \sin \theta} \frac{\partial \Psi}{\partial \phi} \quad (2.12)$$

and

$$u_\phi = \frac{1}{R \sin \theta} \frac{\partial \Phi}{\partial \phi} - \frac{1}{R} \frac{\partial \Psi}{\partial \theta}. \quad (2.13)$$

The divergence and radial component of the curl of (2.11) are

$$\vec{\nabla} \cdot \vec{u} = \nabla^2 \Phi, \quad (2.14)$$

$$\hat{r} \cdot (\vec{\nabla} \times \vec{u}) = -\nabla^2 \Psi. \quad (2.15)$$

Taking the divergence of the momentum equation (2.1) and making the substitution from (2.14) results in

$$\left[ \left( \frac{\partial}{\partial t} + \alpha - \nu \nabla^2 \right) \nabla^2 + \frac{2\Omega}{R^2} \frac{\partial}{\partial \phi} \right] \Phi + 2\Omega \left[ \cos \theta \nabla^2 - \frac{\sin \theta}{R^2} \frac{\partial}{\partial \theta} \right] \Psi = -g \nabla^2 \eta - \nabla^2 U. \quad (2.16)$$

Similarly, the radial component of the curl of the momentum equation with the substitution (2.15) is

$$\left[ \left( \frac{\partial}{\partial t} + \alpha - \nu \nabla^2 \right) \nabla^2 + \frac{2\Omega}{R^2} \frac{\partial}{\partial \phi} \right] \Psi - 2\Omega \left[ \cos \theta \nabla^2 - \frac{\sin \theta}{R^2} \frac{\partial}{\partial \theta} \right] \Phi = 0. \quad (2.17)$$

Equations (2.16) and (2.17) are equivalent to equations 3.7 of Longuet-Higgins (1968), with the dissipative terms added. Equations (2.16) and (2.17) are also equivalent to equations 12 and 13 of Tyler (2011) if one ignores the residual terms ( $R_1$  and  $R_2$ ) and does not make the substitution for  $\eta$  using the continuity equation (see (2.18) below).

Substituting (2.14) into (2.2), the surface displacement is dependent only on  $\Phi$ ,

$$\frac{\partial \eta}{\partial t} + h \nabla^2 \Phi = 0. \quad (2.18)$$

For the response to a westward propagating tidal potential, we can seek wave solutions

of the form

$$\Psi(\theta, \phi, t) = \sum_{m=1}^l \sum_{l=1}^{\infty} [\Psi_l^m e^{i\Omega t} Y_l^m + \Psi_l^{-m} e^{-i\Omega t} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{\infty} [\Psi_l^m e^{i\Omega t} Y_l^m] \right), \quad (2.19)$$

$$\Phi(\theta, \phi, t) = \sum_{m=1}^l \sum_{l=1}^{\infty} [\Phi_l^m e^{i\Omega t} Y_l^m + \Phi_l^{-m} e^{-i\Omega t} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{\infty} [\Phi_l^m e^{i\Omega t} Y_l^m] \right), \quad (2.20)$$

$$\eta(\theta, \phi, t) = \sum_{m=1}^l \sum_{l=1}^{\infty} [\eta_l^m e^{i\Omega t} Y_l^m + \eta_l^{-m} e^{-i\Omega t} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{\infty} [\eta_l^m e^{i\Omega t} Y_l^m] \right) \quad (2.21)$$

where the coefficients  $\Psi_l^m$ ,  $\Phi_l^m$ , and  $\eta_l^m$  are complex, thereby permitting a phase lag between the tidal forcing and the response. The response frequency is assumed to be the same as the forcing frequency.

Using equation (2.18) and the wave solutions for  $\Phi$  (2.20) and  $\eta$  (2.21), we have

$$\eta_l^m = -i \frac{h}{R^2 \Omega} l(l+1) \Phi_l^m. \quad (2.22)$$

Plugging these wave solutions into equations (2.16)-(2.17) and projecting the equations onto each spherical harmonic, we obtain equations describing solutions for the coefficients  $\Psi_l^m$  and  $\Phi_l^m$  that are coupled in  $l$  but separable in  $m$ ,

$$\left[ i\Omega + \alpha + \frac{\nu l(l+1)}{R^2} - \frac{2\Omega i m}{l(l+1)} \right] \Psi_l^m - 2\Omega \left[ \frac{l-1}{l} C_l^m \Phi_{l-1}^m + \frac{l+2}{l+1} C_{l+1}^m \Phi_{l+1}^m \right] = 0, \quad (2.23)$$

$$\left[ i\Omega + \alpha + \frac{\nu l(l+1)}{R^2} - \frac{2\Omega i m}{l(l+1)} - \frac{ighl(l+1)}{\Omega R^2} \right] \Phi_l^m + 2\Omega \left[ \frac{l-1}{l} C_l^m \Psi_{l-1}^m + \frac{l+2}{l+1} C_{l+1}^m \Psi_{l+1}^m \right] = \{ -U_2^1 \delta_{l,2} \delta_{m,1}, -U_{2,W}^2 \delta_{l,2} \delta_{m,2} \} \quad (2.24)$$

with the term on the right hand side of (2.24) corresponding to the forcing potential of interest, either westward obliquity or westward librational eccentricity. The constants  $C_l^m$

are due to differential operators of the spherical harmonics, and these are given by

$$C_l^m = \left( \frac{(l+m)(l-m)}{(2l+1)(2l-1)} \right)^{1/2}. \quad (2.25)$$

Equations (2.23) and (2.24) are effectively equations (3.19) of Longuet-Higgins (1968) with correction of the sign inconsistencies described in Appendix A.1 of Tyler (2011). The differences in the constants between the equations presented here and those of Longuet-Higgins (1968) and Tyler (2011) are primarily due to the use of normalized spherical harmonics throughout this work and a difference in the sign of the tidal potentials. For helpful relations, similar to (3.17) and (3.18) of Longuet-Higgins (1968), we refer the reader to Appendix A.

A similar projection can be obtained for the eastward forcing potentials by changing the direction of the response in (2.19)-(2.21). In this case, we seek wave solutions of the form:

$$\Psi(\theta, \phi, t) = \sum_{m=1}^l \sum_{l=1}^{\infty} [\Psi_l^m e^{-i\Omega t} Y_l^m + \Psi_l^{-m} e^{i\Omega t} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{\infty} [\Psi_l^m e^{-i\Omega t} Y_l^m] \right) \quad (2.26)$$

and similarly for  $\Phi$  and  $\eta$ .

The resulting coupled equations from substituting (2.26) and similar expressions for  $\Phi$  and  $\eta$  into (2.16)-(2.18) are

$$\left[ i\Omega - \alpha - \frac{\nu l(l+1)}{R^2} + \frac{2\Omega i m}{l(l+1)} \right] \Psi_l^m + 2\Omega \left[ \frac{l-1}{l} C_l^m \Phi_{l-1}^m + \frac{l+2}{l+1} C_{l+1}^m \Phi_{l+1}^m \right] = 0, \quad (2.27)$$

$$\left[ -i\Omega + \alpha + \frac{\nu l(l+1)}{R^2} - \frac{2\Omega i m}{l(l+1)} + \frac{ighl(l+1)}{\Omega R^2} \right] \Phi_l^m + 2\Omega \left[ \frac{l-1}{l} C_l^m \Psi_{l-1}^m + \frac{l+2}{l+1} C_{l+1}^m \Psi_{l+1}^m \right] = -U_2^1 \delta_{l,2} \delta_{m,1} - U_{2,E}^2 \delta_{l,2} \delta_{m,2} \quad (2.28)$$

with the terms on the right hand side of (2.28) corresponding to the eastward obliquity tidal potential and eastward librational eccentricity tidal potential.

For the prior wave solutions (e.g. (2.19)-(2.21) or (2.26)), we have not included the  $m = 0$  terms. The solutions for  $\Psi$ ,  $\Phi$ , and  $\eta$  must be real, and because the spherical harmonic  $Y_l^0$  is real, the corresponding coefficients  $\Psi_l^0$ ,  $\Phi_l^0$ , and  $\eta_l^0$  must be real. Because the linear system is separable in degree  $m$ , this only applies specifically for the radial component of

the eccentricity forcing. To solve for this response, we can break the tidal potential (2.8) into a westward and eastward potential in a manner similar to that for the obliquity tide and librational component of the eccentricity tide with a constraint that the total response must be real, i.e.

$$\Psi(\theta, \phi, t) = \sum_{l=1}^{\infty} [\Psi_{l,W}^0 e^{i\Omega t} + \Psi_{l,E}^0 e^{-i\Omega t}] Y_l^0 \in \mathbb{R}, \quad (2.29)$$

and similarly for  $\Phi$  and  $\eta$ . Here the coefficients are still complex and the subscripts  $W$  and  $E$  denoting the westward and eastward response, respectively. It can be shown that for (2.29) to be true, the relation

$$\Psi_{l,W}^0 = (\Psi_{l,E}^0)^* \quad (2.30)$$

must be true, and similarly for  $\Phi$  and  $\eta$ . We can solve either for the westward or eastward response, by substituting  $-U_2^0 \delta_{l,2} \delta_{m,0}$  for the right hand side of either (2.24) or (2.28) with the understanding that

$$\begin{aligned} \Psi(\theta, \phi, t) &= \sum_{l=1}^{\infty} [\Psi_{l,W}^0 e^{i\Omega t} + \Psi_{l,E}^0 e^{-i\Omega t}] Y_l^0 \\ &= 2 \sum_{l=1}^{\infty} \Re(\Psi_{l,W}^0 e^{i\Omega t}) Y_l^0 = 2 \sum_{l=1}^{\infty} \Re(\Psi_{l,E}^0 e^{-i\Omega t}) Y_l^0 \end{aligned} \quad (2.31)$$

and likewise for  $\Phi$  and  $\eta$ .

The linear system defined by equations (2.23) and (2.24) can be easily solved numerically for the coupled coefficients  $\Phi_l^m$  and  $\Psi_l^m$ , repeating for each potential of interest. While the series is infinite, typically the coefficients drop off rapidly with increasing  $l$  (see Figure 1). We exploit this fact in §2.3 to derive analytical scalings that reduce the complexity of implementing ocean tidal effects in thermal evolution models.

### 2.2.3 Derived quantities

For ease of solution, the fluid dynamic equations are expressed in terms of the streamfunction  $\Psi$  and scalar potential  $\Phi$ . However the quantities of interest are those such as kinetic energy or energy dissipation rate. Here we explicitly present the process to move from the spherical harmonic coefficients associated with the solution of the fluid dynamic equations to physical quantities.

Once the coefficients  $\Psi$  and  $\Phi$  have been determined for the ocean response, the kinetic energy of the ocean flow,  $E_{tot}$ , can be calculated from

$$E_{tot} = \frac{\rho}{2} \int_V \vec{u} \cdot \vec{u} dV. \quad (2.32)$$

We ignore the potential energy contribution to the total energy because it is negligible when compared to the kinetic energy. The kinetic energy expressed in terms of  $\Psi$  and  $\Phi$  is

$$E_{tot} = \frac{\rho}{2R^2} \int_V \left[ \left( \frac{\partial \Phi}{\partial \theta} + \frac{1}{\sin \theta} \frac{\partial \Psi}{\partial \phi} \right)^2 + \left( \frac{1}{\sin \theta} \frac{\partial \Phi}{\partial \phi} - \frac{\partial \Psi}{\partial \theta} \right)^2 \right] dV. \quad (2.33)$$

Because the energy is a nonlinear product, we must include both the westward and eastward responses, here denoted with the subscripts W and E, respectively. Expanding (2.33) in terms of the wave solutions from (2.19), (2.20) and (2.26) results in

$$\begin{aligned} E_{tot} = & \frac{\rho h}{2} \int_0^{2\pi} \int_0^\pi \left\{ \left[ \sum_{l=1}^{\infty} \sum_{m=1}^l \left( \Phi_{l,W}^m e^{i\Omega t} \frac{\partial Y_l^m}{\partial \theta} + \Phi_{l,W}^{-m} e^{-i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} + \Phi_{l,E}^m e^{-i\Omega t} \frac{\partial Y_l^m}{\partial \theta} \right. \right. \right. \\ & + \Phi_{l,E}^{-m} e^{i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} + \Psi_{l,W}^m e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} + \Psi_{l,W}^{-m} e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} \\ & \left. \left. + \Psi_{l,E}^m e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} + \Psi_{l,E}^{-m} e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} \right) \right]^2 + \\ & \left[ \sum_{l=1}^{\infty} \sum_{m=1}^l \left( \Phi_{l,W}^m e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} + \Phi_{l,W}^{-m} e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} + \Phi_{l,E}^m e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} \right. \right. \\ & + \Phi_{l,E}^{-m} e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} - \Psi_{l,W}^m e^{i\Omega t} \frac{\partial Y_l^m}{\partial \theta} - \Psi_{l,W}^{-m} e^{-i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} \\ & \left. \left. - \Psi_{l,E}^m e^{-i\Omega t} \frac{\partial Y_l^m}{\partial \theta} - \Psi_{l,E}^{-m} e^{i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} \right) \right]^2 \Big\} \sin \theta d\theta d\phi. \end{aligned} \quad (2.34)$$

By orthogonality of the spherical harmonics (2.5), the non-zero terms are

$$\begin{aligned} E_{tot} = & \rho h \int_0^{2\pi} \int_0^\pi \left\{ \sum_{l=1}^{\infty} \sum_{m=1}^l \left[ \left( \Phi_{l,W}^m \Phi_{l,W}^{-m} + \Phi_{l,E}^m \Phi_{l,E}^{-m} + \Psi_{l,W}^m \Psi_{l,W}^{-m} + \Psi_{l,E}^m \Psi_{l,E}^{-m} \right. \right. \right. \\ & + \left( \Phi_{l,W}^m \Phi_{l,E}^{-m} + \Psi_{l,W}^m \Psi_{l,E}^{-m} \right) e^{i2\Omega t} + \left( \Phi_{l,E}^m \Phi_{l,W}^{-m} + \Psi_{l,E}^m \Psi_{l,W}^{-m} \right) e^{-i2\Omega t} \Big) \\ & \left. \left( \frac{\partial Y_l^m}{\partial \theta} \frac{\partial Y_l^{-m}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial Y_l^m}{\partial \phi} \frac{\partial Y_l^{-m}}{\partial \phi} \right) \right] \Big\} \sin \theta d\theta d\phi. \end{aligned} \quad (2.35)$$

Substituting (A.5) into (2.35) and averaging over an orbital period, we find that the average kinetic energy is

$$E_{tot,avg} = \rho h \sum_{l=1}^{\infty} \sum_{m=1}^l l(l+1) (|\Phi_{l,W}^m|^2 + |\Phi_{l,E}^m|^2 + |\Psi_{l,W}^m|^2 + |\Psi_{l,E}^m|^2), \quad (2.36)$$

where the time-dependent components of (2.35) integrate to zero. This expression makes physical sense: the kinetic energy of the flow depends on the amplitude of the tidal response, but not the phase.

While (2.32) is nonlinear, the solution is separable in  $m$ . For ease in presentation, we calculate the  $m = 0$  component separately. The kinetic energy equation (2.33) can be expanded further in terms of the radial response from (2.31) resulting in

$$E_{rad,tot} = \frac{\rho h}{2} \int_0^{2\pi} \int_0^{\pi} \left\{ \left[ \sum_{l=1}^{\infty} (\Phi_{l,W}^0 e^{i\Omega t} + \Phi_{l,W}^{0*} e^{-i\Omega t}) \frac{\partial Y_l^0}{\partial \theta} \right]^2 + \left[ - \sum_{l=1}^{\infty} (\Psi_{l,W}^0 e^{i\Omega t} + \Psi_{l,W}^{0*} e^{-i\Omega t}) \frac{\partial Y_l^0}{\partial \theta} \right]^2 \right\} \sin \theta d\theta d\phi, \quad (2.37)$$

where the derivatives in longitude vanish because the  $m = 0$  mode is axisymmetric. From the relation (A.5), the non-trivial terms of (2.37) are

$$E_{rad,tot} = \frac{\rho h}{2} \sum_{l=1}^{\infty} l(l+1) \left[ (\Phi_{l,W}^0 e^{i\Omega t} + \Phi_{l,W}^{0*} e^{-i\Omega t})^2 + (\Psi_{l,W}^0 e^{i\Omega t} + \Psi_{l,W}^{0*} e^{-i\Omega t})^2 \right] \quad (2.38)$$

which include both time-dependent and time-independent terms. When averaged over the orbital period, the kinetic energy associated with the radial response is

$$E_{rad,tot,avg} = \rho h \sum_{l=1}^{\infty} l(l+1) (|\Phi_{l,W}^0|^2 + |\Psi_{l,W}^0|^2). \quad (2.39)$$

The total average kinetic energy, including all components, is the sum of (2.36) and (2.39):

$$E_{tot,avg} = \rho h \left[ \sum_{l=1}^{\infty} l(l+1) (|\Phi_{l,W}^0|^2 + |\Psi_{l,W}^0|^2) + \sum_{m=1}^l \sum_{l=1}^{\infty} l(l+1) (|\Phi_{l,W}^m|^2 + |\Phi_{l,E}^m|^2 + |\Psi_{l,W}^m|^2 + |\Psi_{l,E}^m|^2) \right]. \quad (2.40)$$

Additionally, the amount of energy dissipation is related to the flow response. Integrated over an orbital period, the amount of work done by the tides should be equivalent to the amount of frictional energy dissipated in the ocean. To calculate these quantities, we take the time-averaged volumetric integral of the the scalar product of the momentum equation and the horizontal velocity vector. In this section, we present the calculation of both of these quantities; the calculation for viscous dissipation is far more straightforward, while the calculation of the tidal work provides more physical insight. We consider the two linear models for viscous dissipation presented in (2.1). The energy dissipated in the ocean is dependent on the viscous term employed. Tyler (2011) employs a linear drag (Rayleigh dissipation) formulation, in which the global dissipation rate is given by

$$\dot{E}_{lin} = -\rho \int_V (\alpha \vec{u} \cdot \vec{u}) dV. \quad (2.41)$$

This dissipation essentially scales with the kinetic energy given by (2.32). Averaged over an orbital period (cf. (2.40)), the viscous dissipation in terms of  $\Psi$  and  $\Phi$  is

$$\begin{aligned} \dot{E}_{lin,avg} = -2\alpha E_{tot,avg} = -2\alpha \rho h \left[ \sum_{l=1}^{\infty} l(l+1) (|\Phi_{l,W}^0|^2 + |\Psi_{l,W}^0|^2) \right. \\ \left. + \sum_{l=1}^{\infty} \sum_{m=1}^l l(l+1) (|\Phi_{l,W}^m|^2 + |\Phi_{l,E}^m|^2 + |\Psi_{l,W}^m|^2 + |\Psi_{l,E}^m|^2) \right]. \end{aligned} \quad (2.42)$$

For a Navier-Stokes type viscosity, the solution to the integral

$$\dot{E}_{NS} = \rho \int_V [(\nu \nabla^2 \vec{u}) \cdot \vec{u}] dV \quad (2.43)$$

is also similar to (2.32) with an additional prefactor of  $-l(l+1)/R^2$  introduced to the summation by the Laplacian operator. The associated time-averaged energy dissipation is thus,

$$\begin{aligned} \dot{E}_{NS,avg} = \frac{-2\rho h \nu}{R^2} \left[ \sum_{l=1}^{\infty} l^2(l+1)^2 (|\Phi_{l,W}^0|^2 + |\Psi_{l,W}^0|^2) \right. \\ \left. + \sum_{l=1}^{\infty} \sum_{m=1}^l l^2(l+1)^2 (|\Phi_{l,W}^m|^2 + |\Phi_{l,E}^m|^2 + |\Psi_{l,W}^m|^2 + |\Psi_{l,E}^m|^2) \right]. \end{aligned} \quad (2.44)$$

The work done by the tidal potential can be calculated from

$$\begin{aligned}\dot{E} &= -\rho \int_V (\nabla U \cdot \vec{u}) dV = -\rho h \int_S (\nabla U \cdot \vec{u}) dS \\ &= -\rho h R^2 \int_0^{2\pi} \int_0^\pi \left[ \left( \frac{1}{R} \frac{\partial U}{\partial \theta} \right) u_\theta + \left( \frac{1}{R \sin \theta} \frac{\partial U}{\partial \phi} \right) u_\phi \right] \sin \theta d\theta d\phi, \quad (2.45)\end{aligned}$$

The full tidal potential  $U$  is the sum of each of the components presented in §2.2.1,

$$\begin{aligned}U &= U_2^0 (e^{i\Omega t} + e^{-i\Omega t}) Y_2^0 + U_2^1 (e^{i\Omega t} Y_2^1 - e^{-i\Omega t} Y_2^{-1} + e^{-i\Omega t} Y_2^1 - e^{i\Omega t} Y_2^{-1}) \\ &\quad + U_{2,W}^2 (e^{i\Omega t} Y_2^2 + e^{-i\Omega t} Y_2^{-2}) + U_{2,E}^2 (e^{-i\Omega t} Y_2^2 + e^{i\Omega t} Y_2^{-2}). \quad (2.46)\end{aligned}$$

Similar to (2.32), the integral (2.45) is separable in  $m$ . Therefore, for clarity, we present the dissipation associated with radial component ( $m=0$ ) separately.

For the obliquity tide and the librational components of the eccentricity tide, the integral (2.45) in terms of the tidal potentials and response coefficients is

$$\begin{aligned}\dot{E} &= -\rho h \int_0^{2\pi} \int_0^\pi \left\{ \left[ \sum_{m=1}^2 \left( U_{2,W}^m e^{i\Omega t} \frac{\partial Y_2^m}{\partial \theta} + U_{2,W}^{-m} e^{-i\Omega t} \frac{\partial Y_2^{-m}}{\partial \theta} + U_{2,E}^m e^{-i\Omega t} \frac{\partial Y_2^m}{\partial \theta} \right. \right. \right. \\ &\quad \left. \left. \left. + U_{2,E}^{-m} e^{i\Omega t} \frac{\partial Y_2^{-m}}{\partial \theta} \right) \right] \times \left[ \sum_{l=1}^\infty \sum_{m=1}^l \left( \Phi_{l,W}^m e^{i\Omega t} \frac{\partial Y_l^m}{\partial \theta} + \Phi_{l,W}^{-m} e^{-i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} \right. \right. \right. \\ &\quad \left. \left. \left. + \Psi_{l,W}^m e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} + \Psi_{l,W}^{-m} e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} + \Phi_{l,E}^m e^{-i\Omega t} \frac{\partial Y_l^m}{\partial \theta} \right. \right. \right. \\ &\quad \left. \left. \left. + \Phi_{l,E}^{-m} e^{i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} + \Psi_{l,E}^m e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} + \Psi_{l,E}^{-m} e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} \right) \right] \right. \\ &\quad \left. + \left[ \sum_{m=1}^2 \left( U_{2,W}^m e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_2^m}{\partial \phi} + U_{2,W}^{-m} e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_2^{-m}}{\partial \phi} + U_{2,E}^m e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_2^m}{\partial \phi} \right. \right. \right. \\ &\quad \left. \left. \left. + U_{2,E}^{-m} e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_2^{-m}}{\partial \phi} \right) \right] \times \left[ \sum_{l=1}^\infty \sum_{m=1}^l \left( \Phi_{l,W}^m e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} \right. \right. \right. \\ &\quad \left. \left. \left. + \Phi_{l,W}^{-m} e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} - \Psi_{l,W}^m e^{i\Omega t} \frac{\partial Y_l^m}{\partial \theta} - \Psi_{l,W}^{-m} e^{-i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} \right. \right. \right. \\ &\quad \left. \left. \left. + \Phi_{l,E}^m e^{-i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^m}{\partial \phi} + \Phi_{l,E}^{-m} e^{i\Omega t} \frac{1}{\sin \theta} \frac{\partial Y_l^{-m}}{\partial \phi} - \Psi_{l,E}^m e^{-i\Omega t} \frac{\partial Y_l^m}{\partial \theta} \right. \right. \right. \\ &\quad \left. \left. \left. - \Psi_{l,E}^{-m} e^{i\Omega t} \frac{\partial Y_l^{-m}}{\partial \theta} \right) \right] \right\} \sin \theta d\theta d\phi. \quad (2.47)\end{aligned}$$

Using the relation (2.5), the non-trivial components of (2.47) are

$$\begin{aligned} \dot{E} = & -\rho h \int_0^{2\pi} \int_0^\pi \left[ \sum_{m=1}^2 \left( U_{2,W}^m \Phi_{2,W}^{-m} + U_{2,W}^{-m} \Phi_{2,W}^m + U_{2,E}^m \Phi_{2,E}^{-m} + U_{2,E}^{-m} \Phi_{2,E}^m \right. \right. \\ & + \left. \left( U_{2,W}^m \Phi_{2,E}^{-m} + U_{2,E}^{-m} \Phi_{2,W}^m \right) e^{i2\Omega t} + \left( U_{2,E}^m \Phi_{2,W}^{-m} + U_{2,W}^{-m} \Phi_{2,E}^m \right) e^{-i2\Omega t} \right) \times \\ & \left. \left( \frac{\partial Y_l^m}{\partial \theta} \frac{\partial Y_l^{-m}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial Y_l^m}{\partial \phi} \frac{\partial Y_l^{-m}}{\partial \phi} \right) \right] \sin \theta d\theta d\phi. \end{aligned} \quad (2.48)$$

Making use of (A.5) and recalling that the coefficients describing the tides (e.g.  $U_{2,W}^1$ ) are real, (2.48) simplifies to

$$\begin{aligned} \dot{E} = & -2(2)(3)\rho h \sum_{m=1}^2 \left( U_{2,W}^m \Re(\Phi_{2,W}^m) + U_{2,E}^m \Re(\Phi_{2,E}^m) \right. \\ & + \left. (U_{2,W}^m \Phi_{2,E}^{m*} + U_{2,E}^m \Phi_{2,W}^m) e^{i2\Omega t} + (U_{2,E}^m \Phi_{2,W}^{m*} + U_{2,W}^m \Phi_{2,E}^m) e^{-i2\Omega t} \right). \end{aligned} \quad (2.49)$$

Thus, the time-averaged tidal dissipation is

$$\dot{E}_{avg} = -2(2)(3)\rho h \sum_{m=1}^2 \left( U_{2,W}^m \Re(\Phi_{2,W}^m) + U_{2,E}^m \Re(\Phi_{2,E}^m) \right). \quad (2.50)$$

The dissipation associated with the radial component is given by

$$\begin{aligned} \dot{E}_{rad} = & -\rho h \int_0^{2\pi} \int_0^\pi \left\{ \left( U_2^0 (e^{i\Omega t} + e^{-i\Omega t}) \frac{\partial Y_2^0}{\partial \theta} \right) \right. \\ & \left. \left[ \sum_{l=1}^\infty \left( (\Phi_{l,W}^0 e^{i\Omega t} + \Phi_{l,W}^{0*} e^{-i\Omega t}) \frac{\partial Y_l^0}{\partial \theta} \right) \right] \right\} \sin \theta d\theta d\phi. \end{aligned} \quad (2.51)$$

The solution of (2.51) can be used to derive a time-averaged dissipation of

$$\dot{E}_{rad,avg} = -2(3)\rho h U_2^0 (\Phi_{2,W}^0 + \Phi_{2,W}^{0*}) = -2(2)(3)\rho h U_2^0 \Re(\Phi_{2,W}^0). \quad (2.52)$$

The sum of (2.50) and (2.52) is

$$\dot{E}_{tot,avg} = -2(2)(3)\rho h \left[ U_2^0 \Re(\Phi_{2,W}^0) + \sum_{m=1}^2 \left( U_{2,W}^m \Re(\Phi_{2,W}^m) + U_{2,E}^m \Re(\Phi_{2,E}^m) \right) \right]. \quad (2.53)$$

Note that dissipation depends only on the real part of  $\Phi$  and is independent of  $\Psi$ . Two physical consequences follow immediately. First, situations in which there is no surface displacement ( $\eta = \Phi = 0$  as in Tyler (2008)) will result in no dissipation. Second, while the amount of kinetic energy depends on the magnitude of  $\Phi$  (see (2.40)), the energy dissipation rate depends on only the real component of  $\Phi$ , and thus is sensitive to the phase lag between the tidal forcing and the fluid response, similar to viscoelastic tidal dissipation in solid bodies (Kaula, 1964; Peale and Cassen, 1978; Segatz et al., 1988). It will be shown in §2.3 that for a wide range of parameters, the primary role of the viscosity is to set the tidal phase lag without altering the amplitude of the response.

#### 2.2.4 General features of the shallow-water tidally-forced system

In Figure 1, we show a solution for the spectral coefficients from the semi-analytic method for a model global ocean on Europa (see parameters in Tables 1 and 2) including all tidal components. Even though in this case the eccentricity is ten times larger than the assumed obliquity, the dominant tidal component is the westward obliquity response ( $l = 1$ ), as pointed out in Tyler (2008). The spherical harmonic modes are coupled in degree because of the Coriolis force. This coupling is fairly weak, and the response has power primarily in the modes around the large scale tidal forcing. For the westward propagating obliquity response, the responses at degrees 2 and 3 have much smaller amplitudes than that at degree 1. Degrees 2 and 3 dominate the eccentricity response. For degrees  $l > 3$ , all responses are negligible. This separation between degrees 3 and 4 allows us to carry out the scaling arguments presented in §2.3.

A key input parameter to the system is the value of the dissipation coefficient,  $\alpha$  or  $\nu$ . We present the average kinetic energy and energy dissipation for a range of hypothetical viscosities in Figures 2 and 3. The value of the viscosity is important in this model for two main reasons. First, the value of the viscosity determines the behavior of the response to the westward propagating obliquity tide. For the tidal responses not due to the westward propagating obliquity tide, the behavior is fairly straightforward. The average kinetic energy is relatively independent of the value of the dissipation coefficient prescribed for a large range of values ( $\nu \lesssim 10^6$  m<sup>2</sup>/s in Figure 2). For the westward obliquity response, the

behavior is different; there is a parameter regime where the amount of kinetic energy, and thus the flow velocities, are sensitive to the prescribed value of  $\nu$ . Additionally, in this regime, the obliquity response may be less important than the eccentricity response. Second, and of more importance for models of thermal evolution, the rate of energy dissipation depends on the viscosity. Thus, the determination of a satellite’s thermal and orbital history hinges on the prescribed value for the viscosity, which is in reality very poorly constrained. We address this issue in §2.4.

### 2.3 Scaling laws for tidally-driven ocean flows

The shallow-water system presented in §2.2 can be solved using a semi-analytic method with computational ease. The challenge in understanding the ocean response to the tidal forcing is determining how the various input parameters affect the resulting flow and choosing an appropriate value for  $\nu$  or  $\alpha$ . We address the first issue here and discuss an estimate for the effective ocean viscosity in §2.4.

The shallow water system presented in §2.2.2 involves series solutions that are coupled in spherical harmonic degree. However, as can be seen in Figure 1, the power decreases quite rapidly with increasing spherical harmonic degree  $l$ . The tidal responses, previously represented as infinite sums or for numeric purposes as a truncated series, can be reasonably estimated by using a very limited number of modes (typically 2-3) without any significant loss of accuracy. Instead of solving the system of equations given by (2.23) and (2.24) numerically, we now solve the system algebraically.

The benefits of these solutions are multifold. First, the simplifications we present make the underlying physics more transparent than the fully coupled problem. Second, these solutions provide relations for the kinetic energy and energy dissipation in terms of the various input parameters (e.g.  $R$ ,  $g$ ,  $h$ ,  $\Omega$ ), instead of the spherical harmonic coefficients. These relations provide insight into the sensitivity of the solutions to the various input parameters that themselves may have a degree of uncertainty. Additionally, these relations are simple to implement in orbital evolution models and are likely accurate in most scenarios, similar to the familiar solid-body tidal heating equations (Segatz et al., 1988; Wisdom, 2008). These scalings are limited in that they do not address the issue that the ocean’s effective viscosity

is essentially unconstrained, but we will utilize these scalings to inform the estimates of the viscosity that we present in §2.4.

In addition to the tidal forcing, the ocean responds to three forces (cf. momentum equation (2.1)): pressure gradients in the form of surface gravity waves, the Coriolis force, and the drag. For the scalings we suggest below, we need the ratios of these forces. We adopt a Navier-Stokes viscosity such that the role of viscosity can be compared to that of rotation through a rotational Reynolds number  $Re$ ,

$$Re \equiv \frac{\Omega R^2}{\nu},$$

where the characteristic velocity is taken to be the linear rotational speed, not the typical ocean flow velocity, and the characteristic length scale is  $R$ . We are also concerned with the square of the ratio of the linear rotational speed to the surface gravity wave speed,  $\epsilon$ , called the Lamb parameter in Longuet-Higgins (1968),

$$\epsilon \equiv \frac{4\Omega^2 R^2}{gh}.$$

Below, we present scaling arguments which allow us to analytically calculate the ocean response, the average kinetic energy and average energy dissipation rate in the case of Navier-Stokes type dissipation. The algebra in the following section can be tedious; for those uninterested in the details, we summarize all of the scaling results in Table 4. These estimates are accurate to 10% of the full solutions from §2.2.2 if the following limits are satisfied, high Reynolds number,

$$Re \geq 50, \tag{2.54}$$

and low Lamb parameter,

$$\epsilon \leq \frac{4}{15}. \tag{2.55}$$

For typical satellite oceans, where the rotation rate and satellite radius are well-known, these limits should be thought of as low effective viscosity ( $\nu \lesssim 10^6 \text{ m}^2/\text{s}$ ) and thick oceans ( $h \gtrsim 10 \text{ km}$ ), respectively. Because the molecular viscosity of water is very small, the scaling constraint (2.54) should be appropriate for all satellite oceans. Earth lies in a high

Lamb parameter regime because the oceans on Earth are far shallower than those presumed to exist on the icy satellites (Khurana et al., 1998). In the low Lamb parameter regime, the time lag between the forcing potential and the ocean's response is small because the gravity wave speed is fast in comparison to the surface rotational speed; as the tidal potential changes, the gravity waves can rapidly propagate this disturbance.

Under these limits, we now present the analysis for the response to the westward propagating obliquity tide. This is likely to be the dominant ocean response for many satellite parameters (as recognized in Tyler (2008)) but for completeness, we have also included the details related to the other tidal components.

### 2.3.1 Scalings for the response to the westward obliquity tidal potential

Looking at the response to the westward obliquity tide (Figure 1), the dominant component of the power spectrum is the  $l = 1$  term (i.e.  $\Psi_1^1$ ). This response is coupled through the Coriolis force to  $\Phi_2^1$  and the forcing tide. We thus carry out this scaling analysis with the assumption that the response is accurately captured by only the  $l = 1$  and  $l = 2$  terms. The vorticity equation (2.23) for  $\Psi_1^1$  here written in terms of the Reynolds number is

$$\left[ i + \frac{2}{Re} - i \frac{2(1)}{(1)(2)} \right] \Psi_1^1 - 2 \left[ \frac{3}{2} C_2^1 \Phi_2^1 \right] = 0. \quad (2.56)$$

The first imaginary term in (2.56) represents the ocean response while the second imaginary term represents a Rossby-Haurwitz wave. In this instance, these two terms exactly cancel, and we obtain

$$\Psi_1^1 = \frac{3C_2^1}{2} Re \Phi_2^1, \quad (2.57)$$

where the amplitude of the response grows as the viscosity decreases and Reynolds number increases; this solution is the resonance that Tyler (2008) refers to. For other tidal responses (see §2.3.3 and §2.3.4), the imaginary terms do not disappear and in fact, are much larger than the Reynolds number term. These terms limit the response amplitude and, therefore, the overall behavior of the other responses differ dramatically from that of the westward obliquity response.

The amplitude of  $\Psi_1^1$  is related to  $\Phi_2^1$  by the horizontal divergence equation (2.24)

$$\left[ i\frac{2}{3} + \frac{6}{Re} - i\frac{24}{\epsilon} \right] \Phi_2^1 + \frac{3}{2}(C_2^1)^2 Re \Phi_2^1 \approx \frac{-U_2^1}{\Omega} \quad (2.58)$$

when the relation (2.57) is substituted and the contribution from  $\Psi_3^1$  is neglected. In the appropriate scaling limits (large Reynolds number and small Lamb parameter), the dominant terms of (2.58) are

$$\left( \frac{3}{2}(C_2^1)^2 Re - i\frac{24}{\epsilon} \right) \Phi_2^1 \approx \frac{-U_2^1}{\Omega}. \quad (2.59)$$

The total kinetic energy (2.40) of the response is dominated by  $\Psi_1^1$  (Figure 1) and can be approximated as

$$E_{tot} \approx 2\rho h |\Psi_1^1|^2 = \frac{9(C_2^1)^2}{2} \rho h Re^2 |\Phi_2^1|^2, \quad (2.60)$$

when the relation (2.57) is included. Recasting (2.59), we have

$$|\Phi_2^1|^2 \approx \frac{|U_2^1|^2}{\Omega^2 \left| \frac{3}{2}(C_2^1)^2 Re - i\frac{24}{\epsilon} \right|^2} = \frac{4|U_2^1|^2}{\Omega^2 Re^2 \left( 9(C_2^1)^4 + 2304 \left( \frac{1}{\epsilon Re} \right)^2 \right)}. \quad (2.61)$$

Substituting (2.6) and (2.61) into (2.60), the kinetic energy of the response is given as

$$E_{tot} \approx \frac{18(C_2^1)^2 \rho h |U_2^1|^2}{\Omega^2 \left( 9(C_2^1)^4 + 2304 \left( \frac{1}{\epsilon Re} \right)^2 \right)} = \frac{27(C_2^1)^2 \pi}{5} \frac{\rho h \Omega^8 R^{12} \theta_0^2}{(9(C_2^1)^4 \Omega^6 R^8 + 144\nu^2 g^2 h^2)}. \quad (2.62)$$

In similar fashion, the average energy dissipation (2.44) can be approximated as

$$\dot{E} \approx \frac{-2\rho h \nu}{R^2} (1)^2 (2)^2 |\Psi_1^1|^2 = -18(C_2^1)^2 \rho h \Omega Re |\Phi_2^1|^2. \quad (2.63)$$

Substituting (2.6) and (2.61) into (2.63), the energy dissipated by the response is given as

$$\dot{E} \approx \frac{-72(C_2^1)^2 \rho h |U_2^1|^2}{\Omega Re \left( 9(C_2^1)^4 + 2304 \left( \frac{1}{\epsilon Re} \right)^2 \right)} = \frac{108(C_2^1)^2 \pi}{5} \frac{\rho h \nu \Omega^8 R^{10} \theta_0^2}{(9(C_2^1)^4 \Omega^6 R^8 + 144\nu^2 g^2 h^2)}. \quad (2.64)$$

We may also use equation (2.62) to derive an average flow speed  $\bar{u}$ :

$$\bar{u} = \frac{3(C_2^1)|U_2^1|}{\sqrt{\pi}\Omega R \left(9(C_2^1)^4 + 2304\left(\frac{1}{\epsilon Re}\right)^2\right)^{1/2}} = \sqrt{\frac{27}{50}} \frac{\Omega^4 R^5 \theta_0}{(9(C_2^1)^4 \Omega^6 R^8 + 144\nu^2 g^2 h^2)^{1/2}}. \quad (2.65)$$

The scalings (2.62), (2.64) and (2.65) are the general form for high Reynolds number and low Lamb parameter as defined in (2.54) and (2.55). While the scalings are cumbersome, these values are simple to calculate in a numerical code. As mentioned previously, it is clear from Figures 2 and 3 that the response to the westward propagating obliquity tide can be split into two behavioral regimes; which regime applies depends on the product of the Reynolds number and the Lamb parameter,  $\frac{1}{\epsilon Re}$ . We present further simplifications for these two regimes in order to highlight the underlying force balances and behavioral physics.

For high Reynolds numbers ( $\frac{1}{\epsilon Re} \lesssim \frac{1}{80}$ ), the Rossby-Haurwitz wave response dominates the bulk behavior. Ignoring the gravity wave contribution in (2.59),  $\Phi_2^1$  is

$$\Phi_2^1 \approx \frac{-U_2^1}{\Omega \left(\frac{3}{2}(C_2^1)^2 Re\right)} = -5\sqrt{\frac{2\pi}{15}}\nu\theta_0, \quad (2.66)$$

which in turn yields

$$\Psi_1^1 \approx \frac{-U_2^1}{(C_2^1)\Omega} = \frac{-3}{2}\sqrt{\frac{2\pi}{3}}\Omega R^2\theta_0. \quad (2.67)$$

This solution asymptotes to the inviscid solution presented in Tyler (2008). In this regime, the kinetic energy is independent of viscosity,

$$E_{tot} \approx 2\rho h|\Psi_1^1|^2 = 3\pi\rho h\Omega^2 R^4\theta_0^2, \quad (2.68)$$

and we can derive an average flow speed  $\bar{u}$  of

$$\bar{u} = \left(\frac{3}{2}\right)^{1/2} \Omega R\theta_0. \quad (2.69)$$

This result makes intuitive sense: if viscosity is not impeding flow significantly, the flow speed is controlled by the rate at which the tidal bulge is moving around the body. From

(2.44) or (2.53), the associated energy dissipation scaling scales linearly with the viscosity

$$\dot{E} \approx 12\pi\rho h\nu\Omega^2 R^2\theta_0^2 = 8\rho h\nu\bar{u}^2. \quad (2.70)$$

For lower Reynolds numbers ( $\frac{1}{\epsilon Re} \gtrsim \frac{1}{80}$ ), while still remaining in the scaling limits defined by (2.54) and (2.55), the gravity waves dominate the response. Then,

$$\Phi_2^1 \approx \frac{U_2^1}{\Omega \left(\frac{i^{24}}{\epsilon}\right)} = \frac{-i}{4} \sqrt{\frac{2\pi}{15}} \frac{\Omega^3 R^4 \theta_0}{gh} \quad (2.71)$$

and therefore

$$\Psi_1^1 \approx \frac{3}{2} \frac{C_2^1 U_2^1 Re}{\Omega \left(\frac{i^{24}}{\epsilon}\right)} = \frac{-i3}{40} \sqrt{\frac{2\pi}{3}} \frac{\Omega^4 R^6 \theta_0}{\nu gh}. \quad (2.72)$$

In this regime, the viscosity has altered the natural frequency of the system, and thus the response amplitudes no longer behave resonantly. The kinetic energy now depends on the value of the Reynolds number,

$$E_{tot} \approx 2\rho h |\Psi_1^1|^2 \approx \frac{(C_2^1)^2 \pi}{320} \rho h \Omega^2 R^4 \theta_0^2 (\epsilon Re)^2 = \frac{3\pi}{400} \frac{\rho \Omega^8 R^{12} \theta_0^2}{\nu^2 g^2 h} \quad (2.73)$$

with an associated average flow speed of

$$\bar{u} = \left(\frac{3}{800}\right)^{1/2} \frac{\Omega^4 R^5 \theta_0}{\nu gh}. \quad (2.74)$$

In this limit flow speeds decrease as viscosity increases. The associated energy dissipation is now inversely proportional to the viscosity,

$$\dot{E} \approx \frac{3\pi}{100} \frac{\rho \Omega^8 R^{10} \theta_0^2}{\nu g^2 h}. \quad (2.75)$$

The shape of the energy dissipation curve (Figure 3) can be thought of as the product of two effects: the bulk flow velocity and the value of the viscosity. This peak in dissipation rate is similar to that seen for Maxwell viscoelastic dissipation in solid bodies (Ross and Schubert, 1989). For the high Reynolds number regime, the velocities are high but the viscosity is too low to generate significant dissipation. In the lower Reynolds number regime, the viscosity is large and thus can dissipate energy. But in analogy to forced-damped har-

monic oscillators, the viscosity has moved the natural frequency of the system sufficiently away from the forcing frequency that the response no longer has large resonant amplitudes.

### 2.3.2 Effective tidal quality factor $Q$ in a satellite ocean

Throughout this section, we have considered viscous effects through a Navier-Stokes viscosity, primarily because this allows for parameterization in the form of the Reynolds number. These scalings can also be expressed in terms of an effective tidal quality factor  $Q$ , which can in turn be related to the linear drag coefficient  $\alpha$  mentioned in §2.2.  $Q$  is defined as (Murray and Dermott, 1999),

$$Q \equiv \frac{2\pi E_{tot}}{\dot{E} \left( \frac{2\pi}{\Omega} \right)} = \frac{\Omega E_{tot}}{\dot{E}} \quad (2.76)$$

and is related to the linear drag coefficient  $\alpha$  by (Tyler, 2011),

$$\alpha = \frac{\Omega}{2Q}. \quad (2.77)$$

For the westward propagating obliquity tide, by substituting (2.68), (2.70), and (2.73), (2.75), we find that the tidal quality factor is simply

$$Q_{obl,W} \approx \frac{1}{4} \left( \frac{\Omega R^2}{\nu} \right) = \frac{1}{4} Re \quad (2.78)$$

for all regimes. We emphasize that this is an effective  $Q$ , in attempting to maintain consistency with the terminology used in Tyler (2011). This  $Q$  is different from that typically defined for solid-body tidal heating (Peale et al., 1979; Murray and Dermott, 1999) in that the dissipation is referenced to the kinetic energy of the ocean, instead of the stored elastic energy due to the tide.

### 2.3.3 Scalings for the response to the eastward obliquity tide

While the magnitudes of the forcing potentials for the eastward and westward propagating obliquity tides are the same, the response of the ocean is fundamentally different. For the westward response, the kinetic energy and energy dissipation are dominated by the lowest

mode,  $|\Psi_1^1|$ . For the eastward response, the kinetic energy is significant in the lowest three modes with the largest amplitude being  $|\Phi_2^1|$  (see Figure 1). There only exists a single scaling regime, and the approximations made here break down at small Reynolds numbers (see criteria given in (2.54)).

The vorticity equation (2.27) provides the relationships between  $\Psi_1^1$ ,  $\Psi_3^1$  and  $\Phi_2^1$ , here assuming power existing only in the three largest modes and expressed in terms of the Reynolds number:

$$\Psi_1^1 = \frac{3}{2} \frac{C_2^1}{\left(\left(\frac{1}{Re}\right)^2 + 1\right)} \left(\frac{1}{Re} + i\right) \Phi_2^1 \quad (2.79)$$

and

$$\Psi_3^1 \approx \frac{4}{3} \frac{C_3^1}{\left(\left(\frac{12}{Re}\right)^2 + \frac{49}{36}\right)} \left(\frac{12}{Re} + i\frac{7}{6}\right) \Phi_2^1. \quad (2.80)$$

The response coefficients are related to the forcing potential through the horizontal divergence of the momentum equation (2.28), here written in terms of Reynolds number and Lamb parameter,

$$\left[-i\frac{4}{3} + i\frac{24}{\epsilon} + \frac{6}{Re}\right] \Phi_2^1 + C_2^1 \Psi_1^1 + \frac{8}{3} C_3^1 \Psi_3^1 \approx \frac{-U_2^1}{\Omega}. \quad (2.81)$$

For large  $Re$ , the Reynolds number terms in the denominator of (2.79) and (2.80) can be neglected. The dominant terms of (2.81) are

$$\left[\frac{6}{Re} + \frac{3}{2} \frac{(C_2^1)^2}{Re} + \frac{1536}{49} \frac{(C_3^1)^2}{Re} + i\frac{24}{\epsilon}\right] \Phi_2^1 = \left[\frac{9237}{686} \left(\frac{1}{Re}\right) + i\frac{24}{\epsilon}\right] \Phi_2^1 \approx \frac{-U_2^1}{\Omega}, \quad (2.82)$$

after making the substitutions for  $\Psi_1^1$  and  $\Psi_3^1$  and neglecting their imaginary components for small Lamb parameter. The resulting flow is given by

$$|\Phi_2^1| \approx |\Im(\Phi_2^1)| \approx \frac{U_2^1 \epsilon}{24\Omega} = \frac{1}{4} \sqrt{\frac{2\pi}{15}} \frac{\Omega^3 R^4 \theta_0}{gh}, \quad (2.83)$$

$$|\Psi_1^1| \approx |\Im(\Psi_1^1)| \approx \frac{3}{2} C_2^1 |\Phi_2^1| = \frac{3}{2\sqrt{5}} |\Phi_2^1|, \quad (2.84)$$

$$|\Psi_3^1| \approx |\Im(\Psi_3^1)| \approx \frac{8}{7} C_3^1 |\Phi_2^1| = \frac{8}{7} \sqrt{\frac{8}{35}} |\Phi_2^1|. \quad (2.85)$$

Combining (2.83) with the relations (2.84), (2.85) and the kinetic energy equation (2.40),

we find that the kinetic energy is independent of viscosity,

$$\begin{aligned} E_{tot,obl,E} &\approx \rho h [2|\Psi_1^1|^2 + 6|\Phi_2^1|^2 + 12|\Psi_3^1|^2] \approx \rho h \left[ \frac{9}{10}|\Phi_2^1|^2 + 6|\Phi_2^1|^2 + \frac{6144}{1715}|\Phi_2^1|^2 \right] \\ &= \rho h \left( \frac{7191}{686} \right) |\Phi_2^1|^2 \approx \frac{2397\pi}{27440} \frac{\rho \Omega^6 R^8 \theta_0^2}{g^2 h}. \end{aligned} \quad (2.86)$$

The kinetic energy in turn may be used to determine an averaged flow speed  $\bar{u}$ :

$$\bar{u} \approx \sqrt{\frac{2397}{54880} \frac{\Omega^3 R^3 \theta_0}{gh}}. \quad (2.87)$$

The associated energy dissipation calculated from (2.53) and the solution of (2.82) is given by

$$\dot{E} = -12\rho h U_2^1 \Re(\Phi_{2,E}^1) \approx -12\rho h U_2^1 \left( \frac{-3079}{131712} \left( \frac{U_2^1 \epsilon^2}{\Omega Re} \right) \right) = \frac{9237\pi}{6860} \frac{\rho \Omega^6 R^6 \nu \theta_0^2}{g^2 h}. \quad (2.88)$$

The tidal quality factor as defined by (2.76) is

$$Q_{obl,E} = \frac{\Omega \dot{E}_{obl,E}}{E_{tot,obl,E}} \approx \frac{\Omega \left( \frac{9237\pi}{6860} \frac{\rho \Omega^6 R^6 \nu \theta_0^2}{g^2 h} \right)}{\left( \frac{2397\pi}{27440} \frac{\rho \Omega^6 R^8 \theta_0^2}{g^2 h} \right)} = \frac{12316}{799} \left( \frac{\Omega R^2}{\nu} \right) = \frac{12316}{799} Re. \quad (2.89)$$

For completeness, we have included here scalings for the response to the eastward propagating obliquity tide. However, under the assumptions of high Reynolds number and low Lamb parameter, it can be formally shown that the contribution from the eastward obliquity response is negligible when compared to that of the westward propagating response.

In the high Reynolds number regime, the kinetic energy ratio is independent of the ocean viscosity. From §2.3.1 and §2.3.3, the relative contributions to the kinetic energy from the westward and eastward obliquity responses are

$$\frac{E_{tot,W}}{E_{tot,E}} \approx \frac{3\pi \rho h \Omega^2 R^4 \theta_0^2}{\frac{2397\pi}{27440} \frac{\rho \Omega^6 R^8 \theta_0^2}{g^2 h}} = \frac{439040}{799} \left( \frac{1}{\epsilon} \right)^2 \gtrsim 7700. \quad (2.90)$$

In the low Reynolds number regime, this ratio is sensitive to the Reynolds number with

$$\frac{E_{tot,W}}{E_{tot,E}} \approx \frac{\frac{3\pi}{400} \frac{\rho \Omega^8 R^{12} \theta_0^2}{\nu^2 g^2 h}}{\frac{2397\pi}{27440} \frac{\rho \Omega^6 R^8 \theta_0^2}{g^2 h}} = \frac{343}{495} Re^2 \gtrsim 1700. \quad (2.91)$$

Similarly, the ratio of energy dissipation is independent of viscosity in the high Reynolds number regime,

$$\frac{\dot{E}_{obl,W}}{\dot{E}_{obl,E}} \approx \frac{12\pi\rho h\Omega^2 R^2 \nu \theta_0^2}{\frac{9237\pi}{6860} \frac{\rho\Omega^6 R^6 \nu \theta_0^2}{g^2 h}} = \frac{439040}{3079} \left(\frac{1}{\epsilon}\right)^2 \gtrsim 2000 \quad (2.92)$$

and scales inversely with the Reynolds number in the low Reynolds number regime,

$$\frac{\dot{E}_{obl,W}}{\dot{E}_{obl,E}} \approx \frac{\frac{3\pi}{100} \frac{\rho\Omega^8 R^{10} \theta_0^2}{\nu g^2 h}}{\frac{9237\pi}{6860} \frac{\rho\Omega^6 R^6 \nu \theta_0^2}{g^2 h}} = \frac{343}{15395} Re^2 \gtrsim 55. \quad (2.93)$$

Typical satellite parameters (e.g. Tables 1 and 2) usually lie far from the scaling limits used to calculate these ratios, and the ratios presented in equations (2.90)-(2.93) are typically much lower than those for actual satellite parameters. Therefore, we reasonably ignore the contribution of the eastward obliquity response in all obliquity-related scalings.

### 2.3.4 Scalings for the response to the eccentricity tide

The responses to the eccentricity tide is similar to that of the eastward propagating obliquity tide, in that there is a single scaling regime characterized by a kinetic energy profile that is independent of viscosity and energy dissipation, linearly varying with viscosity. Unlike the unequal responses of the eastward and westward propagating obliquity tide, the magnitudes of the responses to each of the three components of the eccentricity tide are comparable. We present scalings for each tidal component separately, as well as including scalings for the total response.

The response to the radial eccentricity tidal potential can be approximated with the lowest three modes,  $\Psi_1^0$ ,  $\Phi_2^0$ ,  $\Psi_3^0$ . From (2.23) and ignoring the contributions from the Reynolds number terms in the denominator, similar to the process used in §2.3.3, the relations between the three modes are

$$\Psi_1^0 \approx 3C_2^0 \left( \frac{2}{Re} - i \right) \Phi_2^0 \quad (2.94)$$

and

$$\Psi_3^0 \approx \frac{4}{3}C_3^0 \left( \frac{12}{Re} - i \right) \Phi_2^0 \quad (2.95)$$

These modes are related to the tidal forcing by

$$\left[ i + \frac{6}{Re} - \frac{i24}{\epsilon} + 2 \left( \frac{C_2^0}{2} \right) \left( 3C_2^0 \left( \frac{2}{Re} - i \right) \right) + 2 \left( \frac{4C_3^0}{3} \right) \left( \frac{4}{3}C_3^0 \left( \frac{12}{Re} - i \right) \right) \right] \Phi_2^0 \approx \frac{-U_2^0}{\Omega}, \quad (2.96)$$

where the dominant real and imaginary terms end up being

$$\left[ \frac{130}{7} \left( \frac{1}{Re} \right) - \frac{i24}{\epsilon} \right] \Phi_2^0 \approx \frac{-U_2^0}{\Omega}. \quad (2.97)$$

The forced response  $|\Phi_2^0|$  is thus,

$$|\Phi_2^0| \approx |\Im(\Phi_2^0)| \approx \frac{U_2^0 \epsilon}{24\Omega} = \frac{1}{4} \sqrt{\frac{\pi}{5}} \frac{\Omega^3 R^4 e}{gh}. \quad (2.98)$$

For large Reynolds numbers, the relations (2.94) and (2.95) are dominated by the imaginary term, such that

$$|\Psi_1^0| \approx |\Im(\Psi_1^0)| \approx 3C_2^0 |\Phi_2^0| = 3\sqrt{\frac{4}{15}} |\Phi_2^0|, \quad (2.99)$$

and

$$|\Psi_3^0| \approx |\Im(\Psi_3^0)| \approx \frac{4}{3} C_3^0 |\Phi_2^0| = 4\sqrt{\frac{1}{35}} |\Phi_2^0|. \quad (2.100)$$

Using the lowest three modes, the average kinetic energy associated with the radial tide can be approximated from (2.40) as

$$E_{tot,rad} \approx \rho h [2|\Psi_1^0|^2 + 6|\Phi_2^0|^2 + 12|\Psi_3^0|^2]. \quad (2.101)$$

Combining the relations (2.94), (2.95) and (2.98) with (2.101), we find that the average kinetic energy is

$$\begin{aligned} E_{tot,rad} &\approx \rho h \left[ 2(3C_2^0 |\Phi_2^0|)^2 + 6|\Phi_2^0|^2 + 12 \left( \frac{4}{3} C_3^0 |\Phi_2^0| \right)^2 \right] \\ &= \rho h \left[ \frac{114}{7} |\Phi_2^0|^2 \right] \approx \frac{57\pi}{280} \frac{\rho \Omega^6 R^8 e^2}{g^2 h}, \end{aligned} \quad (2.102)$$

and thus the associated average velocity is given by

$$\bar{u} \approx \sqrt{\frac{57}{560}} \frac{\Omega^3 R^3 e}{gh}. \quad (2.103)$$

From (2.97), we can also calculate  $\Re(\Phi_2^0)$ ,

$$\Re(\Phi_2^0) \approx \frac{-U_2^0}{\Omega \left(\frac{24}{\epsilon}\right)^2} \left( \frac{130}{7} \left( \frac{1}{Re} \right) \right) = \frac{-65}{84} \sqrt{\frac{\pi}{5}} \frac{\Omega^4 R^4 \nu e}{g^2 h^2}. \quad (2.104)$$

The resulting dissipation from (2.52) is thus

$$\dot{E}_{rad} \approx -12\rho h U_2^0 \Re(\Phi_2^0) \approx \frac{39\pi}{14} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}. \quad (2.105)$$

Combining (2.102) and (2.105), we find that the tidal quality factor associated with the radial response is

$$Q_{rad} \approx \frac{\Omega \left( \frac{57\pi}{280} \frac{\rho \Omega^6 R^8 e^2}{g^2 h} \right)}{\frac{39\pi}{14} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}} = \frac{19}{260} \left( \frac{\Omega R^2}{\nu} \right) = \frac{19}{260} Re. \quad (2.106)$$

The response of the ocean to the librational eccentricity tidal potential is dominated by the lowest two modes,  $\Phi_2^2$  and  $\Psi_3^2$ . To calculate the westward response, we return to the vorticity equation given by (2.23). It can be shown that for large Reynolds numbers

$$\Psi_3^2 \approx 3C_3^2 \left( \frac{12}{Re} - i\frac{2}{3} \right) \Phi_2^2 \quad (2.107)$$

and

$$|\Psi_3^2| \approx |\Im(\Psi_3^2)| \approx 2C_3^2 |\Phi_2^2| = 2\sqrt{\frac{1}{7}} |\Phi_2^2|. \quad (2.108)$$

From (2.24) and including the dominant real and imaginary terms as before, the tidal response is given by

$$\left[ \frac{6}{Re} + \frac{96(C_3^2)^2}{Re} - i\frac{24}{\epsilon} \right] \Phi_2^2 = \left[ \frac{138}{7} \left( \frac{1}{Re} \right) - i\frac{24}{\epsilon} \right] \Phi_2^2 \approx \frac{-U_{2,W}^2}{\Omega}. \quad (2.109)$$

Similar to the response to the radial component, the kinetic energy is related to

$$|\Phi_2^2| \approx |\Im(\Phi_2^2)| \approx \frac{|U_{2,W}^2|^\epsilon}{24\Omega} = \frac{1}{8} \sqrt{\frac{2\pi}{15}} \frac{\Omega^3 R^4 e}{gh} \quad (2.110)$$

while the dissipation is related to

$$\Re(\Phi_2^2) \approx \frac{-U_{2,W}^2}{\Omega \left(\frac{24}{\epsilon}\right)^2} \left( \frac{138}{7} \left( \frac{1}{Re} \right) \right) = \frac{-23}{56} \sqrt{\frac{2\pi}{15}} \left( \frac{\Omega^4 R^4 \nu e}{g^2 h^2} \right). \quad (2.111)$$

The average kinetic energy associated with the response can be approximated from (2.40), (2.108) and (2.110) as

$$\begin{aligned} E_{tot,lib,W} &\approx \rho h [6|\Phi_2^2|^2 + 12|\Psi_3^2|^2] \approx \rho h [6|\Phi_2^2|^2 + 12(2C_3^2|\Phi_2^2|)^2] \\ &= \rho h \left[ \frac{90}{7} |\Phi_2^2|^2 \right] \approx \frac{3\pi}{112} \frac{\rho \Omega^6 R^8 e^2}{g^2 h} \end{aligned} \quad (2.112)$$

with a corresponding average flow speed of

$$\bar{u} \approx \sqrt{\frac{3}{224}} \frac{\Omega^3 R^3 e}{gh}. \quad (2.113)$$

From the relations (2.53) and (2.111), the associated energy dissipation is

$$\dot{E}_{lib,W} \approx -12\rho h U_{2,W}^2 \Re(\Phi_{2,W}^2) \approx \frac{69\pi}{140} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}. \quad (2.114)$$

The associated Q is therefore

$$Q_{lib,W} \approx \frac{\Omega \left( \frac{3\pi}{112} \frac{\rho \Omega^6 R^8 e^2}{g^2 h} \right)}{\left( \frac{69\pi}{140} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h} \right)} = \frac{5}{92} \left( \frac{\Omega R^2}{\nu} \right) = \frac{5}{92} Re. \quad (2.115)$$

The process to calculate the response to the eastward propagating librational component of the eccentricity tide is essentially the same as that for the westward response. The relationship between  $\Phi_2^2$  and  $\Psi_3^2$  as given by (2.27) is

$$\Psi_3^2 \approx \frac{3C_3^2}{4} \left( \frac{12}{Re} + i\frac{4}{3} \right) \Phi_2^2 \quad (2.116)$$

with

$$|\Psi_3^2| \approx |\Im(\Psi_3^2)| \approx C_3^2 |\Phi_2^2| = \sqrt{\frac{1}{7}} |\Phi_2^2|. \quad (2.117)$$

The dominant terms in the horizontal divergence equation (2.28) are

$$\left[ \frac{6}{Re} + \frac{24(C_3^2)^2}{Re} + i \frac{24}{\epsilon} \right] \Phi_2^2 = \left[ \frac{66}{7} \left( \frac{1}{Re} \right) + i \frac{24}{\epsilon} \right] \Phi_2^2 \approx \frac{-U_{2,E}^2}{\Omega}. \quad (2.118)$$

The resulting flow is given by

$$|\Phi_2^2| \approx |\Im(\Phi_2^2)| \approx \frac{|U_{2,E}^2| \epsilon}{24\Omega} = \frac{7}{8} \sqrt{\frac{2\pi}{15}} \frac{\Omega^3 R^4 e}{gh} \quad (2.119)$$

with

$$\Re(\Phi_2^2) \approx \frac{-U_{2,E}^2}{\Omega \left( \frac{24}{\epsilon} \right)^2} \left( \frac{66}{7} \left( \frac{1}{Re} \right) \right) = \frac{11}{8} \sqrt{\frac{2\pi}{15}} \left( \frac{\Omega^4 R^4 \nu e}{g^2 h^2} \right). \quad (2.120)$$

Similar to (2.112), the average kinetic energy is

$$E_{tot,lib,E} \approx \rho h \left[ 6|\Phi_2^2|^2 + 12 (C_3^2 |\Phi_2^2|)^2 \right] = \rho h \left[ \frac{54}{7} |\Phi_2^2|^2 \right] \approx \frac{63\pi}{80} \frac{\rho \Omega^6 R^8 e^2}{g^2 h} \quad (2.121)$$

and the flow speed is

$$\bar{u} \approx \sqrt{\frac{63}{160}} \frac{\Omega^3 R^3 e}{gh} \quad (2.122)$$

The corresponding energy dissipation for the eastward flow (2.53) is

$$\dot{E}_{lib,E} \approx -12\rho h U_{2,E}^2 \Re(\Phi_{2,E}^2) \approx \frac{231\pi}{20} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}. \quad (2.123)$$

The tidal quality factor is

$$Q_{lib,E} \approx \frac{\Omega \left( \frac{63\pi}{80} \frac{\rho \Omega^6 R^6 \nu \theta_0^2}{g^2 h} \right)}{\left( \frac{231\pi}{20} \frac{\rho \Omega^6 R^8 \theta_0^2}{g^2 h} \right)} = \frac{3}{44} \left( \frac{\Omega R^2}{\nu} \right) = \frac{3}{44} Re. \quad (2.124)$$

The three components of the eccentricity-driven flow behave quite similarly; the main difference exists in the pre-factors defining each of the flows. From (2.102), (2.112), and

(2.121), the ratios of the kinetic energies are

$$E_{tot,lib,E} \approx \frac{147}{38} E_{tot,rad} \approx \frac{147}{5} E_{tot,lib,W} \quad (2.125)$$

with a total kinetic energy associated with the eccentricity tide of

$$E_{tot,ecc} \approx \frac{57\pi}{56} \frac{\rho \Omega^6 R^8 e^2}{g^2 h}. \quad (2.126)$$

The energy dissipation behaves similarly with the eastward librational response contributing a bulk of the dissipation with non-negligible contributions from the other components. The relative dissipation rates from (2.105), (2.114), and (2.123) are

$$\dot{E}_{lib,E} \approx \frac{539}{130} \dot{E}_{rad} \approx \frac{539}{23} \dot{E}_{lib,W}. \quad (2.127)$$

The total energy dissipation associated with the eccentricity tide is

$$\dot{E}_{ecc,tot} \approx \frac{519\pi}{35} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}. \quad (2.128)$$

Thus, an approximation for  $Q$  related to all components of the eccentricity tide is

$$Q_{ecc} \approx \frac{\Omega \left( \frac{57\pi}{56} \frac{\rho \Omega^6 R^8 e^2}{g^2 h} \right)}{\frac{519\pi}{35} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}} = \frac{95}{1384} \left( \frac{\Omega R^2}{\nu} \right) = \frac{95}{1384} Re. \quad (2.129)$$

## 2.4 Effective viscosity in a global ocean

The scalings presented in §2.3 provide a description of global ocean behavior; however, to estimate the energy dissipation in the ocean, these scalings require a prescribed value for the effective viscosity. This viscosity is unlikely to be as small as the molecular viscosity, presumably due to turbulence (Pope, 2000), but it is unclear how large an effective viscosity one should adopt (Lumb and Aldridge, 1991). While there is potential to infer from measurements what the effective viscosity is for a global liquid layer, such as for the liquid core of Earth (Smylie, 1999; Mound and Buffett, 2007; Buffett and Christensen, 2007) or Mercury, it is unlikely that we can do so for icy satellite oceans. Also, while these measure-

ments can give some insight into the effects of turbulence, the effective viscosity is likely to be time-variable, changing with flow conditions and/or degree of turbulence (Brito et al., 2004).

Because of the high degree of variability amongst satellites of the outer Solar System both in the past and at present, it is unlikely that a single value for the effective ocean viscosity can be broadly applied. Thus, we approach the question of what the effective viscosity in a satellite ocean is by analogy with the Earth’s oceans. On Earth, there is a long history of estimating dissipation of tides due to bottom boundary friction (Taylor, 1920; Jeffreys, 1921; Munk and MacDonald, 1960). The value for the drag coefficient  $c_D$  was originally empirically-based (Taylor, 1920). Even in more modern numerical ocean models (Jayne and St. Laurent, 2001; Egbert and Ray, 2001), a value  $c_D$  of  $O(0.001)$  is still commonly used. We incorporate this parameterization into the shallow water model for a global ocean. This approach is undeniably simplistic. However, when given the alternative of choosing a value for the effective viscosity where the uncertainty is approximately twelve orders of magnitude (Lumb and Aldridge, 1991), we favor our approach, although there are caveats. We address these in §2.5.5. While we adopt the value  $c_D = 0.002$  in this work, the results and scalings that we present in §2.4.2 are general (and linear) for  $c_D$ , and thus, permit the effect of uncertainties in  $c_D$  to be readily explored.

#### 2.4.1 Numerical method

The momentum equation for an ocean subject to tidal forcing and bottom boundary friction is (cf. (Jayne and St. Laurent, 2001))

$$\frac{\partial \vec{u}}{\partial t} + 2\vec{\Omega} \times \vec{u} = -g\vec{\nabla}\eta - \vec{\nabla}U - \frac{c_D}{h}|\vec{u}|\vec{u}. \quad (2.130)$$

Similar to the process in §2.2.2, we can employ a Helmholtz decomposition and write the divergence and the radial component of the curl of (2.130) as

$$\left[ \left( \frac{\partial}{\partial t} \right) \nabla^2 + \frac{2\Omega}{R^2} \frac{\partial}{\partial \phi} \right] \Phi + 2\Omega \left[ \cos \theta \nabla^2 - \frac{\sin \theta}{R^2} \frac{\partial}{\partial \theta} \right] \Psi = -g\nabla^2 \eta - \nabla^2 U - \left( \frac{c_D}{h} \vec{\nabla} \cdot (|\vec{u}|\vec{u}) \right), \quad (2.131)$$

$$\left[ \left( \frac{\partial}{\partial t} \right) \nabla^2 + \frac{2\Omega}{R^2} \frac{\partial}{\partial \phi} \right] \Psi - 2\Omega \left[ \cos \theta \nabla^2 - \frac{\sin \theta}{R^2} \frac{\partial}{\partial \theta} \right] \Phi = - \left( \frac{c_D}{h} \hat{r} \cdot (\vec{\nabla} \times (|\vec{u}| \vec{u})) \right). \quad (2.132)$$

While the solutions to (2.131) and (2.132) are likely oscillatory, the nonlinearity of the dissipative term requires that we integrate the system numerically. We seek solutions of the form

$$\Psi(\theta, \phi, t) = \sum_{l=1}^{NL} \sum_{m=1}^l [\Psi_l^m Y_l^m + \Psi_l^{-m} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{NL} [\Psi_l^m Y_l^m] \right), \quad (2.133)$$

$$\Phi(\theta, \phi, t) = \sum_{l=1}^{NL} \sum_{m=1}^l [\Phi_l^m Y_l^m + \Phi_l^{-m} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{NL} [\Phi_l^m Y_l^m] \right), \quad (2.134)$$

$$\eta(\theta, \phi, t) = \sum_{l=1}^{NL} \sum_{m=1}^l [\eta_l^m Y_l^m + \eta_l^{-m} Y_l^{-m}] = 2\Re \left( \sum_{m=1}^l \sum_{l=1}^{NL} [\eta_l^m Y_l^m] \right), \quad (2.135)$$

where the coefficients  $\Psi_l^m$ ,  $\Phi_l^m$ , and  $\eta_l^m$  are still complex but are now functions of time. Substituting (2.133)-(2.135) into (2.131), (2.132) and (2.18), and projecting the resulting equations onto individual spherical harmonics, the system of equations to solve is

$$\begin{aligned} \left[ \frac{\partial}{\partial t} + \frac{\nu l(l+1)}{R^2} - \frac{2\Omega i m}{l(l+1)} \right] \Psi_l^m - 2\Omega \left[ \frac{l-1}{l} C_l^m \Phi_{l-1}^m + \frac{l+2}{l+1} C_{l+1}^m \Phi_{l+1}^m \right] \\ = \frac{R^2}{l(l+1)} \left( \frac{c_D}{h} \hat{r} \cdot (\vec{\nabla} \times (|\vec{u}| \vec{u})) \right)_l^m, \end{aligned} \quad (2.136)$$

$$\begin{aligned} \left[ \frac{\partial}{\partial t} + \frac{\nu l(l+1)}{R^2} - \frac{2\Omega i m}{l(l+1)} \right] \Phi_l^m + 2\Omega \left[ \frac{l-1}{l} C_l^m \Psi_{l-1}^m + \frac{l+2}{l+1} C_{l+1}^m \Psi_{l+1}^m \right] \\ = -g\eta_l^m + \frac{R^2}{l(l+1)} \left( \frac{c_D}{h} \vec{\nabla} \cdot (|\vec{u}| \vec{u}) \right)_l^m \\ - 2U_2^0 \cos(\Omega t) \delta_{l,2} \delta_{m,0} - 2U_2^1 \cos(\Omega t) \delta_{l,2} \delta_{m,1} \\ - \left[ (U_{2,W}^2 + U_{2,E}^2) \cos(\Omega t) + (U_{2,W}^2 - U_{2,E}^2) i \sin(\Omega t) \right] \delta_{l,2} \delta_{m,2}, \end{aligned} \quad (2.137)$$

$$\frac{\partial \eta_l^m}{\partial t} - \frac{hl(l+1)}{R^2} \Phi_l^m = 0. \quad (2.138)$$

These equations are coupled in both spherical harmonic degree  $l$  and order  $m$  through the dissipative terms on the right hand side of equations (2.136) and (2.137). We include all

components of the tide in (2.137) because nonlinearity has the ability to couple different spherical harmonic orders, while previously these were separable. Because we have not assumed wave solutions, we now have to solve explicitly for the spherical coefficients of  $\eta$ , whereas we previously assumed the relationship (2.22) between  $\eta$  and  $\Psi$ .

Equations (2.136)-(2.138) are nonlinear and thus cannot be solved with the semi-analytic method previously employed in §2.2.2. Instead of assuming wave solutions as in §2.2.2, we employ a numerical integration method where the time derivatives are explicitly treated using the Adams-Bashforth method. We calculate the nonlinear dissipative terms using a spectral-transform method. The horizontal velocities are calculated on a spherical grid which consists of 100 evenly spaced grid points in longitude and 50 Gauss-Lobatto nodes in colatitude. The frictional dissipation contributions are calculated on this grid, and subsequently, transformed back into their spherical harmonic decomposition. To reduce the required number of spectral coefficients, we include an enhanced Navier-Stokes dissipative term with  $\nu = 1 \times 10^8 \text{ m}^2/\text{s}$  to (2.136) and (2.137) for all coefficients with  $l = \text{NL}$  (the series truncation level) and  $l = \text{NL} - 1$ . We adopt a series truncation level of  $\text{NL} = 30$ . The enhanced dissipative terms reduce the required truncation level by an order of magnitude without affecting the results presented here; for example, the amount of energy dissipated for our model European ocean is approximately  $10^9 \text{ W}$  total, and the amount dissipated from the enhanced dissipation term is less than  $0.1 \text{ W}$ .

We have run a series of numerical integrations varying the seven input parameters (i.e.  $R$ ,  $g$ ,  $h$ ,  $\Omega$ ,  $\theta_0$ ,  $e$ , and  $c_D$ ) to understand the sensitivity of the system to each. These numerical integrations are initialized to either zero or the semi-analytic solution; the steady-state solution is independent of these values. We evolve the equations in time until they produce a constant solution for the kinetic energy and energy dissipation when averaged over an orbital period, similar to the behavior of the linear solution (cf. (2.40) and (2.44)).

### 2.4.2 Scaling arguments

The shallow water system including nonlinear bottom drag settles on a wave solution. While the dissipation processes are not exactly alike, we can assume that the previous linear Navier-Stokes formulation is equivalent, in a time-averaged sense, to the nonlinear bottom

drag with

$$\frac{c_D}{h} |\vec{u}| \vec{u} \sim \nu \nabla^2 \vec{u}. \quad (2.139)$$

Assuming that  $|\vec{u}|$  scales with the average velocity, we find the scalings

$$\nu_{eff,obl} = k_{obl} \frac{c_D}{h} \frac{\Omega^4 R^7 \theta_0}{(9(C_2^1)^4 \Omega^6 R^8 + 144 \nu_{eff,obl}^2 g^2 h^2)^{1/2}} \quad (2.140)$$

by using the relation (2.65) for the obliquity-related response and

$$\nu_{eff,ecc} = k_{ecc} \frac{c_D \Omega^3 R^5 e}{gh^2} \quad (2.141)$$

by using (2.113) for the eccentricity response. From our numerical runs, the least squares fit for  $k_{obl}$  is 0.40 and  $k_{ecc}$  is 0.13 (see Figure 4). The effective viscosity for obliquity (2.140) can be solved quadratically (see Table 4), and displays simpler behavior in the low and high Reynolds number limits discussed in §2.3.1. For a model ocean on Europa, the numerically-derived effective viscosity is  $\nu_{eff,obl} = 3.3 \times 10^3 \text{ m}^2/\text{s}$  for the obliquity-related flow and  $\nu_{eff,ecc} = 1.7 \times 10^2 \text{ m}^2/\text{s}$  for the eccentricity-related flow.

### 2.4.3 Summary

The numerically derived scalings (2.140) and (2.141) provide estimates for the effective viscosity given a set of input parameters. These viscosities can be input into either the semi-analytic system (§2.2.2) or the scaling laws (see Table 4) in order to determine the flow response and energy dissipation rate of a satellite ocean. It is worth noting that the effective viscosities for the obliquity and eccentricity responses scale differently with the input parameters because the velocities scale differently between the two responses. These values for a model European ocean are displayed in Figures 2 and 3 for reference. In the context of thermal-orbital evolution models, where the input parameters change over time, the scalings for the viscosities are critical to better estimating the long-term impact of ocean tidal heating.

## 2.5 Implications for the outer Solar System satellites

With estimates of effective viscosity in hand, it is now possible to complete a survey of the thermal heat budgets for various satellites of the outer Solar System. Specifically, we seek to answer two fairly broad questions: should ocean tidal heating be an important heat source at present for any of the icy satellites; and could ocean tidal heating have been a significant heat source in the past? These questions are difficult to answer in that there are many uncertainties in the parameters that go into the following calculations. Because our goal was to understand broad trends and not to explain specific observations or quantities, we have utilized simplified theoretical models in order to calculate the thermal budgets, with the understanding that these results may diverge from what is observed, as for example in the long-recognized anomalous cases of Mimas or Enceladus (Squyres et al., 1983; Meyer and Wisdom, 2007; Howett et al., 2011).

### 2.5.1 Calculating the available thermal energy

The primary heat sources available to a satellite are radiogenic heat and tidal dissipation, either in the solid ice and rock layers or in a liquid ocean layer. While the term “icy” is frequently used to characterize the large satellites of the outer Solar System, this terminology is a slight misnomer in that a significant fraction of these satellites is comprised of silicates. Thus, the decay of radioactive elements contributes a significant amount to the thermal budget of these satellites. For the radiogenic component of the thermal budget, we assume a chondritic heating rate of  $H = 4.5 \times 10^{-12}$  W/kg (Ellsworth and Schubert, 1983). We calculate an equivalent silicate mass,  $M_{sil}$ , assuming a two-component satellite

$$M_{sil} = \frac{4}{3} \rho_{sil} \left( \frac{\rho_{sat} - \rho_{ice}}{\rho_{sil} - \rho_{ice}} \right) \pi R^3 \quad (2.142)$$

where the silicate density is taken as  $\rho_{sil} = 3500$  kg/m<sup>3</sup>, the ice density is taken as  $\rho_{ice} = 950$  kg/m<sup>3</sup>, and the satellite mean density  $\rho_{sat}$  as presented for various satellites in Table 2.

To estimate the amount of solid body tidal heating, we employ the standard viscoelastic model (Segatz et al., 1988; Ross and Schubert, 1989; Wisdom, 2004). In this model, the

rate of solid body dissipation is given by

$$\dot{E} = \frac{3}{2} \frac{k_2}{Q_{sol}} \frac{(\Omega R)^5}{G} (7e^2 + \sin^2 \theta_0), \quad (2.143)$$

where the satellite structure and dissipation characteristics are parameterized by the tidal Love number,  $k_2$ , and the tidal quality factor,  $Q_{sol}$ , and  $G$  is the gravitational constant. For the unknown satellite parameters,  $k_2$  and  $Q_{sol}$ , we adopt a value of  $Q_{sol}$  of 100 (Goldreich and Soter, 1966), and we estimate the Love number using Kelvin's formula (Love, 1944),

$$k_2 = \frac{3/2}{1 + \frac{19\mu}{2\rho_{sat}gR}} \quad (2.144)$$

for a homogeneous body with rigidity,  $\mu$ , assumed to be  $\mu = 4.0 \times 10^9 \text{ Nm}^{-2}$ . We also calculate the dissipation with a fluid  $k_2$  of 3/2 to understand the energy budget in the limit of a fully deformable body, such as one with an ocean.

In previous works, the dissipation related to the obliquity tide has typically been neglected, primarily because it is assumed that the obliquity is small and also, until recently, there have been no measurements of satellite obliquity. While for most satellites the obliquity solid body tidal dissipation is negligible when compared to the eccentricity driven solid body dissipation, this relationship is reversed in the case of ocean tidal heating.

Titan is currently the only icy satellite with a measured obliquity (Stiles et al., 2008). In order to estimate the contributions to other satellites from the obliquity tide, we have assumed obliquities appropriate for satellites in damped Cassini states with a single orbit precession frequency; interactions with other satellites do not generally change the results significantly (Bills, 2005; Baland et al., 2012). In a Cassini state, the invariable pole, orbit normal, and spin pole remain coplanar as they precess (Colombo, 1966; Peale, 1969). For this coplanar precession to occur, the obliquity must satisfy the relation (Peale, 1969; Ward, 1975; Bills and Nimmo, 2008)

$$\frac{3}{2} [(J_2 + C_{2,2}) \cos \theta_0 + C_{2,2}] p \sin \theta_0 = c \sin(i - \theta_0) \quad (2.145)$$

where  $J_2$  and  $C_{2,2}$  are the degree-2 gravity coefficients,  $c$  is the normalized polar moment,  $i$

is the orbital inclination, and  $p$  is the ratio of the orbital motion to the orbit plane precession and is given by

$$p = \frac{\Omega}{d\Omega_{orb}/dt} \quad (2.146)$$

where  $\Omega_{orb}$  is the longitude of the ascending node. While the orbital parameters are well-established, observations of the gravity coefficients and moment of inertia for many satellites are limited. We estimate the unmeasured gravity coefficients assuming a hydrostatic satellite using the Darwin-Radau relation (Darwin, 1899; Radau, 1885; Hubbard and Anderson, 1978)

$$C_{2,2} = \frac{1}{4} \left( \frac{\Omega^2}{G(\frac{4}{3}\pi\rho_{sat})} \right) \left( \frac{5}{1 + (\frac{5}{2} - \frac{15}{4}c)^2} - 1 \right) \quad (2.147)$$

in the thin shell limit for the moment of inertia (i.e.  $c = 0.67$  (Bills and Nimmo, 2011)).

For the ocean contributions to tidal heating, we estimate the effective viscosities from (2.140) and (2.141) for the obliquity and eccentricity tides separately. We utilize values of  $h = 30$  km and  $c_D = 0.002$  for all satellites. After calculating a satellite-specific effective viscosity, the associated tidal dissipation is calculated from the analytical scalings summarized in Table 4.

Table 1 summarizes all of the common parameters utilized in this analysis, while Table 2 shows the parameters utilized and calculated quantities that are satellite specific. The various contributions to the thermal energy budget are presented for each satellite in Table 3. In Figures 5 and 6, we highlight key results that are tabulated in Table 3. In Figure 5, we present the relative contributions to the ocean tidal dissipation from the eccentricity and obliquity tides. While the obliquity tidal response likely dominates ocean dissipation, there are examples of satellites where the eccentricity response could be more important. The full thermal budgets of many satellites are presented in Figures 6 where the solid-body tidal dissipation is calculated using illustrative values of  $Q = 100$  and  $k_2$  corresponding to a homogenous body or a fully fluid body, respectively. The actual values for these are likely to depend on the details of internal structure and rheology (Ross and Schubert, 1989), but in choosing these two limits, we likely bracket the actual value of the tidal dissipation, while providing insight into the degree of uncertainty associated with the tidal model. It is clear that for most of the satellites the thermal budgets are dominated by two components: ra-

diogenic heating and eccentricity solid-body tidal heating. Triton is likely the only satellite with a large present-day heat contribution from the ocean because of its current lack of orbital eccentricity; Callisto, Titan and Tethys may have some minor ocean contribution.

### 2.5.2 Relative contributions from eccentricity and obliquity

While Tyler (2008, 2011) recognized that a satellite ocean response is typically dominated by the westward obliquity response, this is not always the case. Figure 5 shows the energy dissipated in an ocean calculated with the input parameters  $h = 30$  km,  $c_D = 0.002$ , and the satellite-appropriate parameters from Table 2. For most satellite oceans, the obliquity tidal heating contribution is larger than that from the eccentricity. The reason for this is twofold. First, assuming the obliquity response is in the low viscosity (nearly inviscid) regime, the average velocity will be much higher for the obliquity related flow due to the resonant response (Tyler, 2008), if the obliquity and eccentricity are roughly comparable. Second, because higher velocities equate to higher effective viscosities (see §2.4.2), dissipation is further enhanced in the obliquity response.

Taking the scalings for the energy dissipation from §2.3.1 and §2.3.4 (summarized in Table 4) and the numerical results from §2.4.2, it can be shown, here in the low viscosity regime, that the obliquity likely dominates unless there is a significant disparity between the eccentricity and obliquity:

$$\frac{\dot{E}_{obl}}{\dot{E}_{ecc}} \approx \frac{44800}{519} \left(\frac{1}{\epsilon}\right)^3 \left(\frac{k_{obl}}{k_{ecc}}\right) \left(\frac{\theta_0}{e}\right)^3. \quad (2.148)$$

Recall that for these scalings to be applicable, the Lamb parameter,  $\epsilon$ , must be low (2.55), and additionally,  $\frac{k_{obl}}{k_{ecc}}$  is approximately 3 (cf. §2.4.2). Oceanic energy dissipation is clearly biased towards the obliquity response. Because of this, oceans on satellites with low orbital inclination, and thus, low Cassini state obliquity, are unlikely to experience significant dissipation (Chen and Nimmo, 2011).

### 2.5.3 Global tidal heating pattern

Thus far we have primarily discussed the time- and spatially-averaged energy dissipation rate. While this is an important quantity when thinking about the overall thermal and or-

bital evolution, the spatial distribution of tidal heat can also have implications for spacecraft observables. As suggested for solid-body tidal heating, spatial variations in the heat flux can generate lateral variations in conductive ice shell thickness (Ojakangas and Stevenson, 1989). These variations can potentially generate long-wavelength topography that can be detected through observations of topography or gravity (Schenk and McKinnon, 2009; Nimmo and Bills, 2010). In addition, the spatial pattern of the heating at the base can affect convection in the ice shell (Tobie et al., 2005; Roberts and Nimmo, 2008). The effects of this convection may potentially manifest as geologic features on the surface (Tackley et al., 2001; Besserer et al., 2013).

We present the time-averaged global heating patterns in the ocean for the obliquity response and eccentricity response in Figure 7. The primary power for both responses occurs in degrees  $l = 0, 2$ , and  $4$ , as would be expected from tidally-driven flow. However the relative power in each degree and thus the location of hot and cold spots differ from the case for solid-body tidal heating (Ojakangas and Stevenson, 1989; Beuthe, 2013). Thus, if ocean tidal heating is important, it can potentially be observed and differentiated from solid-body tidal dissipation through observations, for example the presence and spatial pattern of long-wavelength topography.

#### **2.5.4 Triton**

The thermal history of Triton is a puzzle; if satellite ocean tidal heating is significant for any icy satellite, it is most likely to be Triton. Observations of the surface features suggest that Triton has been quite active in the past (Smith et al., 1989; Croft et al., 1995; McKinnon and Kirk, 2007), and this activity could have been as recent as 10 Ma (Schenk and Zahnle, 2007). Eccentricity tides have been evoked for heating mechanisms like frictional shear heating (Prockter et al., 2005) or “thermal blanketing” (Gaeman et al., 2012), but because the eccentricity at present is zero, there is no clear reason why geologic activity should be recent. A significant amount of tidal heating likely occurred during Triton’s capture and subsequent orbital circularization (Ross and Schubert, 1990); however, the likelihood that Triton’s capture is recent is very low (Agnor and Hamilton, 2006; Schenk and Zahnle, 2007) and additionally, the circularization timescale, while dependent on the interior structure, is

likely to be less than 1 Gyr (Ross and Schubert, 1990).

Prior to observations from *Voyager 2*, Jankowski et al. (1989) suggested that solid-body obliquity tidal heating could play a significant role in the thermal history of Triton, specifically if Triton occupied Cassini state 2. From imaging it was shown that this state was unlikely (Smith et al., 1989); however, the role of the obliquity tide on the evolution of Triton should not be discounted. It was recognized that the solid-body tidal heating contribution is small (Jankowski et al., 1989) if Triton is in Cassini state 1; however because of resonance, the ocean can effectively contribute heat at a rate of approximately  $10^{10}$  W, even if this is the case (Figure 6). Due to tidal dissipation, Triton is moving towards Neptune. Thus, obliquity tides and the associated response in a ocean grow over time; this behavior may account for the recent geologic activity on Triton. Coupled thermal-orbital models that include ocean tidal heating could provide further insight into the long term evolution of Triton.

### 2.5.5 Caveats

In estimating the thermal budgets for the outer Solar System satellites (Figure 6), we have used fairly simplistic models in order to calculate the amount of tidal dissipation both in the solid-body and ocean. These models are sufficient to capture and highlight broad trends. Given the general lack of data and large uncertainty in physical parameters, we do not attempt to carry out models of specific bodies in this work. With regard to the solid body dissipation calculated here, it is recognized that the Maxwell viscoelastic model is insufficient to explain the tidal dissipation observed, for example for Enceladus (Meyer and Wisdom, 2007; Howett et al., 2011).

In adopting a simple model for the ocean behavior by analogy to Earth's oceans, we have neglected a significant dissipation mechanism, internal tides, which arise because of compositional stratification (Munk and MacDonald, 1960; Egbert and Ray, 2000). While Europa's ocean, at least, is electrically conductive, presumably due to brine (Khurana et al., 1998; Kargel et al., 2000), it is unclear whether satellite oceans are compositionally stratified (Goodman et al., 2004). The major drivers of compositional variations in Earth's oceans, such as evaporation or addition of fresh water, are likely much less important on icy satellites. Because internal tide generation is largely due to ocean bathymetry (Baines, 1973;

Jayne and St. Laurent, 2001), a property which is completely unconstrained in the context of satellite oceans, we have ignored the effects of a stratified ocean. Additionally, our shallow-water approach cannot capture the effects of convection. Convection will alter the dynamics of the ocean flow to some extent (Chen et al., 2010; Soderlund et al., 2013). However, the magnitude of the tidally-driven velocities are likely comparable to those generated through convection and also, convection has a tendency to destroy stratification. Thus, the shallow-water model may be sufficient to predict the bulk satellite ocean behavior. Unfortunately, due to computational limitations, three-dimensional models that can include convection may not necessarily be more accurate than our shallow-water model, because these models are unable to resolve the processes that presumably dissipate energy in the system (Glatzmaier, 2002).

Throughout this work, we assume a small Lamb parameter; the uncertainty in the Lamb parameter arises from the uncertainty in the ocean depth. The scalings we present in §2.3 are likely not applicable if the ocean depth is less than  $\sim 10$  km. However, if this is the case, the assumption of uniform ocean thickness in equations (2.1) and (2.2) may also not be valid. Long-wavelength topography of either the ice shell above the ocean or the silicate mantle below the ocean can have amplitudes on the kilometer scale (Schenk and McKinnon, 2009; Nimmo and Bills, 2010). The dynamics of the ocean may be far more complex in the case of a very shallow ocean, and the shallow-water scalings we have described do not capture this complexity.

## 2.6 Conclusion

It was recognized by Tyler (2008, 2009, 2011) that tidal dissipation in a global ocean could provide a heat source that had not been previously recognized; however, the amount of dissipation could vary over orders of magnitude depending on the value of the effective viscosity (parameterized as either  $\nu$ ,  $\alpha$ , or  $Q$ ) implemented in the model. We have expanded that work in two ways. First, we provide scalings for the tidal responses, specifically the average kinetic energy and energy dissipation, in terms of the model input parameters. These analytic relations can be easily implemented into thermal-orbital evolution models for many satellites, similar to relations used for solid-body tidal dissipation. Second and more impor-

tantly, we estimate the effective viscosity of a satellite global ocean based on analogy to frictional dissipation in oceans on Earth. Both of these results are summarized in Table 4.

Based on these scaling relations and estimates for the effective viscosity, we find that for almost all satellites the thermal budget is dominated by the radiogenic and solid-body eccentricity tidal heating, as long recognized. For completeness, evolution models can include ocean tidal dissipation; however, this is unlikely to create large deviations from previous results, unless high obliquities existed in the past. In the special case of Triton where its orbit is shrinking, ocean tidal heating may provide a heat source for the apparent recent geologic activity on the surface.

| Parameter      | Value                                                             | First reference |
|----------------|-------------------------------------------------------------------|-----------------|
| $H$            | $4.5 \times 10^{-12} \text{ W kg}^{-1}$                           | §2.5.1          |
| $\rho_{sil}$   | $3500 \text{ kg m}^{-3}$                                          | eq. (2.142)     |
| $\rho_{ice}$   | $950 \text{ kg m}^{-3}$                                           | eq. (2.142)     |
| $Q_{solid}$    | 100                                                               | eq. (2.143)     |
| $G$            | $6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-3} \text{ s}^{-2}$ | eq. (2.143)     |
| $\mu$          | $4 \times 10^9 \text{ N m}^{-2}$                                  | eq. (2.144)     |
| $c$            | 0.67                                                              | eq. (2.145)     |
| $\rho$ (ocean) | $1000 \text{ kg m}^{-3}$                                          | eq. (2.40)      |
| $h$            | 30 km                                                             | eq. (2.2)       |
| $c_D$          | 0.002                                                             | eq. (2.130)     |

Table 1: Constant parameters utilized in thermal energy calculations

|           | $R$ (km) | $\rho_{sat}$ (kg/m <sup>3</sup> ) | $g$ (m <sup>2</sup> /s) | $\Omega$ (rad/s)      | $p$                 | $e$    | $i$ (deg) | $k_2$                 | $J_2$                 | $C_{2,2}$             | $\theta_0$ (deg) |
|-----------|----------|-----------------------------------|-------------------------|-----------------------|---------------------|--------|-----------|-----------------------|-----------------------|-----------------------|------------------|
| Europa    | 1565.0   | 2989                              | 1.31                    | $2.05 \times 10^{-5}$ | $-3.10 \times 10^3$ | 0.0094 | 0.466     | $2.08 \times 10^{-1}$ | $4.36 \times 10^{-4}$ | $1.32 \times 10^{-4}$ | 0.056            |
| Ganymede  | 2631.2   | 1942                              | 1.43                    | $1.02 \times 10^{-5}$ | $-6.77 \times 10^3$ | 0.0013 | 0.177     | $2.42 \times 10^{-1}$ | $1.28 \times 10^{-4}$ | $3.83 \times 10^{-5}$ | 0.031            |
| Callisto  | 2410.3   | 1834                              | 1.24                    | $4.36 \times 10^{-6}$ | $-7.42 \times 10^3$ | 0.0074 | 0.192     | $1.89 \times 10^{-1}$ | $3.27 \times 10^{-5}$ | $1.02 \times 10^{-5}$ | 0.29             |
| Mimas     | 198.2    | 1150                              | 0.064                   | $7.72 \times 10^{-5}$ | $-3.82 \times 10^2$ | 0.0196 | 1.574     | $5.73 \times 10^{-4}$ | $6.17 \times 10^{-2}$ | $1.85 \times 10^{-2}$ | 0.019            |
| Enceladus | 252.1    | 1610                              | 0.11                    | $5.31 \times 10^{-5}$ | $-6.29 \times 10^2$ | 0.0044 | 0.003     | $1.81 \times 10^{-3}$ | $2.09 \times 10^{-2}$ | $6.26 \times 10^{-3}$ | 0.000065         |
| Tethys    | 531.0    | 985                               | 0.15                    | $3.85 \times 10^{-5}$ | $-9.64 \times 10^2$ | 0.0001 | 1.091     | $3.01 \times 10^{-3}$ | $1.80 \times 10^{-2}$ | $5.39 \times 10^{-3}$ | 0.018            |
| Dione     | 561.4    | 1478                              | 0.23                    | $2.66 \times 10^{-5}$ | $-1.56 \times 10^3$ | 0.0022 | 0.028     | $7.56 \times 10^{-3}$ | $5.69 \times 10^{-3}$ | $1.71 \times 10^{-3}$ | 0.00091          |
| Rhea      | 763.5    | 1237                              | 0.26                    | $1.61 \times 10^{-5}$ | $-2.90 \times 10^3$ | 0.0002 | 0.333     | $9.78 \times 10^{-3}$ | $2.50 \times 10^{-3}$ | $7.49 \times 10^{-3}$ | 0.013            |
| Titan     | 2574.73  | 1882                              | 1.35                    | $4.56 \times 10^{-6}$ | —                   | 0.0288 | —         | $2.21 \times 10^{-1}$ | —                     | —                     | 0.32             |
| Miranda   | 235.8    | 1200                              | 0.079                   | $5.15 \times 10^{-5}$ | $-4.58 \times 10^3$ | 0.0013 | 4.338     | $8.84 \times 10^{-4}$ | $2.63 \times 10^{-2}$ | $7.89 \times 10^{-3}$ | 0.010            |
| Ariel     | 578.9    | 1665                              | 0.27                    | $2.89 \times 10^{-5}$ | $-8.30 \times 10^3$ | 0.0012 | 0.041     | $1.02 \times 10^{-2}$ | $5.96 \times 10^{-3}$ | $1.79 \times 10^{-3}$ | 0.00023          |
| Umbriel   | 584.7    | 1399                              | 0.23                    | $1.76 \times 10^{-5}$ | $-1.12 \times 10^4$ | 0.0039 | 0.128     | $7.35 \times 10^{-3}$ | $2.62 \times 10^{-3}$ | $7.87 \times 10^{-4}$ | 0.0012           |
| Titania   | 788.9    | 1714                              | 0.38                    | $8.35 \times 10^{-6}$ | $-8.20 \times 10^3$ | 0.0011 | 0.079     | $1.99 \times 10^{-2}$ | $4.85 \times 10^{-4}$ | $1.46 \times 10^{-4}$ | 0.0060           |
| Oberon    | 761.4    | 1630                              | 0.35                    | $5.40 \times 10^{-6}$ | $-5.30 \times 10^3$ | 0.0014 | 0.068     | $1.68 \times 10^{-2}$ | $2.13 \times 10^{-4}$ | $6.40 \times 10^{-5}$ | 0.022            |
| Triton    | 1353.4   | 2060                              | 0.78                    | $1.24 \times 10^{-5}$ | $-4.27 \times 10^4$ | 0.000  | 156.87    | $8.11 \times 10^{-2}$ | $8.86 \times 10^{-4}$ | $2.66 \times 10^{-4}$ | 0.17             |

Table 2: Satellite specific parameters utilized in thermal energy calculations. The physical parameters on the left half of the table are taken from Schubert et al. (2004) for the Galilean satellites, Thomas (2010), Zebker et al. (2009) and Jacobson et al. (2006) for the Saturnian satellites, Thomas (1988) and Jacobson et al. (1992) for the Uranian satellites, and Thomas (2000) and Jacobson (2009) for Triton. The orbital parameters are taken from JPL satellite ephemerides ([http://ssd.jpl.nasa.gov/?sat\\_elem](http://ssd.jpl.nasa.gov/?sat_elem)). Calculations for the quantities on the right side of the table are described in §2.5.1. The gravity coefficients for the Galilean satellites are from *Galileo* measurements (Schubert et al., 2004); others are calculated from a hydrostatic assumption. The obliquity for Titan is taken to be the observed value (Stiles et al., 2008) and is not a theoretical Cassini state value.

|           | $\dot{E}_{rad}(W)$    | homogeneous $k_2$        |                          | $k_2 = 3/2$              |                          | $\dot{E}_{ocean,ecc}(W)$ | $\dot{E}_{ocean,obl}(W)$ |
|-----------|-----------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|           |                       | $\dot{E}_{solid,ecc}(W)$ | $\dot{E}_{solid,obl}(W)$ | $\dot{E}_{solid,ecc}(W)$ | $\dot{E}_{solid,obl}(W)$ |                          |                          |
| Europa    | $2.02 \times 10^{11}$ | $9.77 \times 10^{11}$    | $1.49 \times 10^9$       | $7.05 \times 10^{12}$    | $1.07 \times 10^{10}$    | $1.47 \times 10^7$       | $3.67 \times 10^9$       |
| Ganymede  | $4.67 \times 10^{11}$ | $8.79 \times 10^9$       | $2.22 \times 10^8$       | $5.46 \times 10^{10}$    | $1.38 \times 10^9$       | $1.66 \times 10^4$       | $1.07 \times 10^9$       |
| Callisto  | $3.20 \times 10^{11}$ | $2.08 \times 10^9$       | $1.35 \times 10^8$       | $1.65 \times 10^{10}$    | $1.08 \times 10^9$       | $8.80 \times 10^2$       | $5.03 \times 10^9$       |
| Mimas     | $4.02 \times 10^7$    | $2.90 \times 10^8$       | $1.17 \times 10^4$       | $7.59 \times 10^{11}$    | $3.05 \times 10^7$       | $2.38 \times 10^7$       | $2.50 \times 10^5$       |
| Enceladus | $2.73 \times 10^8$    | $2.37 \times 10^7$       | $2.26 \times 10^{-1}$    | $1.96 \times 10^{10}$    | $1.87 \times 10^2$       | $2.32 \times 10^4$       | $1.12 \times 10^{-2}$    |
| Tethys    | $1.34 \times 10^8$    | $1.70 \times 10^5$       | $2.36 \times 10^5$       | $8.45 \times 10^7$       | $1.17 \times 10^8$       | $2.57 \times 10^1$       | $3.66 \times 10^6$       |
| Dione     | $2.42 \times 10^9$    | $4.25 \times 10^7$       | $3.14 \times 10^2$       | $8.44 \times 10^9$       | $6.24 \times 10^4$       | $4.46 \times 10^3$       | $2.07 \times 10^2$       |
| Rhea      | $3.31 \times 10^9$    | $1.73 \times 10^5$       | $3.36 \times 10^4$       | $2.65 \times 10^7$       | $5.15 \times 10^6$       | $7.35 \times 10^{-1}$    | $6.87 \times 10^5$       |
| Titan     | $4.11 \times 10^{11}$ | $6.43 \times 10^{10}$    | $3.46 \times 10^8$       | $4.37 \times 10^{11}$    | $2.35 \times 10^9$       | $1.23 \times 10^5$       | $1.06 \times 10^{10}$    |
| Miranda   | $8.50 \times 10^7$    | $6.17 \times 10^5$       | $1.61 \times 10^3$       | $1.05 \times 10^9$       | $2.74 \times 10^6$       | $6.37 \times 10^2$       | $2.69 \times 10^4$       |
| Ariel     | $3.59 \times 10^9$    | $3.00 \times 10^7$       | $4.91 \times 10^1$       | $4.42 \times 10^9$       | $7.23 \times 10^3$       | $1.36 \times 10^3$       | $5.23 \times 10^0$       |
| Umbriel   | $2.32 \times 10^9$    | $2.00 \times 10^7$       | $8.65 \times 10^2$       | $4.08 \times 10^9$       | $1.76 \times 10^4$       | $9.69 \times 10^2$       | $1.82 \times 10^2$       |
| Titania   | $9.71 \times 10^9$    | $4.71 \times 10^5$       | $6.03 \times 10^2$       | $3.55 \times 10^7$       | $4.54 \times 10^4$       | $1.63 \times 10^{-1}$    | $1.00 \times 10^4$       |
| Oberon    | $7.76 \times 10^9$    | $6.10 \times 10^4$       | $6.72 \times 10^2$       | $5.44 \times 10^6$       | $6.00 \times 10^4$       | $5.82 \times 10^{-3}$    | $1.14 \times 10^5$       |
| Triton    | $7.12 \times 10^{10}$ | 0                        | $2.04 \times 10^8$       | 0                        | $3.78 \times 10^9$       | 0                        | $9.53 \times 10^9$       |

Table 3: Energy contributions calculated from parameters presented in Tables 1 and 2 as described in §2.5.1.

| Tidal component                                                       | Average velocity                                                                              | Kinetic energy                                                                        | $\dot{E}$                                                                                  | $\nu$                                                                                                                                                      | Q                      |
|-----------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------|
| Obliquity- westward, ( $\frac{1}{\epsilon Re} \ll \frac{1}{80}$ )     | $\sqrt{\frac{3}{2}} \Omega R \theta_0$                                                        | $3\pi \rho h \Omega^2 R^4 \theta_0^2$                                                 | $12\pi \rho h \Omega^2 R^2 \nu \theta_0^2$                                                 | -                                                                                                                                                          | $\frac{1}{4} Re$       |
| Obliquity- westward, ( $\frac{1}{\epsilon Re} \gtrsim \frac{1}{80}$ ) | $\sqrt{\frac{3}{800}} \frac{\Omega^4 R^5 \theta_0}{\nu g h}$                                  | $\frac{3\pi}{400} \frac{\rho \Omega^8 R^{12} \theta_0^2}{\nu^2 g^2 h}$                | $\frac{3\pi}{100} \frac{\rho \Omega^8 R^{10} \theta_0^2}{\nu g^2 h}$                       | -                                                                                                                                                          |                        |
| Obliquity- eastward                                                   | $\sqrt{\frac{2397}{54880}} \frac{\Omega^3 R^3 \theta_0}{g h}$                                 | $\frac{2397\pi}{27440} \frac{\rho \Omega^6 R^8 \theta_0^2}{g^2 h}$                    | $\frac{9237\pi}{6860} \frac{\rho \Omega^6 R^6 \nu \theta_0^2}{g^2 h}$                      | -                                                                                                                                                          | $\frac{12316}{799} Re$ |
| <b>Obliquity- general</b>                                             | $\sqrt{\frac{27}{2}} \frac{\Omega^4 R^5 \theta_0}{(9\Omega^6 R^8 + 3600\nu^2 g^2 h^2)^{1/2}}$ | $\frac{27\pi \rho h \Omega^8 R^{12} \theta_0^2}{(9\Omega^6 R^8 + 3600\nu^2 g^2 h^2)}$ | $\frac{108\pi \rho h \nu \Omega^8 R^{10} \theta_0^2}{(9\Omega^6 R^8 + 3600\nu^2 g^2 h^2)}$ | $\left( \frac{-\frac{9}{25} \Omega^6 R^8 + \sqrt{(\frac{9}{25} \Omega^6 R^8)^2 + 4(144)(0.40c_D g \Omega^4 R^7 \theta_0)^2}}{2(144)g^2 h^2} \right)^{1/2}$ | $\frac{1}{4} Re$       |
| Eccentricity- radial                                                  | $\sqrt{\frac{57}{560}} \frac{\Omega^3 R^3 e}{g h}$                                            | $\frac{57\pi}{280} \frac{\rho \Omega^6 R^8 e^2}{g^2 h}$                               | $\frac{39\pi}{14} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}$                                 |                                                                                                                                                            | $\frac{19}{260} Re$    |
| Eccentricity- westward, librational                                   | $\sqrt{\frac{3}{224}} \frac{\Omega^3 R^3 e}{g h}$                                             | $\frac{3\pi}{112} \frac{\rho \Omega^6 R^8 e^2}{g^2 h}$                                | $\frac{69\pi}{140} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}$                                | -                                                                                                                                                          | $\frac{5}{92} Re$      |
| Eccentricity- eastward, librational                                   | $\sqrt{\frac{63}{160}} \frac{\Omega^3 R^3 e}{g h}$                                            | $\frac{63\pi}{80} \frac{\rho \Omega^6 R^8 e^2}{g^2 h}$                                | $\frac{231\pi}{20} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}$                                | -                                                                                                                                                          | $\frac{3}{44} Re$      |
| <b>Eccentricity- total</b>                                            | -                                                                                             | $\frac{57\pi}{56} \frac{\rho \Omega^6 R^8 e^2}{g^2 h}$                                | $\frac{519\pi}{35} \frac{\rho \Omega^6 R^6 \nu e^2}{g^2 h}$                                | $0.13 \frac{c_D \Omega^3 R^5 e}{g h^2}$                                                                                                                    | $\frac{95}{1384} Re$   |

Table 4: Summary of scalings presented in §2.3, §2.3.3 and §2.3.4. These scalings are accurate to 10% of the full semi-analytic solution of §2.2.2 when  $Re = \frac{\Omega R^2}{\nu} > 50$  and  $\epsilon = \frac{4\Omega^2 R^2}{g h} \lesssim \frac{4}{15}$ . The values presented for  $\nu$  are numerically obtained (see §2.4.2).

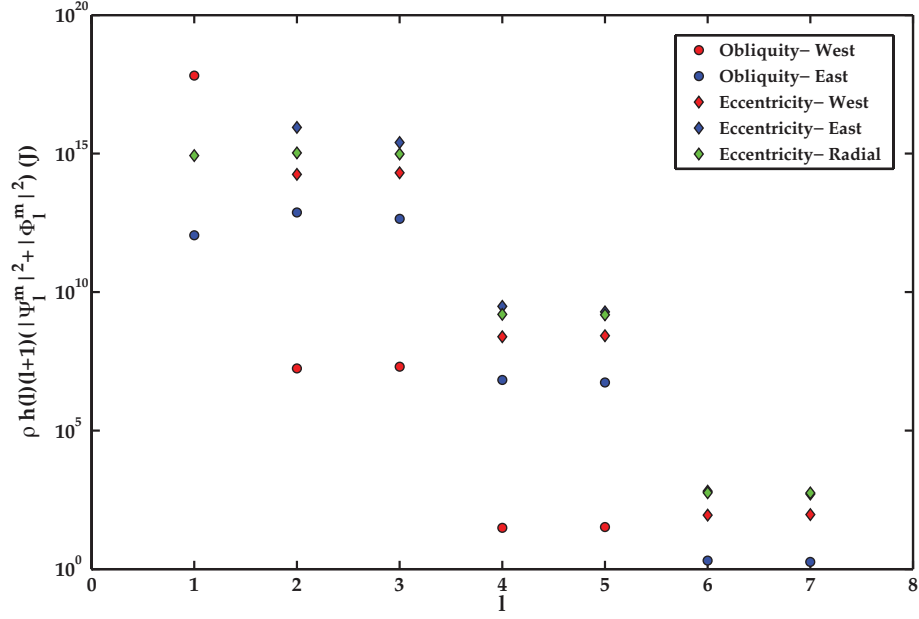


Figure 1: Power spectrum with varying spherical harmonic degree  $l$  of the full semi-analytic solution described in §2.2.2 using parameters relevant to an ocean on Europa (Tables 1 and 2), here with a viscosity of  $\nu = 1.0 \times 10^2 \text{ m}^2/\text{s}$  for both the obliquity and eccentricity tides. The response of each tidal component is separated. Note the significant separation between degrees 1 and 2 in the case of the westward obliquity response and degrees 3 and 4 for all other tidal components, justifying the scaling arguments presented in §2.3, §2.3.3 and §2.3.4.

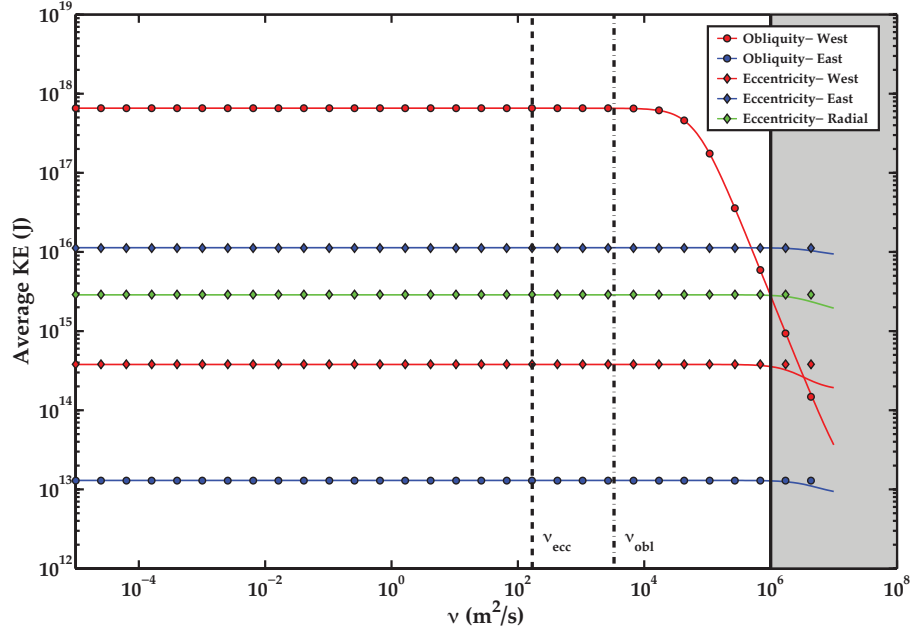


Figure 2: Average kinetic energy associated with each of the tidal responses for an ocean on Europa with parameters from (Tables 1 and 2), where symbols indicate the values calculated from scaling arguments (§2.3, §2.3.3 and §2.3.4) and the solid lines indicate values calculated through the semi-analytic solution (§2.2.2). The shaded region indicates where the Reynolds number is less than 50 for this model ocean and the scaling argument predictions are no longer within 10% of the semi-analytic solution value (see (2.54)). The dashed line represents the effective viscosity for the response to the eccentricity tide and the dot-dashed line represents that for the obliquity tide, both of which are numerically-derived (see §2.4.2) using the same coefficient of drag ( $c_D = 0.002$ ). Note that these effective viscosities are not the same, making the obliquity tide the likely dominant contributor to the ocean energy dissipation (see §2.5.2 for further details).

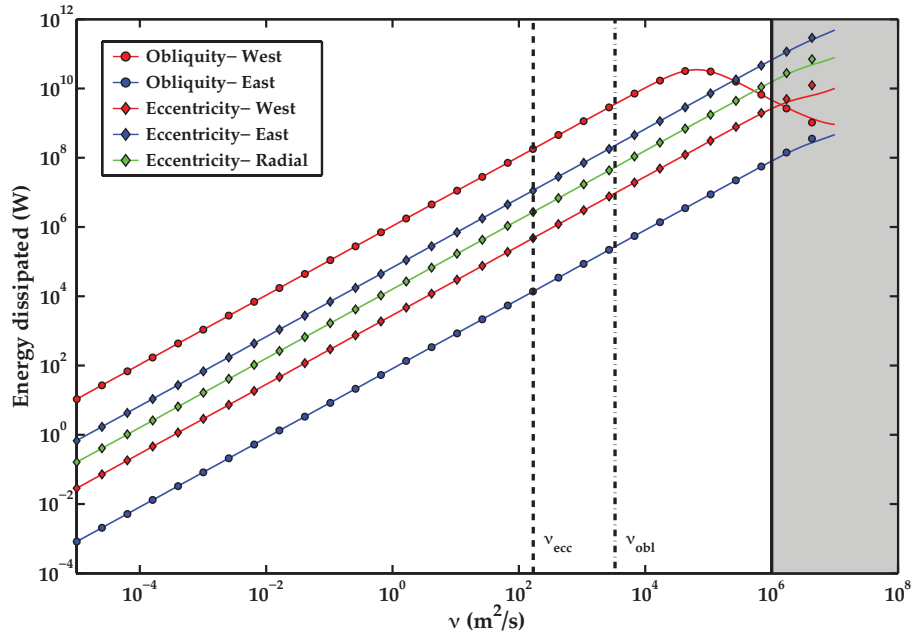
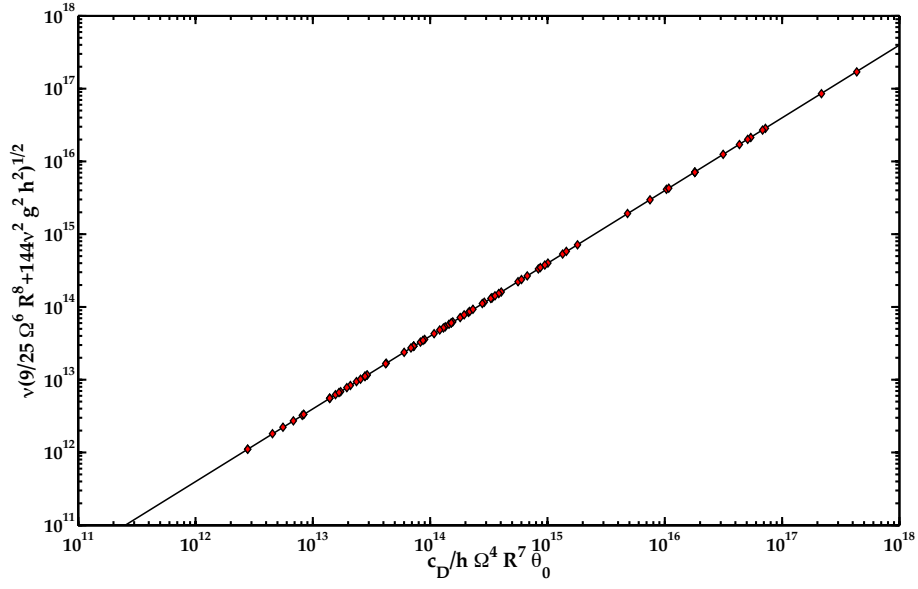
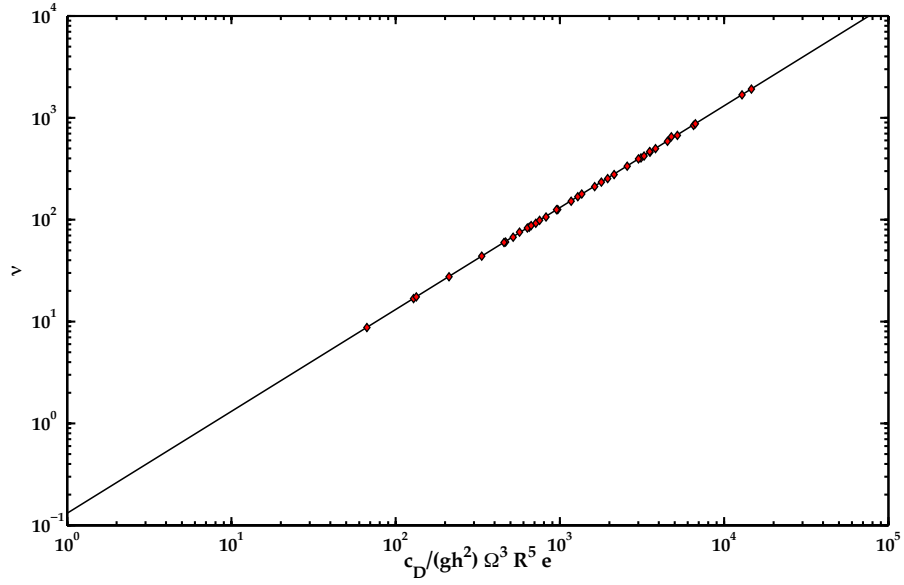


Figure 3: Similar to Figure 2 but for average energy dissipation.



(a) obliquity



(b) eccentricity

Figure 4: Results (diamonds) and linear fits (solid lines) from numerical runs presented in §2.4.2. The slopes are  $k_{obl} = 0.40$  and  $k_{ecc} = 0.13$  for (a) obliquity and (b) eccentricity, respectively.

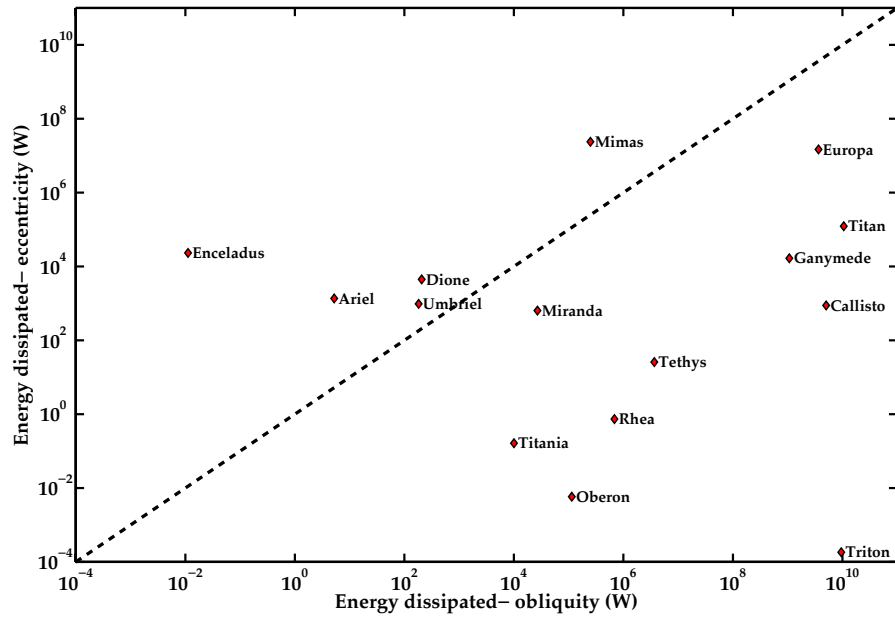
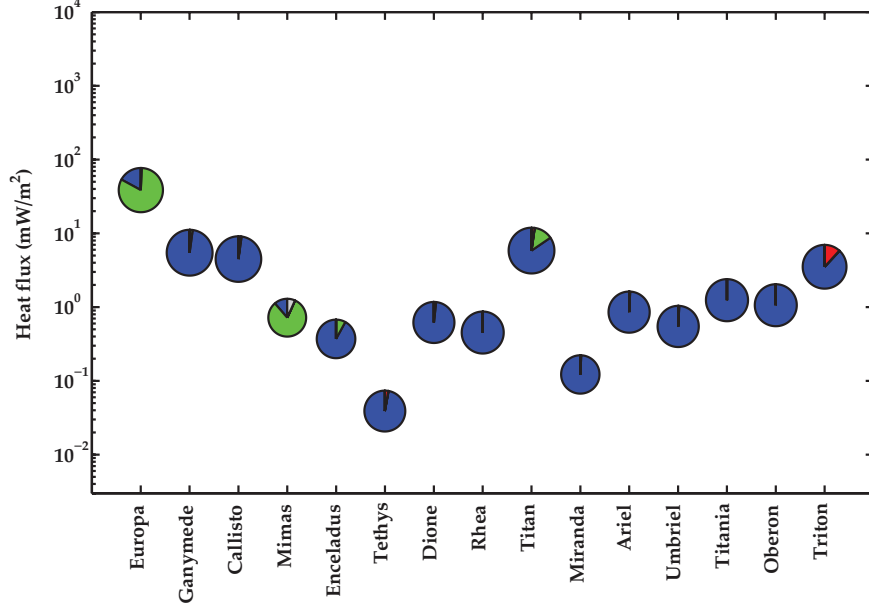
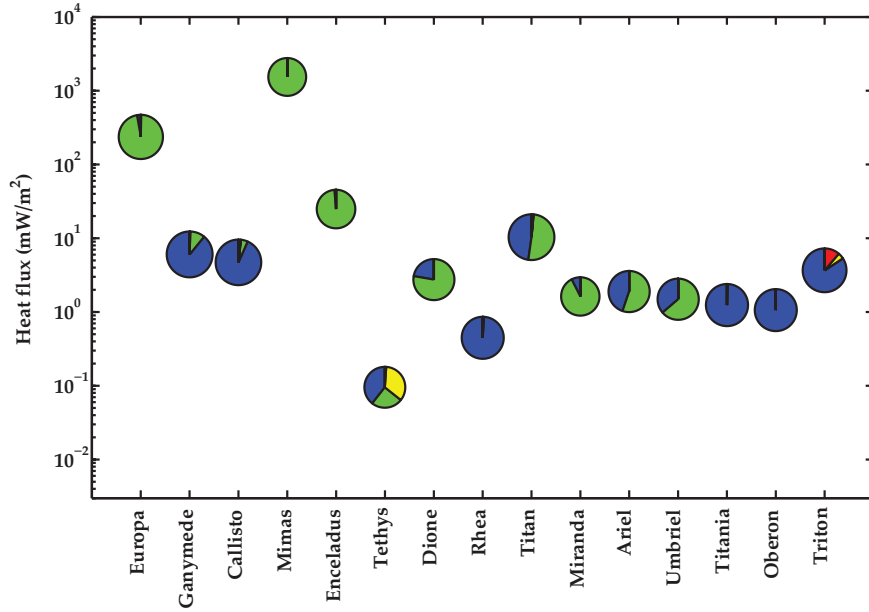


Figure 5: Energy dissipated in various satellite oceans due to the obliquity and eccentricity tides calculated from the scalings summarized in Table 4. The parameters used are  $h = 30$  km,  $c_D = 0.002$ , and satellite parameters from Table 2. For satellites lying to the right of the dotted line, the obliquity response is the dominant contribution to dissipation. Here for plotting purposes, Triton has energy dissipation associated with the eccentricity, but in reality, this value should be zero.

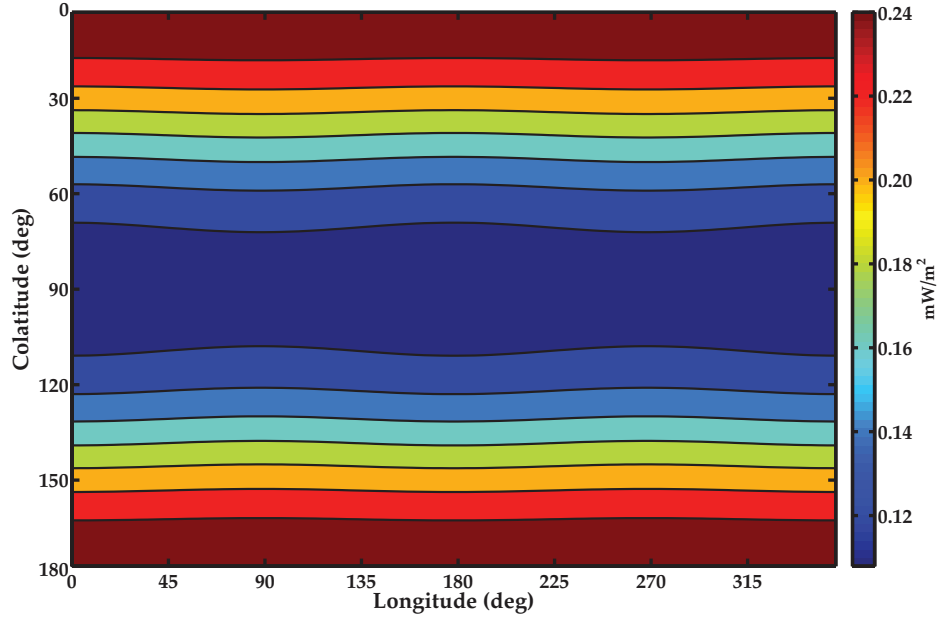


(a) homogeneous  $k_2$

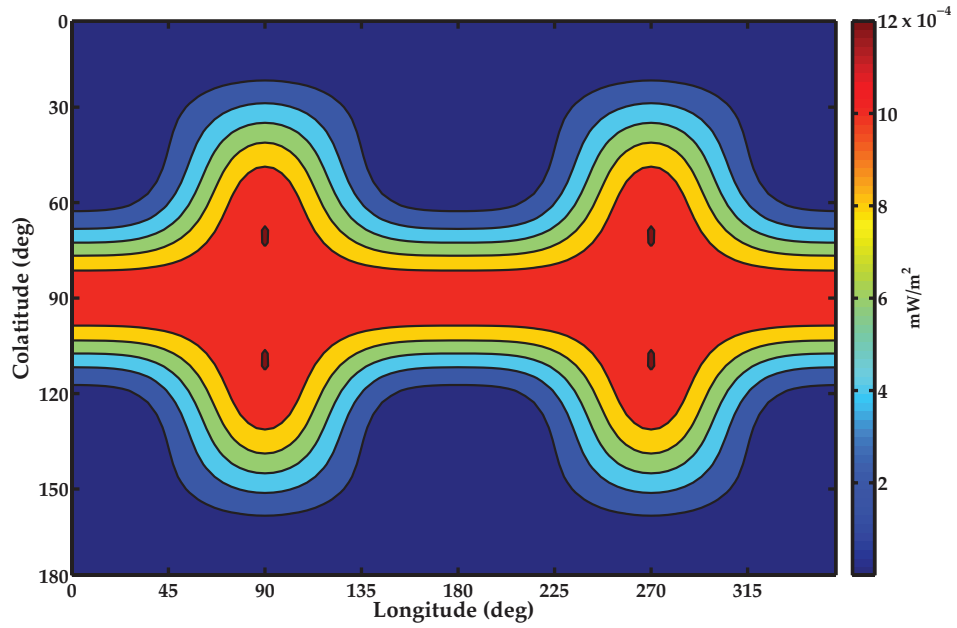


(b)  $k_2 = 3/2$

Figure 6: Thermal heat budget for various outer Solar System satellites assuming solid-body  $Q = 100$ , a bottom drag of  $c_D = 0.002$ , and ocean depth of  $h = 30$  km; solid body tidal heating components are calculated assuming a (a) viscoelastic homogeneous internal structure and (b) completely fluid response. Details of the calculations are presented in §2.5.1, while the numerical quantities are presented in Table 3. Pie charts indicate relative contributions from radiogenic heating (blue), solid body eccentricity tidal heating (green), solid body obliquity tidal heating (yellow), ocean obliquity tidal heating (red), and ocean eccentricity tidal heating (grey). The size of the pie charts are scaled by  $\log(R)$ .



(a) obliquity response



(b) eccentricity response

Figure 7: Time-averaged energy dissipation pattern associated with (a) obliquity tidal response and (b) eccentricity tidal response for a model ocean on Europa. These are calculated from the numerical model presented in §2.4 with  $c_D = 0.002$  and  $h = 30$  km.

### **3 Tidal dissipation in the lunar magma ocean and its effect on the early evolution of the Earth-Moon system**

#### **3.1 Introduction**

The Earth-Moon system has a complex history. The evolution of the lunar orbit involves the interplay of tides arising on both the Earth and the Moon and the subsequent transfer of energy from the orbit and rotation into heat via viscous dissipation; this dissipation process is frequency and time-dependent and can vary in magnitude for many different reasons, such as the configuration of the continents, the properties of the atmosphere, the internal thermal structure, and the presence a liquid magma ocean. While current observations of both the Earth and the Moon are plentiful, the early evolution of the Earth-Moon system, in terms of orbital parameters as well as the dissipative nature of the two bodies, has few constraints.

Following the Moon-forming impact, the early Moon was likely covered by a global magma ocean. Models of magma solidification have a difficult time reconciling the calculated rapid cooling of the ocean ( $\sim 10$  Myr) (Solomon and Longhi, 1977; Elkins-Tanton et al., 2011) with geochronological observations that the ages of different lunar rocks span over  $\sim 200$  Myr (Carlson and Lugmair, 1988; Borg et al., 1999; Nemchin et al., 2006; Nyquist et al., 2010). It has been suggested that tidal dissipation primarily in the crust may have prolonged cooling of these lunar rocks (Meyer et al., 2010).

Tidal dissipation due to viscoelastic solid-body tides has been evoked as a heat source for many bodies in the Solar System (Peale et al., 1979; Squyres et al., 1983; Tobie et al., 2005; Meyer and Wisdom, 2007), the Moon included (Peale and Cassen, 1978; Meyer et al., 2010). For a global satellite ocean, Tyler (2008) suggested that a resonance between the obliquity tide and a Rossby-Haurwitz wave could generate significant ocean flow and turbulent dissipation of the energy contained in this flow could also be a different and, perhaps significant, tidal heat source. In certain instances, this ocean-related obliquity tidal dissipation can be larger than the corresponding solid-body tidal dissipation, see Chapter 2. The obliquity of the Moon is quite variable as the orbit expands. At a semi-major axis of 30 Earth radii, there is a large obliquity excursion due to a transition between Cassini states 1 and 2 (Ward, 1975). Thus, tidal dissipation in the magma ocean may have been a significant

and hitherto unrecognized heat source when the Moon was still molten.

The goal of this paper is to present a model for the coupled thermal-orbital evolution of the early Earth-Moon system, including the effects of tidal dissipation in the lunar magma ocean. We find that if the lunar magma ocean is dissipative (as a result of obliquity tides), then significant inclination damping will occur. Because this damping is related to the magnitude of the obliquity, to allow the Moon to attain its current inclination, the magma ocean must have solidified prior to the large obliquity excursion associated with the Cassini state transition, which occurs near  $30R_E$ . Since the timing of magma ocean cooling is approximately known from geochronological constraints (see summary in Elkins-Tanton et al. (2011)), this argument provides a constraint on the early evolution of the Moon's orbit. The Earth's tidal quality factor  $Q$  controls the outward migration of the Moon, and during the period when the lunar magma ocean exists, we find that  $Q$  of the Earth is likely to be approximately 1000, which is much less dissipative than the current  $Q$  of  $\sim 12$  (Munk and MacDonald, 1960).

### 3.2 Methods

The evolution of the Earth-Moon system is a coupled thermal-orbital problem; the orbital parameters change due to tidal dissipation in the the Moon and the amount of energy dissipated due to tides depends on these evolving parameters. While many papers have looked at the evolution of the Earth-Moon system (Goldreich, 1966; Mignard, 1981; Touma and Wisdom, 1994; Meyer et al., 2010), none of these have explicitly included tidal dissipation in the liquid magma ocean. In the case of a global ocean, the tide due to the obliquity can force a resonant response resulting in dissipation in the ocean that may be greater than that in the viscoelastic crust (Tyler, 2008; Chen et al., 2013). Here we present the details to calculate the evolution of the Earth-Moon orbit, including time-variable tidal dissipation in the Moon due to an evolving magma ocean. We describe the four components of the model in the following sections: an orbital evolution model (§3.2.1), calculation of magma ocean dissipation (§3.2.2), calculation of the Moon's obliquity (§3.2.3), and solidification of the magma ocean (§3.2.4). Figure 9 presents a schematic of how these various components are coupled.

### 3.2.1 Coupled thermal-orbital evolution

Mignard (1979, 1980, 1981) derived equations describing the time-evolution of the lunar orbit. These equations are the averaged form of Lagrange’s planetary equations assuming that the disturbing function involves a constant time lag  $\Delta t$  (Darwin, 1908; Mignard, 1979, 1980, 1981; Touma and Wisdom, 1994) and includes dissipation in both the Earth and the Moon. For simplicity, we ignore evolution of the orbital eccentricity in this analysis; however, eccentricity has likely played an important role in many facets of the Moon’s evolution including inclination pumping during the evection resonance (Touma and Wisdom, 1998), tidal heating of the plagioclase flotation crust (Meyer et al., 2010) and the determination of the Moon’s current shape (Garick-Bethell et al., 2010). The tidal dissipation related to high orbital eccentricity is likely dominated by viscous dissipation in the solid-body and not by that in the magma ocean.

Ignoring orbital eccentricity, the Mignard evolutionary equations are (Mignard, 1981; Meyer et al., 2010):

$$\frac{dX}{dt} = \frac{C_X}{X^7} \left[ -(1 + A_X) + \left( U X^{3/2} \cos i + A_X \cos \theta_0 \right) \right] \quad (3.1)$$

$$\frac{di}{dt} = \frac{-C_i}{X^{13/2}} \left( U + \frac{A_i}{X^{3/2}} \right) \sin i \quad (3.2)$$

$$\frac{dU}{dt} = -C_{U,0} \left( U - \frac{n_\odot}{n_G} \right) + \frac{C_{U,1}}{X^{15/2}} \left( 1 - U X^{3/2} \cos i \right) \cos i - \frac{C_{U,2}}{X^6} U (\sin i)^2 \quad (3.3)$$

$$\begin{aligned} \frac{d\left(\frac{\omega}{n_G}\right)^2}{dt} = & \frac{-C_{\omega,0}}{X^6} \left( \left(\frac{\omega}{n_G}\right)^2 \left(\frac{2 + (\sin i)^2}{2}\right) + U^2 \left(\frac{3(\cos i)^2 - 1}{2}\right) + 2\frac{U}{X^{3/2}} \cos i \right) + \\ & C_{\omega,1} \left( 2\frac{n_\odot}{n_G} U - \left(\frac{\omega}{n_G}\right)^2 - U^2 \right) \end{aligned} \quad (3.4)$$

where  $X = a/R_E$ , the ratio of the semi-major axis  $a$  to the radius of the Earth  $R_E$ ,  $i$  is the orbital inclination,  $\theta_0$  is the Moon’s obliquity,  $\omega$  is the rotational frequency of the Earth,  $n_G = \sqrt{GM/R_E^3}$  is the grazing mean motion,  $n_\odot$  is the mean motion of the Earth around the Sun, and  $U = \omega/n_G \cos(I)$  with the obliquity of the Earth  $I$ . The constants are defined

as:

$$C_X = 6\gamma \frac{m}{M} \frac{m}{\mu} \quad (3.5)$$

$$C_i = \frac{3}{2}\gamma \frac{m}{M} \left(\frac{m}{\mu}\right)^{1/2} \quad (3.6)$$

$$C_{U,0} = 3\frac{\gamma}{\alpha} \left(\frac{M_\odot}{M}\right)^2 \left(\frac{R_E}{a_\odot}\right)^6 \quad (3.7)$$

$$C_{U,1} = 3\frac{\gamma}{\alpha} \left(\frac{m}{M}\right)^2 \left(\frac{m}{\mu}\right)^{1/2} \quad (3.8)$$

$$C_{U,2} = \frac{3}{2}\frac{\gamma}{\alpha} \left(\frac{m}{M}\right)^2 \quad (3.9)$$

$$C_{\omega,0} = 3\frac{\gamma}{\alpha} \left(\frac{m}{M}\right)^2 \left(\frac{m}{\mu}\right) \quad (3.10)$$

$$C_{\omega,1} = \frac{3}{2}\gamma \left(\frac{M_\odot}{M}\right)^2 \left(\frac{m}{\mu}\right) \left(\frac{R_E}{a_\odot}\right)^6 \quad (3.11)$$

with

$$\gamma \equiv k_{2E} n_G^2 \Delta t_E \quad (3.12)$$

where  $m$  is the mass of the Moon,  $M$  is the mass of the Earth,  $M_\odot$  is the mass of the Sun,  $\mu = (1/m + 1/M)^{-1}$  is the reduced mass,  $\alpha$  is the dimensionless moment of inertia of the Earth,  $a_\odot$  is the Earth-Sun distance,  $k_{2E}$  is the potential Love number of the Earth, and  $\Delta t_E$  is the tidal time lag of the Earth. A schematic of the orbit is presented in Figure 8, and the values of the physical parameters are presented in Table 5. We note that we have corrected typographic errors for  $\gamma$ ,  $C_i$ , and  $C_{\omega,1}$  from Meyer et al. (2010). The Mignard tidal parameter  $A$  is defined as

$$A_X = A_i = A \equiv \left(\frac{k_{2M} \Delta t_M}{k_{2E} \Delta t_E}\right) \left(\frac{M}{m}\right)^2 \left(\frac{R_M}{R_E}\right)^5 \quad (3.13)$$

$k_{2M}$  is the potential Love number of the Moon, and  $\Delta t_M$  is the tidal time lag of the Moon and  $R_M$  is the radius of the Moon. This tidal parameter relates the amount of energy dissipation in the Moon to that in the Earth assuming a constant time lag, viscoelastic dissipation model in both bodies. A constant time lag model is not equivalent to a constant  $Q$  tidal model as suggested in equation 17 of Meyer et al. (2010); there are many papers that address this difference (Goldreich, 1966; Touma and Wisdom, 1994; Efroimsky and Williams,

2009; Efroimsky and Makarov, 2013) and thus we do not belabor this error here.

The dissipation inherent in the Mignard evolutionary equations is equivalent to that of solid-body tides (Peale et al., 1979; Segatz et al., 1988; Ross and Schubert, 1989). Tides in the lunar magma ocean may also remove energy from and alter the orbit, but in a different manner: the tides force ocean flow and the kinetic energy of this flow is subsequently dissipated through turbulence (see Chapter 2). To account for this transfer of energy, we propose modifications to the Mignard equations presented in §3.2.1. While it is possible to calculate these modifications via a disturbing function, we derive them from considerations of conservation of energy and angular momentum.

In a Keplerian orbit, the orbital energy is given by

$$E = \frac{-GMm}{2a}, \quad (3.14)$$

and tidal dissipation causes changes in the semi-major axis of the orbit through the relation:

$$\dot{E} = \frac{GMm}{2a^2} \dot{a} \quad (3.15)$$

where the dot designates a time derivative. For an inclined orbit with no eccentricity, the vertical component of the orbital angular momentum (i.e. that parallel to the invariable pole  $\hat{k}$  in Figure 8) is (Chyba et al., 1989),

$$L = m(GM)^{1/2} a^{1/2} \cos i. \quad (3.16)$$

This quantity is conserved and the following relations are true after taking the time derivative of (3.16) and including the relation (3.15)

$$\frac{\dot{a}}{a} = 2 \tan i \frac{\dot{a}}{a} = \frac{2a\dot{E}}{GMm}. \quad (3.17)$$

These relations must hold true independent of the energy dissipation process.

In the Mignard equations, the Mignard tidal parameter conceals the relations (3.17).

Ignoring dissipation in the Earth, the evolution of the semi-major axis from (3.1) is

$$\dot{a} = C_X \left( \frac{R_E}{a} \right)^8 a A_X (\cos \theta_0 - 1) = \frac{2a^2 \dot{E}_M}{GMm} \quad (3.18)$$

where  $\dot{E}_M$  represents dissipation in the Moon. Thus, the tidal parameter  $A_X$  is related to  $\dot{E}_M$  by

$$A_X = \frac{1}{3\gamma} \left( \frac{M}{m} \right) \left( \frac{a}{R_E} \right)^8 \frac{a \dot{E}_M}{GMm (\cos \theta_0 - 1)}. \quad (3.19)$$

It can be shown that equation (3.19) is equivalent to the definition in (3.13) if

$$\dot{E}_M = \frac{-3}{2} k_{2M} \Delta t_M \frac{G^2 M^3 R_M^5}{a^9} \sin^2 i \quad (3.20)$$

under the assumptions that  $i \approx \theta_0$  and small  $i$ , which is typically assumed for the case of solid-body obliquity tides (Peale and Cassen, 1978; Wisdom, 2008).

Similarly, the evolution of the inclination from equations (3.2) and (3.17) can be expressed as

$$\frac{di}{dt} = -C_i \left( \frac{R_p}{a} \right)^8 A_i \sin i = \frac{a \dot{E}_M \cos i}{GMm \sin i} \quad (3.21)$$

and thus,

$$A_i = \frac{-2}{3\gamma} \left( \frac{M}{m} \right) \left( \frac{a}{R_p} \right)^8 \frac{a \dot{E}_M \cos i}{GMm (\sin i)^2}, \quad (3.22)$$

which similarly simplifies to (3.13) for the dissipation given by (3.20). To account for dissipation in the magma ocean, we utilize the generic forms for  $A_X$  and  $A_i$  as presented in (3.19) and (3.22) in the evolutionary equations (3.1) and (3.2) instead of the definition presented in (3.13) because now we are considering dissipation in the magma ocean. We describe how to calculate the appropriate  $\dot{E}_M$  for the magma ocean in the following section.

### 3.2.2 Magma ocean dissipation

Tyler (2008, 2009) suggested that a global liquid ocean within a satellite, specifically Europa and Enceladus, would respond resonantly to obliquity-related tidal forcing and that frictional dissipation of this ocean flow could provide a significant heat source. For most of the outer Solar System satellites, the dominant heat source is unlikely to be ocean dissipation (see Chen et al. (2013)); this result is primarily due to the fact that these satellites formed

locally in a disk (Canup and Ward, 2002), and thus their inclinations and the resultant obliquities are quite small. However, the Moon's inclination is currently much higher than any of the outer Solar System satellites, approximately  $5^\circ$  where other satellites' inclinations are less than  $1^\circ$  (Yoder, 1995), and the Moon likely underwent a Cassini state transition early in its history, where its obliquity could have been much larger than the orbital inclination (Ward, 1975). Therefore, it is reasonable to believe that dissipation in the magma ocean may have contributed a significant amount of heat in the evolution of the Moon. We thus summarize a method of calculating tidal dissipation in this early magma ocean, and point the reader to Chen et al. (2013) for further details of the method.

The behavior of a global magma ocean can be described through the linear shallow-water equations on a sphere:

$$\frac{\partial \vec{u}}{\partial t} + 2\Omega \cos \theta \hat{r} \times \vec{u} = -g\vec{\nabla}\eta - \vec{\nabla}U + \nu\nabla^2\vec{u} \quad (3.23)$$

$$\frac{\partial \eta}{\partial t} + h\vec{\nabla} \cdot \vec{u} = 0 \quad (3.24)$$

where  $\vec{u}$  is the radially-averaged, horizontal velocity vector,  $\Omega$  is the constant rotation rate, which is the same as the orbital mean motion for a synchronous satellite,  $\hat{r}$  is the unit vector in the radial direction,  $g$  is the surface gravity,  $\eta$  is the vertical displacement of the surface,  $h$  is the constant ocean depth and  $\nu$  represents an effective horizontal viscous diffusivity.  $U$  represents the radially-averaged forcing potential due to tides, here for the obliquity tide:

$$U = \frac{-3}{2}\Omega^2 R_M^2 \theta_0^2 \sin \theta \cos \theta (\cos(\phi - \Omega t) + \cos(\phi + \Omega t)). \quad (3.25)$$

For synchronously rotating satellites, the forcing potential due to obliquity is resonant with a Rossby-Haurwitz wave (Tyler, 2008), and thus flow in a global ocean can be significant. The energy dissipation associated with this response is given by:

$$\dot{E}_{obl} = \frac{108\pi}{25} \frac{\rho h \nu \Omega^8 R_M^{10} \theta_0^2}{(9/25 \Omega^6 R_M^8 + 144 \nu^2 g^2 h^2)} \quad (3.26)$$

where  $\rho$  is the magma ocean density and  $\nu$  is a turbulent viscous diffusivity given by the

numerically-derived scaling law (Chen et al., 2013)

$$\nu = \left( \frac{\frac{-9}{25}\Omega^6 R_M^8 + \sqrt{\left(\frac{9}{25}\Omega^6 R_M^8\right)^2 + 4(144)(0.40c_D g \Omega^4 R_M^7 \theta_0)^2}}{2(144)g^2 h^2} \right)^{1/2} \quad (3.27)$$

with an associated bottom drag  $c_D$ .

The energy dissipation in the ocean evolves in time as the Moon's rotation period  $\Omega$ , obliquity  $\theta_0$  and ocean depth  $h$  evolve. Assuming the Moon has remained synchronous, the rotational frequency is equal to the orbital mean motion, which changes with the orbital semi-major axis (cf. equation (3.1)). We assume a Cassini state value for the obliquity and describe the process of determining this value in §3.2.3. We describe the time evolution of the magma ocean depth in §3.2.4.

### 3.2.3 Cassini state obliquity

While the current lunar obliquity can be measured, the obliquity of the Moon in the past is uncertain. For this work, we assume that throughout its evolution the Moon has occupied a Cassini state, where its spin axis, the orbit normal and an invariable pole remain coplanar as they precess (Colombo, 1966; Peale, 1969). The obliquity is given by the angle between the orbit normal and the Moon's spin axis (see Figure 8).

For coplanar precession to occur, the obliquity angle  $\theta_0$  must satisfy the relation (Peale, 1969; Bills and Nimmo, 2008)

$$\frac{3}{2} [(J_2 + C_{2,2}) \cos \theta_0 + C_{2,2}] p \sin \theta_0 = c \sin(i - \theta_0) \quad (3.28)$$

where  $J_2$  and  $C_{2,2}$  are the degree-2 gravity coefficients and  $c$  is the normalized polar moment of inertia of the Moon. The ratio of orbital motion to the orbit plane precession  $p$  is

$$p = \frac{\Omega}{d\Omega_{orb}/dt} \quad (3.29)$$

where  $\Omega_{orb}$  is the longitude of the ascending node.

It is well-known that the currently observed gravity coefficients of the Moon are inconsistent with a hydrostatic tidally distorted body (Jeffreys, 1915; Konopliv et al., 1998, 2001;

Garrick-Bethell et al., 2010). Because we are concerned with the Moon during the early stages of orbital evolution, where it is warm and likely to behave more like a fluid, we make the hydrostatic assumption. For hydrostatic bodies, the normalized moment of inertia  $c$  can be related to the displacement Love number  $h_2$  via the Radau-Darwin relation (Hubbard and Anderson, 1978)

$$c = \frac{2}{3} \left( 1 - \frac{2}{5} \left( \frac{5}{h_2} - 1 \right)^{1/2} \right). \quad (3.30)$$

For a synchronous satellite, the degree-2 gravity coefficients can be related to  $h_2$  by

$$J_2 = \frac{5}{6} \left( \frac{R_M^3 \Omega^2}{Gm} \right) (h_2 - 1) \quad (3.31)$$

$$C_{2,2} = \frac{3}{10} J_2. \quad (3.32)$$

The precession of the orbit arises due to torques from the rotational flattening of the Earth and the Sun. The total precessional frequency is the sum of the two

$$\frac{d\Omega_{orb}}{dt} = \frac{-3}{2} n J_{2E} \left( \frac{R_E}{a} \right)^2 + \frac{-3}{4} \left( \frac{M_\odot}{M+m} \right) n \left( \frac{a}{a_\odot} \right)^3 \quad (3.33)$$

with orbital mean motion  $n$ , which for synchronous satellites is equal to the rotation rate  $\Omega$ , and degree-2 gravity coefficient of the Earth  $J_{2E}$ . The effect of the Earth's flattening is important when the Moon is close to the Earth, i.e.  $a \lesssim 17R_E$  (Goldreich, 1966). For a hydrostatic planet,  $J_{2E}$  can be expressed as

$$J_{2E} = \frac{1}{3} \left( \frac{R_E^3 \omega^2}{GM} \right) k_{2E} \quad (3.34)$$

where  $\omega$  is the Earth's rotation rate and  $k_{2E}$  is the tidal Love number. The physical constants used for these calculations are tabulated in Table 5.

### 3.2.4 Solidification of the ocean

The solidification of the magma ocean is a complex problem, involving fractional crystallization from a chemically- and thermally-evolving reservoir. Our main interest with respect to the solidification process is to determine the ocean depth as a function of time because

the tidal dissipation is dependent on this quantity. Unlike other models, we do not seek to explain the geochemistry and geochronology of the Moon's surface (Carlson and Lugmair, 1988; Borg et al., 1999; Nemchin et al., 2006; Nyquist et al., 2010) and therefore, we choose to model the solidification of the magma ocean as a Stefan problem (Turcotte and Schubert, 2002) with heating to account for the effect of tidal dissipation. The evolution of the radial position of the solid-liquid interface  $r_i$ , in this case between the solid crust and the liquid magma, is

$$\rho L \frac{d(R_M - r_i)}{dt} = -k \left( \frac{\partial T}{\partial r} \right)_{r=r_i} - H \quad (3.35)$$

where  $L$  is the latent heat of fusion,  $k$  is the thermal conductivity of the crust, and  $H$  is the heat flux into the base of the crust. This heat flux is assumed to be solely due to tidal dissipation in the magma ocean such that

$$H \approx \frac{\dot{E}_M}{4\pi R_M^2}. \quad (3.36)$$

Assuming a conductive thermal profile in the solid crust, the temperature gradient at the interface is given by (Turcotte and Schubert, 2002)

$$\left( \frac{\partial T}{\partial r} \right)_{r=r_i} = \frac{-(T_m - T_0)}{(R_M - r_i)} \frac{2}{\sqrt{\pi}} e^{-\lambda_1^2} \frac{\lambda_1}{\text{erf}(\lambda_1)} \quad (3.37)$$

where  $T_m$  is the magma ocean temperature,  $T_0$  is the surface temperature,  $c_p$  is the specific heat capacity of the crust, and  $\lambda_1$  is determined by

$$\frac{e^{-\lambda_1^2}}{\lambda_1 \text{erf}(\lambda_1)} = \frac{L\sqrt{\pi}}{c_p(T_m - T_0)}. \quad (3.38)$$

The approach we have used is by far the simplest model of a solidifying magma ocean; however, as we will discuss in §3.4.2, the conclusions drawn from our model only depend on the time of ocean solidification and not the details of the process.

### 3.3 Results

Our nominal model is initialized at a semi-major axis of  $a = 6.5R_E$ , inclination of  $i = 12^\circ$ , and a rotation period of the Earth of 6 hours; these are estimates of the conditions after

passage through the “eviction” resonance (Touma and Wisdom, 1994, 1998). Figure 10 shows the orbital evolution accounting for tidal dissipation only in the Earth ( $A = 0$ ) for various values of the tidal time lag  $\Delta t_E$ . Changing the time lag does not alter the evolution path; the curves collapse on top of each other with respect to the semi-major axis. The Earth’s time lag sets the rate of the Moon’s outward migration.

Figure 11 shows the fully coupled thermal-orbital evolution of the Earth-Moon system. The most obvious effect of including magma ocean dissipation is the rapid damping of the orbital inclination. The inclination monotonically decreases as a function of time due to dissipation in both the Earth and the Moon (cf. equation (3.2)) unless altered by orbital resonances, which beyond the “eviction” resonance are not thought to have occurred in the Earth-Moon system (Touma and Wisdom, 1998). Thus, the inclination at the end of magma ocean evolution must be higher than the current inclination of  $\sim 5^\circ$ . The inclination damping is controlled primarily by the dissipation in the Moon, which is mainly related to the value of the obliquity. For the parameters used in our model, the Moon’s obliquity is determined by the Cassini state transition at a semi-major axis of  $\sim 30R_E$ . The large obliquity excursion around this transition generates a significant amount of tidal dissipation if a magma ocean is present and this dissipation rapidly damps the inclination. Only models that delay the evolution through the Cassini state transition until after the magma ocean is solidified can maintain an orbital inclination greater than that currently observed. This effect can be achieved through decreasing the Earth’s tidal time lag (Figure 11).

### 3.4 Discussion

As shown in Figure 11, a major constraint for our coupled thermal-orbital evolution model is the current (relatively high) orbital inclination. The damping of the inclination due to dissipation solely from the Earth is weak. As shown in Figure 10, extremely large time lags ( $\sim 1000$  s) can maintain an inclination of greater than  $5^\circ$  over billions of years of evolution. The damping of inclination due to satellite dissipation operates on a different, and much shorter, timescale, especially if a magma ocean is present. In fact, this effect is so significant that we believe it provides two constraints for the early evolution of the Earth-Moon system:

- The Moon’s magma ocean must have solidified prior to passage through the Cassini state transition.
- The early Earth’s effective tidal time lag is likely to be less than  $\sim 1$  s, a value that is orders of magnitude less than the present value of  $\sim 10$  minutes (Munk and MacDonald, 1960), i.e. the Earth was significantly less dissipative in its early history when compared to its present state.

For several reasons, the role of inclination and obliquity has been widely neglected when modeling tidal dissipation in satellites and their orbital evolution. First, the orbital inclinations of the regular satellites of Jupiter and Saturn are small (Yoder, 1995), presumably due to formation (Canup and Ward, 2002). Because of this, the satellite obliquities are assumed to be small because of constraints from imagery (Smith et al., 1989; Stiles et al., 2008) as well as from theoretical Cassini state calculations (Bills, 2005; Bills and Nimmo, 2011). For solid-body tides, the amount of tidal dissipation is given by (Peale and Cassen, 1978; Wisdom, 2008):

$$\dot{E} = \frac{3}{2} \frac{k_{2M}}{Q_M} \frac{(\Omega R_M)^5}{G} (7e^2 + \sin^2 \theta_0). \quad (3.39)$$

In the Solar System, solid-body tidal dissipation is skewed towards the contribution from eccentricity tides for two main reasons: first, by the pre-factor of 7 in this tidal model and second, by the presence of the Laplace resonance in the Jovian system and the Enceladus-Dione resonance in the Saturnian system (Peale et al., 1979; Peale, 1986; Ross and Schubert, 1989; Meyer and Wisdom, 2007), which enhance values of the orbital eccentricity.

It has been long recognized that the Moon has anomalously high inclination, the “mutual inclination problem” (Burns, 1986; Touma and Wisdom, 1998), and has likely undergone a period where the obliquity was extremely high, the Cassini state transition (Ward, 1975). Using a solid-body dissipation model, Peale and Cassen (1978) looked at the Moon and concluded that a high obliquity excursion did not significantly affect the Moon’s long-term thermal evolution. We do not debate this conclusion; however, it was recently only recognized that resonant flow behavior in the ocean due to the obliquity tide (Tyler, 2008) can result in tidal dissipation that is orders of magnitude larger than that predicted by the Maxwell model even at small obliquities (less than  $1^\circ$ ) (Chen et al., 2013).

### 3.4.1 Inclination damping

The major result of our coupled model is that dissipation in the lunar magma ocean causes significant damping in the orbital inclination (Figures 11). The damping of the inclination due to satellite dissipation was given from conservation of angular momentum arguments in equation (3.17) and repeated here:

$$\frac{\dot{a}}{a} = 2 \tan i \frac{di}{dt} = \frac{2a\dot{E}_i}{GMm}. \quad (3.40)$$

We would like to emphasize that the timescale for inclination damping due to dissipation in the satellite exhibits similar behavior to the well-known phenomenon of eccentricity damping (Murray and Dermott, 1999):

$$\frac{\dot{a}}{a} = \frac{2e\dot{e}}{(1-e^2)} = \frac{2a\dot{E}_{ecc}}{GMm}, \quad (3.41)$$

which is recognized to be quite rapid. Inclinations for satellites of the Solar System are relatively small and are likely to have been damped perhaps due to gas drag in the disk, but also by tidal damping, similar to eccentricities. This inclination damping mechanism is quite different from (and much faster than) inclination damping due to dissipation in the primary which is given by (Kaula, 1964; Tittlemore and Wisdom, 1989)

$$\frac{-1}{i} \frac{di}{dt} \approx \frac{1}{4a} \dot{a}. \quad (3.42)$$

From (3.40) and (3.42), we can calculate inclination damping timescales  $\tau_{i,sat}$  and  $\tau_{i,p}$  due to dissipation in the satellite and the primary, respectively. These timescales are

$$\tau_{i,sat} = \frac{i}{\frac{di}{dt}} \approx \frac{i^2 a}{\dot{a}}, \quad (3.43)$$

$$\tau_{i,p} = \frac{i}{\frac{di}{dt}} \approx \frac{a}{\dot{a}}. \quad (3.44)$$

For the inclinations we are concerned with, the factor of  $i^2$  clearly makes the damping timescale more rapid for satellite dissipation. This difference is emphasized by the dramatic change in inclination between our orbital evolution models with and without lunar magma

ocean dissipation (Figure 11).

### 3.4.2 Dissipation in the early Earth

How much dissipation occurred in the early Earth is a mystery. The current tidal quality factor of the Earth is small,  $Q \sim 12$  due mainly to dissipation in the oceans (Munk and MacDonald, 1960; Munk, 1997; Egbert and Ray, 2000), which is equivalent to a tidal time lag of  $\Delta t_E \approx 10$  minutes. Integrating back in time with this present value of  $Q$ , results in an Earth-Moon system age of  $\sim 1.6$  Gyr (Murray and Dermott, 1999; Bills and Ray, 1999); therefore, it is clear that the  $Q$  of the early Earth was not as low as it is present. Dissipation in the Earth is geologically unconstrained prior to 0.6 Ga (Williams, 2000).

From our model, we find that the tidal time lag of the early Earth, specifically when the magma ocean was still present in the Moon, is likely to be less than 1 s. The time lag controls the outwards semi-major axis evolution of the Moon. As the Moon moves outwards from the location of the eviction resonance ( $6 - 7 R_E$ ), its obliquity grows. The magma ocean dissipation scales with the obliquity and thus dissipation also increases as the Moon moves outwards. As dissipation occurs, energy is drawn from the orbit and the orbital inclination is damped. Therefore, it is critical that while the magma ocean is present, the high obliquity excursion related to the Cassini state transition does not occur. Otherwise, the inclination would be damped to a value lower than what is currently observed. Figure 11 shows that the obliquity of the Moon need not be at its maximum for this damping to occur; as an estimate, if the obliquity is larger than  $\sim 1^\circ$  then magma ocean dissipation likely damps the inclination to a smaller than present value. This value for obliquity occurs at  $a \approx 20R_E$  in our models.

Coupled thermal-orbital models are required to capture the details of the early Earth-Moon evolution; however, our conclusions are fairly simple and insensitive to the details of the model. Summarized, these results are: the magma ocean must solidify prior to when the Moon's semi-major axis reaches  $\sim 20R_E$ , otherwise ocean tidal dissipation will damp the orbital inclination, and since the Earth's dissipation controls the outwards migration of the Moon, the Earth during this time period must have been far less dissipative than it is at present. We now present the results from an analytic model (Murray and Dermott, 1999) for

the semi-major axis evolution for different constant values of  $Q$  in the Earth in Figure 12. For a magma ocean that remains liquid for 10-100 Myr (Solomon and Longhi, 1977; Elkins-Tanton et al., 2011), a reasonable estimate for  $Q$  where the semi-major axis does not exceed  $20R_E$  is approximately 1000. Zahnle et al. (2007) suggested that an atmosphere on the early Earth could reduce the tidal dissipation of the Earth, such that  $Q \gtrsim 300$ .

### 3.4.3 Caveats

Through this coupled thermal-orbital model of the early Earth-Moon system, we have suggested that the current orbital inclination of the system provides a constraint on the dissipation behavior of the early Earth. This result is related to the fact that the dissipation of satellite tides is very effective in damping the inclination (cf. equation (3.40)). We provide estimates for the tidal time lag of the early Earth and the threshold value of the semi-major axis where the lunar magma ocean is likely to have been solidified; the exact values of these quantities are certainly model dependent. The main uncertainty of this result is in the calculation of  $\dot{E}$  of the magma ocean. The total amount of energy dissipation scales with the value of the viscous diffusivity  $\nu$ , which is poorly constrained (see discussion in Chen et al. (2013)). For the magma ocean, we have adopted a value of  $c_D = 0.002$ , commonly used in models for bottom friction in terrestrial oceans (Jayne and St. Laurent, 2001; Egbert and Ray, 2001), and mapped this value to a viscous diffusivity through equation (3.27). If the drag coefficient of the magma ocean is in reality smaller than the value used here, the tidal time lag in the Earth can be larger. However, because the values of obliquity are so high during the Cassini state transition, it is difficult to envision a scenario where a magma ocean can be present through the Cassini state transition, while the orbital inclination is not severely damped by ocean dissipation.

## 3.5 Conclusion

In our model for the coupled thermal-orbital evolution of the Moon that includes dissipation produced by the lunar magma ocean, we find that dissipation in the ocean has a strong tendency to damp the orbital inclination. The dissipation is related to the Moon's obliquity. In the early evolution of the system, the obliquity is relatively small, less than  $\sim 1^\circ$ . However,

because of the occurrence of the Cassini state transition at  $\sim 30R_E$ , the obliquity increases rapidly near a semi-major axis of  $\sim 20R_E$  and it is difficult to maintain a high orbital inclination in the presence of enhanced dissipation due to the increasing obliquity. Therefore, our first conclusion is that the magma ocean must solidify prior to  $\sim 20R_E$  (with the exact distance being sensitive to the initial conditions and details of the thermal-orbital model), in order for the inclination to not be damped to a value lower than that which is currently observed,  $i = 5.16^\circ$ .

The evolution of the semi-major axis of the Moon's orbit is controlled in our model by dissipation in the Earth. In our model, we find that the time lag of the early Earth is likely to be quite small (less than  $\sim 1$  second) for magma oceans that solidify prior to a semi-major axis of  $\sim 20R_E$ . This result can be compared to models without magma ocean dissipation, where the time lag can be larger by many orders of magnitude, even up to  $\sim 1000$  s and still have a higher inclination than present. This small time lag loosely translates to a constant tidal  $Q$  for the early Earth of approximately 1000; this value is consistent with the suggestion that the low value of  $Q$  at present is due to enhanced dissipation associated with resonant behavior ocean basins (Webb, 1982). In the early history of the Earth, this was not the case (Bills and Ray, 1999), and additionally, the dissipation was likely lower due to the Hadean atmosphere (Zahnle et al., 2007). This result also suggests that global water oceans were absent (or not significantly dissipative) on the early Earth in the period following the eviction resonance.

| Symbol    | Parameter                                 | Value                                                              |
|-----------|-------------------------------------------|--------------------------------------------------------------------|
| $G$       | gravitational constant                    | $6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ |
| $M$       | mass of the Earth                         | $5.97 \times 10^{24} \text{ kg}$                                   |
| $R_E$     | radius of the Earth                       | 6370 km                                                            |
| $m$       | mass of the Moon                          | $7.25 \times 10^{22} \text{ kg}$                                   |
| $R_M$     | radius of the Moon                        | 1737 km                                                            |
| $M_\odot$ | mass of the Sun                           | $2.0 \times 10^{30} \text{ kg}$                                    |
| $a_\odot$ | Earth-Sun distance                        | $149.6 \times 10^6 \text{ km}$                                     |
| $k_{2E}$  | potential Love number of the Earth        | 0.97                                                               |
| $\alpha$  | normalized moment of inertia of the Earth | 0.33                                                               |
| $g$       | surface gravity of the Moon               | $1.62 \text{ m}^2 \text{ s}^{-1}$                                  |
| $\rho$    | magma ocean density                       | $3000 \text{ kg m}^3$                                              |
| $c_D$     | bottom drag coefficient                   | 0.002                                                              |
| $c$       | normalized moment of inertia of the Moon  | 0.393                                                              |
| $L$       | latent heat of fusion                     | $5.0 \times 10^5 \text{ J kg}^{-1}$                                |
| $k$       | thermal conductivity                      | $3.77 \text{ kg m s}^{-3} \text{ K}^{-1}$                          |
| $T_m$     | temperature of magma ocean                | 1200 K                                                             |
| $T_0$     | temperature of lunar surface              | 280 K                                                              |
| $c_p$     | specific heat capacity                    | $1256 \text{ J kg}^{-1} \text{ K}^{-1}$                            |

Table 5: Physical parameters utilized in the model for Earth-Moon system evolution

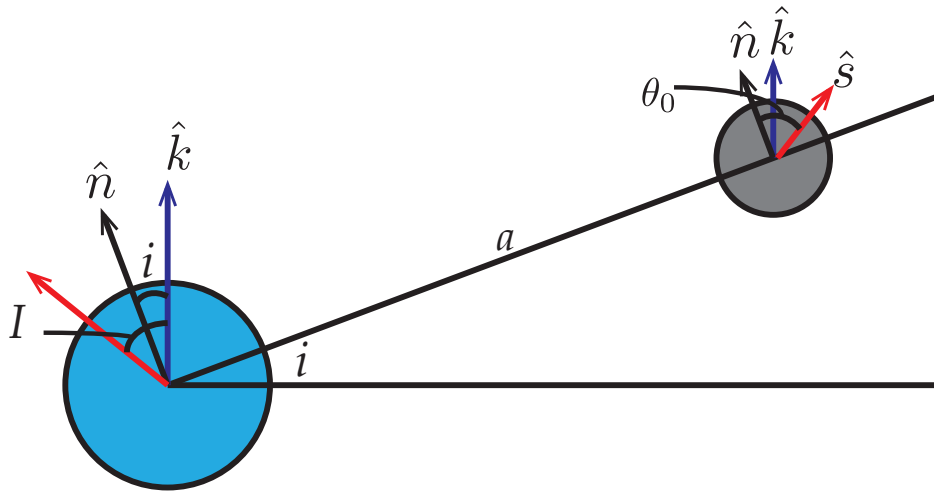


Figure 8: Geometry of the orbit described by the model in §3.2.1. The relevant angles are the inclination  $i$ , the angle between the orbit normal ( $\hat{n}$ ) and the invariable pole ( $\hat{k}$ ), and the Moon's obliquity  $\theta_0$ , the angle between the spin axis ( $\hat{s}$ ) and the orbit normal ( $\hat{n}$ ). The Earth's obliquity is denoted by  $I$ . We assume that the Moon's obliquity occupies a Cassini state where  $\hat{n}$ ,  $\hat{k}$ , and  $\hat{s}$  all remain coplanar while  $\hat{n}$  and  $\hat{s}$  precess.

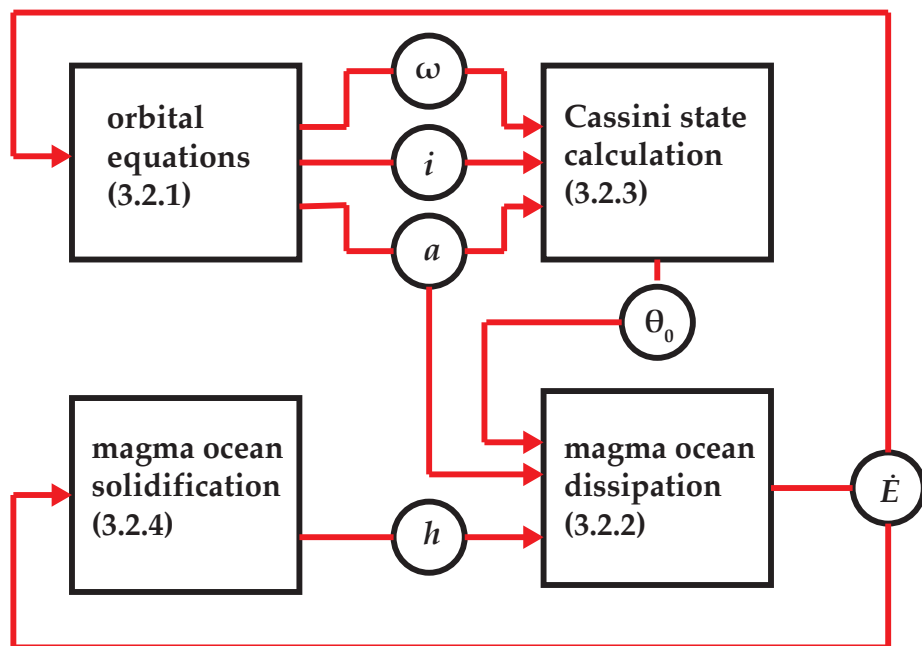


Figure 9: Schematic for the coupled model described in §3.2.

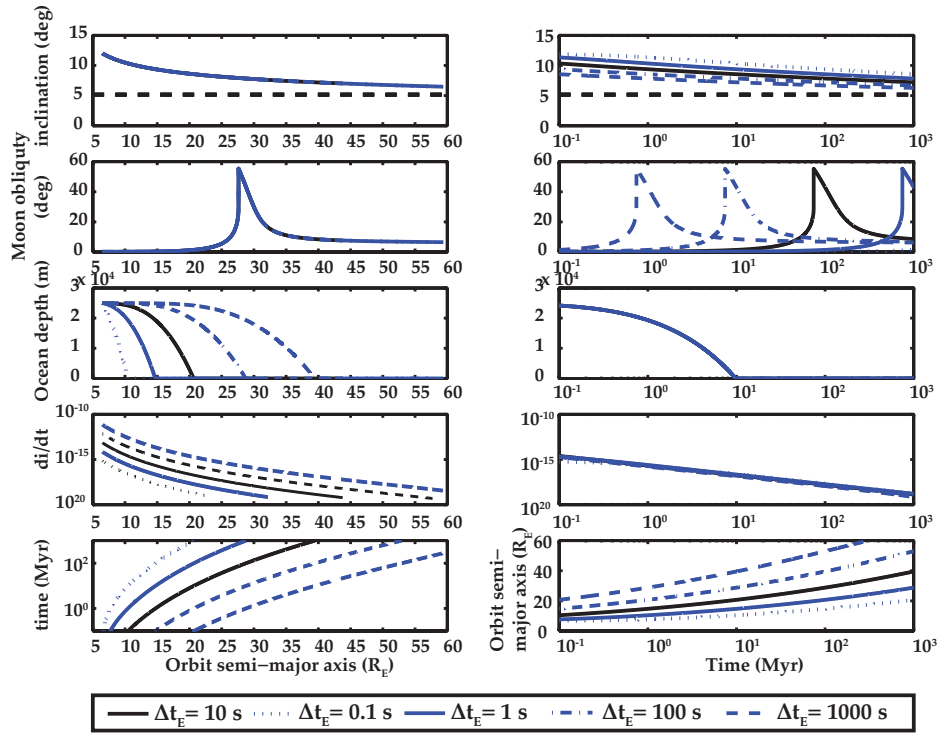


Figure 10: Orbital and thermal evolution for a model without magma ocean dissipation for various tidal time lags of the Earth. Changing the time lag does not change the trajectory of the evolution, but controls the rate of orbital evolution. The dashed line in the top two panels indicates the current value of the orbital inclination. Even for large time lags, the inclination is not damped significantly.

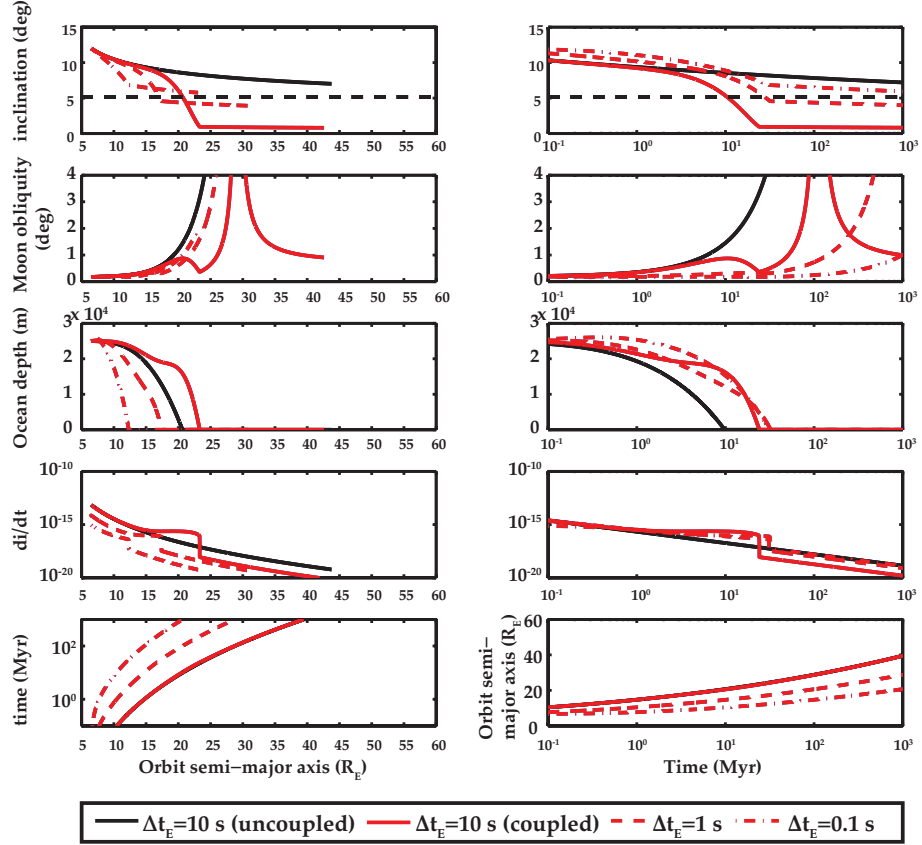


Figure 11: Orbital and thermal evolution for a model without magma ocean dissipation with  $\Delta t_E = 10$  (black line as in Figure 10) and with the fully coupled model including ocean dissipation (red curves). Dissipation in the magma ocean can result in significant inclination damping. The Moon’s obliquity controls this ocean dissipation. The Moon’s obliquity is unlikely to have reached greater than  $\sim 1^\circ$  prior to magma ocean dissipation; otherwise, the inclination would have been damped to a value lower than the presently observed value.

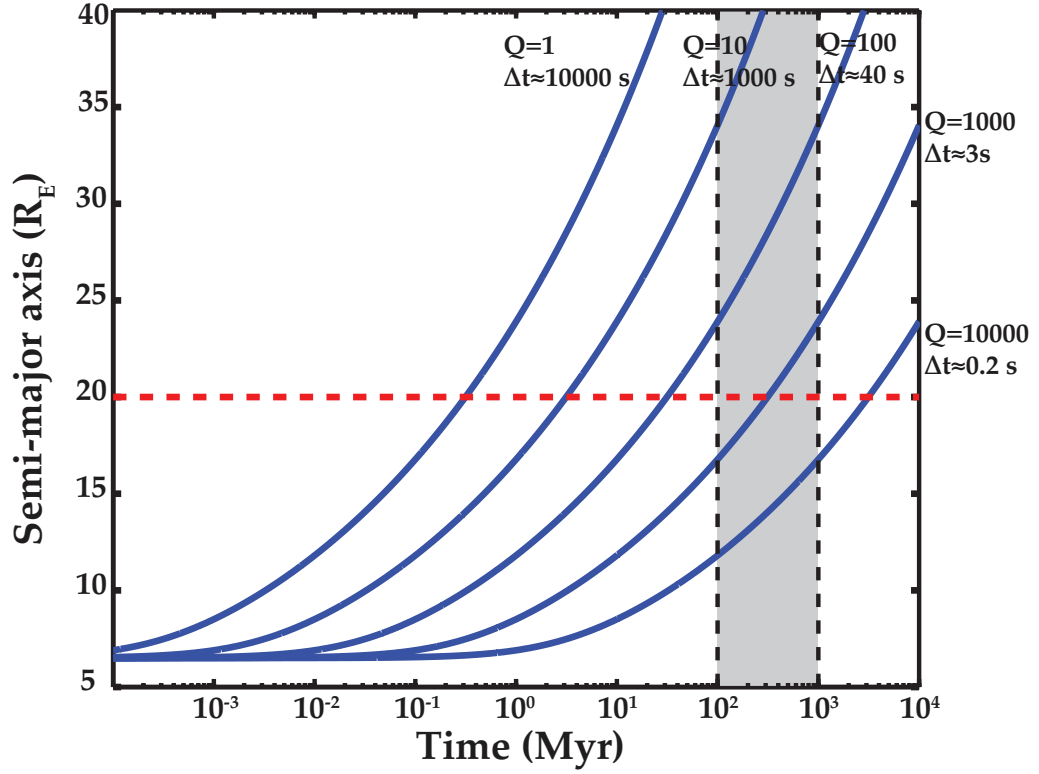


Figure 12: Semi-major axis evolution for an analytic orbital evolution model with only dissipation in the Earth with constant tidal quality factor  $Q$ . The grey shaded region is the timescale for magma ocean solidification. The red dotted line indicates the semi-major axis where the obliquity of the Moon is greater than  $\sim 1^\circ$ ; if the Moon is still molten at this point, dissipation in the ocean is significant enough to damp the inclination below the present value. Thus,  $Q$  of the early Earth is unlikely to be as low as it is at present ( $Q \sim 12$ ). We present estimates for equivalent  $\Delta t_E$  for comparison to the fully coupled thermal-orbital model presented in §3.2.

## 4 Summary

Recent observations from the *Galileo* and *Cassini* spacecraft suggest that global-scale liquid oceans are present and may be common on outer Solar System satellites. It has long been recognized that the presence of such oceans can dramatically enhance the response of satellite solid-body tides, likely causing the geologic activity observed on many satellites. Tyler (2008, 2009, 2011) suggested that tidal dissipation within a global ocean itself may additionally provide a significant amount of heat; however, a key free parameter in his work was the effective viscosity in a satellite ocean. The amount of dissipation scales with the value of the effective viscosity, a quantity that is, for all practical purposes, unconstrained. The goal of this dissertation was to constrain the amount of tidal dissipation occurring in a global satellite ocean and to understand the relative importance of ocean tidal dissipation in the evolution of satellites in the Solar System.

### 4.1 Constraining dissipation in a global scale ocean

In Chapter 2, we described a numerical model for flow in an ocean driven by tides with a nonlinear bottom friction formulation for the viscous term. Using results from this model, we obtained novel scaling relations that map the coefficient of bottom drag, a quantity whose value is well-established, to an equivalent effective turbulent viscosity. This mapping is a way to determine quantities of interest, e.g. the ocean kinetic energy and tidal dissipation, using a computationally inexpensive model, such as that of Tyler (2011), while preserving the constraints provided by using the bottom drag coefficient.

While computationally inexpensive, the semi-analytic method, which was used in Tyler (2011) to determine satellite ocean flow, can obfuscate the underlying physics. For those in the planetary science community who are unconcerned with the details of the fluid dynamics, we derived scaling laws to describe the ocean response in terms of orbital and satellite parameters. We believe that these scalings capture the dominant ocean behavior while also being easier to understand and simpler to implement in other contexts, such as thermal and orbital evolution models.

## 4.2 Applications of ocean tidal heating

We applied the methods developed in Chapter 2 to answering two questions: first, is ocean tidal heating currently an important contributor to the thermal budgets of any of the outer Solar System satellites?; and second, can a coupled thermal-orbital evolution model including magma ocean tides provide constraints on the evolution of the early Earth-Moon system?

### 4.2.1 Thermal budget of satellites in the outer Solar System

For the major satellites of Jupiter, Saturn, Neptune and Uranus, we calculated the amount of thermal energy that could be provided by the following sources: radiogenic heating, solid-body tidal dissipation for both eccentricity tides and obliquity tides, and ocean tidal dissipation for both eccentricity tides and obliquity tides. We find that for a coefficient of bottom drag of  $c_D = 0.002$ , ocean tidal dissipation is unlikely to contribute significantly to the thermal budget of most satellites. Radiogenic heating and solid-body eccentricity tides are the dominant contributors.

The one notable exception is Neptune’s satellite Triton. Images from *Voyager I* suggested that Triton has undergone a recent resurfacing event. However, because the orbit is currently circular, eccentricity tides are unlikely to have caused this resurfacing. A reasonable explanation for this resurfacing is that as Triton’s orbit has evolved inwards over time, obliquity tides have gotten larger. Dissipation associated with an ocean’s tidal response could generate a heating event in the recent past.

### 4.2.2 The evolution of the early Earth-Moon system

We can accurately measure the effects of tides on the present-day Earth-Moon system. It is well-known that tidal dissipation in the Earth primarily occurs in the oceans at present; but the properties of the Earth and the Moon in the deep past are poorly constrained. In Chapter 3, we developed a coupled thermal-orbital model that incorporates the scalings developed for tidal dissipation in a global ocean. In order for our model results to be consistent with the current orbital configuration, the early lunar magma ocean must solidify before the Cassini state transition, where the Moon’s obliquity and magma ocean dissipation can be-

come quite large. The location of this transition is relatively fixed with respect to the orbital semi-major axis ( $\sim 30$  Earth radii). On the other hand, the timing of the transition is not fixed and is instead controlled by the rate of orbital expansion, which in turn is related to dissipation in the Earth. Because magma ocean solidification is thought to take approximately 200 Myr from geochronological constraints, the maximum outwards expansion rate is known and implies that the Earth must be significantly less dissipative during this period. We find that for a constant time lag tidal model, the Earth's time lag when the lunar magma ocean was still present was approximately 1 second. This value is considerably less than the current time lag of approximately 600 s and suggests that the Earth did not have dissipative, global-scale water oceans during the Hadean epoch.

## A Useful spherical harmonic relations

To obtain equations for the spherical harmonic coefficients  $\Psi_l^m$  and  $\Phi_l^m$  in Chapter 2 (e.g. equations (2.23) and (2.24)), we utilized the following spherical harmonic relations:

$$Y_l^{-m} = (-1)^m Y_l^{m*}, \quad (\text{A.1})$$

$$\nabla^2 Y_l^m = \frac{-l(l+1)}{R^2} Y_l^m, \quad (\text{A.2})$$

$$\cos \theta Y_l^m = C_{l+1}^m Y_{l+1}^m + C_l^m Y_{l-1}^m, \quad (\text{A.3})$$

$$\sin \theta \frac{\partial Y_l^m}{\partial \theta} = l C_{l+1}^m Y_{l+1}^m - (l+1) C_l^m Y_{l-1}^m. \quad (\text{A.4})$$

These are similar to equations (3.17) of Longuet-Higgins (1968) and (23)-(25) of Tyler (2011) for normalized Laplace spherical harmonics. In addition, a useful relation when calculating the spatially averaged quantities, such as the kinetic energy, is

$$\int_0^{2\pi} \int_0^\pi \left[ \frac{\partial Y_l^m}{\partial \theta} \frac{\partial Y_{l'}^{m'*}}{\partial \theta} + \frac{1}{\sin^2 \theta} \frac{\partial Y_l^m}{\partial \phi} \frac{\partial Y_{l'}^{m'*}}{\partial \phi} \right] \sin \theta d\theta d\phi = l(l+1) \delta_{l,l'} \delta_{m,m'}. \quad (\text{A.5})$$

Note, these relations are only appropriate for spherical harmonics of degree  $l$  and order  $m$ ,  $Y_l^m$ , defined as

$$Y_l^m(\theta, \phi) \equiv \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos \theta) e^{im\phi} \quad (\text{A.6})$$

employing an additional Condon-Shortley phase factor of  $(-1)^m$  for  $m > 0$ . These spherical harmonics are orthogonal under

$$\int_0^{2\pi} \int_0^\pi Y_l^m Y_{l'}^{m'*} \sin \theta d\theta d\phi = \delta_{l,l'} \delta_{m,m'} \quad (\text{A.7})$$

where  $*$  denotes the complex conjugate and  $\delta$  is the Kronecker delta.

## B Reconciliation of various forms of the Cassini state constraint equation

While the Cassini state constraint equation is relatively-straightforward, there have been inconsistencies to the form of the equation through the literature, likely stemming from a typographic error in Peale (1969). Here, we express the correct constraint equation in terms of both gravity coefficients,  $J_2$  and  $C_{2,2}$  and the principal moments of inertia,  $A$ ,  $B$ , and  $C$ . These are related by

$$C_{2,2} \equiv \frac{1}{4c} \left( \frac{B - A}{C} \right) \quad (\text{B.1})$$

$$J_2 \equiv \frac{1}{c} \left( \frac{C - \frac{1}{2}(A + B)}{C} \right) \quad (\text{B.2})$$

where  $c$  is the normalized polar moment.

The correct Cassini state constraint equation as presented in (2.145) is

$$\frac{3}{2} [(J_2 + C_{2,2}) \cos \theta_0 + C_{2,2}] p \sin \theta_0 = c \sin(i - \theta_0) \quad (\text{B.3})$$

where  $i$  is the orbital inclination, and  $p$  is the ratio of the orbital motion to the orbit plane precession and is given by

$$p = \frac{\Omega}{d\Omega_{orb}/dt} \quad (\text{B.4})$$

where  $\Omega_{orb}$  is the longitude of the ascending node. The equivalent equation written in terms of moments of inertia is

$$\frac{3}{2} p \left( \frac{(C - A)}{C} \right) \sin(\theta_0) \cos(\theta_0) + \frac{3}{8} p \left( \frac{(B - A)}{C} \right) \sin(\theta_0) (1 - \cos(\theta_0)) = \sin(i - \theta_0). \quad (\text{B.5})$$

The constraint equations (B.3) and (B.5) or an equivalent representation appear in Peale (1969) equation 17, Ward (1975), and Bills and Nimmo (2008). The equations presented as the constraint equation in Peale (1969) equation 18, Bills and Nimmo (2011), and Chen and Nimmo (2011) are incorrect.

## C A discrepancy in the Mignard equations for the evolution of the orbital eccentricity

Throughout Chapter 3, we neglected the evolution of the orbital eccentricity. While the Moon likely had a large eccentricity excursion (Touma and Wisdom, 1998; Garrick-Bethell et al., 2010), eccentricity tidal dissipation in the magma ocean is dominated by the obliquity response even for relatively high eccentricities. We would like to note here that the Mignard evolutionary equations as presented in Meyer et al. (2010) may not be correct with respect to the orbital eccentricity.

The Mignard evolutionary equations including eccentricity  $e$  are

$$\frac{dX}{dt} = \frac{C_X}{X^7} \left[ \frac{-f_0}{\beta^{15}} (1 + A) + \frac{f_1}{\beta^{12}} \left( UX^{3/2} \cos i + A \cos \theta_0 \right) \right] \quad (\text{C.1})$$

$$\frac{de}{dt} = \frac{C_e}{X^8} \left[ \frac{-f_3}{\beta^{13}} (1 + A) + \frac{f_4}{\beta^{10}} \left( UX^{3/2} \cos i + A \cos \theta_0 \right) \right] \quad (\text{C.2})$$

where those parameters not defined previously in §3.2.1 are

$$C_e = 3\gamma \frac{m}{M} \frac{m}{\mu}, \quad (\text{C.3})$$

$$\beta \equiv \sqrt{1 - e^2}, \quad (\text{C.4})$$

$$f_0 = 1 + \frac{31}{2}e^2 + \frac{255}{8}e^4 + \frac{185}{16}e^6 + \frac{25}{64}e^8, \quad (\text{C.5})$$

$$f_1 = 1 + \frac{15}{2}e^2 + \frac{45}{8}e^4 + \frac{5}{16}e^6, \quad (\text{C.6})$$

$$f_3 = 9e + \frac{135}{4}e^3 + \frac{135}{8}e^5 + \frac{45}{64}e^7, \quad (\text{C.7})$$

$$f_4 = \frac{11}{2}e + \frac{33}{4}e^3 + \frac{11}{16}e^5. \quad (\text{C.8})$$

Ignoring the orbital inclination, the orbital evolution due to dissipation in the Moon must conserve angular momentum  $L$ , where

$$L = m(GM)^{1/2}(a(1 - e^2))^{1/2}. \quad (\text{C.9})$$

Because this quantity is constant in time, the following relations are true after taking the

time derivative of (C.9) and including the relation (3.15)

$$\frac{\dot{a}}{a} = \frac{2e\dot{e}}{(1-e^2)} = \frac{2a\dot{E}}{GMm}. \quad (\text{C.10})$$

For solid-body eccentricity tides (constant phase lag description), the tidal dissipation in the satellite is

$$\dot{E}_{ecc} = \frac{-21}{2} \frac{k_{2s}}{Q_s} \frac{(nR_s)^5}{G} e^2 = \frac{-21}{2} \frac{k_{2s}}{Q_s} \frac{G^{3/2} M_p^{5/2} R_s^5}{a^{15/2}} e^2. \quad (\text{C.11})$$

Plugging equation (C.11) into the relations in (C.10), we obtain

$$\dot{a} = -21 \left( \frac{k_{2s}}{Q_s} \right) \left( \frac{M_p}{M_s} \right) \left( \frac{R_s}{a} \right)^5 n a e^2 \quad (\text{C.12})$$

and

$$\dot{e} = \frac{-21}{2} \left( \frac{k_{2s}}{Q_s} \right) \left( \frac{M_p}{M_s} \right) \left( \frac{R_s}{a} \right)^5 n e. \quad (\text{C.13})$$

Making the following assumptions:  $i = \theta_0 = 0$ ,  $A \gg 1$ ,  $A \gg UX^{3/2}$ , then the Mignard evolutionary equations for the semi-major axis and eccentricity are

$$\frac{dX}{dt} = \frac{C_X}{X^7} A \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right) = C_X \left( \frac{R_p}{a} \right)^7 A \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right), \quad (\text{C.14})$$

and

$$\frac{de}{dt} = \frac{C_e}{X^8} A \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right) = C_e \left( \frac{R_p}{a} \right)^8 A \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right) \quad (\text{C.15})$$

without making substitutions for the constants  $C_X$ ,  $C_e$ ,  $f_0$ ,  $f_1$ ,  $f_3$ ,  $f_4$ ,  $\beta$  and  $A$ . In making the substitutions for these constants, we are looking to see if the equation pairs (C.12, C.14) and (C.13, C.15) are equivalent in order to insure that the Mignard equations correctly reflect the relations in (C.10).

For clarity, we attempt to make the algebra as transparent as possible when making the constant substitutions. Here we begin with the evolution of the eccentricity and substitute (C.3) for  $C_e$  and (3.12) for  $\gamma$ :

$$\frac{de}{dt} = 3 \left( \frac{M_s}{M_p} \right) k_{2p} n_G^2 \Delta t_p \left( \frac{R_p}{a} \right)^8 A \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right). \quad (\text{C.16})$$

Making the appropriate substitution for  $n_G$ , this becomes:

$$\frac{de}{dt} = 3 \left( \frac{M_s}{M_p} \right) k_{2p} \Delta t_p \frac{GM_p}{R_p^3} \left( \frac{R_p}{a} \right)^8 A \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right) \quad (\text{C.17})$$

$$= 3 \left( \frac{M_s}{M_p} \right) k_{2p} \Delta t_p \frac{GM_p}{a^3} \left( \frac{R_p}{a} \right)^5 A \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right). \quad (\text{C.18})$$

Making the substitution for  $A$  (3.13), the above equation becomes:

$$\frac{de}{dt} = 3 \left( \frac{M_s}{M_p} \right) k_{2p} \Delta t_p \frac{GM_p}{a^3} \left( \frac{R_p}{a} \right)^5 \left( \frac{k_{2s} \Delta t_s}{k_{2p} \Delta t_p} \right) \left( \frac{M_p}{M_s} \right)^2 \left( \frac{R_s}{R_p} \right)^5 \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right), \quad (\text{C.19})$$

which simplifies to

$$\frac{de}{dt} = 3 \left( \frac{M_p}{M_s} \right) k_{2s} \Delta t_s n^2 \left( \frac{R_s}{a} \right)^5 \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right), \quad (\text{C.20})$$

If we make the substitution that

$$Q_s^{-1} = n \Delta t_s, \quad (\text{C.21})$$

then

$$\frac{de}{dt} = 3 \left( \frac{M_p}{M_s} \right) \left( \frac{k_{2s}}{Q_s} \right) \left( \frac{R_s}{a} \right)^5 n \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right). \quad (\text{C.22})$$

Substituting now for  $f_3$  (C.7),  $f_4$  (C.8), and  $\beta$  (C.4),

$$\begin{aligned} \left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right) &= - \left( 9e + \frac{135}{4}e^3 + \frac{135}{8}e^5 + \frac{45}{64}e^7 \right) (1 - e^2)^{-13/2} \\ &\quad + \left( \frac{11}{2}e + \frac{33}{4}e^3 + \frac{11}{16}e^5 \right) (1 - e^2)^{-5} \end{aligned} \quad (\text{C.23})$$

which for small  $e$  to  $O(e^2)$  is

$$\left( \frac{-f_3}{\beta^{13}} + \frac{f_4}{\beta^{10}} \right) \approx -9e + \frac{11}{2}e = \frac{-7}{2}e. \quad (\text{C.24})$$

Plugging (C.24) into (C.22), we have

$$\frac{de}{dt} = \frac{-21}{2} \left( \frac{M_p}{M_s} \right) \left( \frac{k_{2s}}{Q_s} \right) \left( \frac{R_s}{a} \right)^5 n e \quad (\text{C.25})$$

which agrees with (C.13).

For the evolution of the semi-major axis, we substitute (3.5) for  $C_X$  and (3.12) for  $\gamma$  and making the switch from  $X$  to  $a$  and  $R_p$ :

$$\frac{da}{dt} = 6 \left( \frac{M_s}{M_p} \right) k_{2p} n_G^2 \Delta t_p \left( \frac{R_p}{a} \right)^8 a A \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right). \quad (\text{C.26})$$

Making the substitution for  $n_G$ , this becomes:

$$\frac{da}{dt} = 6 \left( \frac{M_s}{M_p} \right) k_{2p} \Delta t_p \left( \frac{GM_p}{R_p^3} \right) \left( \frac{R_p}{a} \right)^8 a A \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right) \quad (\text{C.27})$$

$$= 6 \left( \frac{M_s}{M_p} \right) k_{2p} \Delta t_p \left( \frac{GM_p}{a^3} \right) \left( \frac{R_p}{a} \right)^5 a A \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right). \quad (\text{C.28})$$

Making the substitution for  $A$  (3.13), the above equation becomes:

$$\frac{da}{dt} = 6 \left( \frac{M_s}{M_p} \right) k_{2p} \Delta t_p \left( \frac{GM_p}{a^3} \right) \left( \frac{R_p}{a} \right)^5 a \left( \frac{k_{2s} \Delta t_s}{k_{2p} \Delta t_p} \right) \left( \frac{M_p}{M_s} \right)^2 \left( \frac{R_s}{R_p} \right)^5 \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right), \quad (\text{C.29})$$

which simplifies to

$$\frac{da}{dt} = 6 \left( \frac{M_p}{M_s} \right) k_{2s} \Delta t_s n^2 \left( \frac{R_s}{a} \right)^5 a \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right). \quad (\text{C.30})$$

With the substitution for  $Q_s$  (C.21) then this becomes:

$$\frac{da}{dt} = 6 \left( \frac{M_p}{M_s} \right) \left( \frac{k_{2s}}{Q_s} \right) \left( \frac{R_s}{a} \right)^5 n a \left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right). \quad (\text{C.31})$$

Substituting now for  $f_0$  (C.5),  $f_1$  (C.6), and  $\beta$  (C.4),

$$\left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right) = - \left( 1 + \frac{31}{2} e^2 + \frac{255}{8} e^4 + \frac{185}{16} e^6 + \frac{25}{64} e^8 \right) (1 - e^2)^{-15/2} \quad (\text{C.32})$$

$$+ \left( 1 + \frac{15}{2} e^2 + \frac{45}{8} e^4 + \frac{5}{16} e^6 \right) (1 - e^2)^{-6} \quad (\text{C.33})$$

which for small  $e$  to  $O(e^2)$  is

$$\left( \frac{-f_0}{\beta^{15}} + \frac{f_1}{\beta^{12}} \right) \approx - \left( 1 + \frac{31}{2} e^2 + \frac{15}{2} e^2 \right) + \left( 1 + \frac{15}{2} e^2 + 6 e^2 \right) = \left( \frac{-31}{2} + 6 \right) e^2 = \frac{-19}{2} e^2. \quad (\text{C.34})$$

Plugging (C.34) into (C.31), we have

$$\frac{da}{dt} = -57 \left( \frac{M_p}{M_s} \right) \left( \frac{k_{2s}}{Q_s} \right) \left( \frac{R_s}{a} \right)^5 n a e^2, \quad (\text{C.35})$$

which is not equal to (C.12).

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