LBNL-40347
Preprint

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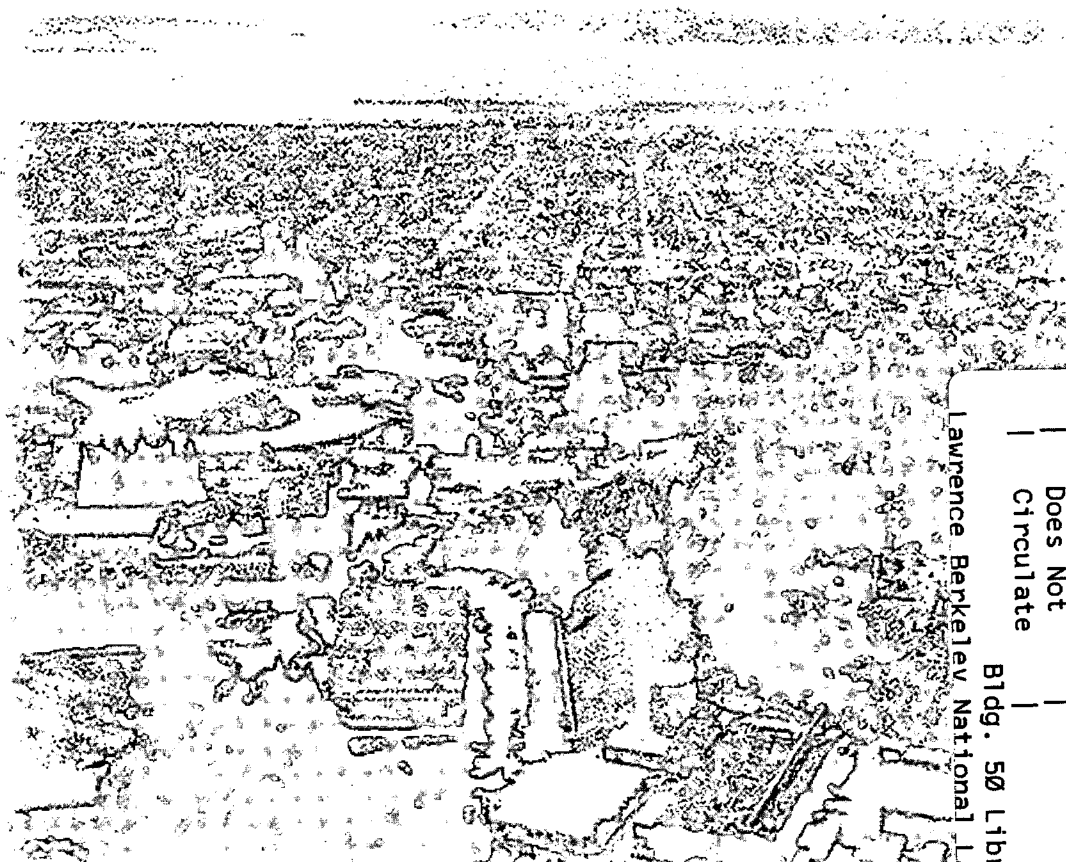
Physics Division

November 1997
Submitted to
Nuclear Physics B



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LBNL-40347

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This work was supported by the Director, Office of Energy Research, Office of Basic Energy Sciences, of the U.S. Department of Energy under Contract Nos. DE-AC03-76SF00098 and FG02-91ER40671, and by the National Science Foundation under Grant No. PHY-9600258.

hep-th/9710206

PUPT-1732
LBNL-40347, UCB-PTH-97/25

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Abstract

When a D-brane wraps around a cycle of a curved manifold, the twisting of its normal bundle can induce chiral asymmetry in its world-volume theory. We obtain the general form of the resulting anomalies for D-branes and their intersections. They are not cancelled among themselves, and the standard inflow mechanism does not apply at first sight because of their apparent lack of factorizability and the apparent vanishing of the corresponding inflow. We show however after taking into consideration the effects of the nontrivial topology of the normal bundles, the anomalies can be transformed into factorized forms and precisely cancelled by finite inflow from the Chern-Simons actions for the D-branes as long as the latter are well defined. We then consider examples in type II compactifications where the twisting of the normal bundles occurs and calculate the changes in the induced Ramond-Ramond charges on the D-branes.

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1 Introduction

In recent studies of string theory, brane configurations play a very important role. The low energy physics of such configurations are that of field theories, which often possess both gauge and global symmetries. In such constructions, some global symmetries, usually the R symmetries that act on the supercharges, originate from the rotation symmetry of the bulk string theory restricted to the normal bundles of the branes. They are gauged in the bulk spacetime and therefore must be free of anomalies, just as the symmetries gauged on the branes. However, there is generically chiral asymmetry with respect to these global symmetries on a D-brane or the intersection of a pair of D-branes, known as an *I-brane.* It brings about pure and mixed anomalies involving these global symmetries in the effective brane worldvolume theory. If this were the only story, such brane configurations would be inconsistent.

The mechanism to cancel the anomaly in an otherwise anomalous theory is to compensate it with an "anomalous" variation of the classical action. An example is the Green-Schwarz mechanism for type I and heterotic string theories [1]. More generally, the anomalous theory can be embedded in a higher dimensional theory. The anomalous variation of the classical action of the bigger theory is localized at ("flows" to) the worldvolume for the anomalous theory and cancels its anomaly, hence the name *anomaly inflow* [2, 3]. More recently it has been applied to derive the Chern-Simons type of actions on D-branes, whose classical variations cancel the Yang-Mills and gravitational anomalies that appear on a certain class of I-branes [4]. However, there are additional anomalies associated with the global R symmetries as mentioned earlier. They exist for generic D-branes and their intersections. If D-branes are wrapped around nontrivial cycles of a curved compactification manifold [5, 6, 7], the anomalies can manifest themselves as nonvanishing variation of the effective action under a local gauge transformation. Such scenarios have appeared in studies of string dualities [8, 9] as well as field theory dualities [10, 11, 12, 13]. They have also found use in studying topological field theories [5, 14, 15]. However, anomaly cancellation for them has not been investigated until now.

In generalizing the inflow method to such cases, one inevitably runs up against a serious obstruction. Factorizability of an anomaly, as defined precisely later, is crucial for it to be cancelled via the inflow mechanism. However, for the additional anomalies we study, factorizability is apparently lost. To recover it we shall encounter a classic result from differential topology*. It allows us to cancel the new anomalies in all cases as long as the D-brane

*This result, the relation between Thom class and Euler class, has also been used in a different context: anomaly analysis for the NS5-branes in type IIA string theory and the 5-branes in M theory [16, 17].

Chern-Simons actions are well-defined.

The D-brane Chern-Simons actions derived in [4] imply that topological defects on D-branes carry their own Ramond-Ramond charges determined by their topological ("instanton") numbers. This observation has far reaching consequences [18, 19, 20]. To cancel the new anomalies that we study, the Chern-Simons actions are modified. This can change the induced Ramond-Ramond charges on a D-brane if it is wrapped around some cycle of a nontrivial compactification manifold.

The plan of this paper is as follows. In section 2 we discuss how the inflow mechanism works. In addition to a review of some known results, we shall uncover subtleties in the choice of the kinetic action for the Ramond-Ramond field that have not been addressed in the literature. We also define carefully the notion of brane current. For describing flat D-branes, it is just a very convenient notation, but in the anomaly cancellation considered later in this paper, it plays an essential role. In section 3 we consider the chiral asymmetry induced by twisting the normal bundle and compute the resulting anomaly. We then point out the apparent obstruction to cancelling such anomaly. In section 4 this difficulty is overcome with the help of some interesting topological information encoded in the brane current. Then in section 5 we give examples where the normal bundles of D-branes are nontrivial and calculate the induced Ramond-Ramond charge. In the appendix we comment on the relevance of brane stability and supersymmetry to our anomaly analysis.

2 The Inflow Mechanism

The inflow mechanism was originally discovered in the context of gauge theory [2], where the action in spacetime has a gauge noninvariant term. Its variation is concentrated on topological defects and cancels the anomalies produced by their chiral fermion zero modes. It was recognized in [4] that this mechanism also applies to the Yang-Mills and gravitational anomalies that arise for a certain class of intersecting D-branes in string theory. In this section, we present systematically the details of the inflow mechanism. Although much of it is a review of the earlier results cited above, there are some salient departures. The most important one being our use of a kinetic action manifestly symmetric with respect to all Ramond-Ramond potentials. Its use is really required by the way the inflow mechanism works for D-branes and turns out to be important for reproducing the correct Ramond-Ramond charge.

As it shall become clear, an anomaly must be factorizable in an appropriate sense in order to be cancelled by inflow. One of the difficulty associated with the anomalies we consider in this paper is their apparent lack of factorizability, and the key to cancelling them involves rewriting them in a factorized form.

2.1 Branes and Currents

Before discussing the detail of the inflow mechanism, we first introduce a notion that is very convenient here and will prove essential later. Usually a brane is introduced into the bulk theory by adding to the bulk action

$$\int_M \mathcal{L}_M$$

where M is the m-dim worldvolume of the brane and $\mathcal{L}_M$ the Lagrangian density governing the dynamics on the brane. One may rewrite this into an integration over total (bulk) spacetime X, with the help of a "differential form" τ_M, defined by

$$\int_M \zeta \equiv \int_X \tau_M \wedge \zeta \tag{2.1}$$

for all rank m form ζ defined on M*. Thus the rank of τ_M is equal to the codimension of M in X. To be precise, (eq. 2.1) defines τ_M as an element in the dual of the space of forms, known to mathematicians as the space of *currents* [21]. Currents are differential-form analogue of distributions; likewise, τ_M is the generalization of Dirac's delta function†. Obviously, τ_M must have singular support on M and integrate to 1 in the transverse space of M.

In (eq. 2.1), the form ζ is allowed to be any form on M. If instead it is restricted to be closed, the same equation only defines a cohomology class $[\tau_M]$, known as the *Poincare dual* of M. It contains topological information about M. τ_M can be defined as a particular representative of $[\tau_M]$ that is supported only on M.

In this paper, we shall call τ_M the brane current associated with a brane wrapped around M, for a very physical reason. For illustration, consider a d-dim gauge theory with a conventional 2-form field strength F. Let M be the worldline trajectory of an electrically charged particle embedded in the total spacetime X. The kinetic term for the gauge field F is

$$S_{\text{gauge}} = -\frac{1}{2}\int_X F \wedge *F. \tag{2.2}$$

*This definition makes sense because any form ζ on M can be extended to be a form on X by a suitable bump function with support on a tubular neighborhood of M. Conversely, if ζ is a form defined on X to start with, pull-back to M is implicit on the LHS of (eq. 2.1), as in similar expressions throughout this paper.

†In this language, a delta function in R^d is really a rank d current that maps a function (0-form) into a number.

The coupling of the potential to the electron is

$$\begin{aligned} S_{\text{matter}} &= -\int_M A \\ &= -\int_X \tau_M \wedge A. \end{aligned} \tag{2.3}$$

Then the equation of motion for A yields

$$\begin{aligned} *j_{ele} &\equiv d * F \\ &= (-1)^d \tau_M. \end{aligned} \tag{2.4}$$

So the usual physical current (source) is related to τ_M by a Hodge $*$ operation. Similarly, if $\hat{M}$ is the $(d-3)$-dim worldvolume of a magnetically charged object, the Bianchi identity would read something like

$$*j_{mag} \equiv dF = \pm \tau_{\hat{M}}.$$

Now return to string theory. Let M be the worldvolume of a D-brane. It couples to the Ramond-Ramond potential C of the appropriate rank just as in (eq. 2.3) but with A replaced by C. Then (eq. 2.4) gives the definition of the brane current τ_M with F replaced by the appropriate Ramond-Ramond field strength H.

On M, the tangent bundle $T(X)$ of the total spacetime X, decomposes into the Whitney sum of $T(M)$ and $N(M)$, the tangent and normal bundles to M respectively. Note that within each fiber of $N(M)$ (eq. 2.4) is just the usual Poisson equation. Its RHS has Dirac's δ-type singular support on the zero section. Thus τ_M can be constructed *locally* as

$$\tau_M \overset{naive}{=} \delta(x^1) dx^1 \wedge \cdots \wedge \delta(x^{\dim N(M)}) dx^{\dim N(M)}, \tag{2.5}$$

where x^μ are Gaussian normal coordinates in the transverse space of M, or equivalently Cartesian coordinates in the fiber of $N(M)$. We emphasize that this expression is naive and in general ill-defined globally.

Now consider the intersection $M_{12} \equiv M_1 \cap M_2$ of two brane-worldvolumes M_1 and M_2. In the literature M_{12} has been called I-brane. For simplicity we shall concentrate on I-branes from intersections at right angle, but the results apply to other cases as well[‡]. The right angle condition implies that on the I-brane $M_1 \cap M_2$, the tangent bundle of the total spacetime $T(X)$ decomposes as follows:

$$\begin{aligned} T(X) &= T(M_1) \cap T(M_2) \oplus T(M_1) \cap N(M_2) \\ &\quad \oplus N(M_1) \cap T(M_2) \oplus N(M_1) \cap N(M_2), \end{aligned} \tag{2.6}$$

[‡]The basic reason is that the relevant quantum numbers of the massless fermions are determined by $T(M_1) \cap T(M_2)$ and $N(M_1) \cap N(M_2)$, which are well defined even for oblique intersections.

where $\cap$ denotes fiberwise set theoretic intersection. It is clear that

$$T(M_{12}) = T(M_1) \cap T(M_2) \tag{2.7}$$

and

$$N(M_{12}) = T(M_1) \cap N(M_2) \oplus N(M_1) \cap T(M_2) \oplus N(M_1) \cap N(M_2). \tag{2.8}$$

Then (eq. 2.5) implies that

$$\begin{aligned} \tau_{M_1} \wedge \tau_{M_2} &= \tau_{M_{12}} \qquad \text{if } N(M_1) \cap N(M_2) = \emptyset, \\ &\overset{naive}{=} 0 \qquad \text{otherwise}, \end{aligned} \tag{2.9}$$

where in the second line we have used the anticommutivity of exterior multiplication. Here again we emphasize that the second equation is naive, because it uses the naive expression (eq. 2.5). The correct statement and its important implication will be given in section 4. Intersections on which $N(M_1) \cap N(M_2) = \emptyset$ are known as *transversal.*

2.2 The Inflow

Suppose the anomaly on an I-brane M_{12} can be written in the following form:

$$I_{12} = \pi \int \tau_{M_{12}} \wedge \left(Y_1 \wedge \tilde{Y}_2 + Y_2 \wedge \tilde{Y}_1\right)^{(1)}, \tag{2.10}$$

where Y_i and $\tilde{Y}_i$, $i = 1, 2$, are some invariant polynomials of the Yang-Mills field strengths and gravitational curvatures defined on M_i. The expression $Z^{(1)}$ denote the Wess-Zumino descent [22, 23] of an invariant curvature polynomial Z: if N is the constant part of Z,

$$Z \equiv N + Z_0,$$

and $Z^{(0)}$ is its secondary characteristic,

$$Z_0 \equiv dZ^{(0)},$$

then the gauge variation of $Z^{(0)}$ is

$$\delta_g Z^{(0)} \equiv dZ^{(1)}.$$

Y_i and $\tilde{Y}_i$ must be defined entirely by the D-branes wrapping M_i. For example, Y_1's dependence on the gravitational curvature from $T(M_1)$ and $N(M_1)$ may be different, but it must not distinguish, say, between the contributions from $T(M_1) \cap T(M_2)$ and $T(M_1) \cap N(M_2)$. In this paper such an anomaly is called factorizable.

To cancel the anomaly (eq. 2.10), one introduces the following ansatz for a Chern-Simons type action on D-branes [4][§]:

$$-\frac{\mu}{2}\sum_i \int_{M_i} N_i C - (-1)^q H \wedge Y_i^{(0)} = -\frac{\mu}{2}\sum_i \int_X \tau_{M_i} \wedge (N_i C - (-1)^q H \wedge Y_i^{(0)}). \tag{2.11}$$

Here q is 1 for IIA and 0 for IIB string theory. i labels the D-brane wrapping worldvolume M_i, whose brane current is τ_{M_i}. N_i is the constant part of Y_i. Anomaly computation in section 3 will show that it is the multiplicity of the D-branes wrapping M_i. μ, rather than $\frac{\mu}{2}$ as one would naively expect, is the brane charge, for reason to be explained shortly. C and H are the formal sums of all the Ramond-Ramond antisymmetric tensor potentials and field strengths respectively. Integration automatically picks out products of forms with the appropriate total rank. In the following we shall often denote by $Z_{(n)}$ the rank n part of any formal sum Z. For example,

$$C = C_{(1)} + C_{(3)} + C_{(5)} + C_{(7)} + C_{(9)}$$

for type IIA string theory and

$$C = C_{(0)} + C_{(2)} + C_{(4)} + C_{(6)} + C_{(8)}$$

for type IIB string theory. It is important to remark that unlike the usual Chern-Simons action, in (eq. 2.11) one cannot use integration by parts to reduce the RHS to the more uniform expression of $-\frac{\mu}{2}\int_{M_i} C \wedge Y_i$. The reason is, as we shall see, $dH_{(n)} \neq 0$, even away from any magnetic D(8-n) brane. So H has corrections to its usual expression of dC:

$$H = dC + \cdots. \tag{2.12}$$

Therefore a brane Lagrangian in the form of $-\frac{\mu}{2}C \wedge Y$ is different from (eq. 2.11) by some additional terms. In fact, only (eq. 2.11) can cancel the factorized anomaly (eq. 2.10).

In a theory that treats electric and magnetic potentials on equal footing, there could be ambiguity in deriving the equations of motion using the conventional kinetic action. Since (eq. 2.11) explicitly involves both electric and magnetic sources, it must be understood to be part of an action that is a manifestly *electro-magnetically symmetric*. The detail of the action and its ramifications are interesting in their own rights and presented in the next subsection. The relevant results can be summarized as follows: given the coupling in (eq. 2.11), with the factor of $\frac{1}{2}$, the equations of motion are

$$d * H = \mu \sum_i \tau_i \wedge Y_i, \tag{2.13}$$

[§]T-duality relates the charge μ for D-branes of different dimensions. With a suitable choice for the unit of length, they are all equal [26].

and the Bianchi identities are

$$dH = -\mu \sum_i \tau_i \wedge \tilde{Y}_i, \tag{2.14}$$

with

$$\tilde{Y}_{j(l)} = -(-1)^{\frac{dim(M_j)-q}{2}} (-1)^{l/2} Y_{j(l)}, \tag{2.15}$$

without any factor of $\frac{1}{2}$! Note Y and $\tilde{Y}$ are in general different. It will become apparent later that the factor $(-1)^{l/2}$ relates $\tilde{Y}$ to Y by complex conjugation of the group representation of the associated Yang-Mills gauge group, while the factor $(-1)^{\frac{dim(M_j)-q}{2}}$ chooses an orientation for the I-brane.

The Bianchi identities (eq. 2.14) impose very strong conditions on the terms represented by $\cdots$ in (eq. 2.12). The minimal expression for H is

$$H = dC - \mu(-1)^q \tau_{M_j} \wedge \tilde{Y}_j^{(0)}, \tag{2.16}$$

where $\tilde{N}_j$ is the constant part of $\tilde{Y}_j$, and $\tilde{Y}_j^{(0)}$ its secondary characteristic (similar notations apply to the untilded Y's). Since the field strengths H are physical observables, they must be invariant under gauge transformations. Thus C must have compensating gauge variations:

$$\delta_g C = \mu \sum_j \tau_{M_j} \wedge \tilde{Y}_j^{(1)}, \tag{2.17}$$

where $\tilde{Y}_j^{(1)}$ is the Wess-Zumino descent of $\tilde{Y}_j$.

Now we can compute the variation of (eq. 2.11) under gauge transformations to be

$$\begin{aligned} \delta_g S &= -\frac{\mu^2}{2} \sum_{ij} \int_X \tau_{M_i} \wedge \tau_{M_j} \wedge \left(\tilde{Y}_j^{(1)} N_i + \tilde{Y}_j (Y_i)^{(1)} \right) \\ &= -\frac{\mu^2}{2} \sum_{ij} \int_X \tau_{M_i} \wedge \tau_{M_j} \wedge \left(Y_i \wedge \tilde{Y}_j \right)^{(1)}. \end{aligned} \tag{2.18}$$

For a particular pair of *distinct* D-brane worldvolume M_1 and M_2, this gives an anomalous variation

$$\begin{aligned} \delta_g S_{12} &= -\frac{\mu^2}{2} \int_X \tau_{M_1} \wedge \tau_{M_2} \left(\tilde{Y}_2^{(1)} N_1 + \tilde{Y}_2 (Y_1)^{(1)} + \tilde{Y}_1^{(1)} N_2 + \tilde{Y}_1 (Y_2)^{(1)} \right) \\ &= -\frac{\mu^2}{2} \int_X \tau_{M_1} \wedge \tau_{M_2} \left(Y_1 \wedge \tilde{Y}_2 + Y_2 \wedge \tilde{Y}_1 \right)^{(1)}. \end{aligned} \tag{2.19}$$

According to the first equation in (eq. 2.9), when $N(M_1) \cap N(M_2) = \emptyset$, this *inflow* precisely cancels the anomaly (eq. 2.10) if

$$\frac{\mu^2}{2} = \pi. \tag{2.20}$$

So the anomaly and inflow analysis also constitutes an independent verification of brane charge computed in [24]. The factor of $\frac{1}{2}$ in (eq. 2.11) relative to (eq. 2.13) and (eq. 2.14) is crucial for agreement¶.

The cases with $N(M_1) \cap N(M_2) = \emptyset$ were considered in [4]. When this does not hold, the second equation in (eq. 2.9) suggests that the inflow (eq. 2.19) vanishes. However, we shall show in section 3 that on the corresponding I-branes there still exist anomalies. Fortunately, in section 4, we shall find the correction to (eq. 2.9) that keeps the inflow finite and cancels the anomaly.

2.3 Electro-magnetically Symmetric Action

In this subsection we derive the equations of motion (eq. 2.13) and justify the relative factor of $\frac{1}{2}$ in (eq. 2.11)‖. As mentioned earlier, this factor is essential for obtaining the correct brane charges required by string duality [24, 26]. The kinetic action for antisymmetric tensors we shall use is the one proposed in [27] for source free situations. After we couple it to sources, it is well suited for (eq. 2.11) because it treats both electric and magnetic potentials on the same footing. The price to pay is the loss of manifest Lorentz invariance — the action has only manifest rotation invariance in the spatial dimensions, although it possesses additional symmetries that reduce on shell to the usual Lorentz transformations [27]. More recently, there has been progress in covariantizing it**. However, for the present discussion the simpler noncovariant version suffices.

First consider just one electro-magnetic dual pair of RR fields $H_{(n)}$ and $\breve{H}_{(d-n)}$, where the subscripts, often omitted, denote the ranks of forms. Their respective potentials are $C_{(n-1)}$ and $\breve{C}_{(d-n-1)}$. Now let

$$C = \Phi + A \tag{2.21}$$

and

$$H = E + B \tag{2.22}$$

so that the components of Φ and E consist of those of C and H respectively with a temporal index, while A and B have only spatial indices. Similarly we can also decompose the spacetime exterior derivative d into the spatial

¶In [4], there was no factor of $\frac{1}{2}$ in the Chern-Simons action, but the total anomaly was computed to be twice as large, so the same value for μ was obtained. We would like to thank the authors of [4] for useful communications regarding this issue.

‖A similar factor of $\frac{1}{2}$ in the coupling to sources has also been suggested recently in [25]. However, the detailed form of the action used there seems to be different.

**See, for example, [28].

exterior derivative ∇ and the temporal part d_t:

$$d = d_t + \nabla, \tag{2.23}$$

with

$$\{d_t, \nabla\} = 0 = d_t^2 = \nabla^2. \tag{2.24}$$

Then

$$\begin{aligned} E &= d_t A + \nabla\Phi. && (2.25)\\ B &= \nabla A. && (2.26) \end{aligned}$$

The analogy with the usual non-manifestly Lorentz covariant formulation of electrodynamics should be clear. The same can be carried out for the dual fields:

$$\begin{aligned} \breve{H} &= \breve{E} + \breve{B}, \\ \breve{C} &= \breve{\Phi} + \breve{A}. && (2.27) \end{aligned}$$

Consider now the action [27]:

$$S_{BE} = -\frac{1}{2}\int (B \wedge \breve{E} - E \wedge \breve{B} + B \wedge *B + \breve{B} \wedge *\breve{B}). \tag{2.28}$$

In the absence of sources, the fields satisfy the following Bianchi identities in light of (eq. 2.24):

$$\begin{aligned} \nabla B &= 0 = \nabla \breve{B}, && (2.29)\\ d_t B + \nabla E &= 0 = d_t \breve{B} + \nabla \breve{E}. && (2.30) \end{aligned}$$

By using the first of them one finds that the equations of motion for Φ and $\breve{\Phi}$ are trivially satisfied — they only enter the action as parts of total exterior derivatives. This implies a larger set of gauge transformations than in the usual formulation:

$$\delta_g A = \nabla\Gamma, \qquad \delta_g \breve{A} = \nabla\breve{\Gamma}; \tag{2.31}$$

$$\delta_g \Phi = \Psi, \qquad \delta_g \breve{\Phi} = \breve{\Psi} \tag{2.32}$$

with independent Γ, $\breve{\Gamma}$, Ψ, and $\breve{\Psi}$. The gauge transformations (eq. 2.32) allow Φ and $\breve{\Phi}$ to be set to 0, corresponding to the usual temporal gauge. Applying (eq. 2.30), the equations of motion for A and $\breve{A}$ are found to be

$$\begin{aligned} \nabla(\breve{E} + *B) &= 0; && (2.33)\\ \nabla(E - (-1)^{n(d-n)} * \breve{B}) &= 0 && (2.34) \end{aligned}$$

respectively: the expressions inside the parenthesis are closed. By using the gauge transformations (eq. 2.31), one can choose a gauge so that they vanish:

$$\begin{aligned} \breve{E} &= -*B; \\ E &= (-1)^{n(d-n)} * \breve{B}. \end{aligned} \tag{2.35}$$

They then give the duality relation between H and $\breve{H}$. Substituting them for the Bianchi identities (eq. 2.29) and (eq. 2.30) one finally recovers the conventional equations of motion for antisymmetric tensors:

$$\begin{aligned} \nabla * E &= 0 = d_t * E + \nabla * B; \\ \nabla * \breve{E} &= 0 = d_t * \breve{E} + \nabla * \breve{B}. \end{aligned} \tag{2.36}$$

Note that although the action (eq. 2.28) is not Lorentz invariant, the equations of motion obtained from it are. Furthermore, one can recover from (eq. 2.28) the conventional action for one of the gauge potential, say A, in temporal gauge by solving the duality equation (eq. 2.35) for its dual $\breve{A}$ and make the gauge choice

$$\Phi = 0 = \breve{\Phi}. \tag{2.37}$$

Now let us put in the sources. In the conventional action formalism, where only one potential is used, the potential remains single valued if just electric sources are present. When there is also magnetic source, the potential can only be defined over patches — it is a connection of a nontrivial bundle [29]. The Bianchi identities must be modified. When one switches to the dual description, the meaning of electric and magnetic sources are interchanged, as are the equations of motion and the Bianchi identities. In the symmetric formalism we use here, because both of the dual pair of potentials are used, some Bianchi identities must be modified whichever type of sources is introduced — there is no longer a meaningful distinction between "electric" and "magnetic" sources. However they are called, the same set of equations for the field strengths must obtain in all three approaches if they are equivalent.

Let the brane current for the the sources be proportional to

$$\lambda = \omega + \sigma, \tag{2.38}$$

$$\breve{\lambda} = \breve{\omega} + \breve{\sigma}, \tag{2.39}$$

with the decomposition into the temporal parts (ω and $\breve{\omega}$) and the spatial parts (σ and $\breve{\sigma}$) understood. They are normalized so that the Bianchi identities are now

$$\begin{aligned} \nabla B &= \breve{\sigma}, \\ d_t B + \nabla E &= \breve{\omega}; \\ \nabla \breve{B} &= \sigma, \\ d_t \breve{B} + \nabla \breve{E} &= \omega. \end{aligned} \tag{2.40}$$

These brane currents also make a contribution, denoted by S_j, to the total action. One can derive the form of S_j by using the modified Bianchi identities (eq. 2.40). The equations of motion for Φ and $\breve{\Phi}$ require the dependence of S_j on them to be

$$S_j = \frac{1}{2} \int \left((-1)^{n+1} \Phi \wedge \sigma + \frac{1}{2} (-1)^{(n+1)(d-n)} \int \breve{\Phi} \wedge \breve{\sigma} + \cdots \right). \tag{2.41}$$

This is necessary for the consistency of the theory and ensures that the gauge transformations (eq. 2.32) continue to hold. Note the factor of $\frac{1}{2}$. It comes from the same factor in (eq. 2.28).

Turning now to the equations of motion for A and $\breve{A}$, we demand that the duality relation (eq. 2.35) holds again. This completely fixes the dependence of S_j on them:

$$S_j = \frac{1}{2} \int \left((-1)^{n+1} A \wedge \omega + \frac{1}{2} (-1)^{(n+1)(d-n)} \int \breve{A} \wedge \breve{\omega} + \cdots \right). \tag{2.42}$$

Now S_j is completely determined and has a Lorentz invariant expression:

$$S_j = \frac{1}{2} \int \left((-1)^{n+1} C \wedge \lambda + (-1)^{(n+1)(d-n)} \breve{C} \wedge \breve{\lambda} \right). \tag{2.43}$$

The conventional equations of motion are again determined from the Bianchi identities (eq. 2.40) and the duality relation (eq. 2.35):

$$\begin{aligned} \nabla * \breve{E} &= (-1)^{n(d-n)} \breve{\sigma}, \\ d_t * \breve{E} + \nabla * \breve{B} &= (-1)^{n(d-n)} \breve{\omega}; \\ \nabla * E &= -\sigma, \\ d_t * E + \nabla * B &= -\omega. \end{aligned} \tag{2.44}$$

When, say, $\breve{\lambda} = 0$, one can recover the conventional action in temporal gauge for C just as for the source free case. The resulting source term is found to be conventionally normalized, i.e. *without* the factor $\frac{1}{2}$. When both an electric brane of charge q_e and a magnetic brane of charge q_m are present, deforming the worldvolume of, say, the electric brane around the magnetic brane by a complete revolution shifts the action (eq. 2.42) by a constant. The electric and magnetic parts of (eq. 2.42) each makes an equal contribution of $\frac{1}{2} q_e q_m$. Requiring $\exp(iS_j)$ to be single-valued reproduces the standard Dirac quantization: $q_e q_m = 2m\pi$.

Finally, we shall write down the electro-magnetically symmetric action for the Ramond-Ramond fields, which is directly relevant for the inflow mechanism. In string theory, a Ramond-Ramond field strength $H_{(n)}$ and its dual $*H_{(n)}$ appear on equal footing. The formal sum H actually includes all electro-magnetic dual pairs of Ramond-Ramond field strengths, and so does $*H$. To find their relation, recall that these field strengths can be defined as follows in terms of the decomposition of bispinors:

$$H_{\mu_1 \dots \mu_n} = S_L^T \Gamma_{\mu_1} \dots \Gamma_{\mu_n} S_R. \tag{2.45}$$

Here S_L has positive $Spin(1,9)$ chirality, while S_R has positive or negative chirality for IIB and IIA string respectively. It is straightforward to infer from this

$$H_{(n)} = (-1)^{(n+q-1)/2} * (H_{(10-n)}). \tag{2.46}$$

Recall that q is 0 for IIB and 1 for IIA theory. These duality relations can be obtained from the action

$$S_{BE} = -\frac{1}{2}\int d^{10}x \sum_n \left((-1)^{(n-q+1)/2} B_{(n)} \wedge E_{(10-n)} + B_{(n)} \wedge *B_{(n)}\right). \tag{2.47}$$

Then if S_j is the Chern-Simons coupling in (eq. 2.11), it can be shown that the Bianchi identities must be (eq. 2.14) and the equations of motion must be (eq. 2.13).

3 Brane Anomalies

As usual, the anomalies on D-branes and I-branes result from the chiral asymmetry of massless fermions on them. These fermions are in one-to-one correspondence with the ground states of the relevant open string Ramond sectors. In the case of N D-branes wrapping M, the relevant open strings start and end on identical but possibly distinct D-brane. Open string quantization* requires that the Ramond ground states be the sections of the spinor bundle lifted from $T(X) = T(M) + N(M)$, tensored with a vector bundle in the $(N, \bar{N})$ representation (adjoint) of the gauge group $U(N)$ on the brane. The latter is dictated by the usual Chan-Paton factors. Because the adjoint representation is real, these fermions are CPT self-conjugate. We shall be interested in perturbative gauge anomalies, so consider $\dim(M)$ to be even. The GSO projection restricts the fermions to have a definite $SO(1,9)$ chirality. If $N(M) = \emptyset$, one is dealing with D9-branes. The worldvolume theory is the super-Yang-Mills part of the type I string theory [24]. It is chiral and anomalous but its anomaly is cancelled by that of the gravitinos and the inflow from the close string sector via the Green-Schwarz mechanism [1].

When $N(M) \neq \emptyset$, the fermions have the quantum number $(+,+) \oplus (-,-)$ under the worldvolume Lorentz group $\mathrm{Spin}(1,p)$ and the spacetime Lorentz

*See the appendix for a discussion of the issue of stability and supersymmetry of brane configurations.

group restricted to $N(M)$: Spin$(9-p)$. The latter is now the global R symmetry of the worldvolume theory. If $N(M)$ is flat, left and right moving fermions as sensed by the worldvolume are treated equally and the theory is nonchiral. However, when $N(M)$ has curvature, chiral asymmetry on the worldvolume is induced. The point is that the worldvolume chiralities of the fermions are correlated with their representations under the global R symmetry. Therefore a distinction arises between $(+,+)$ and $(-,-)$. The resulting perturbative anomaly can be calculated by the family index theorem [30, 31, 32, 33, 34]. For $\dim(M) = 4k+2$, the $(+,+)$ and $(-,-)$ fermions are independent and separately Majorana. The total anomaly associated with them is

$$\begin{aligned} I_{D-brane} &= \frac{2\pi}{2}\int_M \Big(\mathrm{ch}[U(N)_{(N,\bar{N})}] \wedge \hat{A}[M] \\ &\qquad \wedge\big(\mathrm{ch}[S^+_{N(M)}] - \mathrm{ch}[S^-_{N(M)}]\big)\Big)^{(1)}. \end{aligned} \tag{3.1}$$

Here $\mathrm{ch}[E]$ denotes the Chern character of a vector bundle E. $U(N)_{(N,\bar{N})}$ denotes the vector bundle in the $(N,\bar{N})$ representation of the structure group $U(N)$ associated with the N D-branes . $S^\pm_{N(M)}$ is the spin bundle lifted from $N(M)$ with $\pm$ chirality. $\hat{A}$ is the Dirac genus. The factor of $\frac{1}{2}$ in front reflects the reality of the fermions. Since $U(N)$ is unitary,

$$\begin{aligned} \mathrm{ch}[U(N)_{(N,\bar{N})}] &= \mathrm{ch}(F)\wedge \mathrm{ch}(-F^*) \\ &= \mathrm{ch}(F)\wedge \mathrm{ch}(-F), \end{aligned} \tag{3.2}$$

where

$$\mathrm{ch}(F) \equiv \exp(\frac{F}{2\pi}). \tag{3.3}$$

F is the properly normalized Hermitian field strength for the $U(N)$ connection on the D-brane in the *fundamental* representation. Using

$$\mathrm{ch}[S^+_E] - \mathrm{ch}[S^-_E] = \frac{e(E)}{\hat{A}(E)}, \tag{3.4}$$

which holds for any spin and orientable real vector bundle E, one can rewrite the anomaly as

$$I_{D-brane} = \frac{2\pi}{2}\int_M \left(\mathrm{ch}(F)\wedge \mathrm{ch}(-F)\wedge \frac{\hat{A}[T(M)]}{\hat{A}[N(M)]}\wedge e[N(M)]\right)^{(1)}. \tag{3.5}$$

In the special case when $N(M)$ is null, $e[N(M)]$ as well as $A[N(M)]$ is 1.

For $\dim(M) = 4k$, $(+,+)$ and $(-,-)$ are both complex and related by conjugation. Anomaly can be calculated by the contribution from either (but should not be doubly counted) as

$$\begin{aligned} I_{D-brane} &= 2\pi \int_M \left(\mathrm{ch}(F) \wedge \mathrm{ch}(-F) \wedge \hat{A}(M) \wedge \mathrm{ch}[S^+_{N(M)}]\right)^{(1)} \\ &= 2\pi \int_M \Big(\mathrm{ch}(F) \wedge \mathrm{ch}(-F) \wedge \hat{A}(M) \\ &\quad \wedge \frac{1}{2}\Big(\mathrm{ch}[S^+_{N(M)}] + \mathrm{ch}[S^-_{N(M)}] \\ &\quad + \mathrm{ch}[S^+_{N(M)}] - \mathrm{ch}[S^-_{N(M)}]\Big)\Big)^{(1)}. \end{aligned} \tag{3.6}$$

Because $\mathrm{ch}[S^+_{N(M)}] + \mathrm{ch}[S^-_{N(M)}]$ is a sum of Pontrjagin classes, it is made up of forms of ranks in multiples of 4. The same is true $\mathrm{ch}(F) \wedge \mathrm{ch}(-F)$. So only $\mathrm{ch}[S^+_{N(M)}] - \mathrm{ch}[S^-_{N(M)}]$ can contribute in (eq. 3.6) and we obtain (eq. 3.5) again as the expression for the anomaly.

When two D-branes intersect, additional massless fermions arise from the open string sectors with two ends on the two D-branes respectively. Consider a configuration with N_1 D-branes wrapping around M_1 and N_2 around M_2. In the sector with the string starting on M_1 and ending on M_2, the difference in the boundary conditions on the two ends of the string modifies its zero point energy and shifts the moding of some of its worldsheet operators [35, 36]. The result is that the massless fermions are a section of the chiral spinor bundle lifted from

$$T(M_1) \cap T(M_2) \oplus N(M_1) \cap N(M_2),$$

tensored with the $(N_1, \bar{N}_2)$ vector bundle due to their Chan-Paton quantum numbers. The anomaly can be calculated in the same fashion as before:

$$\begin{aligned} I_{I-brane} &= 2\pi \int_{M_{12}} \Big(\mathrm{ch}(F_1) \wedge \mathrm{ch}(-F_2) \wedge \frac{\hat{A}[T(M_1) \cap T(M_2)]}{\hat{A}[N(M_1) \cap N(M_2)]} \\ &\quad \wedge e[N(M_1) \cap N(M_2)]\Big)^{(1)}. \end{aligned} \tag{3.7}$$

Since $(N_1, \bar{N}_2)$ is complex, the fermions are not self-conjugate, and there is no factor of $\frac{1}{2}$ in front. Note that (eq. 3.5)is precisely one half of the special case of (eq. 3.7) with $M_1 = M = M_2$.

Using brane currents and (eq. 2.1), we can rewrite the anomalies (eq. 3.5) and (eq. 3.7) in forms that will prove useful:

$$\begin{aligned} I_{D-brane} &= \pm \frac{2\pi}{2} \int \tau_M \wedge \Big(e[N(M)] \\ &\quad \wedge \mathrm{ch}(F) \wedge \mathrm{ch}(-F) \wedge \frac{\hat{A}[T(M)]}{\hat{A}[N(M)]}\Big)^{(1)}, \end{aligned} \tag{3.8}$$

$$I_{I-brane} = \pm 2\pi \int \tau_{M_{12}} \wedge \Big(e[N(M_1) \cap N(M_2)]$$
$$\wedge \mathrm{ch}(F_1) \wedge \mathrm{ch}(-F_2) \wedge \frac{\hat{A}[T(M_1) \cap T(M_2)]}{\hat{A}[N(M_1) \cap N(M_2)]}\Big)^{(1)}. \quad (3.9)$$

Here we have left their signs undetermined because, being integrals of differential forms, they really depend on choices of orientation that are not yet fixed by any consideration so far. This ambiguity will soon be resolved by the requirement of factorizability.

In [4], the cases in which M_{12} is the transversal intersection of M_1 and M_2, i.e. $N(M_1) \cap N(M_2) = \emptyset$, were considered. Then the expression for I-brane anomaly (eq. 3.7) can be further simplified as

$$I_{I-brane} = \pm 2\pi \int \tau_{M_1} \wedge \tau_{M_2} \wedge \left(\mathrm{ch}(F_1) \wedge \mathrm{ch}(-F_2) \frac{\hat{A}[T(M_1) \cap T(M_2)]}{\hat{A}[N(M_1) \cap N(M_2)]} \right)^{(1)}, \quad (3.10)$$

where we have evaluated $e(\emptyset)$ to be 1 but kept $\hat{A}([N(M_1) \cap N(M_2)]$ for future comparison.

It is easy to check that (eq. 3.10) is factorizable in the sense of (eq. 2.10), with

$$Y_i = \mathrm{ch}(F_i) \wedge \sqrt{\frac{\hat{A}[T(M_i)]}{\hat{A}[N(M_i)]}} \quad (3.11)$$

and

$$\tilde{Y}_j = -(-1)^{\frac{dim(M_j)-q}{2}} \mathrm{ch}(-F_j) \sqrt{\frac{\hat{A}[T(M_j)]}{\hat{A}[N(M_j)]}}. \quad (3.12)$$

Hence this anomaly can be cancelled by the inflow (eq. 2.19). The sign factor in (eq. 3.12) is determined by (eq. 2.15). As promised before, this fixes the choice of orientation for the anomaly, and (eq. 3.10) becomes

$$I_{I-brane} = -\pi \int \tau_{M_1} \wedge \tau_{M_2} \wedge \Big(\Big((-1)^{\frac{dim(M_2)-q}{2}} \mathrm{ch}(F_1) \wedge \mathrm{ch}(-F_2)$$
$$+ \{1 \leftrightarrow 2\} \Big) \wedge \frac{\hat{A}[T(M_1) \cap T(M_2)]}{\hat{A}[N(M_1) \cap N(M_2)]} \Big)^{(1)}. \quad (3.13)$$

After some manipulation one can show that the two terms in the integrand of (eq. 3.13) contribute equally, rather than cancelling each other, to the anomaly:

$$I_{I-brane} = -(-1)^{\frac{dim(M_2)-q}{2}} 2\pi \int \tau_{M_1} \wedge \tau_{M_2} \wedge \Big(\mathrm{ch}(F_1) \wedge \mathrm{ch}(-F_2)$$
$$\wedge \frac{\hat{A}[T(M_1) \cap T(M_2)]}{\hat{A}[N(M_1) \cap N(M_2)]} \Big)^{(1)}. \quad (3.14)$$

(eq. 3.10) is also trivially correct when $N(M_1) \cap N(M_2)$ is nonempty but trivial, because the RHS' of both (eq. 3.7) and (eq. 3.10) vanish. However, (eq. 2.9) would want one to believe that (eq. 3.10) fails for a nontrivial $N(M_1) \cap N(M_2)$ because its RHS would seem to vanish, although the anomaly does not in general. There are similar difficulties for the D-brane anomaly (eq. 3.5). Consider D-branes with worldvolume M. For $N(M) = \emptyset$, the anomaly is that of Type I string theory and cancelled via the Green-Schwarz mechanism [1]. For $N(M) \neq \emptyset$, the closest thing would be (eq. 2.18) with $M_1 = M = M_2$. However, $\tau_M \wedge \tau_M$ naively vanishes.

4 Topology to the Rescue

It is clear from the earlier discussions that factorizability in the sense of (eq. 2.10) is crucial for an anomaly to be cancelled via this inflow method. However, when the relevant normal bundle is nontrivial, it can be shown that the *integrand* of (eq. 3.7) is no longer factorizable because of the Euler class. In other words, it is not factorizable unless $N(M_1) \cap N(M_2)$ is empty. The same can be said about the D-brane anomaly (eq. 3.5). A related puzzle on the other side of the inflow mechanism has also been shown. The second equation in (eq. 2.9) would imply vanishing inflow for M_{12} as long as $N(M_1) \cap N(M_2) \neq \emptyset$, regardless of the twisting of the normal bundle. It could cancel no anomaly, factorized or not.

The origin of all these difficulties can be traced back to the properties of brane currents. Being a physical observable, τ_M must be globally defined over M. However, (eq. 2.5) only makes sense within each coordinate patch, because between patches the transversal coordinates are defined only up to the transition functions for the normal bundle. To it one must add additional terms, which vanish when $N(M)$ is trivial but turn τ_M into a globally defined form when $N(M)$ is not. Therefore if such correction can be found, it must carry topological information about $N(M)$, and from (eq. 2.5) it must have components with indices tangential to M. Mathematicians have found an elaborate construction for this correction [37]. By pulling τ_M back to M, only parts from the correction can survive. It is remarkable that the result is cohomologically the Euler class $e[N(M)]$ of $N(M)$.

Before proceeding further it is convenient to introduce some notations. First observe that τ_M is determined by $N(M)$, because it should be defined as the limit of nonsingular differential forms with shrinking compact supports in the neighborhood of M, which is approximated by the neighborhood of the zero section of $N(M)$. As such τ_M can be defined for any oriented real orientable vector bundle E by taking M to be the zero section E. To emphasize this we define*

$$\Phi[E] \equiv \tau_M \tag{4.1}$$

for any vector bundle $\pi, E \to M$. The important property just mentioned can be written as

$$\tau_M \wedge \tau_M = \tau_M \wedge \Phi[N(M)] = \tau_M \wedge \Big[e[N(M)]\Big] \tag{4.2}$$

where $[e]$ denotes some representative of the cohomology class of e. Another useful property is [37]:

$$\Phi(A \oplus B) = \Phi(A) \wedge \Phi(B). \tag{4.3}$$

This can be seen as Euler class also factorizes under Whitney sum. Now by (eq. 2.6), for the I-brane worldvolume $M_{12} = M_1 \cap M_2$ we have

$$\begin{aligned}
\tau_{M_1} \wedge \tau_{M_2} &= \Phi[T(M_1) \cap N(M_2) \oplus N(M_1) \cap N(M_2)] \\
&\quad \wedge \Phi[N(M_1) \cap T(M_2) \oplus N(M_1) \cap N(M_2)] \\
&= \Phi[T(M_1) \cap N(M_2) \oplus N(M_1) \cap T(M_2) \oplus N(M_1) \cap N(M_2)] \\
&\quad \wedge \Phi[N(M_1) \cap N(M_2)] \\
&= \tau_{M_{12}} \wedge e[[N(M_1) \cap N(M_2)]],
\end{aligned} \tag{4.4}$$

where in the last step we have used (eq. 4.3) again along with (eq. 4.2). This is the correct replacement for the naive equation in (eq. 2.9). Now returning to the I-brane anomaly (eq. 3.9), one notes that as long as $\dim[T(M_1) \cap T(M_2)] + 2 > \dim[N(M_1) \cap N(M_2)]$, one can use the freedom to add local counterterms to choose to make the Wess-Zumino descent on terms *other than* the Euler form. The I-brane anomaly then becomes

$$\begin{aligned}
I_{I-brane} &= \pm 2\pi \int \tau_{M_{12}} \wedge e[N(M_1) \cap N(M_2)] \\
&\quad \wedge \left(\mathrm{ch}(F_1) \wedge \mathrm{ch}(-F_2) \frac{\hat{A}[T(M_1) \cap T(M_2)]}{\hat{A}[N(M_1) \cap N(M_2)]} \right)^{(1)}.
\end{aligned} \tag{4.5}$$

By the same token, only the cohomology class of e is important here. Substituting for (eq. 4.4), one obtains again (eq. 3.10) as the expression for anomaly. But now it is clearly valid even when the normal bundle is non-trivial. Furthermore, the D-brane anomaly can also be written in this form with $M_1 = M_2 = M$, as long as $\dim[T(M)] + 2 > \dim[N(M)]$. When $\dim[T(M_1) \cap T(M_2)] + 2 < \dim[N(M_1) \cap N(M_2)]$, both the anomaly and the inflow vanish. The case of $\dim[T(M_1) \cap T(M_2)] + 2 = \dim[N(M_1) \cap N(M_2)]$ is an intriguing one and we will comment on it shortly. We have shown that

*Actually for our purpose, knowledge of the cohomology class of $\Phi(E)$ is sufficient. It is called the Thom class of E.

except for that case, the inflow (eq. 2.18) not only does not vanish identically but cancels precisely the anomalies (eq. 3.9) and (eq. 3.8).

There is a nice topological characterization of our results. It has emerged that the anomaly, written as an integral over the total spacetime, is always proportional to

$$\tau_{M_1} \wedge \tau_{M_2}. \tag{4.6}$$

Its cohomology class is the Poincare dual of the transversal intersection of M_1 and M_2. Transversal intersection, unlike geometric or set-theoretic intersection, has the property of stability: because there is no common transverse direction, small perturbation can only move the intersection around but never make it disappear. Consider now a *nontransversal* intersection $M_{12} = M_1 \cap M_2$. Because $N(M_1) \cap N(M_2) \neq \emptyset$, a small perturbation in those directions would naively separate them and lift the intersection altogether. This is the meaning of the second line in (eq. 2.9). Such perturbation is given by a global section of $N(M_1) \cap N(M_2)$. However, a global section of a sufficiently twisted vector bundle will necessarily have nonempty zero locus. For $N(M_1) \cap N(M_2)$, this means that M_1 and M_2 cannot be completely separated. Any small perturbation will leave intact some submanifold of M_{12}, the zero locus of the corresponding section of $N(M_1) \cap N(M_2)$, which is now stable. That is precisely the *traversal* intersection of M_1 and M_2. It can be shown that the Poincare dual of the zero locus of an orientable real vector bundle E is none other than $e(E)$. This gives another derivation of (eq. 4.4). For $M_1 = M = M_2$, the story is similar. $e[N(M)]$ is the Poincare dual of the zero locus of $N(M)$. So $\tau_M \wedge \tau_M$ measures the self-intersection of M. To recapitulate, D-brane and I-brane anomalies are associated with transversal intersections, even when the pertinent geometric intersections are not transversal. In light of this, it seems worthwhile to introduce the notion of *transversal I-brane*, whose brane current is simply $\tau_{M_1} \wedge \tau_{M_2}$.

Now turning to the special case of

$$\dim[T(M_1) \cap T(M_2)] + 2 = \dim[N(M_1) \cap N(M_2)].$$

This implies that $\dim[T(M_1)] + \dim[T(M_2)] = 8$, or that the two D-branes make up an electro-magnetic dual pair. An example would be a D-string intersecting with a D5-brane at 0 angle. For $M_1 = M = M_2$, the condition $\dim[T(M)] + 2 = \dim[N(M)]$ means one is dealing with the self-dual D3-brane in IIB theory. For these examples the anomaly (eq. 3.9) is finite but the inflow, even after taking into account the nontriviality of the normal bundles, still seems to vanish. But one should not rush to conclude that anomaly does not cancel for them, because the intersection of electric and magnetic sources introduces an additional subtlety: the Chern-Simons action (eq. 2.11) is no longer well defined. A more powerful approach is needed but will not be pursued in the present work.

5 Induced Brane Charges

An important consequence of the inflow mechanism, besides lending support to the consistency of various brane configurations, is that charges for the bulk Ramond-Ramond fields are induced by the gauge fields and gravitational curvatures as in (eq. 2.13). Let M be the worldvolume of some Dp-branes with gauge field strength F. Consider a m-cycle γ of M^*. Then

$$Q_{\text{ind}} = \int_\gamma \text{ch}(F) \wedge \sqrt{\frac{\hat{A}[T(M)]}{\hat{A}[N(M)]}} \tag{5.1}$$

gives the induced charge, in integral unit, for the Ramond-Ramond $(p + 1 - m)$-form potential. From the viewpoint of the field theory on the Dp-brane, the characteristic class on the RHS measures the topological charge of a gravitational/Yang-Mills "instanton". Let us call it Y as before. Then (eq. 2.13) shows that $\tau_M \wedge Y$ can be thought of as the brane current for a "fat" D$(p-m)$-brane bound to and spread out on the Dp-brane. When the instanton shrink to zero size, Y also acquires Dirac's δ singularity. $\tau_M \wedge Y$ is just like a brane current. One might well wonder if the instanton can be lifted off the brane and become a physical D-brane in its own right. At least for Yang-Mills instantons there has been much evidence in support of this idea: field theory instantons and branes are continuously connected by transitions between different branches of the moduli space of the I-brane field theory [18, 19, 20, 38]. Recently, more complicated configurations involving gravitational curvatures on the D-brane were used to study geometric engineering and realizations of field theory dualities employing brane configurations [10, 13]. In this section we consider specific examples in which the twisting of the normal bundle modifies the induced charge.

As discussed in the appendix, our analysis seems to apply, a posteriori, to nonsupersymmetric brane configurations as well. However, in most applications considered in the literature there are some supersymmetries left so as to have control over radiative corrections. Therefore here we shall only consider Type II compactifications over d-dimensional manifolds S that preserve some supersymmetries. A D-brane wraps around a m-dimensional submanifold M of S can preserve some of the supersymmetries of the compactification provided M satisfies some conditions. Such a M is called a supersymmetric cycle [39]. All supersymmetric cycles have been analyzed and classified in [6, 7]. We shall consider them one by one. We shall also only consider S' with irreducible holonomy because the analysis for the other cases can be reduced to them. The forms in $\hat{A}(N)$ all have ranks in multiples of 4. On the other

*In this section we count in complex unit the dimensions of compactification manifolds S if it is Calabi-Yau and in real units those of other types as well as all submanifolds of S.

hand, to have nontrivial normal bundle, the D-brane must wrap a *proper* submanifold of M. By counting dimensions and ranks, the contribution of $N(M)$ to Q_{ind} comes from the rank 4 component of

$$\frac{1}{\sqrt{\hat{A}(N(M))}}.$$

For convenience we shall group it together with the contribution from $T(M)$ at the same rank, so the characteristic class we shall be computing is

$$\lambda \equiv \frac{p_1[N(M)] - p_1[T(M)]}{48}. \tag{5.2}$$

Let the Chern roots of $T(M)$ be

$$\pm x_i, \quad i = 1 \ldots \frac{m}{2}. \tag{5.3}$$

For the cases considered here m is always even. Let the Chern roots of $N(M)$ be

$$\pm y_j, \quad j = 1 \ldots \lfloor \frac{d-m}{2} \rfloor, \tag{5.4}$$

with an additional 0 if $d - m$ is odd. Then (eq. 5.2) can be written via the splitting principle as

$$-\frac{1}{48}\left(\sum_i x_i^2 - \sum_j y_j^2\right). \tag{5.5}$$

Of particular interest is whether λ and hence Q_{ind} can be expressed purely in terms of x's, information which is encoded in $T(M)$.

The first nontrivial compactification is K3. However, for this case there cannot be any additional contribution to the induced brane charge from a twisted normal bundle, for dimensional reasons mentioned above.

The next case is for S to be a generic Calabi-Yau 3-fold. According to [6] a supersymmetric cycle is either a Lagrangian submanifold (3-cycle) or a Kahler submanifold (2n-cycle) of the Calabi-Yau 3-fold. For the reasons discussed above, only for Kahler 4-cycles does $N(M)$ make a contribution to Q_{ind}. The holonomy of $T(M)$is $U(2)_T$ and that of $N(M)$ is $U(1)_N$. The Calabi-Yau condition requires

$$x_1 + x_2 + y = 0. \tag{5.6}$$

The relevant charge is proportional to

$$\begin{aligned} \lambda &= \frac{p_1[N(M)] - p_1[T(M)]}{48} \\ &= \frac{2x_1x_2}{48} = \frac{2e(T(M))}{48}. \end{aligned} \tag{5.7}$$

The remaining type of Calabi-Yau compactification is over a generic Calabi-Yau 4-fold. It can have three types of supersymmetric cycles: Lagrangian (4-cycle), Kahler (2n-cycle), and Cayley (4-cycle). A special Lagrangian submanifold has the property that the holonomy of its normal bundle is the same as that of its tangent bundle. Therefore the effect of $N(M)$ on the induced charge completely cancels whatever contribution from $T(M)$: $\lambda = 0$.

Among the Kahler (2n)-cycles, 4-cycles and 6 cycles will see contribution from $N(M)$. The holonomy group of $T(M)$ is $U(n)$. The holonomy group of $N(M)$ is $U(4-n)$. The Calabi-Yau condition says that

$$\sum_i x_i + \sum_j y_j = 0. \tag{5.8}$$

Using this we can calculate

$$\begin{aligned} \lambda &= \frac{1}{48}\left(2\left(\sum_{i_1<i_2} x_{i_1}x_{i_2} - \sum_{j_1<j_2} y_{j_1}y_{j_2}\right)\right) \\ &= \frac{2c_2[T_+(M)] - 2c_2[N_+(M)]}{48}. \end{aligned} \tag{5.9}$$

where $T_+(M)$ and $N_+(M)$ are the holomorphic tangent and normal bundles of M respectively, and c_2 denotes the second Chern class. For a Kahler 6-cycle, $c_2[N(M)]$ is 0, so (eq. 5.9) is entirely determined by information encoded in $T(M)$. This is not so for a Kahler 4-cycle, for which (eq. 5.9) reduces to

$$\lambda_{4\text{-cycle}} = \frac{2\Big(e[T(M)] - e[N(M)]\Big)}{48} \tag{5.10}$$

but cannot be expressed in terms of x alone.

Calabi-Yau 4-folds admit one more type of supersymmetric cycles [7]. It is to date the only known case where a single D-brane breaks the supersymmetries of a type II compactification by $\frac{3}{4}$ instead of $\frac{1}{2}$. They are known as Cayley submanifolds [40]. They are 4-dimensional and satisfy the conditions [41, 7]

$$x_1 + x_2 + y_1 + y_2 = 0, \tag{5.11}$$

and

$$x_1 - x_2 = y_2 - y_1. \tag{5.12}$$

These conditions are sufficiently restrictive to imply the vanishing of λ.

There are two other cases of string compactifications: S may be a seven dimensional manifold with $G(2)$ holonomy or an eight dimensional manifold with Spin(7) holonomy [42, 43]. A generic Spin(7) manifold supports only Cayley submanifolds as supersymmetric cycles [7]. It is again 4-dimensional.

With a suitable choice of orientations, the curvature is subject to (eq. 5.11) but not (eq. 5.12). Then (eq. 5.10) follows again [7].

Finally we come to the case of $G(2)$ manifold. It admits two types of supersymmetric cycles [7]. They are known as coassociative (4-cycle) and associative (3-cycle) submanifolds respectively. Only for the coassociative submanifold will Q_{ind} be affected by the gravitational curvature. With a suitable choice of orientations, they satisfy the condition [41] that

$$x_1 + x_2 + y = 0. \tag{5.13}$$

Hence

$$\lambda = \frac{2x_1x_2}{48} = \frac{2e(T(M))}{48}. \tag{5.14}$$

The results in this section are summarized in the following table.

Holonomy of S	Type of M	λ
SU(3)	Kahler 4	$2e(M)/48$
SU(4)	Special Lagrangian	0
SU(4)	Cayley	0
SU(4)	Kahler 4	$2[e(M) - e(N)]/48$
SU(4)	Kahler 6	$2c_2[T_+(M)]/48$
G(2)	Coassociative	$2e(M)/48$
Spin (7)	Cayley	$2[e(M) - e(N)]/48$

Acknowledgment

We would like to thank K. Bardakci, J. de Boer, O. Ganor, M. Green, J. Harvey, A. Hashimoto, K. Hori, S. Kachru, A. Kato, I. Klebanov, M. Krogh, S. Mandelstam, G. Moore, H. Ooguri, E. Sharpe, D. Sorokin, Y. Oz, and B. Zumino for useful discussions. ZY would especially like to thank H. Ooguri for encouragement throughout this work. The research of YKEC was supported by NSF-PHY9600258 and DOEFG02-91ER40671. ZY was supported by DE-AC03-76SF00098.

A Comments on Brane Stability and Supersymmetry

It is appropriate to address the issue of stability of brane configurations and its relevance to the anomaly analysis*. For a generic brane configuration,

*We would like to thank K. Bardakci for useful conversations regarding this issue.

there are forces between nonparallel branes. If they do not cancel, this configuration is not stable. One can no more trust string perturbation theory in an unstable brane configuration than one can trust perturbative expansion around a false vacuum in field theory. Anomaly calculations is in some sense more robust than many other perturbative calculations, but one must still know the correct spectrum of massless fermions in *some* true vacuum to correctly compute the anomaly. Of course this was the original motivation for t'Hooft's anomaly matching conditions. In the above, we have relied on string perturbation when we obtained the massless fermion contents and their quantum numbers. When the brane configuration is unstable, there is no known reason to expect a priori that such analysis captures correctly the spectrum.

On the other hand, supersymmetry is the only general condition under which the forces between branes cancel. If supersymmetry is completely broken in a brane configuration, the latter is *generically* unstable. For N identical D-branes to preserve some supersymmetry in a string compactification, they must wrap around the supersymmetric cycles classified in [6, 7]. Between a pair of D-branes, the pattern of supersymmetry breaking depends on their relative arrangement. For the case of intersection at right angle, some supersymmetries survive provided that [4]

$$\dim[T(M_1) \cap N(M_2)] + \dim[N(M_1) \cap T(M_2)] = 0 \ (\mathrm{mod} 4). \qquad (A.1)$$

The expression on the LHS of this equation is sometime denoted $nd + dn$ in the literature because it is the number of spacetime coordinates for which the boundary condition of the relevant open string is Neuman on one end and Dirichlet on the other. When (eq. A.1) is not satisfied, anomaly calculation based on perturbative string theory does not have to be reliable. For example, if $nd + dn = 2$, it may be shown that the force between the two D-branes is attractive. It is believed that in this case there exists a stable nonmarginal bound state [26]. There seems a priori to be no reason to expect that the correct degrees of freedom of the bound state to be obtained from a perturbative string analysis carried out at the unstable configuration.

On the other hand, (eq. A.1) was not needed in the analysis carried out in this paper. In fact it follows through as long as

$$\dim[T(M_1) \cap T(M_2)] + \dim[N(M_1) \cap N(M_2)] = 0 \ (\mathrm{mod} 2), \qquad (A.2)$$

a condition satisfied by any pair of D-branes that can coexist in the same string theory. This seems to suggest that even for nonsupersymmetric brane configurations, at least the massless fermion contents might be captured correctly.

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Prepared for the U.S. Department of Energy under Contract No. DE-AC03-76SF00098