Title
APPLICATION OF GENERAL THEORETICAL PRINCIPLES TO EXPERIMENTS - Lecture 3. SELECTION RULES FOR DECAYS

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SELECTION RULES FOR DECAYS - LECTURE 3

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Appendix to Section III.

The Angular Momentum of Light

The momentum density of an electromagnetic field is the Poynting vector $\overrightarrow{E \times H}$ [rationalized units; $c = 1$], therefore the angular momentum density is $\vec{r} \times (\overrightarrow{E \times H})$, and the total angular momentum in a volume V is

$$\vec{J} = \int_V d^3r \, \vec{r} \times (\overrightarrow{E \times H}) \quad .$$

Let us specialize to a free wave, that is, our volume is far from all sources. We introduce, then, potentials in the "Coulomb gauge":

$$A_4 = 0 \qquad \vec{E} = -\dot{\vec{A}}$$

$$\vec{\nabla} \cdot \vec{A} = 0 \qquad \vec{H} = \overrightarrow{\nabla \times A} \quad .$$

So $\overrightarrow{E \times H} = -\dot{\vec{A}} \times (\overrightarrow{\nabla \times A}) = -\dot{A}_i \vec{\nabla} A_i + \dot{A}_i \nabla_i \vec{A}$

using indices and the summation convention for the dot product, to facilitate subsequent manipulation. So

$$\vec{J} = \int_V d^3r \left[-\dot{A}_i \, \overrightarrow{r \times \nabla} \, A_i + \dot{A}_i \, \vec{r} \times \nabla_i \vec{A} \right] \, .$$

Let us further specialize to a wave packet which our volume completely encloses, so that we may integrate by parts (Green's theorem) and throw away surface terms. Thus the second term

$$\mathring{A}_i \vec{r} \times \nabla_i \vec{A} \longrightarrow - \nabla_i(\mathring{A}_i \ \vec{r} \times)\vec{A} = -(\nabla_i \mathring{A}_i)\vec{r} \times \vec{A} - \mathring{A}_i(\nabla_i \vec{r}) \times \vec{A} \ .$$

$$= - \vec{\mathring{A}} \times \vec{A} \ .$$

Thus

$$\vec{J} = \int_V d^3r \left[- \mathring{A}_i \ \overrightarrow{r \times \nabla} \ A_i - \vec{\mathring{A}} \times \vec{A} \ \right]$$

which is now in a form to see that the first term is the orbital angular momentum part L, and the second term is spin $\vec{S}$.

In the first term, $\vec{\nabla} A_i = - \mathring{A}_i \vec{e}_3$ where $\vec{e}_3$ is a unit vector in the direction of the wave number $\vec{k}$ (constant for a plane wave) thus

$$\vec{L} = \int_V d^3r \ \mathring{A}_i^2 \ \vec{r} \times \vec{e}_3 = \int_V d^3r \ \ r \times (E^2 \ \vec{e}_3)$$

which is indeed independent of the state of polarization of the field, as an orbital angular momentum should be. Note that $E^2 \vec{e}_3$ is the momentum density in a plane wave, equal to $\vec{E} \times \vec{H}$.

The second term, on the other hand, does not contain r explicitly, so that it is independent of the position of the origin as a spin should be; and it does depend on the polarization. For linear polarization,

$$\vec{\mathring{E}} \ // \ \vec{E} \ (\therefore \vec{\mathring{A}} \ || \ \vec{A}) \ \text{so} \ \vec{\mathring{A}} \times \vec{A} = 0, \ \text{and} \ \vec{S}_{lin} = 0;$$

for circular polarization $\vec{\mathring{E}} \perp \vec{E}$, so that

$$\vec{S}_{circ} = \int_V d^3r \ (- \vec{\mathring{A}} \times \vec{A}) = \int_V d^3r \ (\pm \mathring{A} \ A \ \vec{e}_3) = \int_V d^3r \ (\pm \frac{E^2}{\omega} \ \vec{e}_3)$$

for $\left\{ \begin{matrix} \text{right} \\ \text{left} \end{matrix} \right\}$ hand polarization. Thus for a plane wave, where $\vec{e}_3$ and ω are constant

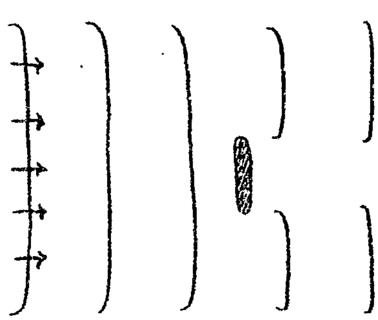
$$S = \pm \frac{1}{\omega} \left[\int_V d^3r \; E^2 \right] \vec{e}_3 = \pm \frac{1}{\omega} \begin{bmatrix} \text{total} \\ \text{energy} \end{bmatrix} \vec{e}_3 \; .$$

If the wave is quantized, its energy $= n\hbar\omega$, so that $\vec{S} = \pm n\hbar\vec{e}_3$: each circularly polarized photon carries a spin $\pm \hbar\vec{e}_3$.

Localization of the Spin: Suppose we have a nearly plane, circularly polarized wave, which has a finite sideways extent: i.e. a plane wave passed thru an aperature (large compared to λ). Then in the central part of the wave, $\overrightarrow{E \times H} = E^2 \vec{e}_3$, so that

$$\vec{J} = \int d^3r \; \vec{r} \times (\overrightarrow{E \times H})$$

of a central part of the wave is purely orbital; the spin (the part of $\vec{J}$ ||
is actually carred by the fringe of the wave. Our integration by parts transformed the representation of the spin to a uniform density $\pm E^2 \vec{e}_3$. But this representation of the spin density has a physical significance. For suppose by inserting an obstacle we absorb light from the central part of the wave. Then by considering the spin of the wave beyond the obstacle, we see that the spin has been absorbed in amount just given by the uniform density formula. And looking at the obstacle microscopically we can see that this is true of the absorption at each atom. Thus we could say that the spin of the wave is carried as



a uniform density $\pm E^2 \vec{e}_3$ in the sense that wherever an energy W is absorbed from any part of the wave, a spin $\pm \frac{W}{\omega} \vec{e}_3$ is absorbed simultaneously.