

Lawrence Berkeley National Laboratory

Recent Work

Title

APPLICATION OF GENERAL THEORETICAL PRINCIPLES TO EXPERIMENTS - Lecture 3.
SELECTION RULES FOR DECAYS

Permalink

<https://escholarship.org/uc/item/1511k0hh>

Author

Goebel, Charles.

Publication Date

1955-12-01

UNIVERSITY OF
CALIFORNIA

*Radiation
Laboratory*

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy
which may be borrowed for two weeks.
For a personal retention copy, call
Tech. Info. Division, Ext. 5545*

BERKELEY, CALIFORNIA

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

UNIVERSITY OF CALIFORNIA

Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

APPLICATION OF GENERAL THEORETICAL PRINCIPLES TO
EXPERIMENTS

SELECTION RULES FOR DECAYS - LECTURE 3

Charles Goebel

December 20, 1955

SELECTION RULES FOR DECAYS

Lecture 3

Charles Goebel

December 20, 1955

Appendix to Section III.

The Angular Momentum of Light

The momentum density of an electromagnetic field is the Poynting vector $\vec{E} \times \vec{H}$ [rationalized units; $c = 1$], therefore the angular momentum density is $\vec{r} \times (\vec{E} \times \vec{H})$, and the total angular momentum in a volume V is

$$\vec{J} = \int_V d^3r \vec{r} \times (\vec{E} \times \vec{H}) .$$

Let us specialize to a free wave, that is, our volume is far from all sources.

We introduce, then, potentials in the "Coulomb gauge":

$$\begin{aligned} A_4 &= 0 & \vec{E} &= -\dot{\vec{A}} \\ \vec{\nabla} \cdot \vec{A} &= 0 & \vec{H} &= \vec{\nabla} \times \vec{A} . \end{aligned}$$

$$\text{So } \vec{E} \times \vec{H} = -\dot{\vec{A}} \times (\vec{\nabla} \times \vec{A}) = -A_i \vec{\nabla} A_i + \dot{A}_i \nabla_i \vec{A}$$

using indices and the summation convention for the dot product, to facilitate subsequent manipulation. So

$$\vec{J} = \int_V d^3r \left[-\dot{A}_i \vec{r} \times \vec{\nabla} A_i + \dot{A}_i \vec{r} \times \nabla_i \vec{A} \right] .$$

Let us further specialize to a wave packet which our volume completely encloses, so that we may integrate by parts (Green's theorem) and throw away surface terms. Thus the second term

$$\begin{aligned} \dot{A}_1 \vec{r} \times \nabla_1 \vec{A} &\rightarrow -\nabla_1 (\dot{A}_1 \vec{r} \times \vec{A}) = -(\nabla_1 \dot{A}_1) \vec{r} \times \vec{A} - \dot{A}_1 (\nabla_1 \vec{r}) \times \vec{A} \\ &= -\dot{A}_1 \vec{r} \times \vec{A} \end{aligned}$$

Thus

$$\vec{J} = \int_V d^3 r \left[-\dot{A}_1 \vec{r} \times \vec{A} - \dot{A}_1 \vec{r} \times \vec{A} \right]$$

which is now in a form to see that the first term is the orbital angular momentum part \vec{L} , and the second term is spin \vec{S} .

In the first term, $\nabla_1 \dot{A}_1 = -\dot{A}_1 \vec{e}_3$ where \vec{e}_3 is a unit vector in the direction of the wave number \vec{k} (constant for a plane wave) thus

$$\vec{L} = \int_V d^3 r \dot{A}_1^2 \vec{r} \times \vec{e}_3 = \int_V d^3 r \vec{r} \times (E^2 \vec{e}_3)$$

which is indeed independent of the state of polarization of the field, as an orbital angular momentum should be. Note that $E^2 \vec{e}_3$ is the momentum density in a plane wave, equal to $\vec{E} \times \vec{H}$.

The second term, on the other hand, does not contain r explicitly, so that it is independent of the position of the origin as a spin should be; and it does depend on the polarization. For linear polarization,

$$\vec{E} \parallel \vec{A} \quad (\therefore \dot{\vec{A}} \parallel \vec{A}) \quad \text{so} \quad \dot{\vec{A}} \times \vec{A} = 0, \quad \text{and} \quad \vec{S}_{\text{lin}} = 0;$$

for circular polarization $\dot{\vec{E}} \perp \vec{E}$, so that

$$\vec{S}_{\text{circ}} = \int_V d^3 r (-\dot{\vec{A}} \times \vec{A}) = \int_V d^3 r (\pm \dot{A} A \vec{e}_3) = \int_V d^3 r (\pm \frac{E^2}{\omega} \vec{e}_3)$$

for $\left\{ \begin{array}{l} \text{right} \\ \text{left} \end{array} \right\}$ hand polarization. Thus for a plane wave, where \vec{e}_3 and ω are

constant

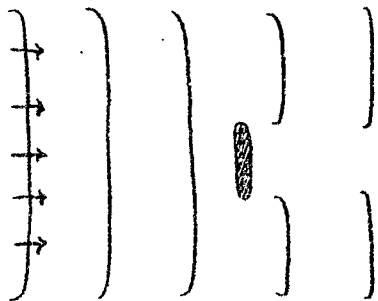
$$S = \pm \frac{1}{\omega} \left[\int_V d^3r E^2 \right] \vec{e}_3 = \pm \frac{1}{\omega} \left[\text{total energy} \right] \vec{e}_3$$

If the wave is quantized, its energy = $n\hbar\omega$, so that $\vec{S} = \pm n\hbar\vec{e}_3$: each circularly polarized photon carries a spin $\pm \hbar\vec{e}_3$.

Localization of the Spin: Suppose we have a nearly plane, circularly polarized wave, which has a finite sideways extent: i.e. a plane wave passed thru an aperture (large compared to λ). Then in the central part of the wave, $\vec{E} \times \vec{H} = E^2 \vec{e}_3$, so that

$$\vec{J} = \int d^3r \vec{r} \times (\vec{E} \times \vec{H})$$

of λ central part of the wave is purely orbital; the spin (the part of $\vec{J} \parallel \vec{e}_3$) is actually carried by the fringe of the wave. Our integration by parts transformed the representation of the spin to a uniform density $\pm E^2 \vec{e}_3$. But this representation of the spin density has a physical significance. For suppose by inserting an obstacle we absorb light from the central part of the



wave. Then by considering the spin of the wave beyond the obstacle, we see that the spin has been absorbed in amount just given by the uniform density formula.

And looking at the obstacle microscopically we can see that this is true of the absorption at each atom. Thus we conclude that the spin of the wave is carried as

a uniform density $\pm E^2 \vec{e}_3$ in the sense that wherever an energy W is absorbed from any part of the wave, a spin $\pm \frac{W}{\omega} \vec{e}_3$ is absorbed simultaneously.