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Superheavy supersymmetry from scalar mass – A parameter fixed points

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Abstract

In supersymmetric models, the well-known tension between naturalness and experimental constraints is relieved if the squarks and sleptons of the first two generations are superheavy, with masses $m_{\text{heavy}} \geq 10 \text{ TeV}$, and all other superpartners are light, with masses $m_{\text{light}} \leq 1 \text{ TeV}$. We show that even if all scalar masses and trilinear A parameters are of order m_{heavy} at some high scale, a hierarchy of $m_{\text{heavy}}^2/m_{\text{light}}^2 \sim 400$ may be generated dynamically through renormalization group evolution. The required high energy relations are consistent with grand unification, or, alternatively, may be realized in moduli-dominated supersymmetry-breaking scenarios. © 2000 Published by Elsevier Science B.V. All rights reserved.

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1. Introduction

Supersymmetry is a well-motivated framework for extending the standard model of strong and electroweak interactions [1,2]. Among its many virtues, weak-scale supersymmetry provides a natural solution to the gauge hierarchy problem, realizes gauge unification without the ad hoc introduction of additional particles, and elegantly explains electroweak symmetry breaking in terms of the large top quark mass.

Given the most general possible set of soft supersymmetry-breaking terms, however, supersymmetric models violate many well-known laboratory con-

straints, particularly those on flavor-changing neutral currents (FCNC) and CP violation. Indeed, studies of the supersymmetric contributions to rare processes place severe constraints on flavor mixing in the sfermion ($\tilde{f}$) mass matrices,

$$m_{fij}^2 \tilde{f}_i^* \tilde{f}_j + \text{h.c.}, \quad \tilde{f} = Q, U, D, L, E, \quad (1)$$

where i, j are generational indices, Q denotes quark SU(2) doublets, U and D up- and down-type quark singlets, L lepton doublets, and E lepton singlets. For example, in the basis in which the fermion mass

matrices are diagonal, the $K_L - K_S$ mass difference requires

$$\left[\frac{10 \text{ TeV}}{m} \right]^2 \left[\frac{\text{Re}(m_{Q,D_{12}}^2/m^2)}{0.1} \right]^2 \lesssim 1, \quad (2)$$

where m denotes the average squark mass [3]. If we assume at most moderate suppressions of the second bracketed term from squark degeneracy or squark-quark alignment, this bound implies $m \gtrsim 10 \text{ TeV}$. The constraint from the CP-violating parameter ϵ_K is even more severe, requiring $m \gtrsim 100 \text{ TeV}$ for $\mathcal{O}(1)$ CP-violating phases. Electron and neutron electric dipole moments provide constraints that are less stringent, but nevertheless important, because they are flavor-conserving and therefore unsuppressed by sfermion degeneracy. For $\mathcal{O}(1)$ CP-violating phases, the electric dipole moments require $m \gtrsim 2 \text{ TeV}$. Other difficulties, such as too rapid proton decay through dimension-five operators in grand unified theories [4] and cosmological problems caused by late-decaying moduli [5], are also alleviated if the superpartner and gravitino masses are heavy.

The above constraints are most easily satisfied if all supersymmetric scalars are at the 10 TeV scale. However, such heavy sfermions are in apparent conflict with naturalness, the requirement that there be no large cancellations in radiative corrections. Naturalness suggests that the superpartner masses should be at most $\sim 1 \text{ TeV}$. This conflict may be avoided, however, by observing [6–10] that the most stringent laboratory constraints apply to quantities associated with the first two generations, while naturalness primarily restricts the third-generation sfermions, which couple to the Higgs sector with large Yukawa couplings. The laboratory and naturalness constraints may be simultaneously satisfied if the scalar masses exhibit an inverted mass hierarchy: third-generation scalars are light with masses $m_{\text{light}} \lesssim 1 \text{ TeV}$, while the first two generation scalars are heavy, with masses $m_{\text{heavy}} \gtrsim 10 \text{ TeV}$.

In this letter we will present a mechanism for generating such an inverted mass hierarchy dynamically, through renormalization group evolution. Our scenario preserves and utilizes the appealing features mentioned above. As will become clear, the scenario is compatible with grand unification, and assumes no additional gauge dynamics or particle content (aside

from a right-handed neutrino). In addition, the large top quark Yukawa coupling, which drives electroweak symmetry breaking, is also used to generate the inverted hierarchy. In our scenario, the inverted hierarchy is no accident: both heavy fermions and light scalars are necessarily associated with large Yukawa couplings.

This approach was first investigated in Ref. [11], and then extended and generalized to the case of unified theories in Ref. [12]. Both of these papers assumed the hierarchy $m \gg A, m_{1/2}$ between the scalar masses, m , on the one hand, and the trilinear couplings A and gaugino masses, $m_{1/2}$, on the other, as would follow, for example, from an approximate $U(1)_R$ symmetry. It was found that the third-generation scalar masses can indeed be exponentially suppressed when renormalization group evolution drives them to a zero-value infrared fixed point. In Ref. [12], hierarchies of $m_{\text{heavy}}^2/m_{\text{light}}^2 \sim 20$ were realized. This solves some of the phenomenological problems discussed previously, but only alleviates others. (See Ref. [12] for a detailed discussion.)

In this study, we relax the restriction on the A parameters and consider the hierarchy $m, A \gg m_{1/2}$. Such a hierarchy emerges naturally, for example, in moduli-dominated supersymmetry-breaking scenarios [13–17]. The fixed-point structure may be analyzed as before, but now including both scalar masses and A parameters. We find that the presence of the A parameters leads to greatly improved results, with possible hierarchies of $m_{\text{heavy}}^2/m_{\text{light}}^2 \sim 400$ and $m_{\text{heavy}} \gtrsim 10 \text{ TeV}$ naturally achieved.

2. Naturalness and fixed points

The radiative generation of electroweak symmetry breaking is one of the most important and appealing features of supersymmetry. At the weak scale, the necessary condition is conveniently expressed by the following (tree-level) equation:

$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2, \quad (3)$$

where m_{H_d} and m_{H_u} are soft Higgs boson parameters, μ is the supersymmetry-preserving Higgs mass, and $\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$ is the usual ratio of Higgs

vacuum expectation values. A theory is regarded as natural if Eq. (3) is free from large cancellations.

The parameters m_{H_u} and m_{H_d} are determined by renormalization group evolution from high energies. Schematically, the one-loop evolution equations for the soft masses are of the form

$$\dot{m}^2 \sim -Ym^2 + \tilde{\alpha}m_{1/2}^2 - YA^2, \quad (4)$$

where (positive) numerical coefficients are suppressed. We have used the following notation: $(\dot{}) \equiv d/dt$, where $t \equiv \ln(Q_0^2/Q^2)$ and Q_0 is the initial renormalization scale; and $\tilde{\alpha} \equiv g^2/16\pi^2 = \alpha/4\pi$ and $Y \equiv h^2/16\pi^2$, where g and h are gauge and Yukawa couplings, respectively. Our conventions for the A parameters are that the trilinear soft terms have the form $Ah\phi_i\phi_j\phi_k$. Clearly, the weak-scale Higgs boson parameters are most strongly affected by the third-generation sfermions because of their large Yukawa couplings. In contrast, the contributions from the first two generations of sfermions are suppressed by their Yukawa couplings, and are very small.¹ From this we conclude that the third generation is intimately related to the naturalness problem, while the first two generations effectively decouple.

Let us now assume $m, A \sim m_{\text{heavy}}$ at some high energy unification scale, and that the third-generation Yukawa couplings are unified throughout their renormalization group evolution. (Splittings of the Yukawa couplings will be taken into account in the numerical results to follow.) Let us also assume the gaugino masses are of order m_{light} , so they can be neglected in the following analysis. (Note that gaugino masses cannot be hierarchically suppressed: $m_{1/2}/\alpha$ is an evolution invariant at one-loop, which, along with Eq. (4), constrains $m_{1/2} \sim m_{\text{light}}$.) The one-loop evolution equations for the third-generation scalar masses and A parameters may be collectively written as

$$\dot{\mathbf{m}}_i^2 = -YN_{ij}\mathbf{m}_j^2, \quad (5)$$

where $\mathbf{m}^2$ is a vector containing both m^2 and A^2 parameters, and N is a matrix of positive constants

determined by color and SU(2) factors. This set of equations is easily solved by decomposing arbitrary initial conditions into eigenvectors of N , each of which then evolves independently. Indeed, if N has eigenvectors $\mathbf{e}_i$ with eigenvalues λ_i , the initial condition

$$\mathbf{m}^2(t=0) = \sum c_i \mathbf{e}_i \quad (6)$$

evolves to

$$\mathbf{m}^2(t=t_f) = \sum c_i \mathbf{e}_i \exp\left[-\lambda_i \int_0^{t_f} Y dt\right]. \quad (7)$$

The existence of zero-value infrared fixed points follows immediately from the observation that eigenvectors with large λ_i are asymptotically and exponentially “crunched” to zero at the low scale. If the initial conditions are dominated by such eigenvectors, the soft masses are rapidly suppressed relative to their initial values. Note that this suppression does not hold in the case of the first two generations because of the size of their Yukawa couplings. The dynamically generated hierarchy relates the observed hierarchy in the fermion sector to the desired (and inverted) hierarchy in the scalar sector, with large (small) Yukawa couplings producing heavy (light) fermions and light (heavy) superpartners. The exact hierarchy depends on various details such as the initial conditions for the soft parameters, the evolution interval, and the Yukawa couplings and their evolution. Below, we will define a suppression factor, or crunch factor, S , which will serve as a quantitative measure of the hierarchy achieved for given parameters in a particular model.

3. The scalar mass hierarchy

In this section, we consider a model with the particle content of the minimal supersymmetric standard model with a (heavy) right-handed neutrino, N , as suggested by unified models based on SO(10) or larger groups. The analysis is similar, in principle, to that of Ref. [12], but is extended to include heavy A parameters; this is crucial to improving the results.

¹ Sfermion masses of the first two generations may be important if the trace of hypercharge times m^2 does not vanish or if their masses are $\gtrsim 20$ TeV. In the latter case, their two-loop effects are generally not negligible [8,9,18,19]. See, however, Ref. [20] for models in which this difficulty is avoided.

The superpotential is given by

$$W = -h_t H_u Q U + h_b H_d Q D + h_\tau H_d L E - h_n H_u L N, \quad (8)$$

where the matter fields Q , U , D , L , E , and N are those of the third generation, H_u and H_d are the two Higgs doublets, and all other Yukawa couplings may be neglected for this analysis. The sfermion mass matrices are assumed to be diagonal (see Section 4), but not necessarily degenerate at the boundary, which we take to be the unification scale, M_G . In this approximation, the evolution equations for the soft parameters are given by

$$\begin{aligned} \dot{m}_{H_u}^2 &= -3Y_t(m_{H_u}^2 + m_U^2 + m_Q^2 + A_t^2) \\ &\quad - Y_n(m_{H_u}^2 + m_L^2 + m_N^2 + A_n^2), \\ \dot{m}_{H_d}^2 &= -3Y_b(m_{H_d}^2 + m_D^2 + m_Q^2 + A_b^2) \\ &\quad - Y_\tau(m_{H_d}^2 + m_L^2 + m_E^2 + A_\tau^2), \\ \dot{m}_Q^2 &= -Y_t(m_{H_u}^2 + m_U^2 + m_Q^2 + A_t^2) \\ &\quad - Y_b(m_{H_d}^2 + m_D^2 + m_Q^2 + A_b^2), \\ \dot{m}_U^2 &= -2Y_t(m_{H_u}^2 + m_U^2 + m_Q^2 + A_t^2), \\ \dot{m}_D^2 &= -2Y_b(m_{H_d}^2 + m_D^2 + m_Q^2 + A_b^2), \\ \dot{m}_L^2 &= -Y_n(m_{H_u}^2 + m_L^2 + m_N^2 + A_n^2) \\ &\quad - Y_\tau(m_{H_d}^2 + m_L^2 + m_E^2 + A_\tau^2), \\ \dot{m}_E^2 &= -2Y_\tau(m_{H_d}^2 + m_L^2 + m_E^2 + A_\tau^2), \\ \dot{m}_N^2 &= -2Y_n(m_{H_u}^2 + m_L^2 + m_N^2 + A_n^2), \\ \dot{A}_t &= -6Y_t A_t - Y_b A_b - Y_n A_n, \\ \dot{A}_b &= -Y_t A_t - 6Y_b A_b - Y_\tau A_\tau, \\ \dot{A}_\tau &= -3Y_b A_b - 4Y_\tau A_\tau - Y_n A_n, \\ \dot{A}_n &= -3Y_t A_t - Y_\tau A_\tau - 4Y_n A_n. \end{aligned} \quad (9)$$

Let us assume further that the SO(10) unification relations $Y_t = Y_b = Y_\tau = Y_n = Y$ and $A_t = A_b = A_\tau = A_n = A$ are realized at M_G , and neglect for the

moment deviations from these relations from renormalization group evolution. Then

$$\dot{\mathbf{m}}^2 = -Y\mathbf{N}\mathbf{m}^2, \quad (10)$$

where $\mathbf{m}^2 = (m_{H_u}^2, m_U^2, m_Q^2, m_D^2, m_{H_d}^2, m_L^2, m_E^2, m_N^2, A^2)^T$ and

$$\mathbf{N} = \begin{bmatrix} 4 & 3 & 3 & 0 & 0 & 1 & 0 & 1 & 4 \\ 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 2 \\ 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 3 & 3 & 4 & 1 & 1 & 0 & 4 \\ 1 & 0 & 0 & 0 & 1 & 2 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 16 \end{bmatrix}. \quad (11)$$

The eigenvectors of $\mathbf{N}$, and their associated eigenvalues are

$$\begin{aligned} \mathbf{e}_1 &: 16, (2, 1, 1, 1, 2, 1, 1, 1, 4), \\ \mathbf{e}_2 &: 8, (2, 1, 1, 1, 2, 1, 1, 1, 0), \\ \mathbf{e}_3 &: 6, (2, 1, 0, -1, -2, 0, -1, 1, 0), \\ \mathbf{e}_4 &: 4, (0, 1, 1, 1, 0, -3, -3, -3, 0), \\ \mathbf{e}_5 &: 2, (0, -1, 0, 1, 0, 0, -3, 3, 0), \end{aligned} \quad (12)$$

along with four eigenvectors of zero eigenvalue. Components of the initial conditions along $\mathbf{e}_1$ and, to a lesser extent, $\mathbf{e}_2$, $\mathbf{e}_3$, $\mathbf{e}_4$, and $\mathbf{e}_5$, are rapidly suppressed during the renormalization group evolution. Note that the eigenvectors $\mathbf{e}_1$ and $\mathbf{e}_2$ are both consistent with minimal SO(10) unification, where $\mathbf{16} = \{Q, U, D, L, E, N\}$, $\mathbf{10} \supset \{H_u, H_d\}$, and the SO(10) group is broken in one step to the standard model group. The remarkably simple boundary condition given by $\mathbf{e}_1$,

$$4m_{16}^2 = 2m_{10}^2 = A^2, \quad (13)$$

also has the largest eigenvalue, 16. (For comparison, the largest eigenvalue in Ref. [12] was only 8.) This eigenvector gives rise to the largest hierarchy.

The above argument neglects many details. We therefore evaluate the renormalization group equations numerically at one-loop. The independent evolutions of the Yukawa couplings are included, and the right-handed neutrino is decoupled at its mass, M_N . We do not include two-loop contributions in our analysis, nor do we include one-loop terms of order m_{light} . Two-loop contributions to the Yukawa evolution equations were considered in Ref. [12], and are

required to establish how large the Yukawa couplings can be before perturbation theory breaks down. Here we use the results of Ref. [12], and consider only regions of parameter space where perturbation theory is valid and two-loop effects are small.

We note that two-loop contributions to the scalar mass equations may be important, at least for the largest hierarchies that we will be able to achieve ($m_{\text{heavy}} \sim 20 \text{ TeV}$) [8,9,18,19]. However, such effects are not more important than one-loop $\mathcal{O}(m_{\text{light}})$ terms that we ignore. A complete two-loop analysis requires the specification of all m_{light} parameters, and is highly model-dependent. Our aim here is only to establish the possibility of a very large radiative hierarchy, so we take a model-independent approach. Interesting issues such as radiative electroweak symmetry breaking, vacuum stability, and the light spectrum rely on the model-dependent $\mathcal{O}(m_{\text{light}})$ terms, and are outside the scope of this analysis.

Given our assumptions, the theory is completely specified by the overall scale, m_{heavy} , and two parameters: h_G , the universal third-generation Yukawa coupling at the unification scale, and M_N , the right-handed neutrino mass. In Fig. 1, we show the evolution of the scalar mass parameters from the unification scale, $M_G \approx 2 \times 10^{16} \text{ GeV}$, to the weak scale, $M_W = 1 \text{ TeV}$, for the boundary condition of Eq. (13), with $m_{16} = 10 \text{ TeV}$, $h_G = 2$ and $M_N = 10^8 \text{ GeV}$. At the weak scale, an inverted mass hierarchy is generated, with all third-generation and Higgs mass parameters $\lesssim 1 \text{ TeV}$, and the first two generation scalar masses (not shown) unsuppressed at $\sim 10 \text{ TeV}$. The hierarchy is generated mostly during the first few decades of evolution, where the Yukawa couplings are large and almost universal, so the approximations made in the analytical arguments above are roughly valid.

As in Ref. [12], we quantify the radiatively generated mass hierarchy by a suppression, or crunching, factor

$$S \equiv \frac{\bar{m}^2(M_G)}{\bar{m}^2(M_W)}, \quad (14)$$

where $\bar{m}^2(Q) \equiv \text{Av}[|m^2(Q)|]$. The average is taken over all scalar degrees of freedom in the theory (but

Fig. 1. The renormalization group evolution of the Higgs and third-generation sfermion masses (solid) and the A term (dotted) in the supersymmetric standard model with a right-handed neutrino, for the boundary conditions of Eq. (13) with $h_G = 2$ and $M_N = 10^8 \text{ GeV}$. First- and second-generation scalar masses (not shown) are approximately evolution invariant. Model-dependent effects of order m_{light}^2 modify solutions in the shaded region, and are not included. The suppression factor for this case is $S = 330$ (see text).

not the A parameters), properly weighted by color and $\text{SU}(2)$ factors. In Fig. 2 we plot S in the (M_N, h_G) plane. We see that S depends strongly on both of these two parameters. In the upper left corner, where $h_G \gtrsim 2$ and $M_N \lesssim 10^8 \text{ GeV}$, the value of S can be as large as 400! This should be compared with the maximal crunch factor $S_{\text{max}} \sim 20$ in Ref. [12]. The large values of S are a result of the A parameter contributions in the one-loop evolution equations. The presence of the A parameters doubles the maximal eigenvalue, λ_i , and therefore squares the maximal S . A crunch factor of $S = 400$ corresponds to $m_{\text{heavy}} \sim 20 \text{ TeV}$. This can solve all FCNC

Fig. 2. The suppression factor S for the supersymmetric standard model with a right-handed neutrino and initial boundary conditions as given by Eq. (13). The parameter M_N is the scale at which the right-handed neutrino decouples, and h_G is the value of the universal Yukawa coupling at the unification scale, $M_G \approx 2 \times 10^{16}$ GeV.

and CP problems, with the exception of ϵ_K , which is still beyond bounds if one assumes $\mathcal{O}(1)$ phases.

4. High Energy Scenarios

The fixed-point mechanism proposed above requires the hierarchy $m, A \gg m_{1/2}$. In addition, to achieve the greatest suppression factors, the scalar mass and A parameters must approximately satisfy the boundary condition of Eq. (13). (Small components along the other eigenvectors are tolerable, especially if their eigenvectors are also crunched, as are those of Eq. (12).) As noted above, the necessary boundary conditions are consistent with grand unification. However, they may also find their origin in string scenarios. In this section we show that both the required hierarchy in soft parameters and the specific boundary condition of Eq. (13) may be achieved in moduli-dominated supersymmetry-breaking scenarios with particular assignments of the modular weights.

The soft supersymmetry-breaking parameters may be viewed as arising from external spurion fields that parameterize the hidden-sector supersymmetry breaking. Generally speaking, soft scalar masses arise

from terms in the Kähler potential and therefore depend on D -type supersymmetry breaking. Gaugino masses and trilinear A parameters arise from the gauge kinetic function and the superpotential and depend on F -type breaking. It is certainly possible that different fields contribute to the different soft terms. In weakly-coupled heterotic string theory, for example, the tree-level gauge kinetic function depends only on the dilaton superfield S . The moduli fields T contribute to the scalar mass terms and the A parameters. If supersymmetry breaking is moduli-dominated, with $F_T \gg F_S$, it is natural to have $m, A \gg m_{1/2}$, as required.²

In moduli-dominated supersymmetry-breaking scenarios, the scalar masses are fixed to certain discrete values, determined by integer-valued modular weights. It may then be possible not only to achieve the correct hierarchy, but also the correct ratios of Eq. (13). General discussions and formulae may be found in Refs. [13–17]. Following Ref. [14,15], for example, and assuming for simplicity that only one (overall) modulus T participates in supersymmetry breaking, one finds at tree-level that

$$m_i^2 = (1 + n_i) m_{3/2}^2,$$

$$A_{ijk} = [3 + n_i + n_j + n_k - 2\delta(T)] m_{3/2}, \quad (15)$$

where n_i is the modular weight of field i , and $\delta(T) = \text{Re} T \partial_T \ln h_{ijk}$ is a quantity of order one if the vacuum expectation value of T and the derivative are of order one in Planck units. (In this case, the one-loop correction to the gaugino mass can be at most $\sim m_{\text{light}}$.) We find, then, that the correct ratios in Eq. (13) are obtained for $n_{Q,U,D,L,E,N} = 0$, $n_{H_u,H_d} = 1$, and $\delta \sim 1$. (This choice also implies a modular weight of -4 for the third-generation Yukawa couplings.) The dependence of S on h_G and δ is given in Fig. 3. While non-negative weights are not characteristic of the most well-studied examples, such weight assignments for the standard model fields may be found in Abelian orbifold models [13].

² The μ and $B\mu$ Higgs mixing parameters may arise from both F - and D -type terms, so their origin is more model-dependent. The naturalness conditions discussed in Section 2 require that they both be $\mathcal{O}(m_{\text{light}})$, and we will assume this to be the case, as a consequence, for example, of an approximate Peccei-Quinn symmetry.

Fig. 3. The suppression factor S for the supersymmetric standard model with a right-handed neutrino and initial boundary conditions given by Eq. (15), with $n_{Q,U,D,L,E,N} = 0$, and $n_{H_u,H_d} = 1$.

Two comments are in order before concluding. Up to this point, we have neglected the off-diagonal scalar masses. Given the large radiative hierarchy, large 1-2 mixings are allowed by all flavor-changing constraints, although the requirement from ϵ_K may impose some restriction if CP-violating phases are $\mathcal{O}(1)$. The 1-3 and 2-3 elements are still bounded, however, by the requirement that the weak-scale theory be tachyon-free. (See Ref. [12] for a more complete discussion of these constraints.) In the string context, it may be that these dangerous mixings are suppressed because different generations have discrete quantum numbers that forbid off-diagonal terms in the Kähler potential [13]. Alternatively, string models often possess additional U(1) groups, and the quantum numbers of different generations may suppress the scalar mass mixings [13]. (Such additional groups, however, may make it more difficult to obtain positive modular weights in the massless spectrum.) Note that while such stringy suppressions may already suppress flavor-violating effects, the superheavy radiative hierarchy possesses additional virtues, in that the 1-2 mixing need not be suppressed, and even flavor-conserving difficulties, such as the electric dipole moments and the Polonyi problem, are solved.

Finally, we note that any tree-level relation such as Eq. (15) receives quadratically divergent one-loop

corrections from non-renormalizable couplings [21–23], which fractionally are of order $\Lambda^2/16\pi^2$, where Λ is the cut-off in Planck units. These corrections are not calculable and can conceivably be large. Since they are loop-suppressed, we expect them to be small, of order a few percent, in which case they do not affect our discussion.

5. Conclusions

In this paper we have examined the possibility that all scalar masses and A parameters are of order some superheavy scale, m_{heavy} , when they are generated. The third-generation sfermions are driven to a scale $m_{\text{light}} \lesssim 1$ TeV by their large Yukawa couplings. For boundary conditions that we have identified, an attractive zero-value fixed point allows hierarchies of $m_{\text{heavy}}^2/m_{\text{light}}^2 \sim 400$. The necessary boundary conditions are consistent with SO(10) and similar grand unified theories. It is also possible that they have a stringy origin in terms of modular weights in moduli-dominated supersymmetry-breaking scenarios.

If we take $m_{\text{heavy}} \sim 20$ TeV, such a scenario leads to a supersymmetric theory which naturally preserves the gauge hierarchy. It also solves many difficulties common to supersymmetric models that follow from flavor- and CP-violating constraints, proton decay,³ and cosmology. Detailed study of sub-TeV issues, such as electroweak symmetry breaking and the low energy spectrum, requires a model-dependent two-loop analysis.

Although we have considered renormalization group evolution below the unification scale, our observations hold more generally. For example, we have verified that the crunch factors found in Ref. [12], in the case of evolution between the Planck and unification scales, are also enhanced by an order of magnitude once the trilinear A parameters are included, just as in the case of evolution below the unification scale discussed here.

³ More specifically, proton decay mediated by first- and second-generation sfermions is highly suppressed; however, recently analyzed processes involving third-generation scalars may still be dangerous [4].

Finally, we note that while this scenario suggests that the scalars of the first and second generation are all well beyond the direct discovery reaches of planned colliders, these theories are not exempt from experimental probes. In particular, this scenario predicts that all gauginos and third-generation scalars are below ~ 1 TeV, so measurements of the non-decoupling superoblique parameters [24–30] should be possible. The superoblique parameters provide indirect measurements of m_{heavy} , or the gravitino mass scale, and hence of the supersymmetry-breaking scale in the hidden sector. While some particles may evade direct detection in hierarchical scenarios, indirect signatures can provide important confirmation and extensive insights regarding the high energy theory.

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References

- [1] H.P. Nilles, Phys. Rep. 110 (1984) 1.
- [2] H. Haber, G. Kane, Phys. Rep. 117 (1985) 75.

- [3] For a summary of these and other flavor and CP violation constraints, see, for example, F. Gabbiani, E. Gabrielli, A. Masiero, L. Silvestrini, Nucl. Phys. B477 (1996) 321.
- [4] T. Goto, T. Nihei, Phys. Rev. D 59 (1999) 115009, hep-ph/9909251.
- [5] G.D. Coughlan, W. Fischler, E.W. Kolb, S. Raby, G.G. Ross, Phys. Lett. B 131 (1983) 59.
- [6] M. Drees, Phys. Rev. D 33 (1986) 1468.
- [7] M. Dine, R. Leigh, A. Kagan, Phys. Rev. D 48 (1993) 4269.
- [8] S. Dimopoulos, G.F. Giudice, Phys. Lett. B 357 (1995) 573.
- [9] A. Pomarol, D. Tommasini, Nucl. Phys. B 466 (1996) 3.
- [10] A.G. Cohen, D.B. Kaplan, A.E. Nelson, Phys. Lett. B 388 (1996) 588.
- [11] J.L. Feng, C. Kolda, N. Polonsky, Nucl. Phys. B 546 (1999) 3.
- [12] J. Bagger, J.L. Feng, N. Polonsky, hep-ph/9905292, Nucl. Phys. B, in press.
- [13] L. Ibanez, D. Lust, Nucl. Phys. B 382 (1992) 305.
- [14] A. Brignole, L. Ibanez, C. Munoz, Nucl. Phys. B 422 (1994) 125.
- [15] A. Brignole, L. Ibanez, C. Munoz, C. Scheich, Z. Phys. C 74 (1997) 157.
- [16] V.S. Kaplunovsky, J. Louis, Phys. Lett. B 306 (1993) 269.
- [17] Y. Kawamura, T. Kobayashi, Phys. Rev. D 56 (1997) 3844.
- [18] N. Arkani-Hamed, H. Murayama, Phys. Rev. D 56 (1997) 6733.
- [19] K. Agashe, M. Graesser, Phys. Rev. D 59 (1999) 015007.
- [20] J. Hisano, K. Kurosawa, Y. Nomura, Phys. Lett. B 445 (1999) 316.
- [21] K. Choi, J.E. Kim, H.P. Nilles, Phys. Lett. B 73 (1994) 1758.
- [22] H.P. Nilles, N. Polonsky, Phys. Lett. B 412 (1997) 69.
- [23] K. Choi, J.S. Lee, C. Munoz, Phys. Rev. Lett. 80 (1998) 3686.
- [24] M.M. Nojiri, K. Fujii, T. Tsukamoto, Phys. Rev. D 54 (1996) 6756.
- [25] H.-C. Cheng, J.L. Feng, N. Polonsky, Phys. Rev. D 56 (1997) 6875.
- [26] H.-C. Cheng, J.L. Feng, N. Polonsky, Phys. Rev. D 57 (1998) 152.
- [27] M.M. Nojiri, D.M. Pierce, Y. Yamada, Phys. Rev. D 57 (1998) 1539.
- [28] S. Kiyoura, M.M. Nojiri, D.M. Pierce, Y. Yamada, Phys. Rev. D 58 (1998) 075002.
- [29] E. Katz, L. Randall, S.F. Su, Nucl. Phys. B 536 (1998) 3.
- [30] U. Mahanta, Phys. Rev. D 59 (1999) 015017, hep-ph/9907230.