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# Reducing Delay while Maintaining Capacity in Mobile Ad-hoc Networks Using Multiple Random Routes

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**Abstract**— We present a different multiuser diversity strategy for packet relaying in mobile wireless ad-hoc network, which permits more than one-copy of a packet being received by relay nodes, thus allowing to decrease the delay on such networks for a fixed number of total nodes  $n$ . We show that the  $\Theta(1)$  throughput is preserved. Also, we find that the average delay and variance scale like  $\Theta(n)$  and  $\Theta(n^2)$  respectively for both one-copy and multi-copy techniques. For finite  $n$ , in single-copy relaying strategy, the delay values are not bounded as a consequence from the tail of the exponential distribution. However, by multi-copy relaying, we cut-off the tail of the exponential distribution of the delay. Accordingly, a bounded delay is obtained.

## I. INTRODUCTION

Recently, Gupta and Kumar [1] showed that the capacity of a fixed wireless network decreases as the number of nodes increases. Then, Grossglauser and Tse [2] presented a two-phase packet forwarding technique for mobile ad hoc networks (MANET), utilizing multiuser diversity, in which a source node transmits a packet to the nearest neighbor, and that relay delivers the packet to the destination when this destination becomes the closest neighbor of the relay. The scheme was shown [2] to increase the capacity of the MANET, such that it attains  $\Theta(1)$  as the total number of nodes  $n$  in the network increases. However, the delay experienced by packets under this strategy was shown to be large and even infinite for a finite  $n$ .

This paper analyzes an improved two-phase packet forwarding strategy for MANETs that attains the  $\Theta(1)$  capacity of the basic scheme by Grossglauser and Tse [2], but provides bounded delay when the number of nodes  $n$  is fixed. Our basic idea is to give a copy of the packet to multiple one-time relay nodes that are within the transmission range of the sender. These relays follow multiple random routes and can reach the destination earlier if compared with only one copy [2].

The remaining of the paper is organized as follows. Section II introduces the network model and explains our relaying strategy presenting the fraction of cells that successfully forward packets. Section III provides the interference computation. Section IV shows that the new relaying scheme attains the  $\Theta(1)$  capacity. Section V analyzes the delay reduction resulting from our

forwarding strategy and presents theoretical and simulation results. Section VI concludes the paper summarizing the main ideas presented.

## II. MODEL

The modeling problem we address is that of a MANET where  $n$  mobile nodes move in a unit circular area (or disk). We consider a time-slotted operation of the system to simplify the analysis, and we assume that the communication occurs among those nodes that are close enough, so that interference caused by other nodes is low, allowing reliable communication. The model is basically the same as the one introduced by Grossglauser and Tse [2], who consider a packet to be delivered from sender to destination via one-time relaying.

The position of node  $i$  at time  $t$  is indicated by  $X_i(t)$ . The nodes are assumed to be uniformly distributed on the disk at the beginning and there is no preferential direction of movement. The trajectories for different users are independent and identically distributed (iid). Nodes are assumed to move according to the *uniform mobility model* [3], in which the steady-state distribution for the mobile nodes is uniform. At each time step, a scheduler decides which nodes are senders, relays, or destinations, in such a manner that the association pair, source-destination, does not change with time. Each node can be a source for one session and a destination for another session. Packets are assumed to have header information for scheduling and identification purposes, and a time-to-live threshold field.

Suppose that at time  $t$  a source  $i$  has data to a certain destination  $d(i)$ . Since nodes  $i$  and  $d(i)$  have a direct transmission only  $1/n$  fraction of time on the average, a relay strategy is required to deliver data  $d(i)$  via relay nodes. We assume that each packet can be relayed in sequence at most once. So a packet passes two phases (see Fig. 1): The packet is transmitted from the source to a relay node during *Phase 1* (time slot  $t_0$ ), and it is delivered to its destination by the relay node during *Phase 2* (time slot  $t$ ). Direct transmission from source to destination is also allowed. Both phases occur concurrently, but *Phase 2* has absolute priority in all scheduled sender-receiver pairs.

We introduce a new packet delivery scheme to reduce the delay by allowing more than one copy of the same packet being received during *Phase 1*, i.e., more than one relay node receives the same copy of the packet. Thus, the chance that a

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copy of this packet reaches its destination in a shorter time is increased compared with using only one relay node as in [2]. If for some reason a relaying node fails to deliver the packet when it is within the transmission range of the destination, the packet can be delivered when another relaying node carrying a copy of the same packet approaches the destination.

In Fig. 1(a) three copies of the same packet are received by adjacent relay nodes  $j$ ,  $p$ , and  $k$  during *Phase 1*. All such relays are located within a distance  $r_o$  from sender  $i$ . At a future time  $t$ , in *Phase 2*, node  $j$  reaches first the destination and delivers the packet. Note that relay node  $j$  is not the closest node to source during *Phase 1* while it reaches the destination first.

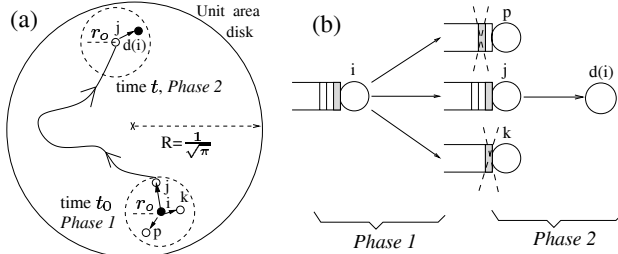


Fig. 1. (a) Three packet copies transmission at *Phase 1*. Node  $j$  is the first to find the destination, and delivers the packet at *Phase 2*. (b) Time-to-live threshold timeout after three packet copies transmission (from a).

One way of avoiding same packet delivery is to assign a sequence number (SN) and time-to-live (TTL) threshold to each packet. Before a packet is delivered to its destination, a handshake can be established between relay and destination to verify that the destination has not received a copy of the same packet. Because we address the network capacity for any embodiment of the multi-copy relaying strategy, we assume in the rest of this paper that the overhead of the relay-destination handshake is negligible. All relays delete the packet copies from their queues after the TTL expires for the packet (see Fig. 1(b)), and the destination of the packet remembers the SN of a packet it receives for a period of time larger than the TTL of the packet to ensure that any handshake for the packet is correct.

Among the total number of nodes  $n$  in the network, a fraction of them,  $n_S$ , is randomly chosen by the scheduler as senders, while the remaining nodes,  $n_R$ , operate as receiving nodes [2]. A sender density parameter  $\theta$  is defined as  $n_S = \theta n$ , where  $\theta \in (0, 1)$ , and  $n_R = (1 - \theta)n$ . In [2] each sender transmits to its nearest neighbor. However, it may be the case that a sender can have more than one receiver node in the feasible transmission range, and the proposed multi-copy relay strategy takes advantage of this by allowing those additional receiving nodes to also have a copy of the packet. These additional packet copies follow different random routes and can find the destination earlier compared to [2], where only one node receives the packet.

If the density of nodes in the disk is  $\rho = \frac{n}{\text{total area}} = \frac{n}{\pi} = n$ , then, for a uniform distribution of nodes, the radius for one sender node is given by

$$1 = \theta \rho \pi r_o^2 = \theta n \pi r_o^2 \implies r_o = \frac{1}{\sqrt{\theta n \pi}}. \quad (1)$$

Thus, the radius  $r_o$  defines a cell (radius range) around a sender. The number of receiving nodes, called  $K$ , for each sender node varies. Referring to the recent work by El Gamal, Mammen, Prabhakar and Shah [4], each cell in our strategy has area  $a(n) = \frac{1}{n_S} = \frac{1}{\theta n}$ . By applying random occupancy theory

[5], the fraction of cells containing  $L$  senders and  $K$  receivers is obtained by

$$\begin{aligned} P\{\text{senders} = L, \text{receivers} = K\} &= \binom{n}{L} \left(\frac{1}{\theta n}\right)^L \left(1 - \frac{1}{\theta n}\right)^{n-L} \binom{n-L}{K} \left(\frac{1}{\theta n}\right)^K \left(1 - \frac{1}{\theta n}\right)^{n-L-K} \\ &\approx \frac{n^L}{L!} \left(\frac{1}{\theta n}\right)^L \left[\left(1 - \frac{1}{\theta n}\right)^{\theta n}\right]^{n/(\theta n)} \frac{n^K}{K!} \left(\frac{1}{\theta n}\right)^K \left[\left(1 - \frac{1}{\theta n}\right)^{\theta n}\right]^{n/(\theta n)} \\ &\approx \frac{1}{L!} \left(\frac{1}{\theta}\right)^L e^{-1/\theta} \frac{1}{K!} \left(\frac{1}{\theta}\right)^K e^{-1/\theta}. \quad (\text{for } n \text{ very large}) \quad (2) \end{aligned}$$

Accordingly, for  $L = 1$ ,  $K \geq 2$ , and  $\theta = \frac{1}{3}$ , we have that  $\frac{1}{\theta} e^{-1/\theta} (1 - e^{-1/\theta} - \frac{1}{\theta} e^{-1/\theta}) \approx 0.12$  fraction of the cells contain one sender and at least two receivers. In addition, for  $\theta = \frac{1}{3}$ , we have that  $(\frac{1}{\theta} e^{-1/\theta})^2 \approx 0.02$  fraction of the cells have one sender and one receiver. In this case, the scheduler does not select these cells for packet transmission, because the delivery delay incurred can last to infinity as we show later. Also, the maximum number of nodes in any cell, with high probability (whp)<sup>1</sup>, is  $O(\frac{\log(n)}{\log(\log(n))})$  [5]. Thus, whp  $K \leq \frac{c \log(n)}{\log(\log(n))} \ll n$  for some constant  $c > 0$ .

### III. INTERFERENCE ANALYSIS

In the previous section, we obtained the fraction of cells that has one sender surrounded by  $K \geq 2$  receiving nodes within  $r_o$ , assuming a uniform distribution of nodes. Suppose that, in any of these cells, one of the  $K$  receiving nodes is at the maximum neighborhood distance  $r_o$ . We want to know how the SIR measured by this receiver behaves as the number of total nodes in the network (and therefore the number of total interferers) goes to infinity. We are interested in determining whether feasible communication between the sender and the farthest neighbor<sup>2</sup> (with distance  $r_o$ ) is still possible, even if the number of interferers grows.

At time  $t$ , node  $j$  is capable of receiving at a given rate of  $W$  bits/sec from  $i$  if [2], [1]

$$\frac{P_i(t) \gamma_{ij}(t)}{N_0 + \underbrace{\sum_{k \neq i} P_k(t) \gamma_{kj}(t)}_I} \geq \beta, \quad (3)$$

where  $P_i(t)$  is the transmitting power of node  $i$ ,  $\gamma_{ij}(t)$  is the channel path gain from node  $i$  to  $j$ ,  $\beta$  is the Signal to Noise and Interference Ratio (SNIR) level necessary for reliable communication,  $N_0$  is the noise power, and  $I$  is the total interference at  $j$ . The channel path gain is assumed to be a function of the distance only, so that  $\gamma_{ij}(t) = 1/|X_i(t) - X_j(t)|^\alpha = 1/r_{ij}^\alpha(t)$  [2], [1], where  $\alpha$  is the path loss parameter, and  $r_{ij}(t)$  is the distance between  $i$  and  $j$ . Given that, for narrowband communication, the interference coming from other nodes generally is much greater than the noise power, the denominator in Eq. (3) is dominated by the interference factor, thus resulting in the signal to interference ratio  $SIR = P_i(t) \gamma_{ij}(t) / I$ .

For a packet to be successfully received, Eq. (3) must be satisfied. Hence, consider a receiver at any location in the network during a given time  $t$ . Its distance from the center  $r'$  is shown in Fig. 2, where  $0 \leq r' < \frac{1}{\sqrt{\pi}} - r_o$ . Let us assume that the sender

<sup>1</sup>With high probability means with probability  $\geq 1 - \frac{1}{n}$  [5].

<sup>2</sup>This represents the worst case scenario, because the other  $K - 1$  neighbors are located either closer or at the same distance  $r_o$  to the sender, so they measure either a stronger or the same SIR level.

is at distance  $r_o$  from this receiver and transmitting at constant power  $P$ , so that the power measured by the receiver is given by  $P_R = \frac{P}{r_o^\alpha}$ .

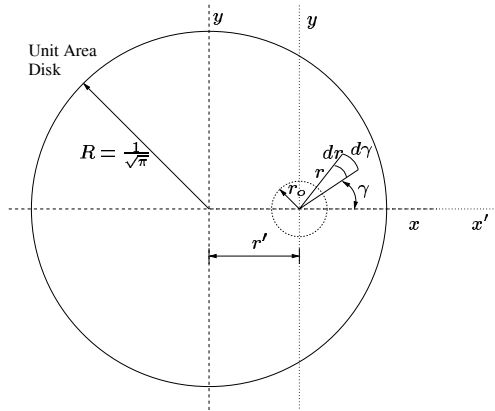


Fig. 2. Snapshot of the unit area disk at a given time  $t$ . The analyzed receiver is located at  $r'$  from the center while the sender is at distance  $r_o$  from the receiver.

To obtain the interference at the receiver caused by all transmitting nodes in the disk, let us consider a differential element area  $r dr d\gamma$  that is distant  $r$  units from the receiver (see Fig. 2). Because the nodes are uniformly distributed in the disk, the transmitting nodes inside this differential element of area generate, at the receiver, the following amount of interference<sup>3</sup>

$$dI = \frac{P}{r^\alpha} \theta \rho r dr d\gamma = \frac{P}{r^{\alpha-1}} \theta n dr d\gamma. \quad (4)$$

For  $\alpha > 2$ , the total interference at the receiver located at distance  $r'$  from the center with total of  $n$  nodes in the network is obtained by integrating Eq. (4) over all the disk area. Hence,

$$\begin{aligned} I_{r'}(n) &= \int_{disk} dI = \int_0^{2\pi} \int_{r_o}^{r_m(r',\gamma)} \frac{P\theta n}{r^{\alpha-1}} dr d\gamma = P\theta n \int_0^{2\pi} \frac{r_m^{2-\alpha}}{2-\alpha} \Big|_{r_o}^{r_m(r',\gamma)} d\gamma \\ &= \frac{P\theta n}{\alpha-2} \int_0^{2\pi} \left\{ \frac{1}{r_o^{\alpha-2}} - \frac{1}{[r_m(r',\gamma)]^{\alpha-2}} \right\} d\gamma. \end{aligned} \quad (5)$$

$r_m$  is the maximum radius for  $r$  and is a function of the location  $r'$  and the angle  $\gamma$ . We use the boundary disk curve (or circumference) equation expressed as a function of the  $x$ -axis and  $y$ -axis shown in Fig. 2, i.e.,  $x^2 + y^2 = (1/\sqrt{\pi})^2$ , and define  $x = x' + r'$ , in which  $x' = r_m \cos\gamma$ , and  $y = r_m \sin\gamma$ . Then we solve for  $r_m$  to obtain  $r_m(r',\gamma) = \sqrt{\frac{1}{\pi} - (r' \sin\gamma)^2} - r' \cos\gamma$ . By substituting this result in Eq. (5) we arrive at

$$I_{r'}(n) = \frac{2P\theta n}{\alpha-2} \underbrace{\int_0^{2\pi} \left( \sqrt{\pi-1} - (r' \sin\gamma)^2 - r' \cos\gamma \right)^{2-\alpha} d\gamma}_{f_\alpha(r')}, \quad (6)$$

in which  $f_\alpha(r')$  = constant for given  $r'$ . Thus, the SIR when  $n \rightarrow \infty$  is given by

$$SIR = \frac{P_R}{I_{r'}(n)} = \frac{\alpha-2}{2} \left[ 1 - \underbrace{\frac{1}{\pi^{\frac{\alpha}{2}} (\theta n)^{\frac{\alpha-2}{2}} f_\alpha(r')}}_{n \rightarrow \infty 0} \right]^{-1} = \frac{\alpha-2}{2}. \quad (7)$$

Hence, the SIR remains constant when  $n$  grows to infinity and this constant does not depend on  $r'$  if  $0 \leq r' < 1/\sqrt{\pi} - r_o$ . If  $r' = 1/\sqrt{\pi}$  the SIR has a greater value (see Fig. 3).

Hence, by having the SIR approaching a constant as  $n$  scales to infinity, the network designer can properly devise the receiver (i.e., design modulation, encoding, etc.) such that Eq. (3) can be satisfied for a given  $\beta$ , allowing reliable (feasible) communication among close neighbors during *Phase 1* and also during *Phase 2*, for those cells that can successfully forward packets.

<sup>3</sup>Because the nodes are considered to be uniformly distributed in the disk and  $n$  grows to infinity, we approximate the sum in Eq. (3) by an integral.

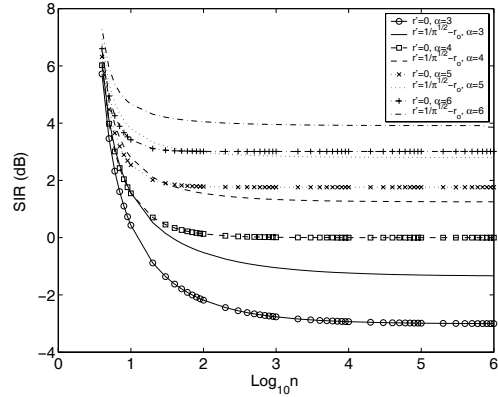


Fig. 3. SIR as a function of  $n$ , for  $3 \leq \alpha \leq 6$  and  $\theta = \frac{1}{3}$ , and the receiver node considered located at the center and at the boundary of the network.

#### IV. SOURCE-DESTINATION THROUGHPUT

We know that the throughput for a one-copy relay is  $\Theta(1)$  [2]. In the case of multi-copy transmission, only one copy is delivered to destination and the others are dropped from the additional relaying nodes. Thus, only one node out of  $K$  nodes actually functions as a relay (as in Fig. 1(b)). Accordingly, only one copy of different packets passes through the two-phase processes. Since the nodes trajectories are iid and they move according to the *uniform mobility model*, the traffic from each source node is uniformly distributed among all nodes [2]. From Eq. (2), each cell employing multi-copy forwarding has throughput of  $\Theta(1) \approx 0.12$ . Therefore, the network transport capacity (i.e., the network throughput) is  $\Theta(n)$ . Consequently, the network throughput of  $\Theta(n)$  is uniformly distributed among all source-destination pairs [4]. Thus, the exact total throughput per source-destination pair ( $\Lambda$ ) is proportional to the fraction of cells that successfully forward packets (i.e., the cells that are selected by the scheduler containing feasible sender-receiver pairs). Then, for one sender and at least  $K$  receivers per cell, we have

$$\Lambda = P\{L=1, \text{ receivers} \geq K\} \approx \frac{1}{\theta} e^{-1/\theta} \left[ 1 - \sum_{k=0}^{K-1} \frac{1}{k!} \left( \frac{1}{\theta} \right)^k e^{-1/\theta} \right]. \quad (8)$$

Hence, for at least two receivers per cell and  $\theta = \frac{1}{3}$ ,  $\Lambda = \frac{1}{\theta} e^{-1/\theta} (1 - e^{-1/\theta} - \frac{1}{\theta} e^{-1/\theta}) \approx 0.12 = \Theta(1)$ . Therefore, the multi-copy forwarding strategy attains the same throughput order as in [2]. Also, for at least one receiver per cell and  $\theta = \frac{1}{3}$ ,  $\Lambda = \frac{1}{\theta} e^{-1/\theta} (1 - e^{-1/\theta}) \approx 0.14$ . Hence, for the case  $K \geq 1$ , Eqs. (2) and (8) give the same throughput value obtained by Tse and Grossglauser [2]. Thus, in the single-copy forwarding strategy [2], although they have  $K \geq 1$ , their scheme selects only the nearest neighbor from the sender amongst the  $K$  receiver nodes.

#### V. DELAY EQUATIONS

Now we find the relationship between the delay value  $d$  obtained for the case of only one copy relaying [2], and the new delay  $d_K$  for  $K \geq 2$  copies transmitted during *Phase 1* in steady-state behavior. Obviously, we have  $d_K \leq d$ .

##### A. Single-Copy Forwarding Case

Assume that *node 1* received a packet from the source during time  $t_0 = 0$ .  $P\{|X_1(s) - X_{dest}(s)| \leq r_o \mid s\}$  is denoted as the probability of relay *node 1* at position  $X_1(s)$  being close enough to the destination node *dest* given that the time interval length is  $s$ , where  $r_o$  is the radius distance given by (1) so that successful delivery is possible. The time interval length  $s$  is the

delivery-delay random variable. Using the results from [6] it can be shown that

$$\begin{aligned} E_U [P\{|X_1(s) - X_{dest}(s)| \leq r_o \mid s\}] &= 1 - e^{-\lambda s}, \\ &\left(1 - \lambda e^{-\lambda} \int_0^s h_{X'}(t) dt \int_0^s e^{\lambda} \int_0^t h_{X'}(u) du h_{X'}(t) dt\right) \\ &= P\{S \leq s\} = F_S(s), \end{aligned} \quad (9)$$

where  $E_U[\cdot]$  means the ensemble average over all possible starting points which are uniformly distributed on the disk.  $F_S(s)$  can be interpreted as the cumulative density function of the delay random variable  $S$ . The function  $h_X(t)$  is the difference from the uniform distribution, such that  $h_X(0) = 0$  and  $|h_X(t)| < 1$  for all  $t$ , and  $X'$  is a point at distance  $r_o$  from the destination. The parameter  $\lambda$  is related to the mobility of the nodes in the disk and can be expressed by [6]

$$\lambda = \frac{2r_o v}{\pi R^2} = \frac{2r_o v}{1} = 2r_o v = \frac{1}{\Theta(n)}, \quad (10)$$

since, from (1), the radius  $r_o$  decreases with  $1/\sqrt{n}$ , as well as  $v$  must decrease with  $1/\sqrt{n}$  [4].

Now,  $h_X(t)$  has to be taken according to the random motion of the nodes [6]. If we consider the *uniform mobility model* [3], then  $h_X(t) = 0 \forall t \geq 0$ . Applying this result in (9) we have

$$E_U [P\{|X_1(s) - X_{dest}(s)| \leq r_o \mid s\}] = 1 - e^{-\lambda s} = F_S(s), \quad (11)$$

which has the probability density function (see Fig. 4):

$$f_S(s) = \frac{dF_S}{ds} = \begin{cases} \lambda e^{-\lambda s} & \text{for } 0 \leq s < \infty \\ 0 & \text{otherwise.} \end{cases} \quad (12)$$

Thus, the delay behaves exponentially with mean  $\frac{1}{\lambda}$  and variance  $\frac{1}{\lambda^2}$ . We conclude from (10), (11), and (12) that the average packet delay is  $\Theta(n)$  and its variance is  $\Theta(n^2)$ .

From (11) and (12), the delay value can last to infinity as a consequence of the tail of the exponential distribution even if the number of total nodes in the network  $n$  is finite.

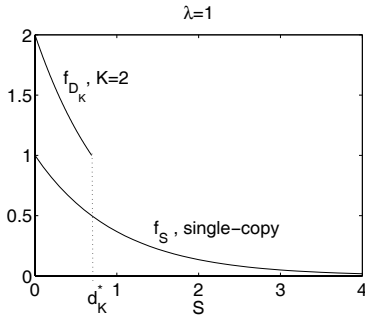


Fig. 4. Delay probability density functions for single-copy and  $K = 2$ , when  $\lambda = 1$ , for the network in steady-state. For single-copy the delay is not bounded as  $S$  can go to infinity.

### B. Multi-Copy Forwarding Case

Now consider that  $K$  copies of the same packet were successfully received by adjacent relaying nodes during *Phase 1* (where  $1 < K \ll n$ ). Let  $P_D(s)$  be the probability of having the first (and only) delivery of the packet at time interval length  $s$ . Hence, given that only one-copy delivery is enforced, and all  $K$  relays are looking for the destination, we have that

$$\begin{aligned} P_D(s) &= P\left\{\bigcup_{i=1}^K [|X_i(s) - X_{dest}(s)| \leq r_o \mid s]\right\} \\ &= \sum_{i=1}^K P\{|X_i(s) - X_{dest}(s)| \leq r_o \mid s\} \\ &\approx K \cdot P\{|X_1(s) - X_{dest}(s)| \leq r_o \mid s\}, \end{aligned} \quad (13)$$

since with the relay-destination handshake, at most one copy can be delivered, implying that the  $K$  relay-destination delivery events are mutually exclusive, and observing that the  $K$  relays are not uniformly spread in the disk right after *Phase 1*, but are close to each other (within  $r_o$ ), and after that, they need some time ( $t_{spread}$ ) to be uniformly spread, and this time interval is a function of the speed of the nodes  $v$ . However, as we show later,  $t_{spread}$  is negligible compared to the maximum delivery delay. Therefore, Eq. (13) follows given that node trajectories are iid.

From (11) and (13) and changing  $s$  by  $d_K$  to indicate the delay for  $K$ -copies forwarded during *Phase 1*, we have for the *uniform mobility model*,

$$\begin{aligned} E_U [P_D(s)] &= E_U \left[ P\left\{ \bigcup_{i=1}^K [|X_i(s) - X_{dest}(s)| \leq r_o \mid s = d_K] \right\} \right] \\ &= P\{D_K \leq d_K\} = F_{D_K}(d_K) \approx K \left(1 - e^{-\lambda d_K}\right), \end{aligned} \quad (14)$$

for a uniform steady-state distribution resulting from the random motion of the nodes.  $F_{D_K}(d_K)$  can be interpreted as the cumulative density function of the delay random variable  $D_K$  for  $K$  relays copies transmission at *Phase 1*.

From (14), the maximum value attained by  $D_K$  is

$$F_{D_K}(d_K^{max}) = 1 \approx K(1 - e^{-\lambda d_K^{max}}) \implies d_K^{max} \approx \frac{1}{\lambda} \log\left(\frac{K}{K-1}\right). \quad (15)$$

Eq. (15) reveals that, for a finite  $n$ , the new delay obtained by multi-copy forwarding is bounded by  $d_K^{max}$  after ensemble averaging over all possible starting points topology uniformly distributed on the disk.

As mentioned above, the exact bounded value must also include the time interval  $t_{spread}$  necessary to have all  $K$  nodes uniformly spread in the disk after *Phase 1*. Because the nodes move with speed  $v = \Theta(\frac{1}{\sqrt{n}})$ , then  $t_{spread} = \Theta(\sqrt{n})$ . Now, from (10) and (15), and since  $K \ll n$ , we have that  $d_K^{max} = \Theta(n)$ . Therefore,  $t_{spread} \ll d_K^{max}$ .

Also, from (10) and (15), since  $K \ll n$ ,  $d_K^{max}$  grows to infinity and no bounded delay is guaranteed if  $n$  scales to infinity.

The probability density function for  $D_K$  is

$$f_{D_K}(d_K) = \frac{dF_{D_K}}{dd_K} \approx \begin{cases} K\lambda e^{-\lambda d_K} & \text{for } 0 \leq d_K \leq d_K^{max} \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

Hence, in the multi-copy forwarding scheme the tail of the exponential delay distribution is cut off (see Fig. 4). The average delay for  $K$ -copies forwarding is then given by

$$E[D_K] = \int_0^{d_K^{max}} d_K f_{D_K}(d_K) dd_K \approx \frac{1}{\lambda} \left[1 - \log\left(\frac{K}{K-1}\right)^{K-1}\right], \quad (17)$$

and the delay variance is  $Var[D_K] \approx \frac{1}{\lambda^2} \{1 - K(K-1)[\log(\frac{K}{K-1})]^2\}$ .

Since  $K \ll n$ , we conclude that the average delay and variance for any  $K$  are fractions of  $\frac{1}{\lambda}$  and  $\frac{1}{\lambda^2}$ , respectively, and they also scale like  $\Theta(n)$  and  $\Theta(n^2)$ . Nevertheless, the number of nodes does not scale to infinity in real MANETs, and for a fixed  $n$  we can obtain significant average and variance delay reductions for small values of  $K$  compared to the single-copy relay scheme. For example, if  $K = 2$  a reduction of more than 69% over the average delay is obtained (i.e., for single-copy Mean =  $\frac{1}{\lambda}$ , for multi-copy ( $K = 2$ ) Mean =  $\frac{0.307}{\lambda}$ ). We also observe that the mean and variance values decrease when  $K$  increases.

### C. Relationship between Delays

We showed that the throughput of our multi-copy scheme is the same order as the one-copy scheme [2]. This capacity is proportional to the probability of a packet reaching the destination. Hence, because only one copy of the packet is actually delivered to the destination for single-copy or multi-copy, their total probabilities can be approximated at their respective delivery time, i.e.,

$$P\left\{\bigcup_{i=1}^K [|X_i(s) - X_{dest}(s)| \leq r_o] | s = d_K\right\} \approx P\{|X_1(s) - X_{dest}(s)| \leq r_o | s = d\}, \quad (18)$$

and so their ensemble averages are

$$E_U[P\left\{\bigcup_{i=1}^K [|X_i(s) - X_{dest}(s)| \leq r_o] | s = d_K\right\}] \approx E_U[P\{|X_1(s) - X_{dest}(s)| \leq r_o | s = d\}], \quad (19)$$

whose solution must be obtained by substituting (9) (for  $s = d_K$  and  $s = d$  respectively) on both sides of (19) and solving for  $d_K$  for the particular model of random motion of nodes. For a steady-state uniform distribution for the motion of the nodes, a simplified solution is obtained by substituting (11) and (14) in (19) and solving for  $d_K$  we have

$$d_K \approx \frac{1}{\lambda} \log\left(\frac{K}{K-1+e^{-\lambda d}}\right). \quad (20)$$

This last equation reveals very interesting properties for the strategy of transmitting multiple copies of a packet during *Phase I*. If  $K = 1$ , then obviously  $d_K = d$ . If we let  $d \rightarrow \infty$ ,  $n$  be finite, and because  $K \ll n$ , then we have

$$d_K^{max} \approx \lim_{d \rightarrow \infty} \frac{1}{\lambda} \log\left(\frac{K}{K-1+e^{-\lambda d}}\right) = \frac{1}{\lambda} \log\left(\frac{K}{K-1}\right) \stackrel{K \geq 2}{\geq} 1 \text{ constant}. \quad (21)$$

Therefore, if  $K$  is strictly greater than one, then the delay obtained in the multi-copy relay scheme is bounded for a finite number of nodes  $n$ , even when the single-copy relay scheme in [2] incurs infinite delays. This is the same asymptotic value already predicted by (15). The time-to-live threshold must be set greater than the worst asymptotic delay ( $K = 2$ ) to allow the packet to be delivered, i.e.,  $d_2^{max} = \frac{\log(2)}{\lambda} < TTL$ .

Fig. 5 shows curves for (20), in which  $\lambda = 0.01$ . The case of single-copy is also plotted. In all cases, except single-copy, the delay  $d_K$  tends to a constant value as  $d$  increases. Hence, for a finite  $n$ , the multi-copy relay scheme can reduce a delay of hours in the single-copy relay scheme to a few minutes or even a few seconds, depending on the network parameter values.

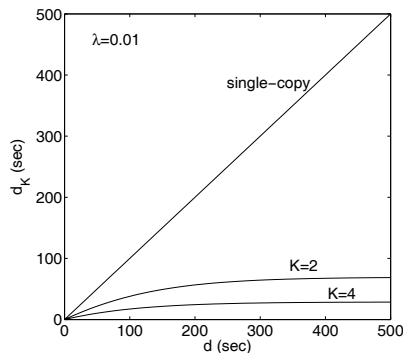


Fig. 5. Relationship between delays  $d_K$  and  $d$  for single-copy,  $K = 2$ , and  $K = 4$ , for a uniform distribution resulting from the random motion of the nodes for the network in steady-state.

### D. Simulation Results

In our simulations we implemented the simplified version of the *random waypoint mobility model* [7] for the random motion of the nodes, using the *BonnMotion* simulator [8]. No pause was used and  $v_{min} = v_{max} = v$  (as it resembles the *uniform mobility model* [3]). Fig. 6 shows the results for 1000 seconds of simulations for  $n = 1000$  nodes,  $v = 0.13m/s$ ,  $r_o = 0.02m$ ,  $K = 2$ , and a unit area disk as the simulation area, which results  $\lambda = 0.0052$ . The theoretical curve from Eq. (20) is also plotted.

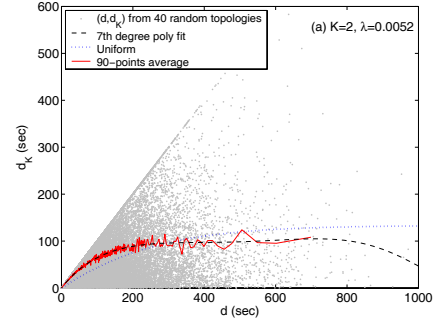


Fig. 6. Simulation results for the *random waypoint mobility model*. Each grey point is a pair  $(d, d_K)$  delay measured for 40 random topologies all plotted together. A 7th degree polynomial fit for all the points and a 90 consecutive points average are plotted for  $K = 2$ . The theoretical curve for the steady-state uniform distribution is also plotted.

## VI. CONCLUSIONS

We have analyzed delay issues for two packet forwarding strategies, namely, the one-copy two-phase scheme advocated by Grossglauser and Tse [2], and a multi-copy two-phase technique. We found that in both schemes the average delay and variance scale like  $\Theta(n)$  and  $\Theta(n^2)$  for  $n$  total users in a mobile wireless ad hoc network. In the case of multi-copies transmission the *multiuser diversity* strategy is preserved by allowing one-time relaying of packets and by delivering only the copy of the packet carried by the node that first reaches the destination close enough so that it successfully delivers the packet. We also show that our technique does not change the order of the magnitude of the throughput capacity in the MANET, while a bounded delay can be guaranteed for finite  $n$ .

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