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Fast Pulsars, Variational Bound, Other Facets of Compact Stars

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Abstract

We derive a limit on the rotation period of a gravitationally bound star that is analogous to Ruffini's mass limit for a neutron star. We discuss the impact of gravitational radiation-reaction instabilities. The condition that self-bound stars can rotate faster is derived. The present status of searches for fast pulsars is reviewed. The composition of neutron stars is discussed in the context of nuclear and hypernuclear constraints. The phase transition from hadron to quark phase in the interior of dense neutron stars is discussed properly accounting for the fact that this first order phase transition involves two conserved charges. Hybrid stars are then described.

1 Introduction

My particular approach to the question of whether strange stars can be identified by pulsar periods is strongly influenced by the following consideration. We know from many aspects of nuclear physics that nuclei are composed of nucleons, not quark matter. Therefore we know that non-strange quark matter lies above the energy per nucleon of nuclear matter. The available energy scale is that of QCD, say 100 MeV. On the other hand, because the energy per nucleon of 3-flavor quark matter at the same baryon density is lower than 2-flavor by the factor $\sim (2/3)^{1/3}$, then strange matter lies below 2-flavor quark matter by about 100 MeV, ie. in the vicinity of the nuclear matter energy per nucleon. Whether this places it above the nucleon mass, or below the energy per nucleon in iron ($\sim$ 930 MeV) is a one percent question! Therefore we cannot look to models of confinement, indeed not even lattice QCD, to answer definitive questions about the difference between strange and neutron stars, or whether indeed the former exist. While it is certainly interesting to explore the predictions of models, they need not portray accurately either the limitations or possibilities for strange stars. Therefore so far as is possible, in trying to distinguish how strange stars and neutron stars can have different limiting rotational frequencies, I will rely only on model independent estimates and minimal constraints.

I will derive a lower bound on the period of neutron stars using minimal constraints and will discuss also the effects of gravitation radiation-reaction instabilities on this bound. Then I will derive the simple condition that strange matter would have to satisfy so that a strange star could rotate with a shorter period. I will give an observational update on fast pulsars, and the sensitivity of present searches. Finally, in a different vein, I will discuss two topics of relevance to the structure and composition of neutron stars which are also relevant to the burning of neutron stars to strange stars, if it is true that strange matter is the absolute ground state of the strong interaction.

2 Rotational Limit on Gravitationally Bound Stars

I derive a limit on the rate of rotation of neutron stars, or more generally, gravitationally bound stars, that is analogous to Ruffini's limit on the maximum mass that a neutron star can have. For this purpose I adopt the following set of minimal constrains: (1) Einstein's equations, (2) causality, (3) the equation of state matches that of Harrison and Wheeler for the low density region, (4) the limiting mass is at least $1.44 M_{\odot}$, so as to be consistent with PSR 1913+16. I vary the equation of state to minimize the Kepler rotation period, using a flexible parameterization of the equation of state described in more detail in ref. [1]. The results are summarized in Fig. 1 where the minimum possible central energy density as a function of pulsar period is shown. A very conservative adjustment of the relativistic Kepler period is included to account for the increase in minimum stable period due to gravity wave instabilities, as $P = 1.1 P_K$. We learn three facts: (1) pulsars with periods greater than 1 ms need have central densities only three times nuclear. Such a star can quite conceivably be a neutron star. (2) Small periods in the sub-millisecond region require that the star's central density be very large, so large at periods below 0.5 ms that it becomes doubtful that nucleons can exist as individual entities. (3) No gravitationally bound star can have a period below about 0.42 ms.

As we shall see in the next section, gravity wave instabilities, accounted for at their typical impact, actually increases the minimum stable period to about 30 percent above Kepler, thus increasing the above bound. Thus very conservatively, an observed period below 0.4 ms would signal that the star cannot be a neutron star.

3 Gravity Wave Instabilities

The Kepler frequency, above which centrifuge overwhelms gravity at the equator of a rotating star, provides only an absolute upper bound on frequency. There is another instability that sets in at lower frequency which therefore provides a more stringent and realistic limit [3]. It originates in counter-rotating surface vibrational modes, which at sufficiently high rotational frequency of the star are dragged forward. In this case, gravitational radiation which inevitably must accompany the aspherical transport of matter, does not damp the modes, but rather drives them [4, 5]. Viscosity plays the important role of damping such gravitational-wave driven instabilities at a sufficiently reduced frequency such that the viscous damping rate and power in gravity waves are comparable [6]. We have found recently that these viscosity modified GR instabilities may set in at a significantly small fraction (60-70%) of the Kepler frequency and therefore set a more realistic upper bound than the latter [7, 8]. In Fig. 2 we show the minimum stable period for the family of stars belonging to a typical equation of state, at two temperatures, corresponding to young hot stars and old cold ones. For reference, the Kepler period for the star at the mass limit is 0.63 ms. Since the fastest rotation will usually be set in the early history of the star, we see that

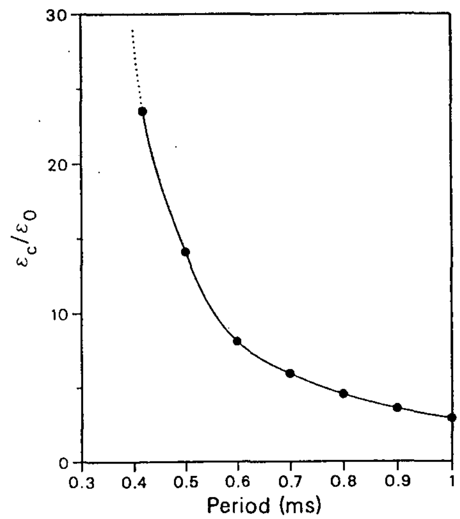

Figure 1: Minimum possible central density of 'neutron' star as a function of limiting stable rotation period.

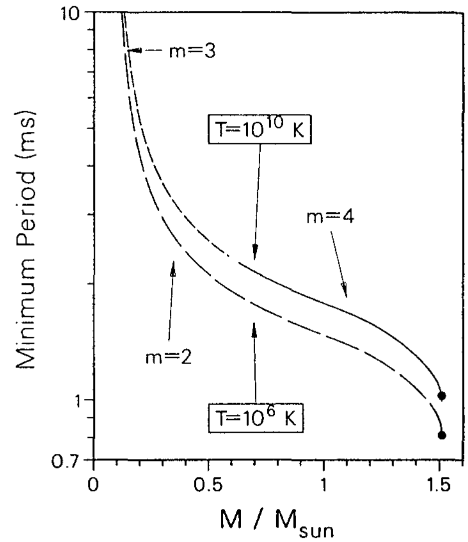

Figure 2: Minimum stable period against gravitational radiation-reaction for hot and cold stars of a typical family of stars [2]

the limit imposed by gravity waves, is considerably more stringent than the Kepler mass-shedding period. Nevertheless we use the very conservative estimate quoted in the previous section in constructing the limiting rotation period of a neutron star in Fig. 1.

4 Limiting Period for Self-Bound Stars

In keeping with our approach of using minimal constraints and model independent estimates, to discuss the rotation of strange stars, or in general self-bound stars, we use no specific model. To do so would allow us to make a statement only about the model. Instead we embed the hypothesis of self-binding within the equation of state in the form, $\epsilon = p/v^2 + \epsilon_n$, where in general v can depend on p and where ϵ_n is the equilibrium energy density, or 'normal' density of the self-bound medium. For small objects, for which gravity is unimportant, the mass is given (for a sphere) by $M = (\frac{4}{3}\pi R^3)\epsilon_n$ so that, as is well known, $R \sim M^{1/3}$, just as for a nucleus. This behavior is entirely different than for a gravitationally bound star where a small mass leads to a large star. Equating centrifuge and gravity, and using the above relation between radius and mass we get, $\Omega_K = \alpha\sqrt{M/R^3} = \alpha\sqrt{4\pi\epsilon_n/3}$, where $\alpha \sim 0.65$ is an empirical factor that takes into account the effects of frame dragging, a phenomenon unique to general relativity. From this we can say that if the equilibrium density of self-bound matter, expressed in terms of the density of normal nuclear matter, ϵ_0, satisfies the inequality

$$\epsilon_n > 1.3\epsilon_0(\text{ms}/P)^2 \tag{1}$$

then the entire family of stars of mass up to the limiting mass can have periods as small as P. The radius mass relation is illustrated in Fig. 3 for several star families.

The dashed lines represents loci such that a star lying below one of them can have period as small as that which labels the locus.

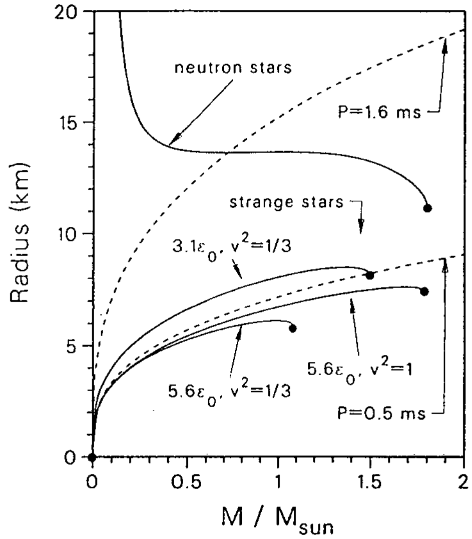

Figure 3: Radius mass relation for typical neutron star and for strange stars having different values of sound speed or equilibrium density.

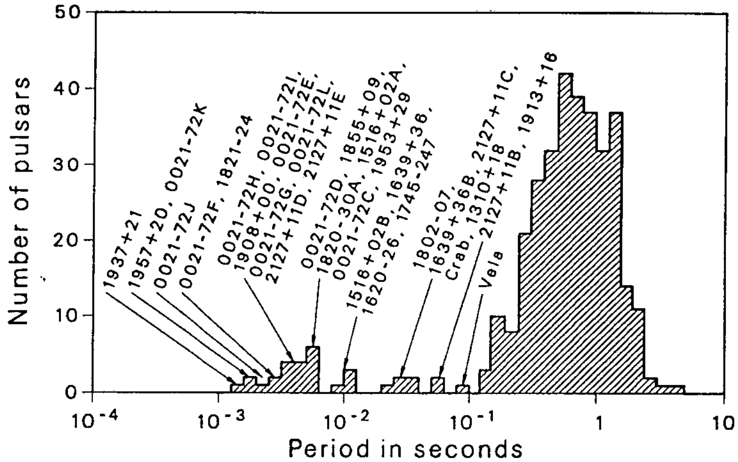

Figure 4: Distribution of pulsar periods. There is a relatively strong attenuation in sensitivity of radio pulsar surveys for periods below about 1 ms.

5 Update on Millisecond Pulsar Discoveries

The first millisecond pulsar was discovered in 1982 [9] and in the next seven years about one a year has been found. The situation has changed radically with the recent discovery of an anomalously large population of millisecond pulsars in globular clusters. Ten have been found during the last year within the cluster 47 TUC [10].

Although some astronomers have optical detectors that are sensitive to extra terrestrial submillisecond pulses, all known millisecond pulsars are seen only at radio frequencies. There are inherent biases in sensitivity of radio astronomy techniques against detection of pulsars with very short periods. These arise because of the different time delay introduced by the interstellar plasma into the different frequencies contained in the emitted pulse and falling within the bandwidth of the radio telescope. Correction for such dispersion is usually done by guessing the column height of plasma to the as yet undiscovered pulsar and Fourier analysing the result for the presence of a periodicity. This is repeated for various column heights until a periodic signal is detected, or until wisdom dictates that one look elsewhere for a pulsar. Thus a bias against detection of very short period signals is introduced. At the present time sensitivity declines sharply below 1 ms [11]. The recent rash of discoveries of millisecond pulsars in globular clusters promises to change the rules of the game however, and we expect sensitivity to pulsars below this period if they exist in near future searches. The reason is that once one or more fast pulsars have been discovered within a given cluster, the dispersion effects can be accurately calibrated, and used to achieve greater sensitivity in subsequent searches within that cluster. It has been realized recently that globular clusters, where the density of stars is $\sim$ 1000 times that in the field of the galaxy, are very favorable environments for the formation (and destruction) of binary systems due to (astronomically) frequent two and three star collisions. Consequently they appear to be the residence of an anomalous population of old cold neutron stars that have been spun up to the millisecond region by accretion from their companions [12].

The distribution of pulsar periods is shown in Fig. 4. Pulsars right down to the roll-off in sensitivity at 1 ms have been found. It could of course be a coincidence that the roll-off in sensitivity of past surveys coincides with the period of the fastest existing pulsars. If not, and pulsars with shorter periods exist, the globular cluster discoveries may soon reveal submillisecond pulsars. As discussed above, on a very conservative basis, we can say that a pulsar with period below $\sim$ 0.4 ms cannot be a neutron star, but can be a self-bound star with equilibrium density satisfying the inequality eq. 1. Taking account of gravity wave instabilities, we doubt that neutron stars can have stable periods below about 1 ms.

6 Composition of Neutron Stars related to Hypernuclei

In numerous discussions of neutron stars, they are treated as if they were pure in neutron, or had only a small admixture of protons and corresponding electrons to make them charge neutral. Such approximations have been made even in topics of relevance to this conference, like the burning of a neutron star to strange star. It is of interest therefore to the subject of strange stars, as well as the subject of neutron stars if strange matter is not the absolute ground state, to study the composition of compact stars with constraints imposed from nuclear and hypernuclear physics [13, 2]. The nuclear properties used to fix the coupling constants of nucleons to mesons in nuclear field theory are the saturation binding $B/A = \epsilon_0/\rho_0 - m = 16$ MeV and density, $\rho_0 = 0.153\ \mathrm{fm}^{-3}$, compression modulus $K = 240 - 300$ MeV, symmetry energy coefficient $a_{sym} = 32$ MeV, and nucleon effective mass at saturation, $m^\star/m = 0.7 - 0.78$. The remaining unknowns not fixed by the above properties are the couplings of the hyperons to the meson fields. They are expressed as a ratio to

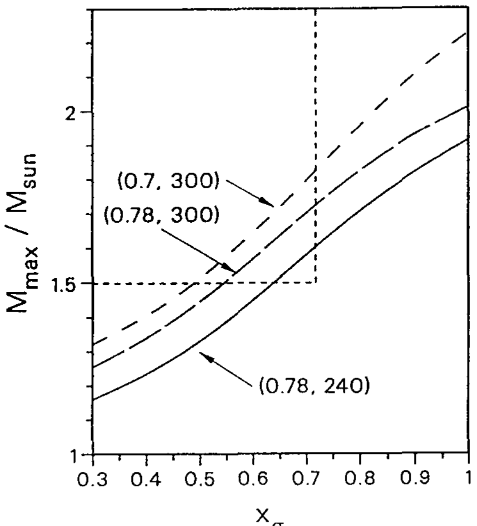

Figure 5: Maximum neutron star mass as function of hyperon scalar coupling with corresponding x_ω chosen so as to yield correct Lambda binding in nuclear matter. Numbers in parentheses are m^*/m and K in MeV.

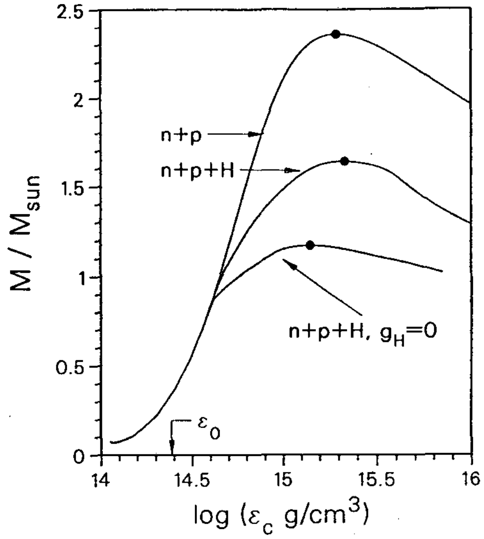

Figure 6: Sequences of neutron stars for the three cases discussed in the text.

the above mentioned nucleon couplings,

$$x_\sigma = g_{H\sigma}/g_\sigma, \quad x_\omega = g_{H\omega}/g_\omega, \quad x_\rho = g_{H\rho}/g_\rho. \tag{2}$$

For their determination the hypernuclear property that is unambiguous is the inferred binding energy of the Λ hyperon in saturated nuclear matter of -28 MeV [14]. We derive now an expression for this binding in our model. From the Weisskopf [15] relation between the Fermi energy and the energy per nucleon of a self-bound system at saturation, $e_F = (\epsilon/\rho)_0$, which is a special case of the Hugenholtz-Van Hove theorem [16], we obtain for the binding energy of the lowest Λ level in nuclear matter,

$$\left(\frac{B}{A}\right)_\Lambda = x_\omega V - x_\sigma S, \tag{3}$$

where $S = g_\sigma \sigma$, $V = g_\omega \omega_0$ are the values of the scalar and vector field strengths at saturation. The Fermi energy of the lowest Lambda level is the $k = 0$ value of the Dirac eigenvalue of the theory, $e_\Lambda(k) = g_{\omega\Lambda}\omega_0 + (k^2 + m_\Lambda^{\star 2})^{1/2}$. Eq. (3) yields a continuous ambiguity in the *pair* of values x_σ, x_ω each pair of which yield the same Lambda binding of -28 MeV. Combined with neutron star masses, the ambiguity is bounded from below by $M_{max} \sim 1.5 M_\odot$. Combined with the reasonable assumption that the hyperon coupling constants are less than those of the nucleon, based both on the observation that the lowest s-state nucleon is bound by approximately twice as much as the Lambda hyperon, and also on the basis of quark counting [17], the ambiguity is bounded from above to be less than $x_\sigma < 0.9$ (which corresponds to $x_\omega < 1$). It is even more stringently bounded from above if the value of x_σ from

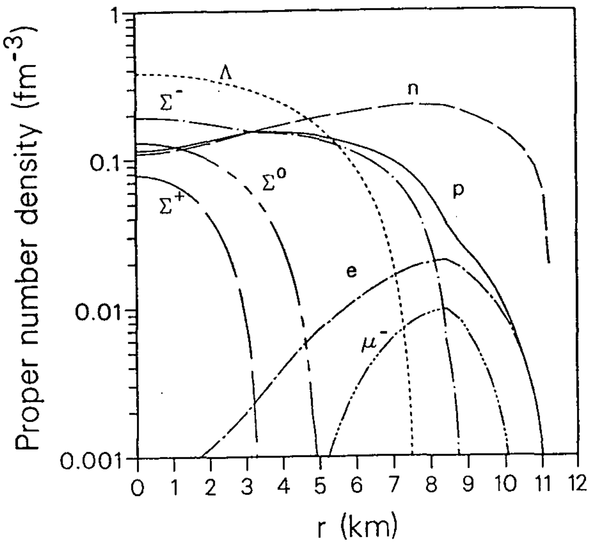

Figure 7: Composition of the maximum mass star for the case discussed in the text.

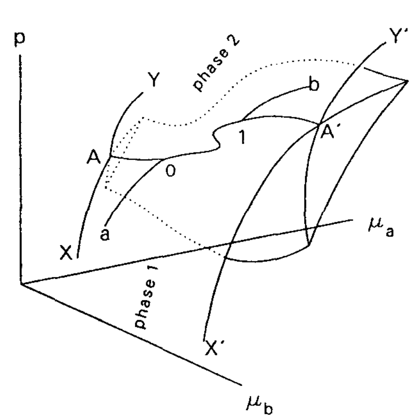

Figure 8: Schematic of pressure as a function of μ_a, μ_b. The physical pressure is the curve a0 (pure phase 1), 01 (mixed phase with concentration χ between 0 and 1), and 1b (pure phase 2). Dotted curves lie below foreground pressure surfaces.

the fit to hypernuclear levels is accepted, uncertain as it is. It was found that $x_\sigma = 0.46 \pm 0.26$, $x_\omega = 0.48 \pm .32$ so that we may take $x_\sigma < 0.72$ [18].

The results of the analysis are shown in Fig. 5. We show several curves of maximum neutron star mass as a function of the ratio of the hyperon to nucleon coupling to the scalar meson, corresponding to different values of K and $m^\star$. This is because of some ambiguity in the empirical values of K, which is taken to lie in the range $240 - 300$ MeV [19, 20, 21, 22], and the effective (Dirac) nucleon mass at saturation density, $m^\star_{sat}/m$, which is taken in the range $0.7 - 0.78$, corresponding to the empirical non-relativistic effective mass in the range $0.74 - 0.83$, which to good approximation [23] has been identified as the Landau effective mass [24]. The correspondence is through the expression $m^\star_{Landau} = [k/(\partial e(k)/\partial k)]_{k_F} = (k_F^2 + {m^\star_{sat}}^2)^{1/2}$. The three curves span the range of uncertainty in these parameters. For each value of of x_σ, the value of x_ω is chosen in each case to yield the Lambda hyperon binding in saturated nuclear matter of -28 MeV. Arbitrarily we set $x_\rho = x_\sigma$. This choice is not a sensitive one, since an alternative choice $x_\rho = x_\omega$ yields essentially the same results. The acceptable ranges of M and x_σ, as discussed above, lie in the boxed area in the upper left of the figure. For the hyperon to nucleon scalar coupling x_σ we find a minimum allowable value of ~ 0.5 from the lower bound on the maximum neutron star mass that is *also* consistent with the Lambda binding in nuclear matter, while hypernuclear levels yield a somewhat uncertain upper bound of ~ 0.72.

Choosing $K = 300$ MeV, $m^\star/m = 0.7$, which we consider to be the best empirical values [19, 20, 21, 22, 23], we show in Fig. 6 the sequence of stars obtained under three different circumstances: (1) Hyperons are neglected; (2) they are taken into account with the coupling $x_\sigma = 0.6$ which falls in the middle of the range discussed above and all particle species are in equilibrium; (3) hyperons are introduced as free baryons, interacting only through the weak interaction so that the system is in equilibrium. For case (2) we show the populations in the maximum mass star in Fig. 7. Integrated over the star the baryon population is 59% neutrons, 17% protons and 24% hyperons. Thus both protons and hyperons are more populous in neutron stars than in the early estimates of a decade ago. The main factors responsible are the symmetry energy and the fact that charge neutrality can be achieved with the presence of hyperons almost to the exclusion of electrons. In this connection note that hyperons carry the conserved baryon charge whereas electrons are present only to the extent that charge neutrality is not otherwise achieved.

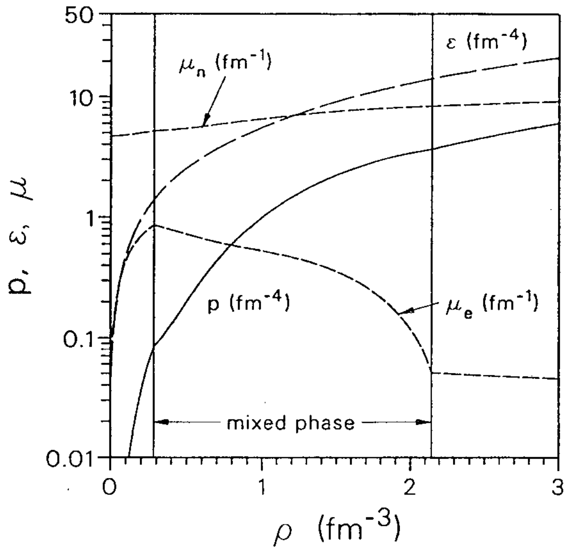

Figure 9: Pressure, energy density and chemical potentials as function of baryon density when there is more than one conserved charge.

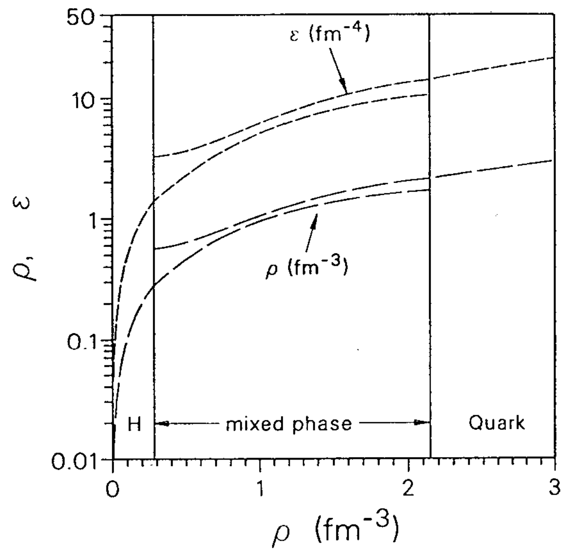

Figure 10: Baryon density and energy density of each phase as a function of average baryon density.

7 Hybrid Stars

Here we discuss the case of compact stars if strange matter is not the absolute ground state but can exist in phase equilibrium with the confined hadronic phase at the elevated pressures found in neutron stars. A star has two conserved charges, baryon and electric. A first order phase transition is in this case quite different than in a simple substance such as the well known gas-liquid transition which has only one conserved attribute. Most importantly, the pressure is not a constant while the body completes the transition from one phase to the other [25] as it is in a simple substance. This has quite important consequences in the presence of an external field, and therefore for the structure of a neutron star as we shall soon see. The reason that the pressure is not a constant at all concentrations of the two phases in the mixed phase is because phase equilibrium is not a point in the pressure chemical-potential plane as it is for a simple substance; instead the pressure is a plane in the two chemical potentials, and phase equilibrium occurs along the intersection of the two parts of the plane that correspond to the two pure phases respectively. This is illustrated schematically in Fig. 8. For a first order phase transition in a neutron star in beta equilibrium (or any other body containing two conserved charges) Gibbs condition of phase equilibrium is

$$p_H(\mu_n, \mu_e, T) = p_Q(\mu_n, \mu_e, T), \tag{4}$$

which as just remarked describes a curve of intersection in the three dimensional space of p, μ_n, μ_e. When the phases are in equilibrium the body consists partly of one phase and partly the other. Let χ lying in the range (0,1) parameterize the proportion of total volume occupied by the quark phase. Charge neutrality in this mixed phase is enforced by

$$(1-\chi)q_H(\mu_n, \mu_e, T) + \chi q_Q(\mu_n, \mu_e, T) + q_l(\mu_e, T) = 0, \tag{5}$$

where q_H, q_Q, q_l are the electric charge densities of the confined hadronic phase, H, quark phase, Q, and of the leptons, $l = e^-, \mu^-$, respectively. The above equation can be thought of as defining μ_e as a particular function, $\mu_e = f(\mu_n, \chi, T)$ of μ_n. When μ_e is determined by this equation then the curve that eqs. (4) describes is the phase equilibrium curve along which the density (controlled by μ_n) and pressure (a function of μ_n and μ_e) vary while the charge remains zero. Note however that eq. (5) couples the hadronic and quark phases for χ lying between the extremes of its range, so that the equations of motion of *both* phases must be solved *simultaneously* with eq. (4) and (5) in the mixed phase. Therefore the solution, meaning the field variables, chemical potentials, pressure and densities, vary as the concentration! This is unlike phase transitions with only one conserved quantity (eg. H_2O in the gas-liquid transition), where the pressure, chemical potentials, and densities of both phases in equilibrium remain constant until the entire body converts from one phase to the other. The energy density and baryon density in the mixed phase are the same linear combinations of the two phases as the charge,

$$\epsilon = (1-\chi)\epsilon_H(\mu_n, \mu_e, T) + \chi\epsilon_Q(\mu_n, \mu_e, T) + \epsilon_l(\mu_e, T) \tag{6}$$

but because of eq. (5), μ_e depends on χ so that the energy and density are not linear functions of the concentration as in a simple substance. Phase equilibrium requires that only the pressure, chemical potentials and temperature are the same in the two phases in contact. Therefore the characteristic discontinuity in density across the interface of the two phases in equilibrium exists

$$\Delta(\chi) = \rho_H(\mu_n, \mu_e, T) - \rho_Q(\mu_n, \mu_e, T), \tag{7}$$

but, in the general case, it too varies as the concentration, whereas in a simple substance it remains a constant. In complex substances, ones with more than one conserved charge, the discontinuity tends to be small because the density of each phase in equilibrium varies as the concentration so that the discontinuity does not amount to the difference in densities at the extremes of the mixed phase, as in the case of a simple substance [25]. For this reason, although the mixed phase will undoubtedly develop geometrical structure, just as is expected of the sub-nuclear liquid-vapor transition [26, 27], the energy associated with it is likely to be small in comparison with the volume energy [25]. Therefore we do not enter into a discussion of it here, since it cannot much effect the global properties on which we concentrate. Parenthetically we remark that the geometrical structure of the mixed phase may be very important for transport phenomena.

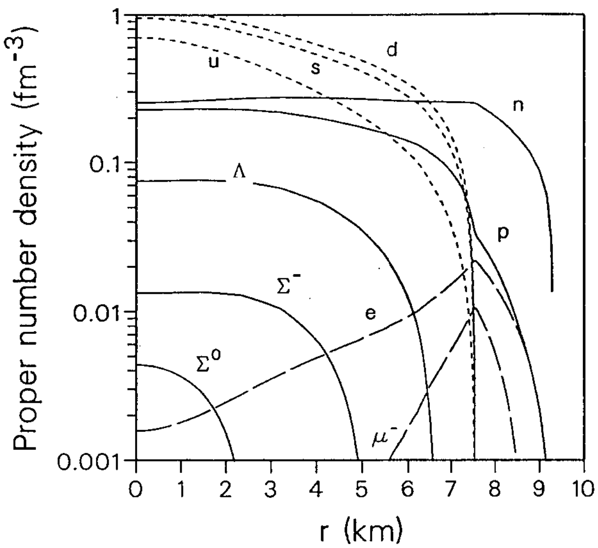

Figure 11: Composition of a hybrid star at the mass limit. Exterior to about 7.6 km is neutron star matter. Interior to this is a mixed phase of confined hadronic matter and quark matter which is overall charge neutral.

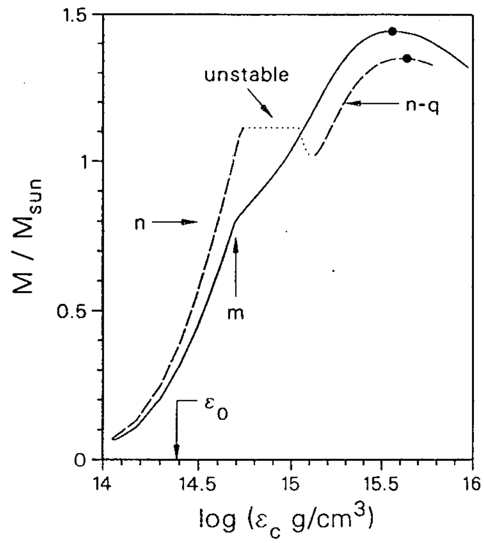

Figure 12: As a function of central density the dashed line interrupted by dots is the family of stars for idealized case having one chemical potential. Regions are; n - pure neutron stars; n-q - quark core and neutron matter mantle stars. Solid curve is family of compact stars in beta equilibrium (two chemical potentials); stars below 'm' are made of neutron star matter; those above have mixed phase cores.

It remains to define what we take to be the description of quark matter. We follow Farhi and Jaffe [28], and choose for simplicity in the present case to set $\alpha_c = 0$.

In Fig. 9 the behavior of the pressure, energy density, and chemical potentials is shown. The monotonic increase of pressure in the mixed phase is in sharp contrast with the behavior in the description involving a single chemical potential, like the well-known example of the gas-liquid transition. It is interesting also to compare the density of hadronic and quark matter in the mixed phase in the two cases. In the present instance this is shown in Fig. 10. The densities are different in the two phases

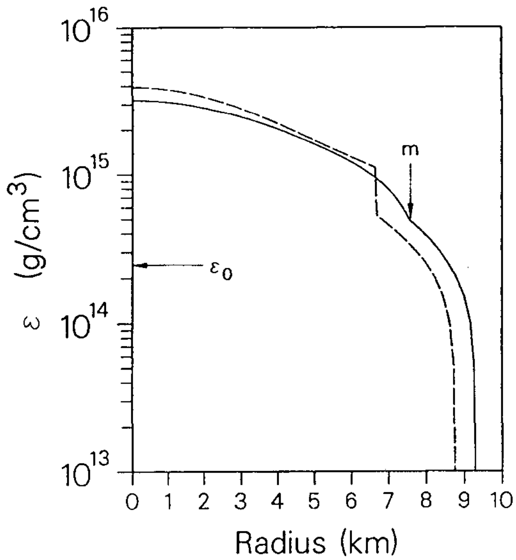

Figure 13: Dashed line is profile of an idealized star with one chemical potential at mass limit of previous figure. Core is pure quark matter and exterior is pure neutron matter. Solid curve is star at mass limit which is in beta equilibrium. The core is in the mixed phase of hadronic and quark matter. The exterior is neutron star matter. Dividing point marked by 'm'.

in equilibrium, but only by a few percent except at the low density end of the mixed phase, and they vary with the concentration. In contrast, when only one charge is conserved the density of each phase is a constant for all concentrations and the density of the quark phase is about a factor two larger than the confined hadronic phase. In a word, a first order phase transition in a system with more than one conserved charge is *smooth* compared to one in which there is a single charge. The populations in the star at the mass limit are shown as a function of Schwarzschild radius in Fig. 11. The entire core of the star out to 7.5 km is in the mixed phase. Populations of quarks are very high within 5 km. The quark matter phase is seen to be negatively charged, while the hadronic matter phase is positively charged. Their charges are almost equal and opposite through most of the star, the lepton populations which complete the charge neutrality being very small.

The masses of stars in two cases is contrasted in Fig. 12. In the one case the phase transition is treated as described here (solid curve). In the other, it is treated as if it were governed by a single chemical potential, that for baryons, as in the case that beta equilibrium is ignored (pure neutron star), or in the case that both phases H, Q in equilibrium are constrained to be individually charge neutral (introduces a discontinuity in μ_e across the phase boundary). Unlike the situation where the star is artificially treated as having a single conserved charge with a consequent constant pressure in the mixed phase and therefore an absence of stable stars with central densities falling in the range of the mixed phase, in the case with two conserved charges, as corresponds to a star in beta equilibrium, there is no unstable range until the normal Oppenheimer mass limit is reached. The density profile of the two cases is compared in Fig. 13.

We have solved both models in the limit of infinite matter. Thus we assume that all significant regions of space occupied by either phase are large in the sense that the volume energy is large compared to the surface energy. We have also neglected the Coulomb rearrangement energy associated with the non-uniform distribution of charge in the mixed phase. In all likelihood geometrical structures of the two phases in equilibrium similar to that discussed for sub-saturation nuclear matter will occur so as to minimize the sum of all these energies. We have neglected the rearrangement energy associated with this structure, though it is clearly an interesting area of investigation. Note that the volume energy against which its importance is to be assessed can be read from Fig. 10 and is seen to be ~ 1 GeV/fm^3. We also note that the difference in the volume energies of the two phases of the mixed phase are nearly the same, so we expect the surface energy to be small. Further, it is a general result that the Coulomb rearrangement energy is half the surface energy and is therefore also small [26]. Therefore although the geometric texture of the mixed phase is likely to be present, the energy associated with creating it is likely to be small. Therefore our approximation of using only the bulk energy to compute the star structure is valid.

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