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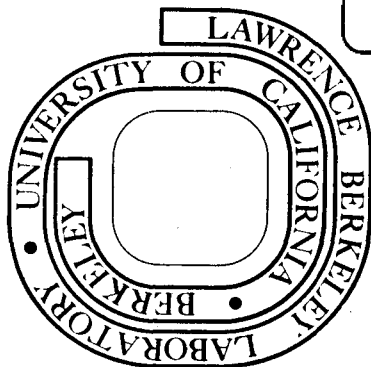
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RAPID MEASUREMENT OF THE EFFECTIVE FIELD
BOUNDARY OF HOMOGENEOUS FIELD MAGNETS[†]

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March 1974

Abstract

It is often sufficient to express the beam optical properties of the fringe field of a magnet by describing its effective field boundary. It is shown how the effective field boundary can be measured very rapidly and accurately.

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1. Introduction

For many beam optical considerations it is convenient to use the concept of the effective field boundary (EFB). This note shows how one can measure very easily the EFB and another quantity closely related to the EFB. We will first define the EFB and discuss some of its properties for the case of a homogeneous field magnet. Then, after describing the proposed measurement technique, the basic concepts will be generalized to also include magnets with non-uniform fields and it will be shown how the equivalent measurements can be made in non-uniform field magnets.

2. Definition and Properties of the EFB of a Uniform Field Magnet

Figure 1 shows schematically the end of a uniform field magnet and the coordinate system used to define the EFB. Its z-coordinate can be expressed through

$$z_Y(x,y) = \left(\int_0^z B_Y(x,y,z) dz \right) / B_0, \quad (1a)$$

with $B_Y(x,y,z)$ describing the y component of the field in the magnet and its fringe field region, and B_0 representing the uniform field level inside the magnet. It is assumed that z_1 is large enough to be in the field free region, and that $z = 0$ is in the uniform field region of the magnet. It has to be pointed out that the requirement of the existence of a region where the field is independent of z is essential to the basic concept of the EFB, and not only to the definition given in eq. (1a). If the field depends slightly on z , one could use as a compensation coil (its function is described below) one

that averages over an appropriate distance in the z direction. But since the concepts do get a little fuzzy in that case, it will not be discussed in detail. Finally, it has to be noted that although we follow the general practice and use the term "the" EFB, "the" EFB is not uniquely defined since its location and shape do depend on the orientation of the z -axis.

It will be useful to use also an alternate expression for z_y : If z_o describes any location that is in the homogeneous field part of the magnet, one can express z_y by

$$z_y(x,y) = z_o + \left(\int_{z_o}^{z_1} B_y(x,y,z) dz \right) / B_o. \quad (1b)$$

A very important property of $z_y(x,y)$ is the easily verifiable fact that $z_y(x,y)$ satisfies the two dimensional Laplace equation. It is consequently also of interest to know the harmonic conjugate to $z_y(x,y)$. That function is completely determined by z_y , except for an arbitrary additive constant. It is easy to show that

$$z_x(x,y) = \left(\int_0^{z_1} B_x(x,y,z) dz \right) / B_o \quad (2)$$

is a harmonic conjugate to z_y , i.e. $z_y + i z_x$, or, more conventionally, $W = z_x - i z_y$ are, in the region of interest, analytic functions of the complex variable $Z = x + iy$.

While the geometric interpretation of z_x is not as straightforward as it is for z_y , z_x can be important for the beam dynamics and helps, of course,

to get a better understanding of the fringe fields. Furthermore, the fact that W is an analytical function of Z can simplify some problems greatly. To quote just one example: Measuring z_x and z_y in the midplane allows calculation of z_x and z_y off the midplane with the very simple methods that are valid for analytic functions of a complex variable, and permits particularly to identify very easily field contributions that are antisymmetric with respect to the midplane of the magnet. Measurement of both z_x and z_y allows also to check the selfconsistency of measurements.

3. Measurement of the EFB of a Homogeneous Field Magnet.

3.1. OUTLINE OF BASIC PROCEDURE

The proposed measurement apparatus consists basically of two flux sensing coils on a common carrier, and an integrator whose input signal is the additively mixed signal from these two coils. One coil is a long one, reaching from the field free region well into the homogeneous field region. It has, except at the very ends, a uniform cross sectional area per unit length (a') in the z direction. Ignoring, for the moment, minor corrections that will be discussed further below, this coil measures the line integral of the magnetic field component parallel to its area vector from one virtual end of the coil to the other. The location of a virtual end is defined as the end of an equivalent coil that has square ends instead of the easier to fabricate rounded ends. The other coil is a coil of small dimensions, located at the homogeneous-field-end of the long coil, with its area vector anti-parallel to the area vector of the long coil. If that coil has an area a_0 (corrected for the signal mixing ratio) equal to $a' \cdot L$, and if the virtual ends

of the long coil are at z_0 (in the homogeneous field) and z_1 (in the field free region), and if the coils are oriented to measure B_y along the z axis, then the flux obtained from the mixed integrated signal is given by

$$\phi_y(x,y) = a' \left(\int_{z_0}^{z_1} B_y(x,y,z) dz - L \cdot B_0 \right)$$

or, with the use of eq. (1b):

$$\phi_y(x,y) = B_0 a' (z_y(x,y) - z_0 - L) . \quad (3)$$

By choosing L to be something of the order $(z_1 - z_0)/2$, one can obviously make $\phi_y = 0$ by moving the coil assembly in the z -direction, obtaining then

$$z_y = z_0 + L. \quad (4)$$

With this null method, the location of the EFB is always at the distance L from the virtual end in the homogeneous field, and can be so marked on the coil carrier.

Turning then the whole assembly by 90° , one obtains a signal that is given by the flux intercepted by the long coil,

$$\phi_x(x,y) = a' \int_{z_0}^{z_1} B_x(x,y,z) dz = a' B_0 z_x(x,y). \quad (5)$$

Contamination of the information about B_x by B_y is avoided because the assembly is in the null position with regard to B_y .

3.2. DISCUSSION AND DETAILS OF THE MEASUREMENT SYSTEM

For some of the procedures that one has to follow with equipment of this kind, it is important to be able to turn the coil assembly accurately by 90° or 180° . Such a "flipper" is most easily adjusted if one employs a design that allows successive application of the same flip angle. By orienting the small coil perpendicularly to B_y and applying the same flip angle four or two times, one can easily see deviations from a flip angle of 360° and apply the corresponding corrections to the flip angle control. For accurate measurement of z_x it is important that the two coils are properly aligned. This is most easily accomplished by looking at the individual coil signals in a magnet that is known from symmetry considerations to have sufficiently small line integrals over B_x . By adjusting the total assembly and the relative angle between both coils to give a zero signal in both coils, one can achieve the desired alignment.

The absolute accuracy for establishing the location of the EFB depends on the measurement of z_0 and $L = a_0/a'$. Since a_0 can be obtained from a flux measurement in a homogeneous field magnet with a field level established by NMR, and the other measurements require absolute geometry determinations, the achievable accuracy is, to say the least, respectable. Relative shape and field level dependent shifts can be determined even more accurately, since integrator drift should not present any serious problem: If one uses an 180° flipper, only short term drift is of interest. One can therefore expect to have drift errors not larger than $\Delta\phi = 10^{-6}$ V sec. Together with a long coil of 2 cm width and 100 turns, one gets an error $\Delta z_y = \Delta\phi / (B_0 a') \approx 5 \mu\text{m}$ at $B_0 = .1$ Tesla.

While the design of the compensation coil, being in a homogeneous field, is not critical, the proper design of the long coil is important since one wants to prevent contamination of the line integrals by their spatial derivatives. We discuss here one specific design that has a well defined geometry and is easy to fabricate. Figure 2 shows a cross section of the proposed design, consisting of two single layers of turns parallel to the $y - z$ - plane, centered with respect to $x = y = 0$. Using the definition (eq. (1a)) for z_y , and taking advantage of the above mentioned properties of $z_x - iz_y$, one can derive easily that

$$(z_y(x,y))_{\text{measured}} = \sum_{\mu=0} \frac{\sin(\mu+1)\alpha}{\sin \alpha} \cdot \frac{2}{(2(\mu+1))!} r^{2\mu} \left(\frac{\partial}{\partial x}\right)^{2\mu} z_y(x,y). \quad (6)$$

Choosing $\alpha = 90^\circ$ eliminates all terms with odd μ and we get

$$(z_y(x,y))_{\text{measured}} = z_y(x,y) - \frac{1}{360} \cdot r^4 \left(\frac{\partial}{\partial x}\right)^4 z_y(x,y) + \dots \quad (7)$$

With a somewhat more complicated design one could cancel the correction term that is proportional to r^4 , but that will seldom, if ever, be necessary.

4. Generalization

For magnets such as quadrupoles or strong focussing bending magnets that have a region where the field has both an x and y component that depend on x and y , but not on z , and has no z component, the description and measurement of the fringe fields discussed above need only be altered slightly. We simply drop the B_0 - normalization used in eqs. (1) and (2), and work directly with the line integrals I_x and I_y :

$$I_y(x,y) = \int_0^{z_1} B_y(x,y,z) dz \quad (8)$$

$$I_x(x,y) = \int_0^{z_1} B_x(x,y,z) dz \quad (9a)$$

$$I_x(x,y) = z_0 B_x(x,y,z) + \int_{z_0}^{z_1} B_x(x,y,z) dz . \quad (9b)$$

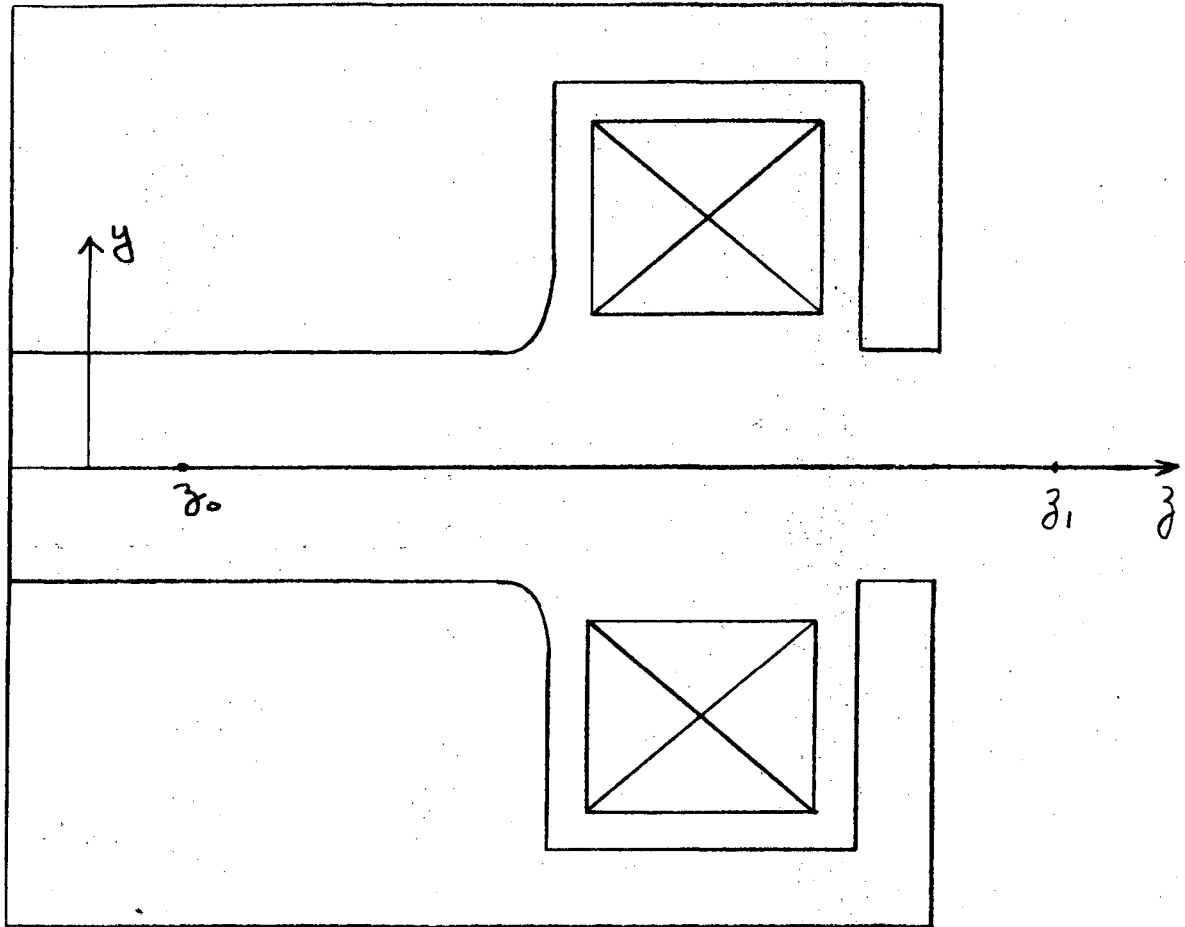
Clearly, $I_x - iI_y$ is an analytic function of $x + iy$, and basically the same procedure is used to measure I_x, I_y as was used for measurement for z_x, z_y . The only significant modification is that one has to look at both the mixed signal from the coils, as well as the compensation coil signal, since the latter is needed to get the first term on the right side of eq. (9b). I_y is obtained by measuring z_y with the null method, and multiplying that value with $B_y(x,y,z_0)$. Regarding the equipment, one has to be more careful with the alignment of the z -axis, and for the compensation coil one has to use one of the well known geometries that allows point-field measurement with small contamination by derivatives.

Figure Captions

Rapid Measurement

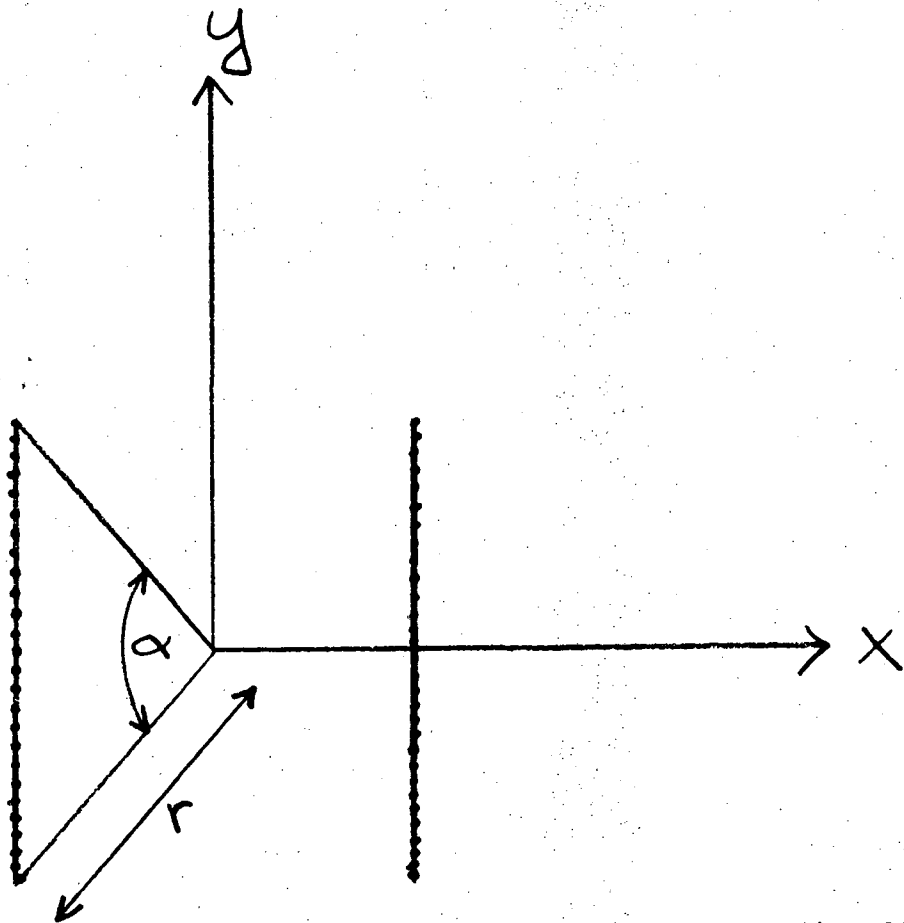
Fig. 1. Cross section through the end of a homogeneous field magnet.

Fig. 2. Cross section through integrating field measurement coil.



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Fig. 1



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Fig. 2

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