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# Origin of Attractive Force of Gravitation

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# Origin of Attractive Force of Gravitation

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Abstract

Asking why the universe has gravity, and not anti-gravity, we present a toy model in which the gravitational constant is replaced by a scalar field, which may have a positive or negative expectation value. It is shown that, in this model, the expansion of the universe drives it to the positive expectation value. We also show that chaotic inflation may occur and that sufficiently small density perturbations can be produced during the inflation.

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Mach's principle asserts that the distribution of matter should determine the inertial system of the universe. Einstein's theory of gravitation has realized this principle in a certain sense. Namely, according to Einstein's theory, the matter determines the metric of space-time for a given gravitational constant (G). If we take the principle more literally and require that the gravitational constant itself should also be determined by the matter content, we are led to theories[1] of the Brans-Dicke type. However, almost all of these theories presented previously have the sign of G fixed. Here, we adopt a stronger principle that even the sign of G should be determined by the matter. The argument for this is that quantum effects of some matter fields may contribute opposite sign to G so that the sign of G may be a variable, not constant.

In this letter, we present a model of gravitation in which the gravitational coupling constant is determined dynamically. Notably, its sign is shown to become positive (gravity) in the expanding universe, although it is possible, in principle, for the constant to take a negative value (anti-gravity). Interesting features of the model are that chaotic inflation[2,3] may occur in an early period of the universe and that small density perturbation[4] $\left(\frac{\delta\rho}{\rho} \leq 10^{-4}\right)$ may be generated.

In our model, there are two ground states with either gravity or anti-gravity, which are stable against small fluctuations. The possible existence of an anti-gravity phase leads to an amazing astrophysical prediction. That is, a star which accidentally rushes into a region with the anti-gravity phase (such a region could exist as a bubble in the gravity phase), would explode and release an enormous amount of energy.

It is worthwhile to mention that our model should be taken as an effective model of a more fundamental theory[5] of gravitation in which the gravitational constant and its sign may vary with time. Our idea is based on the principle that the matter in the universe should determine the metric of the universe, the gravitational constant, and event its sign.

Our model of gravity is described by the Lagrangian,

$$\mathcal{L} = [-\lambda\varphi R + 1/2\ g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)]\sqrt{-g} + \mathcal{L}_m \quad , \tag{1}$$

$$V \equiv h(\varphi^2 - v^2)^2,$$

where R is the Ricci scalar curvature, and $\lambda$ and v are dimensional constants whose signs may be taken to be positive (their signs can be chosen arbitrarily without affecting any physical quantities). $\mathcal{L}_m$ is a Lagrangian of matter, which is assumed not to include the scalar field $\varphi$. This scalar field describes the strength of the gravitational coupling and may take to be either positive or negative. Hereafter, let us call the case of $\varphi > 0$ the gravity phase and the case of $\varphi < 0$ the anti-gravity phase. Evidently, we may choose either of the phases ($\varphi = \pm v$) as a ground state if we switch off the gravitational coupling ($\lambda = 0$); in this case there is a reflection symmetry as $\varphi \to -\varphi$. Switching on the gravitational interaction, we break the symmetry and can tell, in principle, which phase is realized, but it is not trivial to answer the question.

First, let us find solutions for the flat space metric ($g_{\mu\nu} = \eta_{\mu\nu}$, $\eta_{\mu\nu} \equiv (1. - 1. - 1. - 1)$) and examine stability of these solutions against small fluctuations. The equations of motion are given by

$$\varphi(R_{\mu\nu} - 1/2\, g_{\mu\nu}R) + g_{\mu\nu}\Box\varphi - \nabla_\mu\nabla_\nu\varphi = T_{\mu\nu}/\lambda \tag{2.a}$$

and

$$\lambda R + \Box\varphi + V'(\varphi) = 0 \tag{2.b}$$

with $T_{\mu\nu} \equiv \partial_\mu\varphi\, \partial_\nu\, \varphi - g_{\mu\nu}(\frac{1}{2}\, g^{\alpha\beta}\partial_\alpha\varphi\, \partial_\beta\varphi - V) + T^m_{\mu\nu}$ and $\Box\varphi \equiv \frac{1}{\sqrt{-g}}\, \partial_\mu(g^{\mu\nu}\sqrt{-g}\, \partial_\nu\varphi)$ where $T^m_{\mu\nu}$ is the energy momentum tensor of the matter and $\nabla_\mu$ is a covariant derivative. It is easy to see from eq.(2) with $T^m_{\mu\nu} \equiv 0$ that there are two solutions for the flat space metric; one of them ($\varphi = +v$) represents the gravity phase and the other ($\varphi = -v$) represents the anti-gravity phase. In order to examine the stability of these phases, we linearize equations (2). Putting $\varphi = \pm v + \phi$ and $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, it follows that

$$\mp \frac{v}{2}\, \partial^2\gamma_{\mu\nu} + \eta_{\mu\nu}\partial^2\phi - \partial_\mu\partial_\nu\phi = T^m_{\mu\nu}/\lambda$$

$$\frac{\lambda}{2}\, \partial^2\gamma + \partial^2\phi + m^2\phi = 0 \tag{3}$$

with $\gamma_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\, \eta_{\mu\nu}h$, $h \equiv \eta^{\mu\nu}h_{\mu\nu}$, $\gamma \equiv \eta^{\mu\nu}\gamma_{\mu\nu}$ and $m^2 \equiv 8hv^2$. We also have the gauge condition $\partial^\mu\gamma_{\mu\nu} = 0$. Hence, we can obtain from these equations the mass (M) of the field $\phi$,

$$M^2 = \frac{m^2}{1 + \frac{3\lambda}{v}} \quad \text{in the gravity phase}$$

and

$$M^2 = \frac{m^2}{1 - \frac{3\lambda}{v}} \quad \text{in the anti-gravity phase} \tag{4}$$

As we explain in a moment, we must impose a condition of $\lambda/v \ll 1$ for experimental consistency of the gravity phase. Then, it turns out that both phases are stable against small fluctuations.

The above condition comes from the requirement that the model yields weak gravitational fields consistent with experiments. From eq (3), we can derive the following equations for $\gamma_{\mu\nu}$

$$\partial^2 \gamma_{\mu\nu} = -\frac{2}{v\lambda} \tilde{T}_{\mu\nu} \quad ,$$

$$\tilde{T}_{\mu\nu} \equiv T^m_{\mu\nu} - (\eta_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu) \frac{\lambda}{(v + 3\lambda)(\partial^2 + M^2)} T \tag{5}$$

with $T \equiv \eta^{\mu\nu} T_{\mu\nu}$.

The second term on the right hand side in $\tilde{T}_{\mu\nu}$ should be smaller than the first term so that the same weak gravitational fields as in Einstein's model of gravitation are produced. Hence, we must impose the condition,

$$\lambda/v \ll 1 \tag{6.a}$$

We identify

$$(8\pi\lambda v)^{-1} \equiv G \tag{6.b}$$

with the gravitational constant G. Later, we shall see that the value of $\lambda/v$

must be of the order of $10^{-6}$ or less for the cosmological model to yield small density perturbations ($\delta\rho/\rho \leqq 10^{-4}$).

A comment is in order. From eq (6), we find that the constant v is bigger than the Planck mass ($G^{-1}$). This seems to invalidate our model because the existence of such a large dimensional constant apparently forces us to take quantum gravitational effects into account. However, in our model, the constant plays a role only in producing a small dimensionless number $\lambda/v$ and doesn't play any role in producing a large mass (see eq (3) and eq (4)). Our model should be valid as an effective theory only in the energy region less than the Planck mass. Therefore, even if the model possesses such a constant, we have no troubles in actual applications. So far, we have shown that, in the model, there are gravity and anti-gravity phases, both of which are stable against small fluctuations.

Next, let us discuss why the present universe might have chosen the gravity phase because of its expansion. For that purpose, we apply the model to the early period of the universe. We restrict the metric of space-time to the Robertson-Walker metric,

$$dS^2 = dt^2 - a^2(t)[dr^2 + \psi(r)(d\theta^2 + \sin^2\theta d\phi^2)] \tag{7}$$

$$\psi(r) \equiv \begin{cases} \sin r & \dots \text{ closed} \\ r & \dots \text{ flat} \\ \sinh r & \dots \text{ open} \end{cases}$$

where $\psi(r)$ is chosen appropriately for closed, flat and open universes, respectively. We assume, furthermore, that the scalar field $\varphi$ is uniform in space and depends only on time t. Then, the scale factor a(t) and the field $\varphi$ obey the equations:

$$\frac{3\varphi(\dot{a}^2 + k)}{a^2} + \frac{3\dot{a}\dot{\varphi}}{a} = \frac{1}{\lambda}\rho \tag{8.a}$$

$$\ddot{\varphi} + \frac{d}{d\varphi}V(\varphi) - 6\lambda\frac{\ddot{a}a + \dot{a}^2 + k}{a^2} + \frac{3\dot{a}\dot{\varphi}}{a} = 0 \tag{8.b}$$

with $\rho = \frac{1}{2}\dot{\varphi}^2 + V(\varphi) + \rho_{th}$

where k takes the values ±1 or 0 corresponding to closed, open and flat universes, respectively, and $\rho_{th}$ is the thermal energy density of the matter. In (8), dots on a(t) and $\varphi(t)$ mean the derivative with respect to time, t. Eq (8.a) is an equation of gravity and eq (8.b) is the field equation of $\varphi$ (we have not included the thermal contributions in (8.b); see below). Here, we would like to comment that the form of the potential V may be used reliably only under the following condition,

$$hv^4 = V(\varphi = 0) \leq m_p^4 \quad \text{with} \quad m_p^2 \equiv G^{-1} \quad . \tag{9}$$

If the inequality does not hold, quantum gravitational effects would change drastically the form of this potential.

Now, we notice that the energy density $\rho$ in (8.a) is positive definite as long as $\rho_{th} > 0$. This fact leads to the important consequence. That is, at an early time when the field $\varphi$ vanishes ($\varphi = 0$), the value of $\dot{a}\dot{\varphi}$ must be positive. Consequently, it follows that when the universe is expanding ($\dot{a} > 0$) and $\varphi$ is in the neighborhood of the point $\varphi = 0$, $\varphi$ must increase ($\dot{\varphi} > 0$) and the gravity phase is realized naturally. Once $\varphi$ passes through

$\varphi = 0$ and becomes positive, the universe will stay in the gravity phase as long as it expands. This consequence does not depend on the detail of the model. Indeed, without changing the results, we can adopt a more general odd smooth function $f(\varphi)$ as a coefficient of the Ricci scalar curvature ($f(\varphi) \to \varphi$ as $\varphi \to 0$). Furthermore, if the universe is closed ($k = +1$), we can conclude that the anti-gravity phase ($\varphi < 0$) is completely unstable and the gravity phase necessarily appears in time. This is because the positivity of $\dot{\varphi}$ comes from eq (8.a) with $\varphi < 0$ and the universe never stops its expansion ($\dot{a} > 0$) in the anti-gravity phase (in (8.a), the vanishing of the expansion rate, $\dot{a} = 0$, is not consistent with the fact that $\varphi < 0$). Therefore, we find that the model may naturally explain why our universe at present stays in the gravity phase. The gravitational interaction itself breaks the symmetry $\varphi \to -\varphi$ and the expansion of the present universe gives rise to the gravity phase. It should be stressed that the above consequence is derived simply by using gravitational equation (8.a) and the positivity of the energy density.

We shall continue to discuss the application of our model to cosmology, and show that chaotic inflation, which recently has been argued to occur,[2] may arise in our model. According to the argument[2] of Linde, we consider a period when the temperature of the universe is about the Planck energy. In that period, the most probable field configuration is given by the condition of $V \sim m_p^4$. Namely, the configuration $\varphi$ is of order of $v$ (see eq (9)). Since it has been shown[2] that the thermal contributions to the field equation are small during that period, we are able to use the classical equation (8.b). We wish to show that there is a solution such that $\dot{a}/a \sim O(m_p)$ and $\dot{\varphi}/\varphi \sim O(\lambda\sqrt{\lambda/v}) << O(m_p)$; note $\lambda/v << 1$ and $8\pi\lambda v = m_p^2$. Hence, we ignore the second term of $\dot{\varphi}/\varphi$ in (8.a), and using $\rho \sim m_p^4$, we find

$$\dot{a}/a \sim m_p \Rightarrow a = a_0 e^{m_p t} \tag{10}$$

Inserting this result into (8.b) and noting that $\frac{dV}{d\varphi}$ is of order of $\lambda m_p^2$, we obtain

$$\varphi = O(v) + O(\lambda m_p)t \quad . \tag{11}$$

Hence, $\dot{\varphi}/\varphi \sim O(\lambda m_p/v) \sim O(\lambda \sqrt{\lambda/v})$, as expected. Therefore, we find the desired solution which shows that chaotic inflation may occur in our model. The essence for driving chaotic inflation is the flatness[2] of the potential, $V = h(\varphi^2 - v^2)^2$. Indeed, in the previous discussion, we derived two conditions: $hv^4 \leq m_p^4$ and $\lambda/v << 1$. These conditions lead to the flatness of the potential ($h \sim (\lambda/v)^2 << 1$).

The condition $\lambda/v << 1$ can be made more quantitative if we require small density perturbations[2] ($\delta\rho/\rho \leq 10^{-4}$). Following the previous arguments,[2] we are required to have such a small coupling constant h, $h \leq 10^{-12}$. This implies $\lambda/v \leq 10^{-6}$. Therefore, accurate measurements of $\delta\rho/\rho$ in the background radiation could determine to what extent predictions of the model deviate from those of Einstein's model of gravitation in weak gravitational fields (see (5)).

Finally, we must state that we have not succeeded in finding how the closed universe shrinks to a singular point $a = 0$ within a finite time. We guess that the close universe of our model would not collapse into such a singular point, at least not within a finite time.

To summarize, we have shown that in our model of gravitation, which involves both gravity and anti-gravity phases, the universe is driven to choose the gravity phase through its expansion and passes through the period of chaotic inflation, producing reasonably small density fluctuations.

Our model has essentially only one free parameter $\lambda/v$ (see eq (9)), which must be small for producing the weak gravitational fields as in Einstein's theory. Interesting point is that smallness of $\lambda/v$ leads to chaotic inflation. Although we have shown that the gravity phase is realized globally in the universe, some regions with the anti-gravity phase could exist locally as bubbles. Then, if a star were to rush into such a region, the star would explode and release its energy anomalously.

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