## Lawrence Berkeley National Laboratory

Lawrence Berkeley National Laboratory

## Title

Charged and strange hadron elliptic flow in $\mathrm{Cu}+\mathrm{Cu}$ collisions at sqrt sNN $=62.4$ and 200 GeV

## Permalink

https://escholarship.org/uc/item/1679m5ck

## Author

Abelev, Betty
Publication Date
2010-03-25

# Charged and strange hadron elliptic flow in $\mathrm{Cu}+\mathrm{Cu}$ 

 collisions at \$lsqrt\{s_\{NN\}\}\$ = 62.4 and 200 GeVB.I.Abelev<br>(STARCollaboration) March 25, 2010

This work was supported by the Director, Office of Science, Office of Nuclear Science of the U.S. Department of Energy under Contract No. DE-AC0205 CH 11231.

# Charged and strange hadron elliptic flow in $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=62.4$ and 200 GeV 

B. I. Abelev, ${ }^{8}$ M. M. Aggarwal, ${ }^{31}$ Z. Ahammed, ${ }^{48}$ A. V. Alakhverdyants, ${ }^{18}$ I. Alekseev,${ }^{16}$ B. D. Anderson, ${ }^{19}$ D. Arkhipkin, ${ }^{3}$ G. S. Averichev, ${ }^{18}$ J. Balewski, ${ }^{23}$ L. S. Barnby, ${ }^{2}$ S. Baumgart, ${ }^{53}$ D. R. Beavis, ${ }^{3}$ R. Bellwied, ${ }^{51}$ M. J. Betancourt, ${ }^{23}$ R. R. Betts, ${ }^{8}$ A. Bhasin, ${ }^{17}$ A. K. Bhati, ${ }^{31}$ H. Bichsel, ${ }^{50}$ J. Bielcik, ${ }^{10}$ J. Bielcikova, ${ }^{11}$ B. Biritz, ${ }^{6}$
L. C. Bland, ${ }^{3}$ B. E. Bonner, ${ }^{37}$ J. Bouchet, ${ }^{19}$ E. Braidot, ${ }^{28}$ A. V. Brandin, ${ }^{26}$ A. Bridgeman, ${ }^{1}$ E. Bruna, ${ }^{53}$ S. Bueltmann, ${ }^{30}$ I. Bunzarov, ${ }^{18}$ T. P. Burton, ${ }^{3}$ X. Z. Cai, ${ }^{41}$ H. Caines, ${ }^{53}$ M. Calderón de la Barca Sánchez, ${ }^{5}$ O. Catu, ${ }^{53}$ D. Cebra, ${ }^{5}$ R. Cendejas, ${ }^{6}$ M. C. Cervantes, ${ }^{43}$ Z. Chajecki, ${ }^{29}$ P. Chaloupka, ${ }^{11}$ S. Chattopadhyay, ${ }^{48}$ H. F. Chen, ${ }^{39}$ J. H. Chen, ${ }^{41}$ J. Y. Chen, ${ }^{52}$ J. Cheng, ${ }^{45}$ M. Cherney, ${ }^{9}$ A. Chikanian, ${ }^{53}$ K. E. Choi, ${ }^{35}$ W. Christie, ${ }^{3}$ P. Chung, ${ }^{11}$ R. F. Clarke, ${ }^{43}$ M. J. M. Codrington, ${ }^{43}$ R. Corliss, ${ }^{23}$ J. G. Cramer, ${ }^{50}$ H. J. Crawford, ${ }^{4}$ D. Das, ${ }^{5}$ S. Dash, ${ }^{13}$ A. Davila Leyva, ${ }^{44}$ L. C. De Silva, ${ }^{51}$ R. R. Debbe, ${ }^{3}$ T. G. Dedovich,,${ }^{18}$ M. DePhillips, ${ }^{3}$ A. A. Derevschikov, ${ }^{33}$ R. Derradi de Souza, ${ }^{7}$ L. Didenko, ${ }^{3}$ P. Djawotho, ${ }^{43}$ S. M. Dogra, ${ }^{17}$ X. Dong, ${ }^{22}$ J. L. Drachenberg, ${ }^{43}$ J. E. Draper, ${ }^{5}$ J. C. Dunlop, ${ }^{3}$ M. R. Dutta Mazumdar, ${ }^{48}$ L. G. Efimov, ${ }^{18}$ E. Elhalhuli, ${ }^{2}$ M. Elnimr, ${ }^{51}$ J. Engelage, ${ }^{4}$ G. Eppley, ${ }^{37}$ B. Erazmus, ${ }^{42}$ M. Estienne, ${ }^{42}$ L. Eun, ${ }^{32}$ O. Evdokimov, ${ }^{8}$ P. Fachini, ${ }^{3}$ R. Fatemi, ${ }^{20}$ J. Fedorisin, ${ }^{18}$ R. G. Fersch, ${ }^{20}$ P. Filip, ${ }^{18}$ E. Finch, ${ }^{53}$ V. Fine, ${ }^{3}$ Y. Fisyak, ${ }^{3}$ C. A. Gagliardi, ${ }^{43}$ D. R. Gangadharan, ${ }^{6}$ M. S. Ganti, ${ }^{48}$ E. J. Garcia-Solis, ${ }^{8}$ A. Geromitsos, ${ }^{42}$ F. Geurts, ${ }^{37}$ V. Ghazikhanian, ${ }^{6}$ P. Ghosh, ${ }^{48}$ Y. N. Gorbunov, ${ }^{9}$ A. Gordon, ${ }^{3}$ O. Grebenyuk, ${ }^{22}$ D. Grosnick, ${ }^{47}$ B. Grube, ${ }^{35}$ S. M. Guertin, ${ }^{6}$ A. Gupta, ${ }^{17}$ N. Gupta, ${ }^{17}$ W. Guryn, ${ }^{3}$ B. Haag, ${ }^{5}$ A. Hamed, ${ }^{43}$ L-X. Han, ${ }^{41}$ J. W. Harris,,${ }^{53}$ J. P. Hays-Wehle, ${ }^{23}$ M. Heinz, ${ }^{53}$ S. Heppelmann, ${ }^{32}$ A. Hirsch, ${ }^{34}$ E. Hjort, ${ }^{22}$ A. M. Hoffman, ${ }^{23}$ G. W. Hoffmann, ${ }^{44}$ D. J. Hofman, ${ }^{8}$ R. S. Hollis, ${ }^{8}$ H. Z. Huang, ${ }^{6}$ T. J. Humanic, ${ }^{29}$ L. Huo, ${ }^{43}$ G. Igo, ${ }^{6}$ A. Iordanova, ${ }^{8}$ P. Jacobs, ${ }^{22}$ W. W. Jacobs, ${ }^{15}$ P. Jakl, ${ }^{11}$ C. Jena, ${ }^{13}$ F. Jin, ${ }^{41}$ C. L. Jones, ${ }^{23}$ P. G. Jones, ${ }^{2}$ J. Joseph, ${ }^{19}$ E. G. Judd, ${ }^{4}$ S. Kabana, ${ }^{42}$ K. Kajimoto, ${ }^{44}$ K. Kang, ${ }^{45}$ J. Kapitan, ${ }^{11}$ K. Kauder, ${ }^{8}$ D. Keane, ${ }^{19}$ A. Kechechyan, ${ }^{18}$ D. Kettler, ${ }^{50}$ D. P. Kikola, ${ }^{22}$ J. Kiryluk, ${ }^{22}$ A. Kisiel, ${ }^{49}$ S. R. Klein, ${ }^{22}$ A. G. Knospe, ${ }^{53}$ A. Kocoloski, ${ }^{23}$ D. D. Koetke, ${ }^{47}$ T. Kollegger, ${ }^{12}$ J. Konzer, ${ }^{34}$ M. Kopytine, ${ }^{19}$ I. Koralt, ${ }^{30}$ L. Koroleva, ${ }^{16}$ W. Korsch, ${ }^{20}$ L. Kotchenda, ${ }^{26}$ V. Kouchpil, ${ }^{11}$ P. Kravtsov, ${ }^{26}$ K. Krueger, ${ }^{1}$ M. Krus, ${ }^{10}$ L. Kumar, ${ }^{31}$ P. Kurnadi, ${ }^{6}$ M. A. C. Lamont, ${ }^{3}$ J. M. Landgraf, ${ }^{3}$ S. LaPointe, ${ }^{51}$ J. Lauret, ${ }^{3}$ A. Lebedev, ${ }^{3}$ R. Lednicky, ${ }^{18}$ C-H. Lee, ${ }^{35}$ J. H. Lee, ${ }^{3}$ W. Leight, ${ }^{23}$ M. J. LeVine, ${ }^{3}$ C. Li, ${ }^{39} \mathrm{~L} . \operatorname{Li},{ }^{44} \mathrm{~N} . \operatorname{Li},{ }^{52}$ W. Li, ${ }^{41}$ X. Li, ${ }^{40}$ X. Li, ${ }^{34}$ Y. Li, ${ }^{45} \mathrm{Z} . \mathrm{Li},{ }^{52} \mathrm{G} . \operatorname{Lin},{ }^{53}$ S. J. Lindenbaum, ${ }^{27},{ }^{*}$ M. A. Lisa, ${ }^{29}$ F. Liu, ${ }^{52}$ H. Liu, ${ }^{5}$ J. Liu, ${ }^{37}$ T. Ljubicic, ${ }^{3}$ W. J. Llope, ${ }^{37}$ R. S. Longacre, ${ }^{3}$ W. A. Love, ${ }^{3}$ Y. Lu, ${ }^{39}$ G. L. Ma, ${ }^{41}$ Y. G. Ma, ${ }^{41}$ D. P. Mahapatra, ${ }^{13}$ R. Majka, ${ }^{53}$ O. I. Mall, ${ }^{5}$ L. K. Mangotra, ${ }^{17}$ R. Manweiler, ${ }^{47}$ S. Margetis, ${ }^{19}$ C. Markert, ${ }^{44}$ H. Masui, ${ }^{22}$ H. S. Matis, ${ }^{22}$ Yu. A. Matulenko, ${ }^{33}$ D. McDonald, ${ }^{37}$ T. S. McShane, ${ }^{9}$ A. Meschanin, ${ }^{33}$ R. Milner, ${ }^{23}$ N. G. Minaev, ${ }^{33}$ S. Mioduszewski, ${ }^{43}$ A. Mischke, ${ }^{28}$ M. K. Mitrovski, ${ }^{12}$ B. Mohanty, ${ }^{48}$ M. M. Mondal, ${ }^{48}$ B. Morozov, ${ }^{16}$ D. A. Morozov, ${ }^{33}$ M. G. Munhoz, ${ }^{38}$ B. K. Nandi, ${ }^{14}$ C. Nattrass, ${ }^{53}$ T. K. Nayak, ${ }^{48}$ J. M. Nelson, ${ }^{2}$ P. K. Netrakanti, ${ }^{34}$ M. J. Ng, ${ }^{4}$ L. V. Nogach, ${ }^{33}$ S. B. Nurushev, ${ }^{33}$ G. Odyniec,,${ }^{22}$ A. Ogawa, ${ }^{3}$ H. Okada, ${ }^{3}$ V. Okorokov, ${ }^{26}$ D. Olson,,$^{22}$ M. Pachr, ${ }^{10}$ B. S. Page, ${ }^{15}$ S. K. Pal, ${ }^{48}$ Y. Pandit, ${ }^{19}$ Y. Panebratsev, ${ }^{18}$ T. Pawlak, ${ }^{49}$ T. Peitzmann, ${ }^{28}$ V. Perevoztchikov, ${ }^{3}$ C. Perkins, ${ }^{4}$ W. Peryt, ${ }^{49}$ S. C. Phatak, ${ }^{13}$ P. Pile, ${ }^{3}$ M. Planinic, ${ }^{54}$ M. A. Ploskon, ${ }^{22}$ J. Pluta, ${ }^{49}$ D. Plyku, ${ }^{30}$ N. Poljak, ${ }^{54}$ A. M. Poskanzer, ${ }^{22}$ B. V. K. S. Potukuchi, ${ }^{17}$ C. B. Powell, ${ }^{22}$ D. Prindle, ${ }^{50}$ C. Pruneau, ${ }^{51}$ N. K. Pruthi, ${ }^{31}$ P. R. Pujahari, ${ }^{14}$ J. Putschke, ${ }^{53}$ R. Raniwala, ${ }^{36}$ S. Raniwala, ${ }^{36}$ R. L. Ray, ${ }^{44}$ R. Redwine, ${ }^{23}$ R. Reed, ${ }^{5}$ J. M. Rehberg, ${ }^{12}$ H. G. Ritter, ${ }^{22}$ J. B. Roberts, ${ }^{37}$ O. V. Rogachevskiy, ${ }^{18}$ J. L. Romero, ${ }^{5}$ A. Rose, ${ }^{22}$ C. Roy, ${ }^{42}$ L. Ruan, ${ }^{3}$ R. Sahoo, ${ }^{42}$ S. Sakai, ${ }^{6}$ I. Sakrejda, ${ }^{22}$ T. Sakuma, ${ }^{23}$ S. Salur, ${ }^{5}$ J. Sandweiss, ${ }^{53}$ E. Sangaline, ${ }^{5}$ J. Schambach, ${ }^{44}$ R. P. Scharenberg, ${ }^{34}$ N. Schmitz, ${ }^{24}$ T. R. Schuster, ${ }^{12}$ J. Seele, ${ }^{23}$ J. Seger, ${ }^{9}$ I. Selyuzhenkov, ${ }^{15}$ P. Seyboth, ${ }^{24}$ E. Shahaliev, ${ }^{18}$ M. Shao, ${ }^{39}$ M. Sharma, ${ }^{51}$ S. S. Shi, ${ }^{52, \dagger \text { X. H. Shi, }{ }^{41}}$ E. P. Sichtermann, ${ }^{22}$ F. Simon, ${ }^{24}$ R. N. Singaraju, ${ }^{48}$ M. J. Skoby, ${ }^{34}$ N. Smirnov, ${ }^{53}$ P. Sorensen, ${ }^{3}$ J. Sowinski, ${ }^{15}$ H. M. Spinka, ${ }^{1}$ B. Srivastava,,${ }^{34}$ T. D. S. Stanislaus, ${ }^{47}$ D. Staszak, ${ }^{6}$ J. R. Stevens, ${ }^{15}$ R. Stock, ${ }^{12}$ M. Strikhanov, ${ }^{26}$ B. Stringfellow, ${ }^{34}$ A. A. P. Suaide, ${ }^{38}$ M. C. Suarez, ${ }^{8}$ N. L. Subba, ${ }^{19}$ M. Sumbera, ${ }^{11}$ X. M. Sun, ${ }^{22}$ Y. Sun, ${ }^{39}$ Z. Sun, ${ }^{21}$ B. Surrow, ${ }^{23}$ D. N. Svirida, ${ }^{16}$ T. J. M. Symons, ${ }^{22}$ A. Szanto de Toledo, ${ }^{38}$ J. Takahashi, ${ }^{7}$ A. H. Tang, ${ }^{3}$ Z. Tang, ${ }^{39}$ L. H. Tarini, ${ }^{51}$ T. Tarnowsky, ${ }^{25}$ D. Thein, ${ }^{44}$ J. H. Thomas, ${ }^{22}$ J. Tian, ${ }^{41}$ A. R. Timmins, ${ }^{51}$ S. Timoshenko, ${ }^{26}$ D. Tlusty, ${ }^{11}$ M. Tokarev, ${ }^{18}$ V. N. Tram, ${ }^{22}$ S. Trentalange,,${ }^{6}$ R. E. Tribble, ${ }^{43}$ O. D. Tsai, ${ }^{6}$ J. Ulery, ${ }^{34}$ T. Ullrich, ${ }^{3}$ D. G. Underwood, ${ }^{1}$ G. Van Buren, ${ }^{3}$ M. van Leeuwen, ${ }^{28}$ G. van Nieuwenhuizen, ${ }^{23}$ J. A. Vanfossen, Jr., ${ }^{19}$ R. Varma, ${ }^{14}$ G. M. S. Vasconcelos, ${ }^{7}$ A. N. Vasiliev, ${ }^{33}$ F. Videbaek, ${ }^{3}$ Y. P. Viyogi, ${ }^{48}$ S. Vokal, ${ }^{18}$ S. A. Voloshin, ${ }^{51}$ M. Wada, ${ }^{44}$ M. Walker, ${ }^{23}$ F. Wang, ${ }^{34}$ G. Wang, ${ }^{6}$ H. Wang, ${ }^{25}$ J. S. Wang, ${ }^{21}$ Q. Wang, ${ }^{34}$ X. L. Wang, ${ }^{39}$ Y. Wang, ${ }^{45}$ G. Webb, ${ }^{20}$ J. C. Webb, ${ }^{3}$ G. D. Westfall, ${ }^{25}$ C. Whitten Jr., ${ }^{6}$
H. Wieman, ${ }^{22}$ E. Wingfield, ${ }^{44}$ S. W. Wissink, ${ }^{15}$ R. Witt, ${ }^{46}$ Y. Wu, ${ }^{52}$ W. Xie, ${ }^{34}$ N. Xu, ${ }^{22}$ Q. H. Xu, ${ }^{40}$ W. Xu, ${ }^{6}$ Y. Xu, ${ }^{39} \mathrm{Z} . \mathrm{Xu},{ }^{3} \mathrm{~L}$. Xue, ${ }^{41}$ Y. Yang, ${ }^{21} \mathrm{P}$. Yepes, ${ }^{37} \mathrm{~K}$. Yip, ${ }^{3} \mathrm{I}-\mathrm{K}$. Yoo, ${ }^{35} \mathrm{Q}$. Yue, ${ }^{45} \mathrm{M}$. Zawisza, ${ }^{49}$ H. Zbroszczyk, ${ }^{49}$ W. Zhan, ${ }^{21}$ S. Zhang, ${ }^{41}$ W. M. Zhang, ${ }^{19}$ X. P. Zhang, ${ }^{22}$ Y. Zhang, ${ }^{22}$ Z. P. Zhang, ${ }^{39}$ J. Zhao, ${ }^{41}$ C. Zhong, ${ }^{41}$ J. Zhou, ${ }^{37}$ W. Zhou, ${ }^{40}$ X. Zhu, ${ }^{45}$ Y. H. Zhu, ${ }^{41}$ R. Zoulkarneev, ${ }^{18}$ and Y. Zoulkarneeva ${ }^{18}$ (STAR Collaboration)
${ }^{1}$ Argonne National Laboratory, Argonne, Illinois 60439, USA
${ }^{2}$ University of Birmingham, Birmingham, United Kingdom
${ }^{3}$ Brookhaven National Laboratory, Upton, New York 11973, USA
${ }^{4}$ University of California, Berkeley, California 94720, USA
${ }^{5}$ University of California, Davis, California 95616, USA
${ }^{6}$ University of California, Los Angeles, California 90095, USA
${ }^{7}$ Universidade Estadual de Campinas, Sao Paulo, Brazil
${ }^{8}$ University of Illinois at Chicago, Chicago, Illinois 60607, USA
${ }^{9}$ Creighton University, Omaha, Nebraska 68178, USA
${ }^{10}$ Czech Technical University in Prague, FNSPE, Prague, 115 19, Czech Republic
${ }^{11}$ Nuclear Physics Institute AS CR, 25068 Řež/Prague, Czech Republic
${ }^{12}$ University of Frankfurt, Frankfurt, Germany
${ }^{13}$ Institute of Physics, Bhubaneswar 751005, India
${ }^{14}$ Indian Institute of Technology, Mumbai, India
${ }^{15}$ Indiana University, Bloomington, Indiana 47408, USA
${ }^{16}$ Alikhanov Institute for Theoretical and Experimental Physics, Moscow, Russia
${ }^{17}$ University of Jammu, Jammu 180001, India
${ }^{18}$ Joint Institute for Nuclear Research, Dubna, 141 980, Russia
${ }^{19}$ Kent State University, Kent, Ohio 44242, USA
${ }^{20}$ University of Kentucky, Lexington, Kentucky, 40506-0055, USA
${ }^{21}$ Institute of Modern Physics, Lanzhou, China
${ }^{22}$ Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA
${ }^{23}$ Massachusetts Institute of Technology, Cambridge, MA 02139-4307, USA
${ }^{24}$ Max-Planck-Institut für Physik, Munich, Germany
${ }^{25}$ Michigan State University, East Lansing, Michigan 48824, USA
${ }^{26}$ Moscow Engineering Physics Institute, Moscow Russia
${ }^{27}$ City College of New York, New York City, New York 10031, USA
${ }^{28}$ NIKHEF and Utrecht University, Amsterdam, The Netherlands
${ }^{29}$ Ohio State University, Columbus, Ohio 43210, USA
${ }^{30}$ Old Dominion University, Norfolk, VA, 23529, USA
${ }^{31}$ Panjab University, Chandigarh 160014, India
${ }^{32}$ Pennsylvania State University, University Park, Pennsylvania 16802, USA
${ }^{33}$ Institute of High Energy Physics, Protvino, Russia
${ }^{34}$ Purdue University, West Lafayette, Indiana 47907, USA
${ }^{35}$ Pusan National University, Pusan, Republic of Korea
${ }^{36}$ University of Rajasthan, Jaipur 302004, India
${ }^{37}$ Rice University, Houston, Texas 77251, USA
${ }^{38}$ Universidade de Sao Paulo, Sao Paulo, Brazil
${ }^{39}$ University of Science Es Technology of China, Hefei 230026, China
${ }^{40}$ Shandong University, Jinan, Shandong 250100, China
${ }^{41}$ Shanghai Institute of Applied Physics, Shanghai 201800, China
${ }^{42}$ SUBATECH, Nantes, France
${ }^{43}$ Texas A ${ }^{6} M$ University, College Station, Texas 77843, USA
${ }^{44}$ University of Texas, Austin, Texas 78712, USA
${ }^{45}$ Tsinghua University, Beijing 100084, China
${ }^{46}$ United States Naval Academy, Annapolis, MD 21402, USA
${ }^{47}$ Valparaiso University, Valparaiso, Indiana 46383, USA
${ }^{48}$ Variable Energy Cyclotron Centre, Kolkata 700064, India
${ }^{49}$ Warsaw University of Technology, Warsaw, Poland
${ }^{50}$ University of Washington, Seattle, Washington 98195, USA
${ }^{51}$ Wayne State University, Detroit, Michigan 48201, USA
${ }^{52}$ Institute of Particle Physics, CCNU (HZNU), Wuhan 430079, China
${ }^{53}$ Yale University, New Haven, Connecticut 06520, USA ${ }^{54}$ University of Zagreb, Zagreb, HR-10002, Croatia
(Dated: March 26, 2010)


#### Abstract

We present the results of an elliptic flow, $v_{2}$, analysis of $\mathrm{Cu}+\mathrm{Cu}$ collisions recorded with the STAR detector at RHIC at $\sqrt{s_{N N}}=62.4$ and 200 GeV . Elliptic flow as a function of transverse momentum, $v_{2}\left(p_{T}\right)$, is reported for different collision centralities for charged hadrons $h^{ \pm}$, and strangeness containing hadrons $K_{S}^{0}, \Lambda, \Xi, \phi$ in the midrapidity region $|\eta|<1.0$. Significant reduction in systematic uncertainty of the measurement due to non-flow effects has been achieved by correlating particles at midrapidity, $|\eta|<1.0$, with those at forward rapidity, $2.5<|\eta|<4.0$. We also present azimuthal correlations in $p+p$ collisions at $\sqrt{s}=200 \mathrm{GeV}$ to help estimating non-flow effects. To study the system-size dependence of elliptic flow, we present a detailed comparison with previously published results from $\mathrm{Au}+\mathrm{Au}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. We observe that $v_{2}\left(p_{T}\right)$ of strange hadrons has similar scaling properties as were first observed in $\mathrm{Au}+\mathrm{Au}$ collisions, i.e.: (i) at low transverse momenta, $p_{T}<2 \mathrm{GeV} / c, v_{2}$ scales with transverse kinetic energy, $m_{T}-m$, and (ii) at intermediate $p_{T}, 2<p_{T}<4 \mathrm{GeV} / c$, it scales with the number of constituent quarks, $n_{q}$. We have found that ideal hydrodynamic calculations fail to reproduce the centrality dependence of $v_{2}\left(p_{T}\right)$ for $K_{S}^{0}$ and $\Lambda$. Eccentricity scaled $v_{2}$ values, $v_{2} / \varepsilon$, are larger in more central collisions, suggesting stronger collective flow develops in more central collisions. The comparison with $\mathrm{Au}+\mathrm{Au}$ collisions which go further in density shows $v_{2} / \varepsilon$ depend on the system size, number of participants $N_{\text {part }}$. This indicates that the ideal hydrodynamic limit is not reached in $\mathrm{Cu}+\mathrm{Cu}$ collisions, presumably because the assumption of thermalization is not attained.


PACS numbers: 25.75.Ld, 25.75.Dw

## I. INTRODUCTION

At the early stages of relativistic heavy ion collisions, a hot and dense, strongly interacting medium is created. The subsequent system evolution is determined by the nature of the medium. Experimentally, the dynamics of the system evolution has been studied by measuring the azimuthal anisotropy of the particle production relative to the reaction plane $[1 / 3]$. The centrality of the collision, defined by the transverse distance between the centers of the colliding nuclei called the impact parameter, results in an "almond-shaped" overlap region that is spatially azimuthal anisotropic. It is generally assumed that the initial spatial anisotropy in the system is converted into momentum-space anisotropy through re-scatterings [4, 5]. The elliptic flow, $v_{2}$, is the second harmonic coefficient of a Fourier expansion of the final momentum-space azimuthal anisotropy. Due to the selfquenching effect, it provides information about the dynamics at the early stage of the collisions (6) 8]. Elliptic flow can provide information about the pressure gradients, the effective degrees of freedom, the degree of thermalization, and equation of state of the matter created at the early stage. Thus, the centrality and system-size dependence of elliptic flow at different beam energies can be used to study the properties of the matter created in heavy ion collisions [6].

Recently, two important insights have been obtained from the experimental results on $v_{2}$ as a function of transverse momentum, $p_{T}$, in $\mathrm{Au}+\mathrm{Au}$ collisions at RHIC. First, in the low $p_{T}$ region, $p_{T}<2 \mathrm{GeV} / c$, the hadron mass hierarchy predicted by ideal hydrodynamic calcu-

[^0]lations is observed for identified hadrons $\pi, K, K_{S}^{0}, p$, $\Lambda$ and $\Xi$ [9-13]. Even the $\phi$ and $\Omega$, which are believed to have a reduced cross section for hadronic interactions 14 19], are consistent with the mass ordering [13, 20, 21]. Second, in the intermediate $p_{T}$ region, $2<p_{T}<4 \mathrm{GeV} / c, v_{2}\left(p_{T}\right)$ follows a scaling depending on the number of constituent quarks within a given hadron, which can be explained via coalescence models [13, 20, 22]. Quark number scaling suggests that the system is in a partonic state and that the constituent quark degrees of freedom were relevant during the time $v_{2}$ was developed.

STAR's first published paper showed that elliptic flow at RHIC is unexpectedly large 23], comparable to predictions of ideal hydrodynamic calculations [7, 24-26]. This observation is among the evidence favoring the picture of a nearly-perfect liquid [27]. With the assumption of thermalization, ideal hydrodynamic calculations predict that the $v_{2}$ divided by spatial eccentricity, $\varepsilon$, does not depend on the collision centrality 28]. The spatial eccentricity is defined by 29]:

$$
\begin{equation*}
\varepsilon=\frac{\left\langle y^{2}-x^{2}\right\rangle}{\left\langle y^{2}+x^{2}\right\rangle} \tag{1}
\end{equation*}
$$

where $x$ and $y$ are the spatial coordinates in the plane perpendicular to the collision axis. The angle brackets $\rangle$ denote an average weighted by the initial density. However, recent RHIC $v_{2} / \varepsilon$ data for charged hadrons $h^{ \pm}$and strangeness containing hadrons $K_{S}^{0}, \phi, \Lambda, \Xi$ show a trend to increase as a function of the particle density scaled by the system-size [30, 31], lacking the saturation indicated by ideal hydrodynamic calculations 31]. This monotonic increase is a feature of a class of model descriptions that conform to the low-density limit 32]. Whether the thermalization and ideal hydrodynamic limit are reached or not at RHIC is not conclusive. A transport model suggested in Ref. [33] is constructed to link the low-density limit to the ideal hydrodynamic limit. In the micro-
scopic transport picture, the ideal hydrodynamic limit is reached when the mean free path is very small or the cross section is very large. With this transport model approach, the degree of thermalization and the ideal hydrodynamic limit can be addressed [34].

The previous results mainly focus on the centrality dependence of charged hadron and identified hadron $v_{2}$ in $\mathrm{Au}+\mathrm{Au}$ collisions. Since the conditions in $\mathrm{Au}+\mathrm{Au}$ collisions might not hold in smaller systems and at lower beam energies, the system-size and beam-energy dependence of identified hadron elliptic flow will shed light on the properties of partonic collectivity and quark degrees of freedom. Further, the study of $v_{2}$ in collisions of nuclei smaller than $\mathrm{Au}+\mathrm{Au}$ will allow us to test the early thermalization hypothesis in $\mathrm{Au}+\mathrm{Au}$ collisions. To date, there are only a few studies of identified hadron $v_{2}$ in $\mathrm{Cu}+\mathrm{Cu}$ collisions. In this article, we present the results on the azimuthal anisotropy parameter $v_{2}\left(p_{T}\right)$ of $h^{ \pm}, K_{S}^{0}$, $\Lambda, \Xi$ and $\phi$ from $\sqrt{s_{N N}}=62.4$ and $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. As a function of collision centrality, the scaling properties of $v_{2}$ with the transverse kinetic energy $m_{T}-m$ and the number of constituent quarks $n_{q}$ are reported. In the quantity $m_{T}=\sqrt{p_{T}^{2}+m^{2}}, m$ denotes the rest mass of a given hadron. We also discuss system-size dependence in this article.

The rest of the paper is organized in the following way: Section II summarizes the analysis details including data and centrality selections, particle identification and flow methods used for charged hadrons and identified hadrons. In the following, we use $h^{ \pm}, \Lambda$ and $\Xi$ to denote charged hadron, $\Lambda+\bar{\Lambda}$ and $\Xi^{-}+\bar{\Xi}^{+}$, respectively. In Section III we present measurements of $v_{2}$ for $h^{ \pm}$in $\mathrm{Cu}+\mathrm{Cu}$ collisions from different analysis methods. Differences in $v_{2}$ are used to estimate the systematic error. Section IV presents the results and physics discussion of the scaling properties and system-size dependence along with ideal hydrodynamic calculations. Last, a summary is presented in Section V .

## II. EXPERIMENTS AND ANALYSIS

## A. Experiments and data sets

For this article, our data were collected from $\sqrt{s_{N N}}$ $=62.4$ and $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions with the STAR detector during the fifth RHIC run in 2005. In addition data from $\sqrt{s_{N N}}=200 \mathrm{GeV} p+p$ collisions in 2005 were used in the analysis of non-flow contributions. STAR's main Time Projection Chamber (TPC) 35] and two Forward Time Projection Chambers (FTPCs) 36] were used for particle tracking in the central region $(|\eta|<1.0)$ and forward regions $(2.5<|\eta|<4.0)$, respectively. Both the TPC and FTPCs provide azimuthal acceptance over $2 \pi$. Only those events which have the primary vertex position along the longitudinal beam direction $\left(V_{z}\right)$ within 30 cm of the nominal collision point are selected for the analysis. This is done in order to have a more uniform detector
performance within $|\eta|<1.0$. The centrality definition, which is based on the raw charged particle TPC multiplicity with $|\eta|<0.5$, is the same as used previously [37]. After quality cuts, the number of the $60 \%$ most central events is about 24 million for $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions and 10 million for $62.4 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The results from more peripheral collisions are not presented due to trigger inefficiencies at low multiplicity.

The centrality was defined using the number of charged tracks with quality cuts similar to those in Ref. 31]. The $60 \%$ most central events for $v_{2}$ analysis of $h^{ \pm}$were divided into six centrality bins, each spanning an interval of $10 \%$ of the geometric cross section. For $v_{2}$ analysis of $K_{S}^{0}$ and $\Lambda$, centrality bins of $0-20 \%$ and $20-60 \%$ were used. In order to reduce the multiplicity fluctuations in wide centrality bins, we calculated $v_{2}$ in the $10 \%$ wide bins, then combined them using particle yield as the weight.

To select good tracks from primary collisions, charged particle tracks coming from the collision which transversed the TPC or FTPCs were selected by requiring the distance of closest approach to the primary vertex be less than 3 cm . Tracks used for $K_{S}^{0}, \Lambda$ and $\Xi$ reconstruction were not subject to this cut. We required that the TPC and FTPCs had a number of hits used for reconstruction of the tracks of the particles $>15$ and $>5$, respectively. For the TPC and FTPCs the ratio of the number of fit hits to maximum possible hits was $>0.52$. An additional transverse momentum cut ( $0.15<p_{T}<2 \mathrm{GeV} / c$ ) was applied to the charged tracks for the event plane determination.

## B. Particle identification

We utilized the topology of decay as measured with the TPC inside the magnetic field to identify $K_{S}^{0}, \phi$, $\Lambda(\bar{\Lambda})$ and $\Xi^{-}\left(\bar{\Xi}^{+}\right)$. We used the decay channels: $K_{S}^{0} \rightarrow \pi^{+}+\pi^{-}, \phi \rightarrow K^{+}+K^{-}, \Lambda \rightarrow p+\pi^{-}\left(\bar{\Lambda} \rightarrow \bar{p}+\pi^{+}\right)$, $\Xi^{-} \rightarrow \Lambda+\pi^{-}\left(\bar{\Xi}^{+} \rightarrow \bar{\Lambda}+\pi^{+}\right)$. Similar to the previous analysis in Ref. [31], topological and kinematic cuts were applied to reduce the combinatorial backgrounds. Figure 1 shows the invariant mass distributions for (a) $K_{S}^{0}$, (b) $\phi$, (c) $\Lambda$ and (d) $\Xi$ for selected $p_{T}$ bins in $\sqrt{s_{N N}}$ $=200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu} 60 \%$ most central collisions. The cuts used for $\mathrm{Cu}+\mathrm{Cu}$ collisions are similar to those for $\mathrm{Au}+\mathrm{Au}$ collisions in Ref. [31]. The combinatorial backgrounds were estimated from the fourth order polynomial fits for $K_{S}^{0}$ and $\Lambda$ [31]. The invariant mass distribution for $\phi$ is after subtraction of combinatorial background estimated using event mixing 38], the remaining combinatorial backgrounds were estimated by a first order polynomial fit [20]. For $\Xi$, the background was estimated by rotating the transverse momentum of the daughter $\Lambda$ by $180^{\circ}$. This operation breaks the correlation between the $\Lambda$ and the other daughter particle. The resulting invariant mass distributions provide a good approxima-


FIG. 1: (Color online) Invariant mass distributions for (a) $K_{S}^{0}\left(1.2<p_{T}<1.4 \mathrm{GeV} / c\right)$, (b) $\phi\left(1.0<p_{T}<2.0 \mathrm{GeV} / c\right)$, (c) $\Lambda\left(1.4<p_{T}<1.6 \mathrm{GeV} / c\right)$ and (d) $\Xi\left(1.25<p_{T}<1.75 \mathrm{GeV} / c\right)$ in $\sqrt{s_{N N}}=200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu} 60 \%$ most central collisions. The solid curves represent the fits to the invariant mass distributions: Gaussians plus fourth-order polynomials for $K_{S}^{0}, \Lambda$ and $\Xi$, Breit-Wigner plus a linear function for $\phi$. The dotted curves are the estimated backgrounds: the fourth order polynomials for $K_{S}^{0}$ and $\Lambda$, a linear function for $\phi$, and a rotation method described in the text for $\Xi$. For clarity, the invariant mass distributions for $K_{S}^{0}, \Lambda, \phi$ and $\Xi$ are scaled by $1 / 50000,1 / 130000,1 / 5000$ and $1 / 8000$, respectively. The error bars are shown only for the statistical uncertainties.
tion of the true background distribution. The detailed description of the method can be found in Refs. 12, 13].

## C. Flow methods

Anisotropic flow, which is an anisotropy in the particle production relative to the reaction plane, results in correlations among particles and can be studied by the analysis of these correlations. At the same time these correlations are affected by other effects which are not related to the orientation of the reaction plane. Such are commonly referred to as non-flow, and are due, e.g., to resonance decays and jet production. Different methods used to measure anisotropic flow are affected by non-flow effects in different ways, and are used in this analysis to evaluate the systematic uncertainty of the measurements.

## 1. Event Plane method with TPC event plane

The Event Plane method [2] uses the anisotropic flow itself to determine the event plane (the estimated reaction plane), which can be done for each harmonic. The second harmonic flow vector, $Q_{2}$, of the event is constructed using the TPC tracks $i$ in the event with their azimuthal angle, $\phi_{i}$, according to Eq. (2) and Eq. (3). In order to maximize the resolution of the flow effect, the weights $w_{i}$ are set equal to $p_{T}$ up to $2 \mathrm{GeV} / c$.

$$
\begin{equation*}
Q_{2} \cos \left(2 \Psi_{2}\right)=Q_{2 x}=\sum_{i} w_{i} \cos \left(2 \phi_{i}\right) \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
Q_{2} \sin \left(2 \Psi_{2}\right)=Q_{2 y}=\sum_{i} w_{i} \sin \left(2 \phi_{i}\right) \tag{3}
\end{equation*}
$$

Elliptic flow is first calculated with respect to the event plane angle $\Psi_{2}$ as shown in Eq. (4), which is called the observed $v_{2}$. The angle brackets indicate an average over all particles in all events. However, tracks used for the $v_{2}$ calculation are excluded from the calculation of the flow vector to remove auto-correlation effects. Then the observed $v_{2}$ is corrected by the event plane resolution (the denominator in Eq. (51) ) to obtain $v_{2}$ relative to the event plane.

$$
\begin{align*}
& v_{2}^{\mathrm{obs}}=\left\langle\cos \left[2\left(\phi-\Psi_{2}\right)\right]\right\rangle  \tag{4}\\
& v_{2}=\frac{v_{2}^{\mathrm{obs}}}{\left\langle\cos \left[2\left(\Psi_{2}-\Psi_{r}\right)\right]\right\rangle} \tag{5}
\end{align*}
$$

The results are denoted as $v_{2}\{\mathrm{TPC}\}$ in the following.
Since the reaction plane is unknown, the denominator in Eq. (5) is still not calculable. As shown in Eq. (6), we estimate the event plane resolution by the correlations between the azimuthal angles of two subset groups of tracks, called subevents A and B. In this analysis, we use two random subevents with equal numbers of particles. In Eq. (6) $C$ is a constant calculated from the known multiplicity dependence of the resolution [2].

$$
\begin{equation*}
\left\langle\cos \left[2\left(\Psi_{2}-\Psi_{r}\right)\right]\right\rangle=C \sqrt{\left\langle\cos \left[2\left(\Psi_{2}^{A}-\Psi_{2}^{B}\right)\right]\right\rangle} \tag{6}
\end{equation*}
$$

In the case of low resolution $(\leq 0.2)$, such as for the FTPC event plane, $C$ approaches $\sqrt{2}$.

The reaction plane azimuthal distribution should be isotropic in the laboratory frame. Thus, the event plane azimuthal distribution has to be flat if the detectors have ideal acceptance. Since the detectors usually have nonuniform acceptance, a procedure for flattening the event plane distribution is necessary. For the event plane reconstructed from TPC tracks, the $\phi$ weight method is an effective way to flatten the distribution. $\phi$ weights are generated by inverting the $\phi$ distributions of detected tracks for a large event sample. The detector acceptance bias is removed by applying the $\phi$ weight at the $\phi$ of each track to that track. The $\phi$ weights are folded into the weight $w_{i}$ in Eq. (2) and Eq. (3). Independent corrections were applied to each centrality selection in $10 \%$ increments and in 2 bins in the primary vertex position along the longitudinal beam direction $\left(V_{z}\right)$. The corrections were done on a run by run basis (around $50 k$ events).

## 2. Event Plane method with FTPC event plane

The $\eta$ gap between two FTPCs sitting at two sides of the collision in the forward regions can be used to reduce non-flow effects due to short-range correlations. The basic procedures are similar to those for the Event Plane method with the TPC event plane. There are three steps: estimate the event plane with FTPC tracks, calculate $v_{2}$ with respect to the event plane, and obtain the real $v_{2}$ by correction to the real reaction plane. Equations (2)- (6) can be applied, except that: i) the sums in Eq. (2) and Eq. (3) go over FTPC tracks instead of TPC tracks, and ii) two subset groups of tracks are classified according to the sign of $\eta$. The tracks with $-4<\eta<-2.5$ and $2.5<\eta<4$ are called East subevent and West subevent, respectively. Hence, the resolution in Eq. (6) is calculated by the correlation between the azimuthal angles $\Psi_{2}^{\text {East }}$ and $\Psi_{2}^{\text {West }}$. The average in Eq. (4) runs over the TPC tracks as before. The result of this procedure is denoted as $v_{2}\{\mathrm{FTPC}\}$.

Due to the serious loss of acceptance for FTPCs due to partially non-functioning readout electronics, the number of tracks detected by the best sector is about 6 times greater than for the worst one. The result is that the $\phi$ weight method is not enough to generate a flat event plane distribution. Thus, further small corrections are applied after $\phi$ weight corrections using the shift method [39]. Equation (7) shows the formula for the shift correction. The averages in Eq. (7) are taken from a large sample of events. In this analysis, the correction is done up to the twentieth harmonic. This was done in order to make the $\chi^{2}$ divided by the number of degrees of freedom of a flat fit to the event plane azimuthal angle distribution to be less than 1. The distributions of $\Psi_{2}^{\text {East }}$ and $\Psi_{2}^{\text {West }}$ are separately flattened and then the full-event event plane distribution is flattened. Accordingly, the observed $v_{2}$ and resolution are calculated using the shifted (sub)event plane azimuthal angles.

$$
\begin{align*}
\Psi^{\prime} & =\Psi+\sum_{n} \frac{1}{n}[-\langle\sin (2 n \Psi)\rangle \cos (2 n \Psi) \\
& +\langle\cos (2 n \Psi)\rangle \sin (2 n \Psi)] \tag{7}
\end{align*}
$$

## 3. Scalar Product method

The Scalar Product method 30, 40] is similar to the Event Plane method, and gives $v_{2}$ as:

$$
\begin{equation*}
v_{2}\left(p_{T}\right)=\frac{\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle}{2 \sqrt{\left\langle Q_{2}^{A} Q_{2}^{B *}\right\rangle}} \tag{8}
\end{equation*}
$$

where $u_{2, i}=\cos \left(2 \phi_{i}\right)+i \sin \left(2 \phi_{i}\right)$ is a unit vector of the $i$ th particle, $Q_{2}=\sum_{k} u_{2, k}$ is the flow vector with the sum running over all other particles $k$ in the event. The superscript * denotes the complex conjugate of a complex number. $A$ and $B$ denote the two subevents. In the case that $Q_{2}$ is normalized to a unit vector, Eq. (8) reduces to the Event Plane method. In the Scalar Product method, one can use a different (re-centering) technique 41] to correct for detector effects, which presents an alternative to the weighting and shifting procedures described in Sec. IIC 1-2 above. The Scalar Product method is applied to the $v_{2}$ measurement of charged hadrons.

## D. $\quad v_{2}$ versus $m_{i n v}$ method

For $v_{2}$ of the identified particles $K_{S}^{0}, \phi, \Lambda$ and $\Xi$, the $v_{2}$ versus $m_{\text {inv }}$ method is used [31, 42]. Since $v_{2}$ is additive, one can write the total $v_{2}^{\mathrm{Sig}+\mathrm{Bg}}$ as a sum of Signal and Background contributions weighted by their relative yields:

$$
\begin{array}{r}
v_{2}^{\mathrm{Sig}+\mathrm{Bg}}\left(m_{\mathrm{inv}}\right)=v_{2}^{\mathrm{Sig}} \frac{\mathrm{Sig}}{\mathrm{Sig}+\mathrm{Bg}}\left(m_{\mathrm{inv}}\right)+ \\
v_{2}^{\mathrm{Bg}}\left(m_{\mathrm{inv}}\right) \frac{\mathrm{Bg}}{\mathrm{Sig}+\mathrm{Bg}}\left(m_{\mathrm{inv}}\right) \tag{9}
\end{array}
$$

This method involves the calculation of $v_{2}^{\mathrm{Sig}+\mathrm{Bg}}$ as a function of $m_{\text {inv }}$ and then fitting the distribution using Eq. (9) with measured relative yields and parameterizations of $v_{2}^{\mathrm{Sig}}$ and $v_{2}^{\mathrm{Bg}}$. The $\frac{\mathrm{Bg}}{\mathrm{Sig}+\mathrm{Bg}}\left(m_{\mathrm{inv}}\right)$ distribution is the Bg divided by $(\mathrm{Sig}+\mathrm{Bg})$. The $\frac{\mathrm{Sig}}{\mathrm{Sig}+\mathrm{Bg}}\left(m_{\mathrm{inv}}\right)$ distribution is simply calculated by $1-\frac{\mathrm{Bg}}{\operatorname{Sig}+\mathrm{Bg}}\left(m_{\text {inv }}\right)$. The term $v_{2}^{\mathrm{Bg}}$ is parameterized as a linear function in order to take care of the non-constant $v_{2}^{\mathrm{Bg}}$ value as a function of $m_{\text {inv }}$. The fit result $v_{2}^{\mathrm{Sig}}$ is the final observed $v_{2}$. Why this method works well for measuring signal $v_{2}$ is explained as follows: a set of data points is used in the fit over a wide $m_{\mathrm{inv}}$ region for Sig and Bg . Data points far from the mass peak constrain $v_{2}^{\mathrm{Bg}}\left(m_{\text {inv }}\right)$, since pure Bg is expected in


FIG. 2: (Color online) Charged hadron azimuthal correlations as a function of $p_{T}$ in $\sqrt{s_{N N}}=200 \mathrm{GeV} 60 \%$ most central $\mathrm{Cu}+\mathrm{Cu}$ collisions (closed squares) compared to those from $\sqrt{s_{N N}}=200 \mathrm{GeV} p+p$ collisions (open squares). Flow vector calculated from (a) TPC tracks, (b) FTPC tracks. The error bars are shown only for the statistical uncertainties.
this region. Under the peak, the $v_{2}^{\mathrm{Sig}+\mathrm{Bg}}\left(m_{\text {inv }}\right)$ is dominated by the Sig distribution. Finally, the $v_{2}$ signal is extracted by the fitting method shown in Eq. (9).

Note that the subtraction procedure used to extract the $v_{2}$ signal for a given identified particle is independent of the flow correlations. The $v_{2}$ distributions of the overall signal and background are evaluated by one of the flow analysis methods discussed in Sec. IIC 173. In this paper, the Event Plane method with the FTPC event plane is applied for $K_{S}^{0}, \phi, \Lambda$ and $\Xi$.

## E. Non-flow contribution for various methods

The method of determining $v_{2}$ using cumulants of various orders has been shown to eliminate non-flow correlations. However, the method is useful only for large values of flow and multiplicity. For the relatively low values of flow and multiplicity seen in $\mathrm{Cu}+\mathrm{Cu}$ collision, the non-flow correlations have been estimated, as described below.

The Event Plane method with the TPC event plane is sensitive to non-flow effects. Particles of interest tend to correlate with particles used in the flow vector calculation due to short-range non-flow correlations. Also, particles of two random sub-events tend to have those correlations. Thus, non-flow exists in both the observed $v_{2}$ (Eq. (4))


FIG. 3: (Color online) (a) Charged hadron $v_{2}\left(p_{T}\right)$ in $\sqrt{s_{N N}}$ $=200 \mathrm{GeV} 0-60 \% \mathrm{Cu}+\mathrm{Cu}$ collisions. Open circles, closed circles, open squares and closed squares represent the results of $v_{2}$ as function of $p_{T}$ measured by the TPC flow vector $\left(v_{2}\{\mathrm{TPC}\}\right)$, the FTPC flow vector $\left(v_{2}\{\right.$ FTPC $\left.\}\right)$, the TPC and FTPC flow vector with subtracting the azimuthal correlations in $p+p$ collisions ( $\left.v_{2}\{A A-p p, \mathrm{TPC}\}, v_{2}\{A A-p p, \mathrm{FTPC}\}\right)$. (b) The ratio of the results for the various methods described in (a). The error bars are shown only for the statistical uncertainties.
and the resolution (Eq. (6)). To reduce non-flow effects due to short-range correlations, we take advantage of the large $\eta$ gap between the two FTPCs sitting at the two sides of the collision in the forward regions. Non-flow is reduced by the $\eta$ gap between the TPC and FTPCs, but this may not be large enough to remove all non-flow correlations. Thus, we investigate these effects by comparing the azimuthal correlations measured in $\mathrm{Cu}+\mathrm{Cu}$ to those in $p+p$ collisions, where all correlations are assumed to be of non-flow origin [43]. Taking into account the non-flow contribution, the numerator of Eq. (8) can be written as follows 30, 43]:

$$
\begin{align*}
\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle & =\left\langle\sum_{k} \cos \left[2\left(\phi_{p_{T}}-\phi_{k}\right)\right]\right\rangle  \tag{10}\\
& =M v_{2}\left(p_{T}\right) \overline{v_{2}}+\text { nonflow }
\end{align*}
$$

where $\phi_{p_{T}}$ is the azimuthal angle of particles from a given $p_{T}$ bin ( $u_{2, i}^{*}$ in Eq. (8) ) and the sum goes over all tracks $k$ in an event used to determine the flow vector ( $Q_{2}$ in Eq. (8)). The angled brackets denote averaging over the events. The first term in the right-hand side of Eq. (10)


FIG. 4: (Color online) Charged hadron $v_{2}\{$ FTPC $\}$ (closed circles) and $v_{2}\{A A-p p$, FTPC $\}$ (open circles) as function of $p_{T}$ in $\sqrt{s_{N N}}=200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions for centrality bins: (a) $50-60 \%$, (b) $40-50 \%$, (c) $30-40 \%$, (d) $20-30 \%$, (e) $10-20 \%$ and (f) $0-10 \%$. The percentages refer to fraction of most central events. The error bars are shown only for the statistical uncertainties.


FIG. 5: Ratios of $v_{2}\{A A-p p, \mathrm{FTPC}\} / v_{2}\{\mathrm{FTPC}\}$ for charged hadron as function of $p_{T}$ in $\sqrt{s_{N N}}=200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions for centrality bins: (a) $50-60 \%$, (b) $40-50 \%$, (c) $30-40 \%$, (d) $20-30 \%$, (e) $10-20 \%$ and (f) $0-10 \%$. The percentages refer to fraction of most central events.
represents the contribution from elliptic flow. $v_{2}\left(p_{T}\right)$ is the value of elliptic flow at a given $p_{T} . \overline{v_{2}}$ is the elliptic flow on average for all particles used in the sum of Eq. (10). The multiplicity of particles contributing to the sum is denoted by $M$. All other correlations subject to non-flow go to the second term in the right-hand side of Eq. (10). It is assumed that the quantity $\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle$ in $p+p$ collisions can be used to estimate the non-flow in $A A$ collisions 40, 43].

$$
\begin{equation*}
M v_{2}\left(p_{T}\right) \overline{v_{2}}=\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle_{A A}-\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle_{p p} \tag{11}
\end{equation*}
$$

Dividing both sides by $2 \sqrt{\left\langle Q_{2}^{A} Q_{2}^{B *}\right\rangle_{A A}}$ as in Eq. (8) gives

$$
\begin{equation*}
v_{2}\{A A-p p\}\left(p_{T}\right)=\frac{\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle_{A A}-\left\langle Q_{2} u_{2, i}^{*}\left(p_{T}\right)\right\rangle_{p p}}{2 \sqrt{\left\langle Q_{2}^{A} Q_{2}^{B *}\right\rangle_{A A}}} \tag{12}
\end{equation*}
$$

because $2 \sqrt{\left\langle Q_{2}^{A} Q_{2}^{B *}\right\rangle_{A A}}=2 \sqrt{(M / 2) \overline{v_{2}}(M / 2) \overline{v_{2}}}=M \overline{v_{2}}$.
Comparing $p+p$ and $A A$ collisions, one might expect some changes in particle correlations: there could be an increase in correlations due to a possible increase of jet multiplicities in $A A$ collisions or, conversely, some decrease due to the suppression of high $p_{T}$ back-to-back correlations [44]. On the other hand, $A A$ collisions exhibit long $\eta$ range correlations (the "ridge") 45, 46], which are not seen in $p+p$ collisions and the origin of which is under


FIG. 6: (Color online) Charged hadron $v_{2}$ integrated over $p_{T}$ and $\eta$ vs. centrality for the various methods described in the text in $\sqrt{s_{N N}}=200 \mathrm{GeV}$ and $62.4 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The error bars are shown only for the statistical uncertainties.

TABLE I: Summary of systematic errors of $v_{2}$ due to the reconstruction procedure of strange hadrons in $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$.

|  | $K_{S}^{0}$ |  | $\Lambda$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Centrality | Background Cut criteria | Background Cut criteria |  |  |
| $0-60 \%$ | $1 \%$ | $2 \%$ | $1 \%$ | $2 \%$ |
| $0-20 \%$ | $1 \%$ | $2 \%$ | $1 \%$ | $4 \%$ |
| $20-60 \%$ | $4 \%$ | $1 \%$ | $5 \%$ | $1 \%$ |

investigation 47. Thus it is difficult to make an accurate estimate of non-flow contributions. The fact that at high $p_{T}\left(p_{T}>5 \mathrm{GeV} / c\right)$ the $p+p$ results are very close to central $\mathrm{Au}+\mathrm{Au}$ [40, 43] suggests that the uncertainties are relatively small. In the following we estimate the systematic uncertainties arising from non-flow contributions. We use $v_{2}\{A A-p p, \mathrm{TPC}\}$ and $v_{2}\{A A-p p, \mathrm{FTPC}\}$ to denote $v_{2}\{A A-p p\}$ calculated with TPC and FTPC flow vectors, respectively.

## III. SYSTEMATIC UNCERTAINTIES

Non-flow is one of the largest uncertainties in elliptic flow measurements. As we mentioned in Section IIC this effect can be investigated by comparing the azimuthal correlations measured in $\mathrm{Cu}+\mathrm{Cu}$ collisions to those in $p+p$ collisions. The event average of the sum of the correlations is given by Eq. (10).

Figure 2 shows the azimuthal correlation, Eq. (10), as a function of $p_{T}$ for the $0-60 \%$ centrality range in
$\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$, compared to $p+p$ collisions. As we can see, the azimuthal correlations in $\mathrm{Cu}+\mathrm{Cu}$ collisions, shown as solid squares, increase with $p_{T}$ and then saturate above $2 \mathrm{GeV} / c$ while those in $p+p$ collisions, shown as open squares, monotonically increase with $p_{T}$ in the case of the TPC flow vector. With the flow vector determined from FTPC tracks the azimuthal correlations around midrapidity in $p+p$ collisions are small when $p_{T}$ is less than $4 \mathrm{GeV} / c$. It means that one strongly reduces the non-flow effects with the FTPC flow vector relative to the one seen with the TPC flow vector.

In order to illustrate the sensitivity to non-flow for the various flow analysis methods, we first analyzed $h^{ \pm}$elliptic flow in the $60 \%$ most central $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. As shown in Fig. 3 (a), the fact that $v_{2}\{\mathrm{TPC}\}$ is significantly larger than $v_{2}\{\mathrm{FTPC}\}$ indicates a larger non-flow effect in $v_{2}\{\mathrm{TPC}\}$. With the large $\eta$ gap between West and East FTPCs, non-flow effects due to the short-range correlations are reduced in $v_{2}\{\mathrm{FTPC}\} . v_{2}\{\mathrm{FTPC}\}$ saturates at $p_{T} \sim 2.5 \mathrm{GeV} / c$ and then falls off slightly up to $p_{T} \sim 4 \mathrm{GeV} / c$. In order to estimate the remaining non-flow effects in $v_{2}\{\mathrm{FTPC}\}$, we subtract the azimuthal correlations of $p+p$ collisions from those in $\mathrm{Cu}+\mathrm{Cu}$ collisions according to Eq. (12). In Fig. 3 (a),$v_{2}\{A A-p p, \mathrm{FTPC}\}$ is close to $v_{2}\{\mathrm{FTPC}\}$ in the region $p_{T}<4 \mathrm{GeV} / c$. To quantitatively illustrate non-flow systematic uncertainties, Fig. 3 (b) shows the ratios of $v_{2}\{A A-p p, \mathrm{FTPC}\}$ to $v_{2}\{\mathrm{FTPC}\}$, $v_{2}\{A A-p p, \mathrm{TPC}\}$ to $v_{2}\{A A-p p, \mathrm{FTPC}\}$ and $v_{2}\{\mathrm{FTPC}\}$ to $v_{2}\{\mathrm{TPC}\}$ as a function of $p_{T} \cdot v_{2}\{\mathrm{FTPC}\} / v_{2}\{\mathrm{TPC}\}$ shows that non-flow in $v_{2}\{\mathrm{TPC}\}$ increases from $20 \%$ at $p_{T} \sim 0.8 \mathrm{GeV} / c$ to $40 \%$ at $p_{T} \sim 3.5 \mathrm{GeV} / c$. Based on the comparison between $v_{2}\{A A-p p$, FTPC $\}$ and $v_{2}\{$ FTPC $\}$, the residual non-flow in $v_{2}\{\mathrm{FTPC}\}$ is less than $10 \%$ below $p_{T} \sim 4 \mathrm{GeV} / c$. We also checked the $v_{2}\{A A-p p\}$ calculated with the TPC flow vector. Beyond $p_{T} \sim 3 \mathrm{GeV} / c$,


FIG. 7: (Color online) Charged hadron $v_{2}$ as function of $p_{T}$ for $50-60 \%$ (solid circles), $40-50 \%$ (solid squares), $30-40 \%$ (solid triangles), $20-30 \%$ (open circles), $10-20 \%$ (open squares) and $0-10 \%$ (open triangles) in $\sqrt{s_{N N}}=200 \mathrm{GeV}$ and 62.4 $\mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The error bars are shown only for the statistical uncertainties.


FIG. 8: (Color online) (a) Charged hadron $v_{2}$ as a function of $p_{T}$ in $\mathrm{Cu}+\mathrm{Cu}$ collisions. The results from $\sqrt{s_{N N}}=200$ GeV and 62.4 GeV are presented by open symbols and closed symbols, respectively. (b) Ratios of the $v_{2}\left(p_{T}\right)$ from $\sqrt{s_{N N}}=$ 62.4 GeV to 200 GeV . The error bars are shown only for the statistical uncertainties.
$v_{2}\{A A-p p, \mathrm{TPC}\}$ seems systematically lower, but within errors it is similar to $v_{2}\{A A-p p$, FTPC $\}$. This shows that most of the non-flow is eliminated by subtracting the azimuthal correlation in $p+p$ collisions, validating our
earlier assumption.
To illustrate the centrality dependence of the systematic uncertainties, Fig. 4 shows $v_{2}\{$ FTPC $\}$ and $v_{2}\{A A-$ $p p$, FTPC $\}$ as a function of $p_{T}$ for six centrality bins. Ratios of $v_{2}\{A A-p p, \mathrm{FTPC}\}$ to $v_{2}\{\mathrm{FTPC}\}$ for each centrality bin are shown in Fig. 5 from (a) the most peripheral bin $50-60 \%$ to (f) the most central bin $0-10 \%$. For each centrality bin, the ratio falls off slightly as $p_{T}$ increases. For the two peripheral bins $50-60 \%$ and $40-50 \%$, the ratios drop faster than in the other bins, indicating larger non-flow contributions in $v_{2}\{\mathrm{FTPC}\}\left(p_{T}\right)$ in peripheral $\mathrm{Cu}+\mathrm{Cu}$ collisions. Figure 6 shows charged hadron $v_{2}$ integrated over $p_{T}\left(0.15<p_{T}<4 \mathrm{GeV} / c\right)$ and $\eta(|\eta|<1.0)$ vs. centrality for the various methods. It is clear that $v_{2}\{\mathrm{TPC}\}$ is much higher than for the other methods, especially for the peripheral collisions.

To summarize the non-flow systematics we employed the Scalar Product method with TPC and FTPC flow vectors for $h^{ \pm}$in $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. The results for the $60 \%$ most central events are shown in Fig. 3. $v_{2}\{\mathrm{TPC}\}$ has large non-flow contributions while $v_{2}\{$ FTPC $\}$ eliminates most of the non-flow. In what follows, we will report our results in term of $v_{2}\{\mathrm{FTPC}\}$. For simplicity $v_{2}$ denotes $v_{2}\{$ FTPC $\}$ except when the flow method is explicitly specified. With the assumption of pure non-flow effects in $p+p$ collisions, we use $v_{2}\{A A-p p$, FTPC $\}$ to estimate non-flow systematic errors in $v_{2}\{\mathrm{FTPC}\}$. Ratios of $v_{2}\{A A-p p, \mathrm{FTPC}\}$ to $v_{2}\{$ FTPC $\}$ are shown for the $60 \%$ most central events in Fig. 3 (b) and six centrality bins in Fig. 5. The ratios show that non-flow effects increase with $p_{T}$ for all centrality bins and non-flow effects are larger in more peripheral bins. In order to estimate the non-flow systematic error in $v_{2}\{\mathrm{FTPC}\}$, we fitted a constant to the ratio $v_{2}\{A A-p p, \mathrm{FTPC}\} / v_{2}\{\mathrm{FTPC}\}$ in the $p_{T}$ range $(0$,


FIG. 9: (Color online) $v_{2}$ of $K_{S}^{0}$ (open circles), $\Lambda$ ( solid squares), $\Xi$ (solid triangles) and $\phi$ (open stars) as a function of $p_{T}$ for (a1) $0-60 \%$, (a2) $0-20 \%$, (a3) $20-60 \%$ and as a function of $m_{T}-m$ for (b1) $0-60 \%$, (b2) $0-20 \%$, (b3) $20-60 \%$. For comparisons, the results from ideal hydrodynamic calculations 49, 50] are also shown. At a given $p_{T}$, from top to bottom, the curves represent $\pi, K, p, \phi, \Lambda, \Xi$ and $\Omega$. When $p_{T}$ is converted to $m_{T}-m$, this mass hierarchy is reversed in the model results. All data are from $\sqrt{s_{N N}}=200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The error bars are shown only for the statistical uncertainties.
$4 \mathrm{GeV})$. We take the numerical value of this constant as the estimate of the systematic uncertainty. The resulting non-flow systematic error is minus $5 \%$ for $0-10 \%$, $10-20 \%, 20-30 \%$ and $30-40 \%$ collisions and minus $10 \%$ for $40-50 \%$ and $50-60 \%$ collisions. Although for $K_{S}^{0}, \phi, \Lambda$ and $\Xi v_{2}$ the non-flow effects may be different, as we don't have enough statistics to repeat the analysis, we simply assume a similar magnitude for the non-flow systematic error.

The other systematic uncertainties in the $v_{2}$ analysis procedure are studied as follows. We estimate the systematic errors from the shifting method for the FTPC event plane by comparing $v_{2}$ using different maximum harmonics in Eq. (77) and find the systematic errors are less than $1 \%$. The systematic errors in $K_{S}^{0}$ and $\Lambda v_{2}$ resulting from the background uncertainty and topological cut criteria are estimated using the Event Plane method. The uncertainty due to the background subtraction is estimated as the relative differences in $v_{2}$ from fitting the background using second and fourth order polynomials. The systematic uncertainty is also estimated by varying the cut parameters. The systematic errors for $K_{S}^{0}$ and $\Lambda$ from the background uncertainty and the cut criteria are summarized in Table [] From Ref. 31], the estimated systematic uncertainty of $\Lambda$ from feed-down is less than
$2 \%$.

## IV. RESULTS AND DISCUSSION

## A. Charged hadrons

Flow results for charged hadrons were determined using the Scalar Product method Eq.(8) with the flow vector derived from the FTPC tracks. A comparison to $v_{2}\{A A-p p$, FTPC $\}$ Eq. (12) was used to estimate the systematic error. Figure 7 shows $v_{2}\left(p_{T}\right)$ of $h^{ \pm}$for six centrality bins from $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200$ and 62.4 GeV . For a given centrality bin, $v_{2}\left(p_{T}\right)$ initially increases with $p_{T}$. At higher $p_{T}\left(p_{T}>2 \mathrm{GeV} / c\right)$, $v_{2}$ appears to saturate or decrease. $v_{2}\left(p_{T}\right)$ in more peripheral collisions increases faster and reaches higher values as expected for the larger eccentricity.

At low $p_{T}$, the increase of $v_{2}\left(p_{T}\right)$ with $p_{T}$ is consistent with predictions from ideal hydrodynamic calculations, which will be shown in Fig. 9 for identified particles. The model predicts that $v_{2}$ continues increasing beyond $p_{T} \sim$ $2 \mathrm{GeV} / c$. The observed saturation or decrease of $v_{2}\left(p_{T}\right)$ indicates that the model is not valid in this region. One expects that the model should be valid up to higher $p_{T}$ in


FIG. 10: (Color online) Same as Fig. 9 (a1) and (b1), but expanded for the low $p_{T}$ and $m_{T}-m$ regions. The data points with large errors have not been plotted. At a given $p_{T}$, from top to bottom, the curves represent the ideal hydrodynamic calculations for $\pi, K, p, \phi, \Lambda, \Xi$ and $\Omega[49,50]$. When $p_{T}$ is converted to $m_{T}-m$, this mass hierarchy is reversed in the model results. All data are from $0-60 \% \mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. The error bars are shown only for the statistical uncertainties.
a system with larger densities and larger volumes. This was observed in $200 \mathrm{GeV} \mathrm{Au}+\mathrm{Au}$ collisions [48] where $v_{2}\left(p_{T}\right)$ of $h^{ \pm}$saturated at higher $p_{T}$ in more central collisions. However, we do not observe the strong centrality dependence of saturation $p_{T}$ for $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions.

Figure 8 shows the comparison of $v_{2}$ for $h^{ \pm}$from $\sqrt{s_{N N}}$ $=62.4$ and $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The $p_{T}$ dependence of $v_{2}$ at the two energies is similar.

## B. Identified hadrons

The Event Plane method Eqs. (2)-(5) with event plane determined from the FTPC tracks was applied to $K_{S}^{0}, \Lambda$, $\Xi$ and $\phi$. The results are shown in Fig. 9 for the $60 \%$ most central events and also for the $0-20 \%$ and $20-60 \%$ centrality bins at midrapidity $|y|<1$. Due to limited statistics, $\Xi$ and $\phi$ are only shown for the $0-60 \%$ bin. The results from ideal hydrodynamic calculations 49, 50] for each centrality bin are shown for $\pi, K, p, \phi, \Lambda, \Xi$ and $\Omega$, which are displayed by the curves from top (bottom) to bottom (top) for the $p_{T}\left(m_{T}-m\right)$ dependence.

The gross features of $p_{T}$ dependence and hadron-type dependence are similar to those observed in 200 GeV $\mathrm{Au}+\mathrm{Au}$ collisions [31]. At low $p_{T}$, the hadron mass hierarchy (at a given $p_{T}$, the heavier the hadron, the smaller $v_{2}\left(p_{T}\right)$ ) is reproduced by ideal hydrodynamic calculations. (See Fig. 10 (a) for clarity.) Multi strange-quark hadrons $\Xi$ and $\phi$, which participate less in later hadronic
interactions than do single strange-quark hadrons $K_{S}^{0}$ and $\Lambda$, have sizable $v_{2}$. In particular, $\Xi$ is consistent with the mass ordering shown in Fig. 9 (a1). While the model can roughly reproduce the magnitude of the data for the $60 \%$ most central events sample, there is an obvious disagreement in centrality selected data. The model underpredicts $v_{2}\left(p_{T}\right)$ in the $0-20 \%$ bin while it over-predicts the data in the $20-60 \%$ bin. Effects not included in the model which may be relevant are geometrical fluctuations in the initial conditions and finite viscosity effects. It is unclear whether these effects can account for the difference between the model and data.

At higher $p_{T}$, the hydrodynamic model mass ordering breaks. $v_{2}\left(p_{T}\right)$ appears to depend on hadron type: $v_{2}\left(p_{T}\right)$ is grouped by mesons and baryons, with magnitude depending on the number of quarks within the mesons or baryons. Over the entire $p_{T}$ region, both the data and the model exhibit the same qualitative centrality dependence as observed for $200 \mathrm{GeV} \mathrm{Au}+\mathrm{Au}$ collisions 31]: the more peripheral the collision, the larger the $v_{2}$ values. Compared to the results for $200 \mathrm{GeV} \mathrm{Au}+\mathrm{Au}$ collisions 31], the splitting of $K_{S}^{0}$ and $\Lambda v_{2}\left(p_{T}\right)$ is smaller in both the mass ordering region and the hadron-type dependence region. This indicates smaller collective flow in $\mathrm{Cu}+\mathrm{Cu}$ than $\mathrm{Au}+\mathrm{Au}$ collisions, which will be seen more clearly in Section IV D

The transverse kinetic energy scaling first observed in $\mathrm{Au}+\mathrm{Au}$ collisions is also tested in Fig. 9, The results in Fig. 9 (a1)-(a3) are re-plotted as a function of the transverse kinetic energy $m_{T}-m$ in Fig. 9 (b1)-(b3). The


FIG. 11: (Color online) $v_{2} / n_{q}$ versus $p_{T} / n_{q}$ (panels (a1)-(a3)) and ( $\left.m_{T}-m\right) / n_{q}$ (panels (b1)-(b3)), where $n_{q}$ is the number of constituent quarks in the hadron. The parametrization Eq. (13) fitted to the data is shown as the dashed curves. All data are from $\sqrt{s_{N N}}=200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The error bars are shown only for the statistical uncertainties.
quantity $m$ denotes the rest mass of a given hadron. In the low $m_{T}-m$ region, $v_{2}\left(m_{T}-m\right)$ is a linearly increasing function and independent of hadron mass. Transverse kinetic energy scaling holds in the region $m_{T}-m<0.8$ $\mathrm{GeV} / c^{2}$, as observed in $\mathrm{Au}+\mathrm{Au}$ collisions 31, 51]. Calculations using ideal hydrodynamics are shown in each panel as a function of $m_{T}-m$. Contrary to the mass ordering as a function of $p_{T}$, the model shows the reversed mass ordering as a function of $m_{T}-m$ : the heavier the hadron, the larger the $v_{2}\left(m_{T}-m\right)$ value. The results of $K_{S}^{0}, \Lambda$ and $\Xi$ exhibit $m_{T}-m$ scaling in each centrality bin, while the model does not show any scaling. Since no pion results are available, the scaling test of the data is not conclusive. All these effects can be seen more clearly in Fig. 10

## C. Quark-number scaling

In $\mathrm{Au}+\mathrm{Au}$ collisions in the intermediate $p_{T}$ region, 2 $<p_{T}<4 \mathrm{GeV} / c$, the baryon-meson grouping of $v_{2}\left(p_{T}\right)$ follows the number-of-quark scaling: the $v_{2}$ of all hadrons fall onto a universal curve once $v_{2}$ and $p_{T}$ are divided by the number of quarks, $n_{q}$, in a given hadron [31, 52]. The observed scaling can be explained by the coalescence or recombination models [22, 53, 54], indicating
the constituent quark degree of freedom has been manifested before hadronization takes place. $n_{q}$ scaling is tested for various centrality bins in $200 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions: $v_{2}\left(p_{T}\right)$ and $v_{2}\left(m_{T}-m\right)$ scaled by $n_{q}$ are shown in Fig. 11 (a1)-(a3) and (b1)-(b3), respectively. The $n_{q^{-}}$ scaling formula of Ref. [55], which can be written as

$$
\begin{equation*}
\frac{f_{v_{2}}\left(p_{T}\right)}{n_{q}}=\frac{a}{1+e^{-\left(p_{T} / n_{q}-b\right) / c}}-d \tag{13}
\end{equation*}
$$

has been fitted to the data both in $p_{T}$ and $m_{T}-m$ for each centrality bin. In this formula, $a, b, c$ and $d$ are fit parameters, $n_{q}$ is the number of quarks. The $n_{q}$ scaling is observed for $p_{T} / n_{q}>0.8 \mathrm{GeV} / c$, whereas it is seen for the entire $\left(m_{T}-m\right) / n_{q}$ region. Below $\left(m_{T}-m\right) / n_{q} \sim 0.4$ $\mathrm{GeV} / c^{2}$, the $m_{T}-m$ scaling which was established at low $p_{T}$, now scaled by $n_{q}$ leads to the combined ( $m_{T}-$ $m) / n_{q}$ scaling. The universal $n_{q}$ scaling of $v_{2}$ suggests the manifestation of early partonic dynamics in both $\mathrm{Au}+\mathrm{Au}$ and $\mathrm{Cu}+\mathrm{Cu}$ collisions.

## D. Centrality and system-size dependence

The centrality and system-size dependence of $v_{2}$ is related to the physics of the system created in high energy

TABLE II: Participant eccentricity $\varepsilon_{\text {part }}\{2\}$ and number of participants $N_{\text {part }}$ from the Monte Carlo Glauber model [29, 57 and Color Glass Condensate (CGC) model [58 61] calculations in $\mathrm{Au}+\mathrm{Au}$ and $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. The quoted errors are total statistical and systematic uncertainties added in quadrature.

|  | Centrality | $\varepsilon_{\text {part }}\{2\}($ CGC $)$ | $\varepsilon_{\text {part }}\{2\}$ (Glauber) | $N_{\text {part }}$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Au}+\mathrm{Au}$ | $0-80 \%$ | $0.338 \pm 0.002$ | $0.302 \pm 0.004$ | $126 \pm 8$ |
|  | $0-10 \%$ | $0.148 \pm 0.001$ | $0.123 \pm 0.003$ | $326 \pm 6$ |
|  | $10-40 \%$ | $0.353 \pm 0.001$ | $0.296 \pm 0.009$ | $173 \pm 10$ |
|  | $40-80 \%$ | $0.554 \pm 0.002$ | $0.533 \pm 0.018$ | $42 \pm 7$ |
| $\mathrm{Cu}+\mathrm{Cu}$ | $0-60 \%$ | $0.336 \pm 0.009$ | $0.350 \pm 0.008$ | $51 \pm 2$ |
|  | $0-20 \%$ | $0.230 \pm 0.010$ | $0.235 \pm 0.008$ | $87 \pm 2$ |
|  | $20-60 \%$ | $0.434 \pm 0.003$ | $0.468 \pm 0.016$ | $34 \pm 1$ |
|  | $0-10 \%$ | $0.187 \pm 0.002$ | $0.197 \pm 0.002$ | $99 \pm 2$ |
|  | $10-20 \%$ | $0.281 \pm 0.002$ | $0.279 \pm 0.008$ | $75 \pm 2$ |
|  | $20-30 \%$ | $0.360 \pm 0.003$ | $0.369 \pm 0.009$ | $54 \pm 1$ |
| $30-40 \%$ | $0.428 \pm 0.002$ | $0.458 \pm 0.017$ | $38 \pm 1$ |  |
|  | $40-50 \%$ | $0.490 \pm 0.002$ | $0.550 \pm 0.021$ | $26 \pm 1$ |
| $50-60 \%$ | $0.555 \pm 0.004$ | $0.643 \pm 0.031$ | $17 \pm 1$ |  |



FIG. 12: (Color online) $v_{2}$ scaled by participant eccentricity as a function of $p_{T}$ in $\sqrt{s_{N N}}=200$ and $62.4 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. The error bars are shown only for the statistical uncertainties.
nuclear collisions. In the ideal hydrodynamic limit the centrality dependence of elliptic flow is mostly defined by the elliptic anisotropy of the overlapping region of the colliding nuclei, and in the low-density limit by the product of the elliptic anisotropy and the multiplicity. Thus, the centrality and system-size dependence of elliptic flow
should be a good indicator of the degree of equilibration reached in the reaction [28].

For a study of the centrality dependence of $v_{2}\left(p_{T}\right)$ in $\mathrm{Cu}+\mathrm{Cu}$ collisions together with $\mathrm{Au}+\mathrm{Au}$ collisions, we divide $v_{2}\left(p_{T}\right)$ by the initial spatial anisotropy, eccentricity, to remove this geometric effect. The participant eccen-


FIG. 13: (Color online) Centrality dependence of $v_{2}$ scaled by number of quarks and participant eccentricity $\left(v_{2} /\left(n_{q} \varepsilon_{\mathrm{part}}\{2\}\right)\right.$ ) for $K_{S}^{0}$ (left) and $\Lambda$ (right) as a function of $\left(m_{T}-m\right) / n_{q}$ in $0-10 \%, 10-40 \%$ and $40-80 \% \mathrm{Au}+\mathrm{Au}$ collisions (open symbols) 31] and $0-20 \%$ and $20-60 \% \mathrm{Cu}+\mathrm{Cu}$ collisions (solid symbols) at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. Curves are the results of $n_{q}$-scaling fits from Eq. (13) normalized by $\varepsilon_{\text {part }}\{2\}$ to combined $K_{S}^{0}$ and $\Lambda$ for five centrality bins. At a given $p_{T}$, from top to bottom, the curves show a decreasing trend as $N_{\text {part }}$ decreases. The error bars are shown only for the statistical uncertainties.
tricity is the initial configuration space eccentricity of the participants which is defined by [56]

$$
\begin{equation*}
\varepsilon_{\mathrm{part}}=\frac{\sqrt{\left(\sigma_{y}^{2}-\sigma_{x}^{2}\right)+4\left(\sigma_{x y}^{2}\right)}}{\sigma_{y}^{2}+\sigma_{x}^{2}} \tag{14}
\end{equation*}
$$

In this formula, $\sigma_{x}^{2}=\left\langle x^{2}\right\rangle-\langle x\rangle^{2}, \sigma_{y}^{2}=\left\langle y^{2}\right\rangle-\langle y\rangle^{2}$ and $\sigma_{x y}=\langle x y\rangle-\langle x\rangle\langle y\rangle$, with $x, y$ being the position of the participating nucleons in the transverse plane. The root mean square of the participant eccentricity

$$
\begin{equation*}
\varepsilon_{\text {part }}\{2\}=\sqrt{\left\langle\varepsilon_{\text {part }}^{2}\right\rangle} \tag{15}
\end{equation*}
$$

is calculated from the Monte Carlo Glauber model 29, 57] and Color Glass Condensate (CGC) model [58 61]. (See Table $\amalg$ for $\varepsilon_{\text {part }}\{2\}$.) Since the FTPC event plane is constructed from the hadrons which have their origin in participant nucleons and the FTPC event plane resolution is less than 0.2 , what we actually measure is the root mean square of $v_{2}$ with respect to the participant plane 62]. In this case, $\varepsilon_{\text {part }}\{2\}$ is the appropriate measure
of the initial geometric anisotropy taking the event-byevent fluctuations into account [62 64]. Figure 12 shows the centrality dependence of $v_{2}\left(p_{T}\right) / \varepsilon_{\text {part }}\{2\}$ for $h^{ \pm}$in 200 and $62.4 \mathrm{GeV} \mathrm{Cu}+\mathrm{Cu}$ collisions. For a given centrality bin, $v_{2}\left(p_{T}\right) / \varepsilon_{\text {part }}\{2\}$ initially increases with $p_{T}$ and then flattens or falls off at higher $p_{T}$. After the geometric effect is removed, the ordering of the distributions as a function of centrality, observed in Fig. 77 is reversed: the more central the collision, the higher the $v_{2}\left(p_{T}\right) / \varepsilon_{\text {part }}\{2\}$. This suggests that the strength of collective motion is larger in more central collisions.

To further study the centrality dependence of strange hadron $v_{2}$, we normalized the $n_{q}$-scaled values by $\varepsilon_{\text {part }}\{2\}$ and plotted them as a function of $\left(m_{T}-m\right) / n_{q}$. The centrality dependence of $K_{S}^{0}$ and $\Lambda$ results are shown in Fig. 13. The full symbols show from top to bottom the results from $0-20 \%$ and $20-60 \%$ centrality $\mathrm{Cu}+\mathrm{Cu}$ collisions. For comparison, the results from 200 $\mathrm{GeV} \mathrm{Au}+\mathrm{Au}$ collisions [31] are shown by open symbols in Fig. 13. The results in $\mathrm{Au}+\mathrm{Au}$ collisions are slightly different ( $\sim 10 \%$ larger) from the previous published results 31], which were calculated directly from the wide


FIG. 14: (Color online) Number of quarks and participant eccentricity scaled $v_{2}\left(v_{2} /\left(n_{q} \varepsilon_{\text {part }}\{2\}\right)\right)$ of identified particles as a function of $\left(m_{T}-m\right) / n_{q}$ in $0-80 \% \mathrm{Au}+\mathrm{Au}$ collisions (open symbols) 31 and $0-60 \% \mathrm{Cu}+\mathrm{Cu}$ collisions (closed symbols) at $\sqrt{s_{N N}}=200 \mathrm{GeV}$. Circles, squares and triangles represent the data for $K_{S}^{0}, \Lambda$ and $\Xi$, respectively. The error bars are shown only for the statistical uncertainties.
centrality bins. From top to bottom, the results are from $0-10 \%, 10-40 \%$ and $40-80 \%$ centrality bins. For clarity, $K_{S}^{0}$ and $\Lambda$ results are shown in different panels. Curves represent $n_{q}$-scaling fits from Eq. (13) normalized by $\varepsilon_{\text {part }}\{2\}$ to the combined data of $K_{S}^{0}$ and $\Lambda$ for five centrality bins. For a given centrality, $K_{S}^{0}$ and $\Lambda$ results follow a universal curve, which means partonic collective flow is explicitly seen in the measured scaling with $n_{q}$ and $\varepsilon_{\text {part }}\{2\}$. For a given collision system, the stronger partonic collective flow is apparent as higher scaled $v_{2}$ value in more central collisions. To study the systemsize dependence of the scaling properties, the results from $0-60 \%$ centrality $\mathrm{Cu}+\mathrm{Cu}$ and $0-80 \% \mathrm{Au}+\mathrm{Au}$ collisions are shown in Fig. 14. The stronger collective motion in $\mathrm{Au}+\mathrm{Au}$ compared to $\mathrm{Cu}+\mathrm{Cu}$ collisions becomes obvious although the constituent quark degrees of freedom have been taken into account in both systems.

In the ideal hydrodynamic limit where dynamic thermalization is reached, the mean free path is much less than the geometric size of the system. The geometric size of the system and the centrality dependence of flow is totally governed by the initial geometry (eccentricity) [28]. As there is no universal scaling with the eccentricity among either different collision centralities or different collision system sizes, this indicates that the ideal hydrodynamic limit is not reached in $\mathrm{Cu}+\mathrm{Cu}$ collisions, presumably because the assumption of thermalization is not attained. In addition, $v_{2} /\left(n_{q} \varepsilon_{\text {part }}\{2\}\right)$ shows an increasing trend as a function of $N_{\text {part }}$ (See Fig. 13). Table $\Pi$ lists the values of eccentricity and $N_{\text {part }}$ for the
used centrality bins in $\mathrm{Au}+\mathrm{Au}$ and $\mathrm{Cu}+\mathrm{Cu}$ collisions. This suggests that the measured $v_{2}$ is not only dependent on the initial geometry, but also on $N_{\text {part }}$.

Theoretical analyses found that the centrality and system-size dependence of $v_{2}$ can be described by a simple model based on eccentricity scaling and incomplete thermalization. Within these models the lack of perfect equilibration allows for estimates of the effective parton cross section in the quark-gluon plasma and of the shear viscosity to entropy density ratio $(\eta / s)$ [33, 65]. Thus, the $v_{2}$ results from $\mathrm{Cu}+\mathrm{Cu}$ collisions reported in this paper should allow extraction of $\eta / s$ and extrapolation to the ideal hydrodynamic limit.

## V. SUMMARY

We present STAR results on midrapidity elliptic flow $v_{2}$ for charged hadrons $h^{ \pm}$and strangeness containing hadrons $K_{S}^{0}, \Lambda, \Xi$ and $\phi$ from $\mathrm{Cu}+\mathrm{Cu}$ collisions at $\sqrt{s_{N N}}$ $=62.4$ and 200 GeV at RHIC. The centrality dependence of $v_{2}$ for different system-sizes as a function of the transverse momentum $p_{T}$ is presented. To estimate the systematic uncertainties, we studied various measurement methods. Below $p_{T} \sim 4 \mathrm{GeV} / c$, non-flow correlations are reduced with the event plane constructed from hadrons produced in the forward regions $(2.5<|\eta|<4.0)$. We obtained an estimate of the systematic uncertainties due to remaining non-flow contributions based on correlations measured in $p+p$ collisions.

For a given centrality bin, $p_{T}$ and hadron-type dependences of strange hadron $v_{2}$ are similar to those found in $\mathrm{Au}+\mathrm{Au}$ collisions [31]: (i) in the low $p_{T}$ region, $p_{T}<2 \mathrm{GeV} / c$, the hadron mass hierarchy is observed as expected in ideal hydrodynamic calculations: at fixed $p_{T}$, the larger the hadron mass, the smaller the $v_{2}$. (ii) in the intermediate $p_{T}$ region, $2<p_{T}<4 \mathrm{GeV} / c$, $v_{2}$ as a function of either $p_{T}$ or $m_{T}-m$ follows a scaling with the number of constituent quarks $n_{q}$. Larger $v_{2} /\left(n_{q} \varepsilon_{\text {part }}\{2\}\right)$ values are seen in more central collisions, indicating stronger collective flow developed in more central collisions. The comparison with $\mathrm{Au}+\mathrm{Au}$ collisions which go further in density shows eccentricity scaled $v_{2}$ values depend on the system size $\left(N_{\text {part }}\right)$. This suggests that the ideal hydrodynamic limit is not reached in $\mathrm{Cu}+\mathrm{Cu}$ collisions, presumably because the assumption of thermalization is not attained.
ence Grid consortium for providing resources and support. This work was supported in part by the Offices of NP and HEP within the U.S. DOE Office of Science, the U.S. NSF, the Sloan Foundation, the DFG cluster of excellence 'Origin and Structure of the Universe' of Germany, CNRS/IN2P3, STFC and EPSRC of the United Kingdom, FAPESP CNPq of Brazil, Ministry of Ed. and Sci. of the Russian Federation, NNSFC, CAS, MoST, and MoE of China, GA and MSMT of the Czech Republic, FOM and NWO of the Netherlands, DAE, DST, and CSIR of India, Polish Ministry of Sci. and Higher Ed., Korea Research Foundation, Ministry of Sci., Ed. and Sports of the Rep. Of Croatia, Russian Ministry of Sci. and Tech, and RosAtom of Russia.

## VI. ACKNOWLEDGMENTS

We thank the RHIC Operations Group and RCF at BNL, the NERSC Center at LBNL and the Open Sci-
[1] S. A. Voloshin and Y. Zhang, Z. Phys. C 70, 665 (1996).
[2] A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C 58 1671 (1998).
[3] S. A. Voloshin, A. M. Poskanzer and R. Snellings, nuclex/0809.2949.
[4] Y. Lu, M. Bleicher, F. Liu, Z. Liu, H. Petersen, P. Soresen, H. Stöcker, N. Xu and X. Zhu, J. Phys. G 32, 1121 (2006).
[5] H. Sorge, Phys. Rev. Lett. 78, 2309 (1997).
[6] H. Sorge, Phys. Lett. B 402, 251 (1997); Phys. Rev. Lett. 82, 2048 (1999).
[7] J.-Y. Ollitrault, Phys. Rev. D 46, 229 (1992).
[8] D. Teaney, J. Lauret, and E. Shuryak, Phys. Rev. Lett. 86, 4783 (2001).
[9] J. Adams et al., (STAR Collaboration), Nucl. Phys. A 757, 102 (2005).
[10] K. Adcox et al., (PHENIX Collaboration), Nucl. Phys. A 757, 184 (2005).
[11] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 87, 182301 (2001).
[12] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 92, 052302 (2004).
[13] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 95, 122301 (2005).
[14] A. Shor, Phys. Rev. Lett. 54, 1122 (1985).
[15] H. van Hecke, H. Sorge, and N. Xu, Phys. Rev. Lett. 81, 5764 (1998).
[16] S. A. Bass et al., Phys. Rev. C 60, 021902 (1999); A. Dumitru, S. A. Bass, M. Bleicher, H. Stocker, and W. Greiner, Phys. Lett. B 460, 411 (1999); S. A. Bass and A. Dumitru, Phys. Rev. C 61, 064909 (2000).
[17] Y. Cheng et al., Phys. Rev. C 68, 034910 (2003).
[18] S. F. Biagi et al., Nucl. Phys. B 186, 1 (1981).
[19] R. A. Muller, Phys. Lett. 38B, 123 (1972).
[20] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. Lett. 99, 112301 (2007).
[21] S. Afanasiev et al. (PHENIX Collaboration), Phys. Rev. Lett. 99, 052301 (2007).
[22] D. Molnar and S. A. Voloshin, Phys. Rev. Lett. 91, 092301 (2003).
[23] K. H. Ackermann et al. (STAR Collaboration), Phys. Rev. Lett. 86, 402 (2001).
[24] P. Kolb, J. Sollfrank, and U. Heinz, Phys. Rev. C 62, 054909 (2000).
[25] P. Kolb, J. Sollfrank, and U. Heinz, Phys. Lett. B 459, 667 (1999).
[26] D. Teaney and E.V. Shuryak, Phys. Rev. Lett. 83, 4951 (1999).
[27] T. D. Lee, Nucl. Phys. A 750, 1 (2005).
[28] S. A. Voloshin and A. M. Poskanzer, Phys. Lett. B 474, 27 (2000).
[29] M. Miller and R. Snellings, nucl-ex/0312008.
[30] C. Adler et al. (STAR Collaboration), Phys. Rev. C 66, 034904 (2002).
[31] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 77, 054901 (2008).
[32] H. Heiselberg and A. M. Levy, Phys. Rev. C 59, 2716 (1999).
[33] H. J. Drescher, A. Dumitru, C. Gombeaud, and J. Y. Ollitrault, Phys. Rev. C 76, 024905 (2007).
[34] STAR Collaboration, paper in preparation.
[35] K. H. Ackermann et al. (STAR Collaboration), Nucl. Instrum. Methods A 499, 624 (2003).
[36] K. H. Ackermann et al. (STAR Collaboration), Nucl. Instrum. Methods A 499, 713 (2003).
[37] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 89, 202301 (2002).
[38] J. Adams et al., Phys. Lett. B 612, 181 (2005); C. Adler
et al., Phys. Rev. C 65, 041901(R) (2002).
[39] J. Barrette et al. (E877 Collaboration), Phys. Rev. C 56, 3254 (1997).
[40] J. Adams et al. (STAR Collaboration), Phys. Rev. C 72, 014904 (2005).
[41] I. Selyuzhenkov and S. Voloshin, Phys. Rev. C 77, 034904 (2008).
[42] N. Borghini and J. Y.Ollitrault, Phys. Rev. C 70, 064905 (2004).
[43] J. Adams et al. (STAR Collaboration), Phys. Rev. Lett. 93, 252301 (2004).
[44] C. Adler et al. (STAR Collaboration), Phys. Rev. Lett. 90, 082302 (2003).
[45] L. Molnar for the STAR Collaboration, J. Phys. G 34, S593 (2006).
[46] B. Alver et al. (PHOBOS Collaboration), Phys. Rev. Lett. 104, 062301 (2010).
[47] B. I. Abelev et al. (STAR Collaboration), arXiv:0912.3977.
[48] Y. Bai for the STAR Collaboration, the proceedings of Strange Quark Matter 2008, 2008.
[49] P. Huovinen and P.V. Ruuskanen, Ann. Rev. Nucl. Part. Sci. 56, 163 (2006).
[50] P. Huovinen, private communication, 2008.
[51] A. Adare et al. (PHENIX Collaboration), Phys. Rev. Lett. 98, 162301 (2007); A. Taranenko, J. Phys. G 34, S1069 (2007).
[52] B. I. Abelev et al. (STAR Collaboration), Phys. Rev. C 75, 054906 (2007).
[53] R. J. Fries, B. Müller, C. Nonaka and S. A. Bass, Phys. Rev. Lett. 90, 202303 (2003).
[54] R. C. Hwa and C. B. Yang, Phys. Rev. C 67, 064902 (2003).
[55] X. Dong, S. Esumi, P. Sorensen, N. Xu, and Z. Xu, Phys. Lett. B 597, 328 (2004).
[56] B. Alver et al. (PHOBOS Collaboration), Phys. Rev. Lett. 98, 242302 (2007).
[57] M. L. Miller, K. Reygers, S. J. Sanders, P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007).
[58] A. Adil et al., Phys. Rev. C 74, 044905 (2006).
[59] H. J. Dresher and Y. Nara, Phys. Rev. C 75, 034905 (2007).
[60] H. J. Dresher and Y. Nara, Phys. Rev. C 76, 041903 (2007).
[61] T. Hirano and Y. Nara, Phys. Rev. C 79, 064904 (2009).
[62] J. Y. Ollitrault, A. M. Poskanzer and S. A. Voloshin, Phys. Rev. C 80, 014904 (2009).
[63] S. Voloshin, nucl-th/0606022.
[64] B. Alver et al. (PHOBOS Collaboration), Phys. Rev. C 77, 014906 (2008).
[65] C. Gombeaud, and J. Y. Ollitrault, Phys. Rev. C 77, 054904 (2008).


[^0]:    *Deceased
    ${ }^{\dagger}$ Corresponding author.
    E-mail address: sss@iopp.ccnu.edu.cn (S. S. Shi)

