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Abstract

The Monty Hall Dilemma (MHD) is a well-known cognitive illusion. It is often claimed that one reason for the incorrect answers is that people apply the equiprobability principle: they assume that the probability of the two remaining options must be equal. An alternative explanation for assigning the same probabilities to options is that they had the same prior probabilities and people perceive no significant change. Standard MHD versions do not distinguish these possibilities, but a version with unequal prior probabilities could. Participants were given an unequal probabilities version of MHD and told that either the high or low probability option had been eliminated. This affected participants' choices and their posterior probabilities. Only 14% of participants' responses were consistent with applying the equiprobability principle, but 51% were consistent with a "no change" principle. Participants were sensitive to the implications of the prior probabilities but did not appear to use Bayesian updating.

Keywords: Monty Hall Dilemma, probabilistic reasoning, cognitive illusions, cognitive reflection

Introduction

In the Monty Hall Dilemma (MHD) participants are presented with a scenario in which there are three doors, one of which conceals a prize. The participant initially chooses one and then some mechanism (a game show host in the original version) opens one of the other two doors to show that it does not conceal the prize. This mechanism never opens a door with the prize behind it, knows where the prize is, and it always operates. The participant is then offered the choice of staying with the original choice or switching to the other unopened door. Almost all participants say they would stay with their first choice and that the probability of winning is 50% (Granberg & Brown, 1995), however the correct choice is to switch and the probability of winning is 2/3. When Marilyn vos Savant published the MHD in her magazine column and gave its correct answer she reports having received thousands of letters with 92% of the general public disagreed with her, but so did 65% of letters with university addresses (vos Savant, 1997). As such the MHD has proved to be one of the best examples of a cognitive illusion (Piattelli-Palmarini, 1994) and it has been the subject of a number of research studies (see Krauss & Wang, 2003; Tubau, Aguilar-Lleyda & Johnson, 2015, for reviews).

Factors driving the illusion

Tubau et al. (2015) point out that it has been observed in many empirical studies that when participants are given the MHD they display a strong tendency to see the two remaining doors as equally likely to conceal the prize. Stibel, Dror, and Ben-Zeev (2009) found that even in the 100-door version of the MHD in which 98 doors were opened, participants still tended to say that the last two remaining doors had a 50% chance each of concealing the prize. Although Tubau et al. point out that a number of factors have been argued to contribute to the illusion in MHD, one factor that it has been argued is a strong factor is misapplication of the *equiprobability principle* (Falk, 1992; Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni, 1999; Falk & Lann, 2008). Once participants see the options as equally probable they then choose to "stay" due to either illusion of control (Granberg & Dorr, 1998) or anticipation of regret (Gilovich, Medvec, & Chen, 1995).

The equiprobability principle suggests that in the absence of any apparent reason to differentiate options, all options will be assigned the same probability (Johnson-Laird, et al, 1999). So if there are just two apparently identical options they must each have an equal 50% chances of being correct. The equiprobability principle has often been observed when people are faced with uncertain options. For example, Fischhoff, Parker, Bruine de Bruin, Palmgren, Dawes and Manski (2000) found that a large number (over 20%) of US 16 year olds estimated a 50% chance of dying in the next year. Tversky and Kahneman (1974) report that when participants were presented with a party made up of people with one of two professions, but given a description of an individual that the representativeness heuristic could not allocate to one of the groups, they said the probability of him being in either group was 50% regardless of base-rate. Even when Burns and Wieth (2004) presented a variation of the MHD that induced 51% participants to see that it was better to switch, most still said the probability of wining was 50%. Johnson-Laird et al. present ample evidence of the equiprobability principle being applied to a number of situations, but it appears to be particularly strong in the MHD. However the frequency of the 50% answer may not be as strong as evidence of application of equiprobability as has been assumed because there is more than one path to this answer.

Burns and Wieth (2004) suggested that a major barrier to correct reasoning about the MHD is that participants fail to understand the causal structure underlying it. Therefore they see the host's action in opening a door as having not having changed the underlying probabilities. Thus given that the two unopened doors had equal probabilities before any door was opened they may still have equal probabilities. Occasionally participants have report that each of the two doors now has a 33% chance of concealing the prize, making this reasoning transparent, although mathematically incoherent. However participants may be applying this reasoning to conclude that the ratio of the two probabilities of the remaining doors has not changed, and thus the probability of winning after switching is 50%. Thus participants could come to the same answer (i.e., that each door has the same 50% chance of concealing the prize) in two different ways: 1) by misapplying the equiprobability principle; 2) by assuming no change. With the standard version of the MHD it is difficult to distinguish which form of reasoning led to the 50% answer, but a modification used by Granberg (1999) could.

MHD with unequal probabilities

Granberg (1999) tried to probe how people use conditional probabilities in the MHD by giving them an alternative version of the MHD with four doors and unequal probabilities. The Bayesian analysis shows that whether staying or switching has the highest probability of winning depends on which door is initially selected and which door is opened. The optimal strategy for this version is to first select the least likely alternative then switch away from it after a door is opened. Granberg gave participants 60 trials of either equal or unequal probability 4-door versions of the MHD. On the first trial only 11% switched in the equal-probability condition but even fewer (7%) switched in the unequal-probability condition, despite there being a 75% chance of winning (on average) if switching in the later condition. Over 60 trials the switch rates improved in both conditions but at similar rates. Participants in unequal-probability condition increasingly utilized the optimal strategy, but still only used it on an average of about 30% of trials in their last block of 10. Granberg saw this as evidence that participants were satisficing (Simon, 1955).

Granberg (1999) did not ask participants about what they thought were the probabilities that they would win by switching, but by not doing so an opportunity was lost. Few participants would be expected to get these probabilities correct, given how poor participants are at getting the percentage correct in standard version of the MHD (Burns & Wieth, 2004, found only 2% did so). However their incorrect answers could be windows into their reasoning. In the 3-door equal-probabilities version of the MHD the same percentage answer could result from either form of the erroneous reasoning identified above. If a participant was applying the equiprobability principle then they would say there is a 50%, but they would answer the same if they thought that Monty had changed nothing (except for cutting down the options). However in an unequal-probabilities version applying the equiprobability principle would still yield a 50% chance of winning, but if they applied the "no change" principle then the probability of winning would be a function of the unequal probabilities. Thus an unequal probability version of the MHD can be used to probe to what extent are participants' errors due to either of these principles.

The current experiment

In the experiment presented here participants were given a three-door version of the MHD but told that the probability that Door A concealed the prize was 30%, the probability that Door B did was 60% and the probability that Door C did was 10%. (These probabilities were chosen because they yielded different correct answers to staying or switching depending on which door was opened, and because they allowed easy calculation of the percentages corresponding to reasoning I expected from participants.) They were then told that they had initially selected Door A (fixing the first choice eliminated the strategic considerations that were not the focus of this experiment), then that one of the other doors had been opened. Figure 1a shows diagrammatically the scenario for the condition in which Door B was the one unopened, and in Figure 1b for when Door C was unopened.

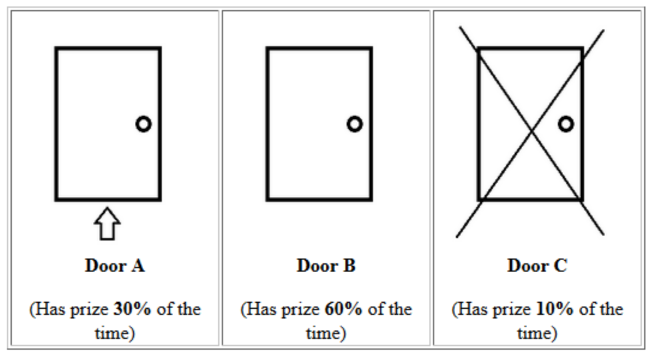

Figure 1a: Diagrammatic depiction of the scenario presented to participants in the Door-B unopened condition.

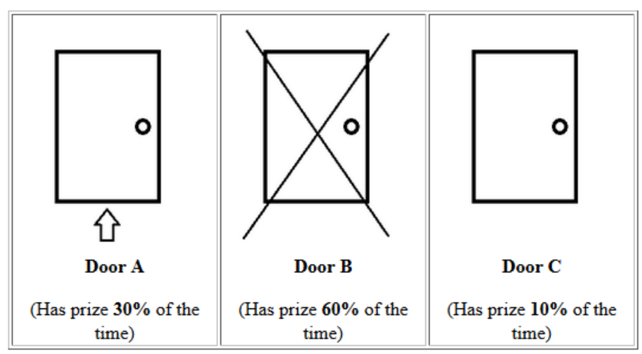

Figure 1b: Diagrammatic depiction of the scenario presented to participants in the Door-C unopened condition.

Participants were then asked whether they would stay with their first choice or switch to the unopened door, and to express their probability of winning if they switched. They were also asked if they had ever seen a question like this before. There were two conditions: *Door-B* in which participants were told that Door B was unopened; and *Door-C* in which participants were told that Door C was unopened.

Bayes' Theorem yields the correct answer to the problems in the two conditions. In the Door-B condition the optimal choice is to switch because there is a 80% chance of winning if they switch, but in the Door-C condition participants should stay because they would have only a 40% chance of winning if they switched (these answers were confirmed by simulations assuming that if the prize was behind Door A then there was an equal chance of opening either Door B or C). If participants apply the equiprobability principle then they should indicate a 50% chance of winning by switching in both conditions. If they are applying the "no change" principle then in the Door-C conditions they may say 25% (.1/(.1+.3)) and in the Door-B conditions 67% (.6/(.6+.3)). However they may apply a cruder version of the "no change" principle and just repeat the prior probabilities of 10% for the Door-C condition or 60% for the Door-B condition.

Tubau et al. (2015) suggest that use of the equiprobability principle may be due to lack of cognitive reflection, so it may be possible to analyses reasoning about the MHD in terms of dual-system models (see Stanovich, 2011). Applying "no change" and repeating prior probabilities may be the least reflective response. However "no change" but updating the probabilities to maintain the same ratio may be the most reflective. Thus responses to the unequal probability MHD may also be used to probe how reflective was the thinking employed.

The goal of the experiment was to analyze the type of reasoning participants used in an unequal-probabilities version of the MHD and thus to investigate what factors are behind the cognitive illusion. An improved understanding of the MHD can improve understanding of how people reason about probabilities.

Method

Participants

A total of 373 participants completed the experiment as part of a class at the University of Sydney. Of these, 105 indicated that they had seen a similar question before, so they were excluded from this analysis. The remaining sample of 268 consisted of 160 women and 108 men.

Materials and procedure

During a class, participants were presented with the task as part of a set of tasks completed on a computer. They were told to read the following questions carefully and answer all the questions. If there were any they did not know the answer to, then they were instructed to guess.

"Pretend you are on a game show, where you are allowed to choose one of three closed doors. Behind one door is a prize (a car), and behind each of the other doors is a goat. After you have chosen a door, the door remains closed for the time being. The game show host, Monty Hall, who knows what is behind the doors, now has to open one of the two unchosen doors and reveal a goat. After he shows you a goat, he asks you to decide whether you want to stay with your first choice or switch to the remaining unopened door."

"By watching the show many times you have calculated that there is a pattern to where the prize is initially placed. Door A has the prize 30% of the time, Door B has the prize 60% of the time and Door C has the prize only 10% of the time." Participants were then instructed to pretend they had first chosen Door A and that Monty Hall then opens <u>Door B</u> (if in the Door-C condition, otherwise <u>Door C</u>) and reveals a goat. Now he asks you whether you want to stick with your first choice (Door A) or switch to the <u>Door C</u> (or <u>Door B</u> if in the Door-B condition). Participants were also shown Figure 1a or Figure 1b, which ever was relevant to their condition.

They were then asked "Would you choose to switch doors or stay with your original door (Door A)?" and "What do you think is the chance of winning the prize if you switch doors (to <u>Door B</u>) [<u>Door C</u> in Door-C condition]?

Participants also completed the Cognitive Reflection Task (CRT) of Frederick (2005) which consists of three problems that require participants to reflect on the answers rather than give the obvious ones. It is used to assess the extent to which participants are reasoning reflective and thus using System 2 rather than relying on System 1 (in terms of Stanovich, 2011).

Results

Choice

After eliminating participants who reported having seen the question before and two with missing responses, there were 115 in the Door-B condition and 151 in the Door-C condition. Table 1 shows the number of participants choosing to stay or to switch depending on which door was left unopened. There was a large effect of condition, $\chi^2(1) = 74.35$, $p < .001$.

This result shows that participants were sensitive to the implication of the prior probabilities of the unopened door, with 86% correctly switching when the unopened door had a high prior probability and 66% correctly staying when the unopened door had a low prior probability. This result already argues that the equiprobability principle is not being commonly applied.

Table 1: Number of participants in each condition (Door-B in which the high probability door is left unopened and Door-C in which the low probability door is left unopened) deciding to stay or switch.

		Choice	
		Stay	Switch
Door-B unopened	(60%)	16	99
Door-C unopened	(10%)	101	50

A slightly surprising aspect of Table 1 was how high the switch rate was for the Door-C condition, given that so few participants switch in standard versions of the MHD despite it being the correct response. This may be due to the initial choice of Door A being allocated to participants rather than being a true choice, which could reduce both the illusion of control (Granberg & Dorr, 1998) and the anticipation of regret (Gilovich et al, 1995) factors that have been seen as driving the excessive number of "stay" decision in the standard MHD. Consistent with this is that previous studies have found that eliminating participants' first choice increased switch rates substantially (Tubau & Alonso, 2003).

The large effect of the prior probabilities is also surprising in light of the lack of an effect of unequal prior probabilities observed by Granberg (1999) on his first trial. Possibly this was because the differences in probabilities for Granberg's four doors were quite small: .1, .2, .3 and .4. It is also possible that because Granberg's first trial was the first of 60 participants were using it to explore rather than thinking deeply about the right choice to make.

Percent

Participants in the Door-B condition gave higher mean percentage (*M* =. 61, *SD* = .16) chances of winning by switching than did those in the Door-C condition (*M* = .41, *SD* = .21), $t(264) = 8.85$, $p < .001$. Consistent with the choice data, participants thought they had a better chance of winning by switching if the higher probability door was left unopened.

Table 2 shows the number of participants giving each percentage response depending of their condition and their choice. Very few participants gave the correct percentages, only 4/266. That these four actually calculated the Bayesian posterior probabilities is thrown into doubt by the observation that two participants generated these responses even when they were incorrect for their condition. Critically, Table 2 reveals that relatively few participant gave the equiprobability response of *50%* when asked how likely they were to win if they switched. In total only 37/266 (14%) of participants did so. Equiprobable responses were no more likely in the Door-C than the Door-B condition, $\chi^2(1) = 0.475$, $p = .47$. As expected based on results from the standard MHD, equiprobability was associated with more "stay" decisions, $\chi^2(1) = 5.76$, $p = .016$.

There was strong evidence that participants followed the principle that nothing had changed. A total of 93/266 (35%) of participants gave responses consistent with them calculating the ratio of the prior probabilities of the opened and unopened door (66% when Door B was unopened; 25% when Door C was unopened). As expected, the *66%* response was much more common from participants in the Door-B than the Door-C condition, $\chi^2(1) = 18.9$, $p < .001$, and the reverse was true for *25%* responses, $\chi^2(1) = 19.2$, $p < .001$. There is also evidence of participants using a cruder version of the "no change" principle and simply giving the prior probabilities as the posterior probabilities (*10%* in Door C; *60%* for Door B). A total of 50/266 (19%) did this. Again as expected, the *60%* response was much more common from participants in the Door-B than the Door-C condition, $\chi^2(1) = 31.4$, $p < .001$, and the reverse was true for *10%* responses, $\chi^2(1) = 20.1$, $p < .001$.

Although only generated by 9 participants, the most common percent response that is not apparently associated with applying equiprobability or a "no change" principles was *70%*. This answer could be due to participants adding together the prior probabilities of the opened and unopened doors. Such addition is sometimes proposed as a way to explain the correct answer to the standard version of the MHD. The unequal probability version of the MHD illustrates why this is a poor way to explain the MHD, but it is interesting that a small number of participants appeared to reasoning using it. If we see the 6 participants responding 30% as some sort of inversion of the *70%* reasoning, this leaves only 10 out of 266 other responses which cannot in some way be linked to the "sum", equiprobability, or "no change" approaches.

Table 2: Frequencies of different values of reported percent chances of winning if participant switched, split by whether the participants decided to stay or switch and their condition (Door-B or Door-C not opened). Two categories cover ranges of responses rather than precise responses.

Percent chance of win if switch	Stay		Switch	
	Door C (.10) not opened	Door B (.60) not opened	Door C (.10) not opened	Door B (.60) not opened
0-9%	0	0	0	2
10%	19	0	5	0
11-24%	1	0	0	1
25%	22	0	1	0
30%	2	0	2	1
33%	25	2	5	2
40%	3	0	0	1
50%	16	7	7	7
60%	1	3	1	23
65-67%	8	4	26	52
70%	3	0	1	5
75%	0	0	0	1
80%	0	0	1	1
100%	0	0	1	2
Totals	101	16	50	99

A surprising aspect of the data in Table 2 was that a high number of participants in the Door C condition who switched gave *66%* as the percentage chance of winning. This is consist with their choice but given that the prior probability of this door concealing the prize was 10% and its posterior probability was 40%, it is hard to infer the reasoning behind this answer. It may just be due to participants who were confused about the problem. Alternatively it could be that participants who had an intuition to switch, but were unable to calculate a probability fell back to using a plausible sounding percentage for a problem proposing three entities. Consistent with this is the similar number of participants giving 33% as the answer, particularly when deciding to stay. Another possibility arises from the fact that two-thirds is the correct answer to the standard MHD, so it is possible that some participants who said that they have not seen a similar problem actually had seen the standard MHD and then repeated its solution.

Cognitive Reflection Task (CRT)

Participants' responses to the CRT were scored by counting how many of the three problems they gave the correct answer to. Overall the participants did quite poorly given that the maximum score is 3.0 but their mean score was 0.55 (SD = 0.86).

There appeared to be an association between CRT scores and how participants responded. Those who generated the equiprobability percent of 50% did not differ from other participants on CRT scores (M=0.43, SD=0.83, n=37 verse M=0.57, SD=0.86, n=229), $t(264) = 0.92$, $p = .36$. However participants repeating the prior probability, either *10%* or *60%* depending on their condition, had *lower* CRT scores than other participants, (M=0.30, SD=0.61, n=50 verse M=0.61, SD=0.89, n=216), $t(264) = 2.34$, $p = .020$. Furthermore participants maintaining the ratio of prior probabilities, either *25%* or *66%* depending on their condition, had *higher* CRT scores than other participants (M=0.72, SD=0.96, n=116 verse M=0.42, SD=0.74, n=150), $t(264) = 2.92$, $p = .004$. So it appears that participants' responses to the unequal probabilities MHD reflect not only what principles they applied, but also how reflective was their thinking.

Discussion

The equiprobability principle has often been presented as a strong driver of the illusion behind the MHD, yet there has been no attempt to test this hypothesis beyond the observation that most participants when given standard versions of the MHD report that that they would have a 50% chance of winning if they switched. To the extent that participants use similar reasoning when faced with the equal and unequal probability versions of the MHD, the finding here that only 14% of participants gave equal probability answers throws into doubt that the equiprobability principle is a strong driver of this cognitive illusion. In a way, it is also a powerful demonstration of just how strong the equiprobability principle can be for some individuals, given that it appeared to be followed by some participants even when it not only led to the wrong answer but it had to be invoked out of thin-air (i.e., there are no mentions of 50% and percentages presented are unequal). So this principle may explain the illusion of the MHD for some people, but they appear to be a small minority.

The experiment's findings represent strong evidence that what leads participants to think that each remaining door is equally probable after Monty has revealed one is that there has be no change to these doors, so they maintain their equal status. Overall 54% of participants gave percentages in which they repeated the prior probability or they maintained the ratio of the prior probabilities. This is consistent with Burns and Wieth's (2004) argument that the main obstacle to correctly seeing the need for conditional reasoning about the MHD is failure to recognize that Monty's actions change things beyond just removing an option. Burns and Wieth argued that the MHD is difficult because it has a causal structure that people have difficulty recognizing and thus they fail to see that Monty's actions have causal consequences. We demonstrated this effect by showing higher switch rates when the MHD was presented as a competition. The finding here using the unequal probability MHD show directly that a belief that there has been "no change" could be driving many people's failures to reason correctly about the MHD.

The current results show almost no evidence of Bayesian updating by participants. They are sensitive to prior probabilities but they do not appear to recognize the problem as one involving conditional probabilities. The more reflective thinkers (as measured by CRT scores) recognize that the posterior probabilities are different from the prior probabilities, so have updated their probabilities after a door was opened. However they have only accounted for the missing door, not taken into account the new conditional probabilities.

The unequal probabilities versions of the MHD have the potential to be useful tools for examining the reasoning people use in the MHD because they have the potential to reveal this reasoning through differential answers. Almost all participants gave percent responses which could be interpreted as revealing their underlying reasoning. Questions concerning both the MHD and probabilistic reasoning more generally might be fruitful explored using appropriate versions of the unequal probabilities MHD.

Does this experiment really address the MHD?

Reviewers of this paper raised some interesting issues regarding whether any experiment using an unequal-probabilities MHD can tell us anything about how people reason in the common equal probabilities version of the MHD.

One claim was that the current experiment had nothing to do with the MHD because participants were told their initial choice and told what door was opened, whereas the common description of the MHD allows people to make their own first choice. However assigning to participants their first choice has been the case in most empirical studies of the MHD going back to the first published study by Granberg and Brown (1995). Doing so has not eliminated the strong bias to decide to stay, so it does not seem to be a critical factor.

The question was also raised as to whether assigning unequal prior probabilities to the doors in itself invalidates the experiment as an examination of reasoning in the MHD. However, that the probabilities have to be equal has not been presented as a critical aspect of the MHD. If it is, then why it is critical needs to be explained. The MHD is a conditional probability problem and Bayes' Law can be as effectively applied to the MHD with unequal probabilities as to a MHD with equal probabilities. From the point of view of conditional probability, it is the equal probabilities presentation that is a special case of a more general problem. The reasoning described as being behind the MHD does not seem to rely on the prior probabilities being equal, although it may simplify the calculations.

An interesting claim made was that the unequal probabilities MHD cannot possibly address the equiprobability principle because it does not have equal prior probabilities. As pointed out earlier the equiprobability principle applies when there is no apparent reason to differentiate the options, thus the presentation of unequal prior probabilities may prevent it from being applied. The probabilities of the unopened doors are also not equal in the equal probabilities version, but people often don't perceive that to be the case. However this interpretation of the principle makes it somewhat circular, that is, it becomes a statement that people judge probabilities to be equal when they don't perceive them as unequal. Such an interpretation limits the explanatory value of the equiprobability principle. (Johnson-Laird et al [1999] also use the equiprobability principle to generating mental models, so perhaps the principle is better seen as a step in reasoning than as a result.) The finding in the current experiment that some participants thought the doors to be equally likely despite the prior probabilities being unequal suggests that a stronger version of the equiporbability principle is used by some people.

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