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Effective Cosmological Constants from Supergravity

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EFFECTIVE COSMOLOGICAL CONSTANTS FROM SUPERGRAVITY

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A derivation of the Randall-Sundrum's model as an effective description of the dynamics of the low-energy spectrum of string theories is given. The geometry can be described in terms of effective cosmological constants in the bulk and on the brane when the dilaton coupling vanishes and these cosmological constants are quantized.

1 Compact and non-compact extra dimensions

Despite the fact that the Standard Model (SM) of electroweak and strong interactions is a renormalizable quantum field theory and thus can be extrapolated *a priori* to any energy scale, there are phenomenological as well as theoretical evidences for a physics beyond this model. Notably a physics that would describe gravity at a quantum level. So far string theory is the only consistent theory in this direction and at low energy the lightest string excitations can be described as an effective quantum field theory such as the SM nevertheless with a caveat regarding the space-time dimension because (super)string theories require ten or eleven dimensions. And since the works of Kaluza and Klein, we know that, if there exists some extra-dimensions to our universe, an infinity of massive states will be associated to each usual 4D field. Because these KK modes have not yet been observed, necessarily their masses must be beyond the experimental range of energies resolved in accelerators ($\sim 1 \text{ TeV}$). That is why the size of extra-dimensions cannot exceed such a ridiculously tiny scale ($\sim 1 \text{ TeV}^{-1} \sim 10^{-19} \text{ m}$). However recent progresses in string theories¹ have corrected this old scenario suggesting that the Standard Model gauge interactions are confined to a four dimensional hypersurface while gravity can still propagate in the whole *bulk* space-time. Since the gravity has not yet been tested for energy beyond 10^{-4} eV , the bounds on the size of extra-dimensions are now much lower ($\sim 1 \text{ mm}$)² and this lack of experimental data allows for a modification of gravitational interactions at submillimetric distances. However this analysis was not yet complete essentially because it assumes a particular factorizable geometry associated to the

higher-dimensional space-time being a direct product of a 4D space-time with a compact space. Recently this last assumption has been overcome³ unwarping a very rich potential of physical effects. The most exciting one reveals the non-incompatibility between non-compact extra-dimensions and experimental gravity. The toy model constructed by Randall and Sundrum (RS) considers a 3-brane embedded in a 5D bulk at energy low enough to model the dynamics of all the fields but the metric itself by a negative cosmological constant in the bulk, Λ_{bk} and a positive cosmological constant or tension on the brane, Λ_{br} . If these two quantities are adjusted such that:

$$\Lambda_{bk} = -\kappa_5^2 \Lambda_{br}^2 / 6 \quad (1)$$

there exists a solution to Einstein equations with a $\mathbb{Z}_2$ symmetry in the transverse coordinate, y , and a 4D Poincaré invariance on the brane:

$$ds^2 = e^{-2\sqrt{-\kappa_5^2 \Lambda_{bk}/6}|y|} dx_4^2 + dy^2 \quad (2)$$

The fluctuation modes around this background contains a normalizable bound state which can be interpreted as the usual 4D graviton. Of course, there still exists an infinite tower of KK modes, even a continuum spectrum without gap, but the shape of their wave functions is such that they almost do not overlap with the 4D graviton and thus maintain the deviations to the Newton's law in limits which are still very far from experimental bounds.

When some matter characterized by its energy density, ρ_{br} , is introduced on the brane, the previous configuration is no longer static and the brane will expand. It has been shown⁴ that the bulk contribution exactly cancels the non-standard extrinsic expansion of the embedded brane⁵ and a usual FRW cosmology is recovered up to small corrections.

2 Effective cosmological constants

How interesting the RS model can be, at the present stage it seems lacking in generality and suffers from apparently *ad hoc* fine-tuning between the cosmological constants λ_{bk} and λ_{br} . It has been proposed⁶ that these cosmological constants could describe the dynamics of a scalar field. We have addressed this question at a more fundamental point of view taking into account the low-energy spectrum of string excitations which are described by 10D supergravity. The bosonic spectrum contains the metric, a scalar field (the dilaton) and numerous differential forms and the effective action takes the general form:

$$S_{eff}^{sugra} = \int d^D x \sqrt{|g|} \left(\frac{1}{2\kappa^2} \mathcal{R} - \frac{1}{2} \partial_\mu \Phi \partial^\mu \Phi - \sum_n e^{\alpha_n \Phi} F_{n+2}^2 \right), \quad (3)$$

The coefficients α_n are explicitly determined by a string computation. The *bulk* effective action can couple to some *branes* and the total action is:

$$S = S_{eff}^{sugra} + S_{eff}^{branes} \quad (4)$$

which the equations of motion for the metric, the dilaton and the forms are derived from. There exists a solution to these equations of motion with particular symmetry namely a Poincaré invariance in $d_{||} = p + 1$ dimensions identified as the longitudinal dimensions of a p -brane and also a rotational invariance in the $d_{\perp} = D - d_{||}$ dimensional transverse space. It is expressed as $(\mu, \nu = 0 \dots d_{||} - 1$ and $i, j = 1 \dots d_{\perp})$:

$$ds^2 = H^{2n_x} \eta_{\mu\nu} dx^{\mu} \otimes dx^{\nu} + H^{2n_y} \eta_{ij} dy^i \otimes dy^j ; \quad (5)$$

where H is a only function of the radial distance, $r = \sqrt{y^i y^j \eta_{ij}}$, in the transverse space. The consistency of the whole set of equations of motion determines the powers n_x and n_y :

$$n_x = -\frac{2(d_{\perp} - 2)\kappa^2}{(D - 2)\mathcal{A}_{WZ}^2} \quad n_y = \frac{2d_{||}\kappa^2}{(D - 2)\mathcal{A}_{WZ}^2} \quad (6)$$

and the coefficient $\mathcal{A}_{WZ}$ which has to be related to the dilaton coupling by:

$$\mathcal{A}_{WZ}^2 = 2\kappa^2 \frac{d_{||}(d_{\perp} - 2)}{d_{||} + d_{\perp} - 2} + \frac{\alpha_p^2}{2} . \quad (7)$$

The function H is harmonic in the transverse space:

$$H = l + \frac{Q}{r^{d_{\perp}-2}} \quad (8)$$

where l is an arbitrary dimensionless constant and Q is a constant with a dimension $d_{\perp} - 2$ in length that can be expressed in terms of κ_D^2 .

In some cases, the geometry associated to these solutions can be described *à la* RS in terms of cosmological constants in the bulk and on the brane. In these cases, the Einstein tensor should be derived from two constants Λ_{bk} and Λ_{br} :

$$G^{\mu\nu} = -\kappa^2 \left(\Lambda_{bk} + \lambda_{br} \frac{\delta^{d_{\perp}}(y)}{\sqrt{g_{\perp}}} \right) g^{\mu\nu} \quad G^{ij} = -\kappa^2 \Lambda_{bk} \left(2 \frac{y^i y^j}{R^2} - g^{ij} \right) \quad (9)$$

The Einstein tensor takes this form if and only if the dilaton coupling vanishes^a and thus Λ_{bk} and Λ_{br} are given by:

$$\Lambda_{bk} \propto -N^{2/(2-d_{\perp})} \kappa_D^{-2D/(D-2)} \quad \Lambda_{br} \propto N \kappa_D^{-2d_{||}/(D-2)} \quad (10)$$

^aNote that the vanishing value of the dilaton coupling is compatible with the supersymmetry for special case only such as the D3-brane of *IIB* supergravity for example.

where N is the number of coincident branes stacked at the origin.

In the case of co-dimension one brane, the numerical factors in (10) lead to the RS fine-tuning. However, according to (6), this case requires an imaginary Wess–Zumino coupling, $\mathcal{A}_{WZ}$, which would destroy unitarity. This result is also suggested by the difficulty to embed the RS scenario in a supersymmetric theory ⁶.

The generalization to higher co-dimension overcomes this difficulty but the Planck scale on the brane diverges meaning that gravity decouples on the brane. However, these solutions can be used to construct new configurations in a method quite similar to RS by cutting the region near infinity and gluing a copy to the region near the origin ⁷: gravity is then localized on a *jump* brane surrounding the SM brane.

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