Coordinates, Structure, and Classical Mechanics

A review of Jill North's *Physics, Structure, and Reality*

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1 Introduction

Jill North's 2009 paper "The 'Structure' of Physics: A Case Study" (North, 2009) sparked a renewed interest in a collection of questions about structure and equivalence in physics. When are two theories are equivalent? What is the structure that a physical theory posits? Under what conditions does one posit less structure than another? And how can coordinates and symmetries tell us about the structure and equivalence of theories? The paper is already as influential as any paper in the literature on structure and equivalence in physics. It is essential reading for philosophers of physics interested in these issues.

North's new book *Physics, Structure, and Reality* is now essential reading as well

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(North, 2021). The book expands upon and clarifies her earlier work, while also introducing many novel and exciting ideas about structure and equivalence. It will not only be of interest to philosophers of physics currently working on these topics. It is written in such a way that even the most technical claims about mathematics and physics that North makes will be accessible to any philosopher. Most anyone working in philosophy of science or metaphysics will find something of great interest in the book.

The book is an attempt to answer perhaps *the* central question in philosophy of physics: "How do we figure out the nature of the world from a mathematically formulated physical theory?" (p. 1). North assumes a brand of scientific realism which entails that "we should take the mathematical structures of our best physical theories seriously in telling us about the nature of the physical world" (p. 4). That is one of the main themes of the book. Chapters 2 and 3 describe the kind of structure that North is interested in and provide some tools we might use to figure out exactly what (and how much) structure a theory posits. One of the novel tools presented in these chapters involves an appeal to the kinds of coordinates that the theory privileges. Since we usually think of coordinates as an 'arbitrary' or 'representationally inert' part of a theory, it is surprising that we can use them to glean information about the underlying structure of the theory. This is, indeed, another one of the themes of the book: "the role of coordinate systems in physics is more subtle and complicated than usually acknowledged" (p. 9). Chapter 6 further clarifies the kind of realism about structure that North accepts and defends it against objections.

This discussion of structure is interesting in its own right, but North also applies its results to recent debates in philosophy of physics. She argues in Chapter 5 that the debate between substantivalists and relationalists about spacetime can be recast as a debate about structure. Doing so revives the debate by bringing it back into contact with current physics; it also yields a novel argument for substantivalism. Chapters 4 and 7 deal directly with the topic of equivalence of physical theories. North argues in Chapter 4 that, contrary to the standard view, the Newtonian and Lagrangian formulations of classical mechanics are inequivalent theories, in virtue of the fact that they posit different amounts of structure. This bring up another overarching theme of the book: "cases of mere notational variants in physics are harder to come by than people usually think" (p. 6). North explains in Chapter 7 why she commits to a strict standard of equivalence. She argues that we should "take [...] a theory's 'metaphysical aspects' seriously" (p. 11). And taking these aspects seriously leads one to "frequently see a non-equivalence between theories or formulations where others see equivalence" (p. 12).

Many of the arguments in the book are compelling. North's explanations of how the topics of the book — equivalence, interpretation, metaphysics, coordinates, structure, etc. — relate to one another ring true. She displays an impressive knowledge of the recent philosophy of physics literature. At its best, North's presentation of the requisite technical material is clear and engaging. Section 2.4 is particularly impressive in this regard. She illustrates how one mathematical object might have more structure than another by patiently walking through an extended example that involves adding levels of structure to a set — first topological structure, then differentiable structure, then affine structure, then metric structure, and so on. Even some of the small off-hand remarks she makes about structure are subtle and enlightening — for example, her claim that "[t]he number of relations defined on a space is [...] not the final word on how much structure there is, for we must take into account the natures or definitions of the relations themselves" (p. 118).

The book contains much to admire and learn from, and there is more valuable material in it than I have space to discuss. So in what follows, I will focus on just two of the main topics. The first is North's idea that we can use coordinates as a window into the structure that a theory posits. The second is North's argument for the inequivalence of Lagrangian and Newtonian mechanics. One virtue of the book is that it is accessible to general philosophers of science and metaphysicians, despite much of the material being quite technical at its core. North makes a conscious effort "to minimize explicit use of mathematics and technicality as much as possible" (p. 13). The result will certainly help to initiate more philosophers into these debates. It is sometimes the case, however, that minimizing technicality obscures some of the important mathematical subtleties that underlie the bold philosophical claims. My aim in what follows is to draw out some of these subtleties.

2 Coordinates and structure

We begin with North's method of uncovering the structure that a theory posits. Her basic idea is that the coordinate systems that a theory privileges provide us with a window into the underlying structure of the theory: "[a] preference for certain coordinates, in the sense that the laws take a simple or natural form in them, is indicative of, it is evidence for, underlying structure" (p. 112). This method is interesting in its own right, but it is also used throughout the book to draw surprising conclusions — for example, that Newtonian and Lagrangian mechanics are inequivalent theories. It is therefore worth discussing in detail.

North's core idea is that a piece of structure can be indirectly characterized by singling out a class of privileged coordinates. We will illustrate this idea with her example of the 2-dimensional Euclidean plane (p. 17–26). One is familiar with the usual way of directly characterizing the metric structure of the Euclidean plane: we simply define the standard metric tensor on $\mathbb{R}^2$. North's idea is that although "the metric tensor more directly encapsulates the geometry of the plane" (p. 23), we can also indirectly characterize this structure by pointing to a particular collection of 'privileged coordinates'. North reminds us that "[f]rom among all the coordinate systems we can use for the plane, there is a particularly nice kind, the Cartesian coordinate systems, which have straight, mutually orthogonal coordinate axes, and whose numerical values reflect the relative locations of the points in a particularly clear manner" (p. 18). These Cartesian coordinates are the privileged ones for the Euclidean plane. They are the ones in which the Euclidean metric takes a particularly "simple form" (p. 25). North points out that if we know that these are the privileged coordinates, we can use this fact to uncover the structure of the space:

> Now think of all the different Cartesian coordinate systems we can use for the plane, and think of the similarities and differences among them. [...] There are some things that all these coordinate systems agree on, despite their disagreement on such things as the coordinate values of a given point or the differences between the x or y coordinate values of distinct points. All Cartesian coordinate systems will agree on the distance [...] between two points [...] We say that the distance between any two points is invariant under, or unchanged by, such changes in coordinates (p. 18–9)

The fact that all of the privileged coordinates agree on the distance between points tells us that the plane comes equipped with metric structure. As North puts it, metric structure is "part of the intrinsic, objective nature of this space" (p. 19). This example demonstrates that in some cases singling out a collection of privileged coordinates on a space suffices to 'characterize' or 'determine' or 'implicitly define' some structures on it — those structures that are agreed upon by all of the privileged coordinates. North puts this basic idea as follows: "There are [...] two ways of characterizing a given structure, and two corresponding routes to learning about it. A structure can be characterized more directly, as in the case of the Euclidean plane and the metric tensor. Or it can be characterized less directly, by means of the coordinate systems we can use for the space and the features that are invariant under transformations of them" (p. 23).

Many parts of this idea merit further attention, but I will examine here the sense in which singling out a collection of coordinates suffices to characterize a structure. In particular, I will consider the following 'conjecture schema'. In what follows, we let (M, g_{ab}) and (M, g'_{ab}) be manifolds with Riemannian metric.[1]

Conjecture. *If* (M, g_{ab}) *and* (M, g'_{ab}) *have the same privileged coordinates, then* $g_{ab} = g'_{ab}$.

If true, this conjecture would capture a sense in which one can, at least in the case of a metric, use collections of coordinates to 'implicitly define' a piece of structure on a manifold. By singling out a collection of coordinates as privileged, one would be pointing to a unique metric. If the conjecture were true for arbitrary structures, rather than merely metrics on the manifold, then this would allow North to move seamlessly from claims about theories 'privileging coordinates' (by their laws taking a particularly

[1]See Malament (2012) for preliminaries on differential geometry.

"simple or natural form" (p. 112) in those coordinates) to claims about the structures that those theories posit. If the conjecture is false, then coordinates are not a perfect guide to underlying structure. Providing the collection of privileged coordinates would not uniquely pick out a structure; the privileged coordinates would not tell us everything about that structure. The truth of the conjecture would, at least in the case of a metric, rule out this possibility. North's discussion provides another way of putting this idea. She asks: "What is the nature of [the two-dimensional Euclidean plane]? One way of getting at an answer to this question, though it is somewhat indirect, is to consider the different kinds of coordinate systems we can use" (p. 17). If the above conjecture is false, then the privileged coordinates would not in general provide a complete answer to that question. The conjecture is therefore an important plank in the project of using coordinates to learn about structure.

Before assessing whether or not it is true, however, we need to make the conjecture precise. In particular, we need to say exactly what the 'privileged coordinates' are for a manifold with metric. North's idea is that these are the coordinates in which the structure looks 'nice' or 'simple' or 'natural'. We begin with the following attempt, which North explicitly suggests (p. 22). Let (U, φ) be a coordinate chart on (M, g_{ab}) i.e. U is a subset of M and $\varphi : U \to \mathbb{R}^n$ is a smooth injective map that assigns 'coordinate values' to each point in U. We will say that (U, φ) is **orthonormal** on (M, g_{ab}) if $g_{ab} = \sum_{i=1}^{n} d_a u^i d_b u^i$ in the region U, where the scalar functions u^i are the coordinate functions of ϕ on U. This is one natural way to make precise the idea that g_{ab} looks nice in the coordinate patch (U, φ). One can easily verify that the orthonormal coordinates on Euclidean space suffice to characterize the Euclidean metric in the sense of our conjecture: if any metric on $\mathbb{R}^n$ has the same orthonormal coordinates as the

Euclidean metric does, then that metric is just the Euclidean metric.

Unfortunately, this is not the case for arbitrary metrics. The conjecture is false if we take orthonormal coordinates to be the privileged ones.[2]

Lemma. *Let (M, g_{ab}) be a metric with an orthonormal coordinate chart (U, φ). Then g_{ab} is flat on U.*

Proof. One can easily show that the coordinate derivative operator associated with the chart (U, φ) (Malament, 2012, 1.7.11) is compatible with g_{ab} on U. Since the coordinate derivative operator is flat (Malament, 2012, p. 72), this immediately implies that g_{ab} is flat on U. □

Proposition 1. *It is not the case that if (M, g_{ab}) and (M, g'_{ab}) have the same orthonormal coordinates, then $g_{ab} = g'_{ab}$.*

This result immediately follows from the lemma above. If g_{ab} and g'_{ab} are both flat on no region of M, then they both have no orthonormal coordinates, but they are not necessarily equal. If we consider orthonormal coordinates to be the privileged ones for a manifold with metric, then distinct metrics can have the same (empty) collection of privileged coordinates. We might therefore try to recover the conjecture by considering more coordinates to be privileged. We will say that a coordinate chart (U, φ) on (M, g_{ab}) is **diagonal** if $g_{ab} = \sum_{i=1}^{n} \alpha^i d_a u^i d_b u^i$ in the region U for some smooth scalar functions α^i on U. This is another way to make precise the idea that the metric looks 'nice' or 'simple' in the coordinate chart. Unfortunately, this makes the conjecture false again.

[2]North implies as much when she remarks that a "defining feature [of a space that] is flat and Euclidean [is that] there is a coordinate system in which the metric takes the simple Pythagorean form", i.e. an orthonormal coordinate chart in our sense (p. 22).

Proposition 2. *It is not the case that if* (M, g_{ab}) *and* (M, g'_{ab}) *have the same diagonal coordinates, then* $g_{ab} = g'_{ab}$.

Proof. Let (M, g_{ab}) be Euclidean space and consider $(M, \Omega^2 g_{ab})$, where $\Omega : M \to \mathbb{R}$ is some smooth scalar function that is not everywhere 1. It is easy to verify that both metrics admit the same diagonal coordinates, but by construction they are not equal. □

Of course, this does not show that the conjecture is false for all ways of characterizing privileged coordinates for a manifold with metric. It only shows that the conjecture is false for two of the most natural such ways. Further work is required to assess the more general status of the conjecture.[3] But there is another degree of freedom we have when considering the conjecture. One can formulate corresponding conjectures for geometric structures other than a metric. We conclude with one case where a structure is perfectly characterized by a natural class of privileged coordinates. A **symplectic form** Ω_{ab} on a $2n$-dimensional smooth manifold M is a smooth tensor field that is closed, non-degenerate, and antisymmetric. We will say that the coordinate chart (U, φ) is **symplectic** if

$$\Omega_{ab} = \frac{1}{2} \sum_{i=1}^{n} d_a u^i d_b u^{i+n} - d_b u^i d_a u^{i+n}$$

in the region U. The symplectic coordinates are those in which the symplectic form takes a particularly natural and simple form; they are analogous to the orthonormal coordinates for a metric. Darboux's theorem states that for every point p in the symplectic manifold (M, Ω_{ab}) there exists a symplectic chart (U, φ) with p in U (Abraham and Marsden, 1978, 3.2.2). This immediately yields the following result.

[3]In particular, one wonders whether 'normal coordinates' could serve as the variety of privileged coordinates that make the conjecture true.

Proposition 3. *If symplectic manifolds* (M, Ω_{ab}) *and* (M, Ω'_{ab}) *have the same symplectic coordinates, then* $\Omega_{ab} = \Omega'_{ab}$.

Proof. Let p be in M. By Darboux's theorem there exists a coordinate chart (U, φ) around p that is symplectic with respect to Ω_{ab}. By assumption this is also symplectic with respect to (M, Ω'_{ab}). And this implies that $\Omega_{ab} = \Omega'_{ab}$ at p. □

One wonders whether versions of the conjecture are true for other structures as well, in addition to symplectic forms. The structures for which a version of the conjecture holds are those structures that *can* be characterized in North's 'indirect' manner, by singling out the privileged coordinates. The more such structures there are, the better the prospects for using coordinates as a window into the underlying structure that a theory posits. Ensuing work on these questions about coordinates and structure will be fruitful, and that is a testament to the clarity and creativity that North's discussion brings to the topic.[4]

3 Classical mechanics

We turn to the second topic of this review: the case study of classical mechanics that North presents in Chapter 4. North previously argued that the Hamiltonian and Lagrangian formulations of classical mechanics are, contrary to the standard view, inequivalent theories (North, 2009). A centerpiece of her new book is her argument for the claim that the standard formulation of Newtonian mechanics is inequivalent to

[4]See also Wallace (2019), which contains similar claims about coordinates and structure.

Lagrangian mechanics. This dissents from "the usual view [that] Lagrangian and Newtonian mechanics are wholly equivalent theories, mere notational variants, differing at most in calculational ease" (p. 107). North argues for the following claim.

C. Standard Newtonian mechanics posits more structure than Lagrangian mechanics.

This claim then implies that the two theories do not 'say the same thing' about the world. They disagree in the same way that, for example, the Newtonian and Galilean theories of spacetime disagree (p. 61). And moreover, insofar as we should prefer theories that posit less structure, C implies that we have reason to prefer the Lagrangian formulation.

North's main argument for C relies on the following two premises.[5]

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[5]North presents another argument for C later in chapter 4, which claims that "[t]he Lagrangian statespace has a general structure of which the Newtonian statespace is a special kind. The flat structure and Euclidean metric of the Newtonian statespace is a special case of the arbitrary curved structure and Riemannian metric of the Lagrangian statespace" (p. 116). For the purposes of this review, one brief remark on the argument will suffice. North argues for the claim that the Newtonian statespace comes equipped with a Euclidean metric by appealing to P1. Since Newton's equations privilege Cartesian coordinates, "the statespace on which these equations are defined, in particular the configuration space that represents the physical space the system moves around in, must admit of such coordinates. This means that the base space is an intrinsically flat ($3n$-dimensional) Euclidean space, with a Euclidean metric — the kind of space on which we can lay down Cartesian coordinates" (p. 114). In what follows, I will argue that P1 stands in need of further support. And without such support, this second argument will not go

P1. Newton's law $F = ma$ privileges Cartesian coordinates.

P2. The Euler-Lagrange equations do not privilege any kind of coordinates.

In brief, North argues that "the Lagrangian equations are invariant under a wider range of coordinate transformations [than Newton's law], which indicates that they require less structure" (p. 116). The dynamical laws of Lagrangian mechanics — the Euler-Lagrange equations — privilege fewer coordinates than the dynamical law $F = ma$ of Newtonian mechanics does. P1 and P2 support C by way of North's method of using coordinates to learn about structure. If a theory privileges fewer coordinate systems, that will indicate to North that the theory posits more structure, since there will be more features that are agreed upon by all of the coordinate systems in this smaller class. For the purposes of this discussion we will simply grant that North's method of using coordinates to learn about structure is successful. At the very least, the basic idea behind the inference of C from P1 and P2 is clear. P2 indicates that the Euler-Lagrange equations "do not distinguish or recognize differences among different coordinate systems [which] means that they do not require or presuppose the mathematical structure that would underlie a distinguished or preferred type of coordinate system" (p. 109). P1 indicates that Newton's law does presuppose the mathematical structure that underlies these privileged coordinates. And so C follows.

We begin with North's argument for P1: "Newton's law [...] prefers Cartesian coordinates, a preference that's revealed by the change in form of the equation in non-Cartesian coordinates, and by how the theory treats systems naturally characterized in terms of such coordinates" (p. 104–5). North points out that the Newtonian equation

through.

of motion $F = ma$ “when expressed in a different coordinate system, does not always have the same mathematical form it did in the original coordinate system: the equation needn’t have the same form when expressed in terms of the new coordinate system as it did in the old” (p. 110). She carefully works through the special case of Cartesian and polar coordinates (p. 96), and shows that Newton’s law does indeed take a simpler form in Cartesian coordinates than it does in polar coordinates. North concludes that standard Newtonian mechanics privileges Cartesian coordinates.

North’s idea behind P2 — the claim that “Lagrangian mechanics eliminates the favoritism for any type of coordinate system” (p. 102) — is that the Euler-Lagrange equations ‘take the same form’ in any generalized coordinates. The mathematical fact that underlies this claim requires a bit of background. Suppose that we are modeling a system of n particles in Lagrangian mechanics. The statespace for this system is the $6n$ dimensional tangent bundle TQ of configuration space Q. A point in TQ represents the positions and velocities of all of the particles in the system. The Lagrangian $L : TQ \to \mathbb{R}$ of the system encodes the system’s ‘activity’ or ‘liveliness’. Insofar as the Lagrangian is sufficiently well behaved, it gives rise to a vector field X_L^a on TQ. The field X_L^a tells us how the system will evolve. Given a point in TQ representing the initial condition of the system, there is a unique integral curve of X_L^a through that point. The state of the system evolves along that integral curve; one can picture the state of the system ‘flowing’ along the vector field X_L^a.

We can now state the following mathematical fact. Let (U, φ) be a coordinate chart on the $3n$-dimensional configuration space Q with coordinate functions $q^1, \ldots, q^{3n}$. These coordinates on M naturally induce a coordinate chart $(TU, T\varphi)$ on the tangent bundle

TQ.[6] We use the notation $q^1, \ldots, q^{3n}, \dot{q}^1, \ldots, \dot{q}^{3n}$ to denote the coordinate functions for this induced chart. Now if we are given an integral curve $(u(t), v(t))$ of X_L^a, then the Euler-Lagrange equations

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}^i}\right) = \frac{\partial L}{\partial q^i}$$

hold in this induced chart $(TU, T\varphi)$ for each $i = 1, \ldots 3n$ (Abraham and Marsden, 1978, 3.5.17). This captures the sense in which the Euler-Lagrange equations do not privilege any coordinate system. They take the same form in any coordinate chart $(TU, T\varphi)$ on TQ.

I have two main concerns with this argument for C. In brief, it seems that Newtonian mechanics posits less structure than North suggests, while Lagrangian mechanics posits more. My first concern is with the argument for P1. North is correct that Newton's law privileges particular coordinate systems, but it is not the Cartesian ones. North works through the special case of polar coordinates and Cartesian coordinates, but it is worth working through the general case here. Consider standard Newtonian mechanics formulated on Galilean spacetime $(\mathbb{R}^4, h^{ab}, t_a, \nabla)$, where the derivative operator ∇ is the 'standard' derivative operator on $\mathbb{R}^4$.[7] This theory says that if a particle has mass m,

[6] The subset $TU = \cup_{p \in U} T_p Q$ of TQ is the union of the tangent spaces for all the points in U. The map $T\varphi : TU \to \mathbb{R}^{6n}$ is defined by mapping (p, v) to $(q^1(p), \ldots, q^{3n}(p), \dot{q}^1(v), \ldots, \dot{q}^{3n}(v))$, where the real numbers $\dot{q}^i(v)$ are such that $v = \sum_i \dot{q}^i(v) \frac{\partial}{\partial u^i}$, i.e. they are the components of v in the basis on T_pQ induced by the chart (U, φ).

[7] The derivative operator ∇ is uniquely determined by the condition that $\nabla_n (\frac{\partial}{\partial x^i})^a = \mathbf{0}$ for each standard coordinate function x^i. That is just to say that it is the coordinate derivative operator for the standard coordinates on $\mathbb{R}^4$. The other structures on Galilean

then it will traverse a smooth timelike curve whose tangent field ξ^a satisfies $t_a\xi^a = 1$ and

$$F^a = m\xi^n \nabla_n \xi^a \tag{1}$$

where F^a is the vector field representing the net force acting on the particle. This equation is just $F = ma$, expressed using the derivative operator ∇; the vector field $\xi^n \nabla_n \xi^a$ is just the acceleration field of the particle. North points out that this equation takes a different form in different coordinates. Indeed, one can compute that in a coordinate patch (U, φ) on Galilean spacetime, equation (1) takes the form

$$F^a = m\Big(\sum_{i=1}^{4}\sum_{j=1}^{4} \overset{j}{\xi} \frac{\partial \overset{i}{\xi}}{\partial u^j} (\frac{\partial}{\partial u^i})^a + \sum_{i=1}^{4}\sum_{j=1}^{4} \overset{i}{\xi}\overset{j}{\xi} (\frac{\partial}{\partial u^j})^n \nabla_n (\frac{\partial}{\partial u^i})^a\Big)$$

where $u^1, \ldots, u^4$ are the coordinate functions for (U, φ) and we write $\xi^n = \sum_{i=1}^{4} \xi^i (\frac{\partial}{\partial u^i})^n$ in these coordinates. Notice that the right-hand term of the acceleration field will vanish if $\nabla_n (\frac{\partial}{\partial u^i})^a = \mathbf{0}$, i.e. if the coordinate curves in the patch (U, φ) are straight according to ∇. This means that, as North clearly points out, Newton's equation of motion will take 'a different form' in polar coordinates — where the coordinate curves are not straight according to ∇ — than it does in standard Cartesian coordinates — where the coordinate curves are straight according to ∇. In the first case, the right-most term of the acceleration field will not vanish, while in the second case it will. In Cartesian coordinates $F = ma$ looks simpler, nicer, and more natural than it does in coordinates whose coordinate curves 'bend' according to ∇. North argues that this "reveals that the [Newton's law] does distinguish or recognize differences among different coordinate

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spacetime are defined in the usual way. See Malament (2012) for details.

systems — it does not say the same thing regardless. This, in turn, means that the law requires or presupposes the mathematical structure that underlies the preferred type of coordinate system" (p. 110). North claims that the preferred type of coordinate system is the Cartesian coordinates, and the mathematical structure that underlies it is a Euclidean metric.

This last claim, however, does not follow. The right-hand term in the acceleration field will vanish for any coordinate system whose coordinate curves are straight according to ∇, not just for the Cartesian ones. Newton's equation (1) will take the same nice form in any 'straight' coordinates. The class of privileged coordinate systems is therefore broader than merely the Cartesian ones, since Cartesian coordinates have straight coordinate axes that are also mutually orthogonal. Note that there is a sense in which this is not surprising. Newton's law explicitly appeals to 'straightness structure' in the form of the covariant derivative operator that appears in its acceleration term. So it is natural that the straight coordinates are the privileged ones. Indeed, it is a mark in favor of North's method of using coordinates to investigate structure that these end up being the privileged ones. But as far as the above argument is concerned, this means that P1 is false. Crucially, it means that Newton's law privileges *more* coordinates than what North suggests, which means that it gives rise to *less* structure than what North suggests, since a broader class of privileged coordinates will agree on fewer features. In particular, the coordinates with 'straight' coordinate curves will not all have the same coordinate metric.[8] And that is a problem for North's argument. It means that

[8]Note that Newtonian mechanics set in Galilean spacetime does posit a kind of Euclidean metric structure h^{ab} on space, but it is not in virtue of the fact that the structure is presupposed by $F = ma$ in the way that North's method requires. And "the structure

Newtonian mechanics posits less structure than P1 suggests it does.

My second concern has to do with P2. Recall that the Euler-Lagrange equations hold in the coordinates $(TU, T\varphi)$ on TQ that are induced by coordinates (U, φ) on Q. Not all coordinates on TQ are of this form, and indeed, one can show that the Euler-Lagrange equations do not hold in all coordinate systems on TQ. One way to see this is simply by examining the equations themselves. They do not make sense in an arbitrary coordinate system on TQ, since they explicitly appeal to the coordinate functions q^i and $\dot{q}^i$, which are only defined when we are working in coordinates $(TU, T\varphi)$. So the Euler-Lagrange equations do privilege a particular kind of coordinate system, contrary to what P2 asserts.

In sum, my first concern with the argument was that Newton's equations privilege more coordinates than P1 suggests, and my second concern was that the Euler-Lagrange equations privilege fewer coordinates than P2 suggests. Even so, North may argue that Newton's equations still privilege *fewer* coordinate systems than the Euler-Lagrange equations do. But one then wonders what sense of 'fewer' is meant. The two most natural explications will not work. First, it seems unlikely that the set of coordinates privileged by Newton's equations has a smaller cardinality than the set privileged by the Euler-Lagrange equations. (One would guess that both sets have the same cardinality as the real numbers.) And second, it cannot be that the former set is a proper subset of the latter, since the former are coordinates on a spacetime, while the latter are coordinates on a statespace; elements of the one set of coordinates simply are not elements of the other. We are therefore left without a compelling argument for C.

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required by the theories' dynamical laws" (p. 111) is the kind of structure that counts for North.

It should be clear, however, that North's book opens up entirely new and promising lines of inquiry. It covers a remarkable amount of material. (I have not even touched on North's general discussion of equivalence in Chapter 7 nor her contribution to the debate on substantivalism and relationalism in Chapter 5, both of which will receive much attention.) And it does so in an original and engaging manner. The ideas put forward in the book will have an impact in metaphysics, general philosophy of science, philosophy of mathematics, and philosophy of physics. They will be discussed and debated for years to come.

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