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Quantum signatures in nonlinear gravitational waves

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The effective quantum field theory description of gravity, despite its non-renormalizability, allows for predictions beyond classical general relativity. As we enter the age of gravitational wave astronomy, an important and timely question is whether measurable quantum predictions that depart from classical gravity, analogous to quantum optics effects which cannot be explained by classical electrodynamics, can be found. In this work, we investigate quantum signatures in gravitational waves using tools from quantum optics. Squeezed-coherent gravitational waves, which can exhibit sub-Poissonian graviton statistics, can enhance or suppress the signal measured by an interferometer, a characteristic effect of quantum squeezing. Moreover, we show that Gaussian gravitational wave quantum states can be reconstructed from measurements over an ensemble of optical fields interacting with a single copy of the gravitational wave, thus opening the possibility of detecting quantum features of gravity beyond classical general relativity.

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1 Introduction

Gravitational wave (GW) detectors have opened a new window into the astrophysical universe, enabling the first direct observations of gravitational radiation from collisions of compact objects [1]. From a quantum mechanical point-of-view, interferometers such as LIGO and VIRGO act as field quadrature detectors for GWs [2, 3], much like homodyne detectors in quantum optics [4]. It is therefore natural to ask whether GW interferometers could be used to probe the quantum mechanical nature of gravity [5] in analogy to quantum optics [6].

It is worth taking a moment to specify the meaning of *quantum mechanical nature of gravity* in this context. While a complete theory of quantum gravity is at this time unrealized, there are no conceptual problems in quantizing the Einstein-Hilbert action with the understanding that it is an *effective* theory, valid well below a cut-off scale assumed to be roughly of the order of the Planck scale by power counting arguments. This approach is well-suited to studying potential quantum effects of gravitational waves in the range of energies and frequencies current experiments are sensitive to. For a recent review see for instance [7].

Recently, the effect of quantum mechanical noise associated to graviton fluctuations of GWs on a model detector has been calculated [8] using the effective quantum field theory description of gravitational waves, and the associated modification of Newton's law discussed in [9]. It was pointed out that the gravity-induced fluctuations – *noise* – in the motion of test masses associated

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to squeezed GW states are enhanced by an exponential factor in the squeezing parameter [3, 10], giving rise to renewed hopes of detecting quantum signatures of the gravitational field in future detectors. These findings lead to the central questions of this work: (i) are there any quantum GW states, in the specific context of quantum Einstein-Hilbert effective field theory, that would produce exponential enhancement factors in a *signal* (apart from the gravity-induced fluctuations) detectable by GW detectors and (ii) could we use the characteristics of such signals to discriminate different quantum GW states? As we will show, both questions can be positively answered.

It is crucial to stress that our focus, at this stage, is not on the actual production of such quantum GW states in astrophysical processes, but rather on their *in principle* detectability using present-day or near-future experimental setups. As it turns out, our main candidate quantum GW states, *squeezed-coherent states*, do have potential production mechanisms in astrophysics. Squeezed GWs are expected to be generated in highly nonlinear situations and at present proposed sources are inflationary cosmology [11–13] and the final stages of black hole evaporation [14], with GW squeezing recently bounded using LIGO observations [15]. It has also been suggested that highly nonlinear perturbed black holes might lead to the production of noncoherent quantum states of GWs; see [8] and [10].

Squeezed-coherent states, in contrast to the simpler coherent states, can be thought of as inherently quantum mechanical, exhibiting interference in phase space [16] and the possibility of sub-Poissonian statistics [4]. Measuring characteristic features produced by such states would offer an empirical path to probe quantum mechanical effects related to gravity. As we will show, squeezed-coherent GWs with modest – *i.e.* order one – squeezing parameters can produce enhanced or suppressed signals in an optical interferometer with sensitivity on the order of present-day GW detectors, in addition to the induced fluctuations associated to quantum uncertainty. This provides a signature of the quantum nature of the waves; the precise value of the enhancement or suppression depends on the peculiarities of the source that produces the waves and are controlled by a phase parameter associated

to the state. Furthermore, measurements over an ensemble of optical fields populating the interferometer can be used to completely reconstruct Gaussian quantum states of GWs, opening the possibility of witnessing non-classical features of gravity, such as sub-Poissonian graviton number statistics. Note that contrary to their optical counterparts, squeezed-coherent states of gravitons are expected to propagate through the universe nearly unperturbed [17], carrying information about their sources with preserved coherence.

Overall, our findings integrate part of an emerging subfield of fundamental physics in which ideas from quantum optics offer new paths to uncover unexplored natural phenomena, with other key examples including the search for beyond the standard model physics using atomic and molecular physics [18], fifth forces with quantum optomechanics [19–21], quantum-enhanced searches for light dark matter [22–25], and nonlinear optics and optomechanics [26, 27]. Quantum gravity itself is not exempt. Of particular interest to us are experimental proposals seeking to measure quantum effects associated to gravity, as well as to measure the gravitational field of quantum matter in table-top experiments [28–35]. While very important, these proposals rely on challenging quantum engineering [36], whereas we consider an alternative observational approach, of seeking for natural phenomena in which quantum effects of gravity are manifest.

This paper is organized as follows: in Sec. 2 we briefly describe a model interferometric GW detector and its interaction with canonically quantized weak gravitational field perturbations. In Sec. 3 we describe the interaction between GW states and a simplified toy model detector consisting of an ensemble of optical qubits. Sec. 4 is devoted to squeezed-coherent GWs and in Sec. 5 we describe how Gaussian GW quantum states can be reconstructed from optical measurements. Sec. 6 describes the interaction between optical coherent states (such as the laser field of a GW interferometer) and squeezed-coherent GWs. Finally, in Sec. 7 we discuss future perspectives on the generation and detection of nonlinear quantum GWs. For details on the calculations throughout the text, we refer to the Appendices.

2 Mode-detector interaction

We consider the interaction of a weak GW with a single mode cavity electromagnetic field as a model for our interferometric GW detector. The model captures the essential features of the effects we aim to discuss. Detectors like LIGO, a variant of the Michelson interferometer, can be formally mapped to a Fabry-Pèrot cavity [37]. The treatment in this work is the same as in [2, 29, 31, 38, 39] and [3] for the canonical quantization of weak metric perturbations around a flat background and we refer to those works for further details on the formalism while we briefly summarize the main results necessary for our discussion.

The interaction between a cavity electromagnetic field and quantized metric perturbations in the tranverse traceless gauge can be formally derived from the Einstein-Hilbert action coupled to the Maxwell stress-energy tensor (see [2] for more details). For a single mode electromagnetic cavity with photon annihilation (creation) operator a ($a^\dagger$) in the presence of $+$ polarized GWs propagating along the $\boldsymbol{n} = \hat{z}$ direction, perpendicular to the cavity axis (chosen to be along the $\hat{x}$ direction), the interaction reads

$$H_I = -\frac{\omega}{4}a^\dagger a \int \frac{d\boldsymbol{k}}{\sqrt{(2\pi)^3}} \left(\sqrt{\frac{8\pi G}{k}} \mathfrak{b}_{\boldsymbol{k}} + h.c. \right) \quad (1)$$

where $k = |\boldsymbol{k}| = \Omega_k$ is the GW frequency for the mode $\boldsymbol{k}$, the operator $\mathfrak{b}_k^\lambda$ ($\mathfrak{b}_{\boldsymbol{k}}^{\lambda\dagger}$) is the canonical graviton annihilation (creation) operator and ω is the cavity frequency.
For our purposes, it is convenient to introduce a discrete form of (1). Following the standard procedure in quantum optics [40] we introduce a quantization volume V and note that $\left[\sqrt{8\pi G/k}\right] = L^{3/2}$, where L denotes the dimension of length. Further, $[d\boldsymbol{k}] = L^{-3}$, which implies the graviton annihilation and creation operators have dimension $[\mathfrak{b}_{\boldsymbol{k}}] = L^{3/2}$. Define $\mathfrak{b}_{\boldsymbol{k}} = \sqrt{V}b_{\boldsymbol{k}}$, where $b_{\boldsymbol{k}}$ is a dimensionless quantity. The discrete limit $d\boldsymbol{k} \to 1/V$, $(2\pi)^{-3/2}\int \to \sum$ then leads to

$$H_I = -\frac{\omega}{4}a^\dagger a \sum_{\boldsymbol{k}} \left(\sqrt{\frac{8\pi G}{kV}} b_{\boldsymbol{k}} + h.c. \right). \quad (2)$$

Defining the single graviton strain for the mode $\boldsymbol{k}$ as $f_{\boldsymbol{k}} = \sqrt{8\pi G/(kV)}$, then the coupling con-

stant of a GW mode $\boldsymbol{k}$ with the cavity electromagnetic field is $g_{\boldsymbol{k}} = \omega f_{\boldsymbol{k}}/4$, which has dimension of frequency. It is also convenient to introduce the dimensionless coupling $q_{\boldsymbol{k}} = g_{\boldsymbol{k}}/\Omega_k$. Therefore, taking into account the previous interaction term, the total discretized Hamiltonian reads $H = H_0 + H_I$, with H_0 the free-field Hamiltonian for the cavity and GW modes:

$$H_0 = \omega a^\dagger a + \sum_{\boldsymbol{k}} \Omega_k b_{\boldsymbol{k}}^\dagger b_{\boldsymbol{k}} . \quad (3)$$

The time evolution generated by the total Hamiltonian can be calculated exactly and is given by

$$U(t) = e^{-iHt} = \prod_{\boldsymbol{k}} U_{\boldsymbol{k}}(t) , \quad (4)$$

where the unitary operator in the interaction picture reads:

$$U_{\boldsymbol{k}}(t) = e^{iB_k(t)(a^\dagger a)^2} e^{q_{\boldsymbol{k}}a^\dagger a(\gamma_k b_k^\dagger - \gamma_k^* b_k)} , \quad (5)$$

with $\gamma_k = (1 - e^{-i\Omega_k t})$ and $B_k(t) = q_{\boldsymbol{k}}^2(\Omega_k t - \sin\Omega_k t)$ and the accompanying GW states evolve according to the free evolution [3, 41]

$$|\Psi(t)\rangle = \prod_{\boldsymbol{k}} e^{-ib_{\boldsymbol{k}}^\dagger b_{\boldsymbol{k}}\Omega t}|\Psi\rangle . \quad (6)$$

See Appendix A for further details. Note that the $B_k(t)$ term is quadratic in $q_{\boldsymbol{k}}$, which leads to effects of order G in the exponent. As shown in [3], this term results in sub-leading effects for various states of interest, such as the vacuum, coherent, squeezed vacuum and thermal states of GWs. From now on, we will therefore approximate (5) as

$$U_{\boldsymbol{k}}(t) \approx e^{q_{\boldsymbol{k}}a^\dagger a(\gamma_k b_k^\dagger - \gamma_k^* b_k)} , \quad (7)$$

where γ_k incorporates the time dependence. Physically, the meaning of (7) is clear: it corresponds to a GW-induced *phase operator* acting on the electromagnetic mode, with the acquired phase proportional to the GW quadrature. Conversely, (7) acts on the GW mode as a *displacement* operator, proportional to the number of photons contained in the electromagnetic field.

We are interested in the dynamics of an optical field interacting with a single GW mode, but notice the unitary operator (4) couples the detector to *all* GW modes. As it turns out [3], the effect of additional modes populated by the vacuum or

thermal states at the expected temperature for a cosmic GW background yields negligible corrections to the dynamics of a single-mode GW populated by a macroscopic mean number of gravitons, such as the ones detectable by LIGO. Therefore, from now on we will make a single-mode approximation, but for estimates on the corrections due to the additional 'empty' modes see Appendix B.

3 Interaction between a quantum GW and a qubit

With the operator (7) we can calculate the time evolution of arbitrary states of the optical field interacting with the relevant modes of quantum GWs. To illustrate that, consider the simplified example of an optical qubit spanned by the vacuum and single photon states $\{|0\rangle, |1\rangle\}$. For simplicity, we drop the index $\boldsymbol{k}$ and unless specified consider only a single GW mode. Time evolution of the optical qubit in the presence of a GW in the state $|\Psi(t)\rangle$ is then

$$
\begin{aligned}
(\alpha|0\rangle + \beta|1\rangle)\,|\Psi(t)\rangle &\to \\
\alpha|0\rangle|\Psi(t)\rangle &+ \beta|1\rangle D(q\gamma)|\Psi(t)\rangle\,, \quad (8)
\end{aligned}
$$

where $D(q\gamma)$ is the displacement operator acting on the GW mode and q, γ are the dimensionless coupling and time-evolution function as defined in the previous section, specialized for the mode of interest. Notice the states $|\Psi(t)\rangle$ and $D(q\gamma)|\Psi(t)\rangle$ are in general not orthogonal. Tracing out the GW mode, we find the time-dependent qubit density matrix,

$$
\rho(t) = \begin{pmatrix} |\alpha|^2 & \mathcal{I}_1(t)\alpha\beta^* \\ \mathcal{I}_1(t)^*\alpha^*\beta & |\beta|^2 \end{pmatrix} \quad (9)
$$

where,

$$
\mathcal{I}_1(t) = \langle\Psi(t)|D(q\gamma)|\Psi(t)\rangle\,. \quad (10)
$$

Dynamics of the qubit in the Bloch sphere is governed by the inner product $\mathcal{I}_1(t)$, which is a complex number with $|\mathcal{I}_1(t)| \le 1$.

The modulus of $\mathcal{I}_1(t)$ is associated to GW induced decoherence processes: it multiplies the off-diagonal terms of the density matrix, and as such it increases the von Neumann entropy of the optical mode whenever $|\mathcal{I}_1(t)| < 1$. An increase in entropy can be understood as a consequence of noise in the system. For certain situations, such as the case of a multi-mode GW thermal state
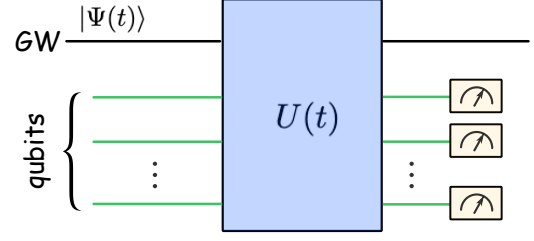

Figure 1: Conceptual experiment of a quantum GW interacting with a simplified toy model detector consisting of an ensemble of optical qubits. The GW state induces an open quantum system dynamics on the qubits, and noise spectroscopy can in principle be used to discriminate between different quantum GW states and measure quantum properties of the gravitational field.

at temperature T, Markovian approximations can be made and it is possible to show that interaction of the qubit with the GW field leads to a decoherence rate Γ proportional to $k_B T \left(\Delta E/E_{\mathrm{pl}}\right)^2$, where ΔE is the energy difference between qubit states and E_{pl} is the Planck energy [42]; see Appendix C for a derivation.

In general the time evolution comprises a combination of shrinkage of the Bloch vector, recoherence, and a unitary evolution given by the phase of $\mathcal{I}_1(t)$ resulting in precession of the Bloch vector. Casting the qubit density matrix as a linear combination of Pauli matrices σ_i in the standard form

$$
\rho(t) = \frac{\mathbb{1} + \vec{r}(t) \cdot \vec{\sigma}}{2}\,, \quad (11)
$$

we can explicitly calculate the Bloch vector $\vec{r}(t)$ for GWs in different quantum states. Considering as the qubit initial state $\rho(0) = |+\rangle\langle+|$, the Bloch vectors of qubits interacting with GWs in the vacuum ($|0\rangle$), coherent ($|h\rangle$) and squeezed ($|r\rangle$) states are respectively given by,

$$
\vec{r}_{|0\rangle}(t) = e^{-q^2(1-\cos\Omega t)}\left(\cos\omega t, \sin\omega t, 0\right), \quad (12)
$$

$$
\begin{aligned}
\vec{r}_{|h\rangle}(t) = e^{-q^2(1-\cos\Omega t)} \times \\
\left(\cos\left(\omega t + \Delta\varphi_0(t)\right), \sin\left(\omega t + \Delta\varphi_0(t)\right), 0\right), \quad (13)
\end{aligned}
$$

$$
\vec{r}_{|r\rangle}(t) = e^{-8e^{2r}q^2\sin^4\left(\frac{\Omega t}{2}\right)}\left(\cos\omega t, \sin\omega t, 0\right), \quad (14)
$$

where the phase $\Delta\varphi_0(t)$ in the coherent case is (see the discussion in the next sections),

$$
\Delta\varphi_0(t) = 2q|h|\sin\Omega t\,. \quad (15)
$$

In deriving (12), (13) and (14) we have considered a squeezed state with vanishing squeezing angle and have taken the limit $r \gg 1$, for simplicity,

but generalization to arbitrary values of r and squeezing phase is straightforward. The coupling strength q governing the interaction between the optical and GW fields is small [3], which guarantees that the backreaction of the detector upon the gravitational field can be neglected [2]. The prefactors in the Bloch vectors account for periodic shrinking and as discussed are associated to gravitational induced *noise*, while the phase $\Delta\varphi_0(t)$ in the coherent case causes a periodic precession of the qubit state analogous to the phase signal imprinted upon a coherent optical mode by a GW. As we will soon show, the phase $\Delta\varphi_0(t)$ is also present in the case of a GW interacting with a coherent electromagnetic state and corresponds to the signal measured by interferometric GW detectors. Notice also the appearance of the exponential enhancement factor e^{2r} in the shrinking factor associated to squeezed GW states, as predicted in [3, 8, 10].

Trajectories of the Bloch vector are hence dependent on the quantum GW state. Different GW quantum states can *in principle* be discriminated by qubit noise spectroscopy [43]. One could then conceive a conceptual experiment as shown in Figure 1, in which an ensemble of qubits is left to interact with a GW state for some time, after which measurements on the qubits are performed. Properties of the statistical distribution of measured states will then contain information on the GW quantum state. This scheme offers a path to search for quantum gravitational effects using interferometric detectors and to discriminate between different quantum states of the GW field. The conceptual experiment shown in Figure 1 can be understood as a quantum mechanical version of the proposals in [8, 10] where, in the context of a classical detector consisting of two test masses separated by a certain distance, it was shown that fluctuations associated to GWs in different quantum states lead to modifications of the geodesic deviation equation by addition of a stochastic Langevin term whose noise correlators are specific to the quantum GW state. Observation of the stochastic trajectories of the test masses could then reveal quantum features of the GW field. Analogously, observation of the stochastic *quantum* trajectories of an ensemble of optical states as dictated by (12) - (14) would allow for the discrimination of different quantum states of GWs. Notably, the authors of [8, 10] have also

shown that when the GW is in a squeezed state with squeezing parameter r, statistical properties of the trajectories of test masses such as the two-point position correlation function and consequently the position power spectral density depend on an enhancement factor approximatelly given by e^{2r}, as we also found in the Bloch vector (14).

4 Squeezed-coherent GW states

We have considered thus far coherent and squeezed states of the GW field. We turn our attention to single-mode *squeezed-coherent states*, defined as

$$|\xi, h\rangle = D(h)S(\xi)|0\rangle, \qquad (16)$$

where $S(\xi)$ is the squeezing operator, $h = |h|e^{i\theta}$ and $\xi = re^{i\phi}$. Note that the displacement and squeezing operators do not commute but we can convert squeezed-displaced waves (SD) into displaced-squeezed states (DS) via the operator identity,

$$D(h)S(\xi) = S(\xi)D(h\cosh r + h^* e^{2i\phi}\sinh r). \qquad (17)$$

In other words, one form can always be brought into the other through a suitable redefinition of constants. We denote gravitational coherent states by $|h\rangle = D(h)|0\rangle$, see Appendix A for definitions and useful formulae.

Squeezed-coherent states can be non-classical in the sense that their graviton number statistics can become sub-Poissonian. To see that, define the Fano factor,

$$F = \Delta N^2 / \langle N \rangle,$$

where $\Delta N^2, \langle N \rangle$ are the variance and mean number of gravitons, respectively. Classical states are obtained by mixing coherent states and always have $F \geq 1$ [40] – i.e. classical states are super-Poissonian – while a Fano factor smaller than one – sub-Poissonian statistics – can only be explained by quantum theory. The Fano factor is related to the field second-order autocorrelation function $g_{GW}^2 = \langle b^\dagger b^\dagger bb \rangle / \langle b^\dagger b \rangle^2$ as

$$g_{GW}^2 = 1 + (F - 1)/\langle N \rangle, \qquad (18)$$

so $F < 1$ implies $g_{GW}^2 < 1$. For a squeezed-coherent state as defined in (16) with $\phi = 2\theta$, the

variance and mean graviton numbers are given by [4],

$$\Delta N^2 = |h|^2 e^{-2r} + 2\sinh^2 r \cosh^2 r \,, \quad (19)$$

$$\langle N \rangle = |h|^2 + \sinh^2 r \,. \quad (20)$$

In the limit that $|h|^2 \gg 2e^{2r}\sinh^2 r \cosh^2 r$ we have

$$F = \frac{\Delta N^2}{\langle N \rangle} \approx e^{-2r} < 1 \,. \quad (21)$$

Note that for a squeezing of order $r \approx 1$, the above approximation is valid whenever $|h|^2 \gg 50$. In this sense squeezed-coherent states can display non-classical behavior even in the macroscopic limit of large numbers of quanta. While this is also the case in electromagnetic quantum optics [44], photons strongly interact with matter, so these states tend to be short-lived [45]. In contrast, GWs interact very weakly with their environment. If squeezed-coherent GW states are produced in nature, they could travel the universe nearly unperturbed [46].

5 GW state reconstruction and non-classical observables

We would now like to show how quantum information contained in a GW can be obtained from an experiment as depicted in Figure 1, where a single copy of the GW interacts with multiple copies of an optical state and subsequent measurements are performed on the optical ensemble. As it turns out, if the GW state is Gaussian – as is the case for squeezed-coherent GWs – we could in principle completely reconstruct the gravitational quantum state and consequently perform non-classicality witness measurements and also obtain the gravitational second-order coherence g_{GW}^2.

Consider a single mode GW initially in the state $|\Psi\rangle$ interacting with the optical wavefunction,

$$|\Phi\rangle = a_0|0\rangle + a_1|1\rangle + a_2|2\rangle + ... \quad (22)$$

The total initial state is $|\Phi\rangle|\Psi\rangle$. Time evolution according to (7) reads,

$$U|\Phi\rangle|\Psi\rangle = a_0|0\rangle|\Psi\rangle + a_1|1\rangle D(-q\gamma)|\Psi\rangle$$
$$+ a_2|2\rangle D(-2q\gamma)|\Psi\rangle + ... \quad (23)$$

where we have absorbed the free dynamics of the GW in the displacement operator, $\langle\Psi(t)|D(q\gamma)|\Psi(t)\rangle = \langle\Psi|D(-q\gamma^*)|\Psi\rangle$.

The reduced density matrix of the optical mode carries information on the initial GW state. Denote the reduced optical density matrix components in the Fock basis as $\rho_{nm}(t) = \langle n|\rho(t)|m\rangle$. The density matrix is an observable and we can have access to all of its components – say by state tomography – provided we let several copies of the initial optical state interact with the GW. Here, we will be interested in the $\rho_{0n} = \langle 0|\rho|n\rangle$ components, given by

$$\rho_{0n} = a_0 a_n^* \langle\Psi|D(-nq\gamma^*)|\Psi\rangle \,, \quad (24)$$

where the $\{a_m\}$ coefficients are known, determined by the initial optical state $|\Phi\rangle$. Therefore, we have access to the quantities $\langle\Psi|D(-nq\gamma^*)|\Psi\rangle$ for different values of n. Define,

$$\mathcal{I}_n(t) = \langle\Psi|D(-qn\gamma^*)|\Psi\rangle =$$
$$\mathrm{Tr}\left(\sigma D(-nq\gamma^*)\right) = \mathrm{Tr}\left(\sigma e^{-qn\left(\gamma^* b^\dagger - \gamma b\right)}\right) \,, \quad (25)$$

where $\sigma = |\Psi\rangle\langle\Psi|$ is the initial GW density matrix and $\gamma^* = 1 - e^{it'}$, with t' denoting the rescaled dimensionless time parameter $t' = \Omega t$.

Recall that the *quantum characteristic function* $\chi(\eta, \eta^*)$ is defined as

$$\chi(\eta, \eta^*) = \mathrm{Tr}\left(\sigma e^{\eta b^\dagger - \eta^* b}\right) = C(\eta, \eta^*)e^{-|\eta|^2/2} \,, \quad (26)$$

where η, η^* are complex variables and $C(\eta, \eta^*)$ is the *normally ordered* quantum characteristic function,

$$C(\eta, \eta^*) = \mathrm{Tr}\left(\sigma e^{\eta b^\dagger} e^{-\eta^* b}\right) \,. \quad (27)$$

This function contains all the information of a quantum state σ [47], and thus can be seen as an alternative representation to the density matrix. It is related to the graviton-number generating function [48], from which all normally ordered moments of the graviton number operator can be obtained, in particular the graviton second-order correlation function g_{GW}^2. If we can partially reconstruct $C(\eta, \eta^*)$ from data, we could obtain information on the graviton-number statistics of the corresponding GW state, or use measured moments of the GW field quadratures to witness nonclassicality of the gravitational field.

Observe that (25) corresponds to the quantum characteristic function evaluated at the contours

in the complex plane given by $qn\left(e^{it'} - 1\right)$, so we expect that some information on the initial GW quantum state can be retrieved from measurements of the reduced optical density matrix. Since we only have partial access to $C(\eta, \eta^*)$ in the complex domain, it is not clear how much of the information can be retrieved. It turns out that if the GW state is Gaussian, the initial GW state and consequently all of its expectation values can be completely recovered from measurements of $\mathcal{I}_n(t)$.

The essential idea is to consider time derivatives of $\mathcal{I}_n(t)$ evaluated at $t' = 0$. For different values of n, the time derivatives provide relations between normally ordered expectation values, from which we can reconstruct the desired first and second-order field correlation functions necessary for specifying the Gaussian state. Define the time derivatives as

$$\alpha_\nu(n) \equiv \frac{\partial^\nu \mathcal{I}_n(t')}{\partial t'^\nu}\Big|_{t'=0} \tag{28}$$

and the field quadratures,

$$X = b + b^\dagger \,, \tag{29}$$
$$Y = i\left(b - b^\dagger\right) \,. \tag{30}$$

Direct computation shows that the moments are given by,

$$\langle qX \rangle = -i\alpha_1(1) \,, \tag{31}$$

$$i\langle qY \rangle = \frac{1}{2}\alpha_2(2) - 2\alpha_2(1) \,, \tag{32}$$

$$\langle q^2 X^2 \rangle = \alpha_2(1) - \frac{1}{2}\alpha_2(2) \,, \tag{33}$$

$$\langle qXqY \rangle - \frac{1}{2}\langle[qX, qY]\rangle = \frac{2}{3}\alpha_3(1) - \frac{1}{12}\alpha_3(2) + \frac{1}{2}\alpha_1(1) \,, \tag{34}$$

$$\langle Y^2 \rangle = 2 + 4\langle N \rangle - \langle X^2 \rangle \,, \tag{35}$$

where $\langle N \rangle$ is the mean graviton number, which can be obtained by measuring the amplitude of the GW. Alternatively, $\langle Y^2 \rangle$ can be obtained from the other moments by normalization of the corresponding Wigner function associated to the covariance matrix. We recall that the time derivatives are with respect to rescaled time, and the commutator $[qX, qY] = -2q^2 i$ is known. We refer to Appendix D for more details on this calculation.

From the moments $\langle X \rangle, \langle Y \rangle, \langle X^2 \rangle, \langle XY \rangle, \langle Y^2 \rangle$ we can reconstruct the GW covariance matrix, which for Gaussian states is equivalent to knowledge of the full quantum state [49]. Furthermore, photon-number statistics, in particular the g^2_{GW} function, can be determined from knowledge of the covariance matrix [50]. The intracavity optical states can be determined by the leaking cavity fields via the input-output formalism [51].

All correlation functions above are multiplied by powers of the dimensionless coupling q. We can express the moments (31)-(35) in the continuum limit by defining free field quadratures for a fixed GW polarization as

$$h = \int \frac{dk}{\sqrt{(2\pi)^3}}\left(\sqrt{\frac{8\pi G}{k}}\mathfrak{b}_k + h.c.\right) \,, \tag{36}$$

$$\bar{h} = i\int \frac{dk}{\sqrt{(2\pi)^3}}\left(\sqrt{\frac{8\pi G}{k}}\mathfrak{b}_k - h.c.\right) \,. \tag{37}$$

Note that in retrieving the continuum limit, we write $h, \bar{h}$ as integrals over all modes, including the empty modes discussed in Appendix B. We have the limits,

$$qX \to (\omega/4\Omega)h \,, \tag{38}$$
$$qY \to (\omega/4\Omega)\bar{h} \,, \tag{39}$$

and

$$\langle[qX, qY]\rangle \to (\omega/4\Omega)^2 \langle\left[h, \bar{h}\right]\rangle = -2i(\omega/4\Omega)^2 \,. \tag{40}$$

Altogether, for a single-mode GW state interacting with an optical density matrix initially in state $\rho(0) = |\Phi\rangle\langle\Phi|$ we find the relations:

$$\langle h \rangle = -\left(\frac{4i}{\omega}\right)\frac{\partial \mathcal{I}_1}{\partial t}\,, \tag{41}$$

$$\langle \bar{h} \rangle = \left(\frac{4i}{\omega\Omega}\right)\left(\frac{1}{2}\frac{\partial^2 \mathcal{I}_2}{\partial t^2} - 2\frac{\partial^2 \mathcal{I}_1}{\partial t^2}\right)\,, \tag{42}$$

$$\langle h^2 \rangle = \left(\frac{16}{\omega^2}\right)\left(\frac{\partial^2 \mathcal{I}_1}{\partial t^2} - \frac{1}{2}\frac{\partial^2 \mathcal{I}_2}{\partial t^2}\right)\,, \tag{43}$$

$$\langle h\bar{h} \rangle = \left(\frac{16}{\omega^2\Omega}\right)\left(\frac{2}{3}\frac{\partial^3 \mathcal{I}_1}{\partial t^3} - \frac{1}{12}\frac{\partial^3 \mathcal{I}_2}{\partial t^3}\right) + \left(\frac{8\Omega}{\omega^2}\right)\frac{\partial \mathcal{I}_1}{\partial t} - i\,, \tag{44}$$

$$\langle \bar{h}^2 \rangle = 2 + 4\langle N \rangle - \langle h^2 \rangle\,, \tag{45}$$

where $\mathcal{I}_n(t) = \rho_{0n}/a_0 a_n^*$ and the proportionality constants a_0, a_n^* are determined by the initial optical state $|\Phi\rangle$. Using these relations, we can obtain the quantum correlation functions of the spacetime metric perturbations associated to a single GW in the transverse traceless gauge, directly from experimentally accessible data.

6 Optical coherent states

Consider now the interaction between an optical coherent state and a squeezed-coherent GW. Let the transverse cavity electric field be defined as $\vec{\mathcal{E}} = \mathcal{E}\hat{z}$, where $\mathcal{E} \equiv \sqrt{\frac{\omega}{V_c}}\left(a + a^\dagger\right)/\sqrt{2}$, with V_c the cavity mode volume. The mean electric field associated to the optical coherent state $|\alpha\rangle$ interacting with an initially independent pure GW state $|\Psi(t)\rangle$ reads

$$\langle \mathcal{E}(t) \rangle = \sqrt{\frac{\omega}{V_c}}\left(\frac{\mathcal{I}_1(t)\alpha + \mathcal{I}_1(t)^*\alpha^*}{\sqrt{2}}\right)\,, \tag{46}$$

where $\mathcal{I}_1(t)$ is given in (10).

For calculation of the expectation value (46), it is more convenient to write the squeezed-coherent state as a displacement followed by the squeezing operator. By use of (17) we then write the squeezed-coherent state as,

$$|\Psi(t)\rangle = S(r, \phi(t))D(h(t))|0\rangle\,, \tag{47}$$

where $S(r, \phi(t))$ is the single-mode squeezing operator in the interaction picture with squeezing parameter r and angle $\phi(t) = \phi_0 - \Omega t$, and $D(h(t))$ is a displacement operator with amplitude $h(t) = |h|e^{-i\Omega t}$.

The $\mathcal{I}_1(t)$ factor can be rewritten,

$$\langle \Psi(t)|D(q\gamma)|\Psi(t)\rangle =$$
$$= \langle h|S^\dagger(r, \phi)D(q\gamma)S(r, \phi)|h\rangle$$
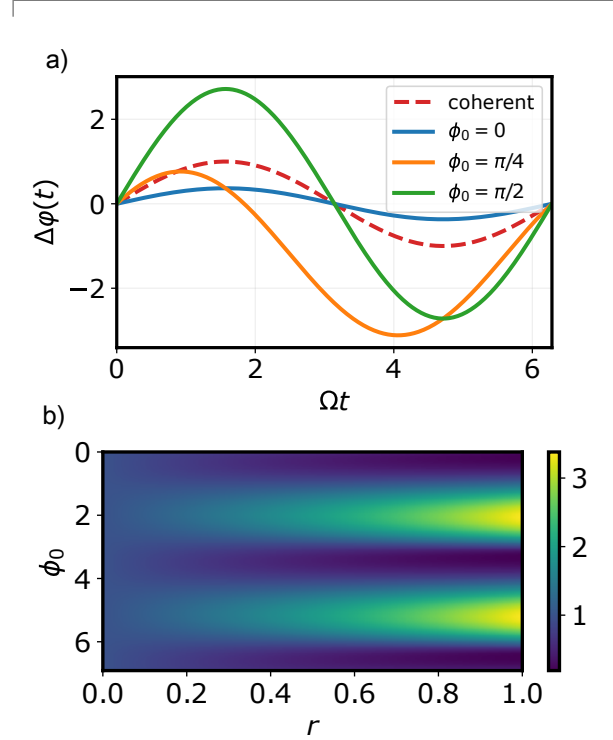$$= \langle h|D(\kappa)|h\rangle\,, \tag{48}$$

a)

b)

Figure 2: a) Normalized relative phase of the optical field as a function of time, in units of GW strain $\omega f/\Omega$. Time is displayed in units of the GW period $2\pi/\Omega$. Different values of the squeezing angle ϕ_0 are considered, exhibiting enhancement ($\phi_0 = \pi/2$, green curve), suppression ($\phi_0 = 0$, blue curve) and an intermediate case ($\phi_0 = \pi/4$, orange curve). Note that in the intermediate case the phase profile is asymmetric and exhibits a characteristic form which can be used to single out squeezed-coherent states from plain coherent states (red dashed curve). For this plot the squeezing parameter is taken to be $r = 1$. b) Maximum phase value (in units of $\omega f/4\Omega$) as a function of squeezing parameter r and angle ϕ_0 showing enhancement or suppression of the GW signal, depending on characteristics of the source.

where κ denotes

$$\kappa = q\left(\gamma \cosh r + \gamma^* e^{2i\phi_0}e^{-2i\Omega t}\sinh r\right)\,. \tag{49}$$

The above expression can be decomposed in terms of linear and quadratic contributions in q. As in the qubit case, quadratic contributions yield real-valued factors and are associated with gravity-induced decoherence, which we neglect in view of the discussions in Appendix B. Considering only terms linear in q the relevant contribution coming from (48) is

$$\exp\left(\kappa|h|e^{i\Omega t} - \kappa^*|h|e^{-i\Omega t}\right) . \qquad (50)$$

This term yields a time-varying phase $e^{i\Delta\varphi}$ in the mean coherent electric field, dependent on the squeezing parameter r and angle ϕ_0,

$$\Delta\varphi = 2|h|q\big(\sin\Omega t \cosh r + \\ + \sin\left(2\phi_0 - \Omega t\right)\sinh r - \sin 2\phi_0 \sinh r\big). \qquad (51)$$

This phase factor can be thought of as a continuous-variable version of the precession of the Bloch vector discussed previously, and corresponds to the phase signal measured by interferometric GW detectors.

The squeezing phase ϕ_0 is a property of the squeezed state. In electromagnetic quantum optics, for example, it is controlled by the phase of the pump beam driving the optical parametric oscillator [52]. The phase (51) tells us that depending on the characteristics of a source of squeezed-coherent GWs, the signal in a GW detector can be *enhanced* or *suppressed* depending on ϕ_0. In particular, if $\phi_0 = \pi/2$, we have

$$\Delta\varphi = 4|h|q\, e^r \sin\Omega t\,, \qquad (52)$$

and the signal is enhanced by a squeezing factor e^r. The enhancement can be traced back to the quantum nature of squeezed states, and we remind its connection to the enhancement of quantum noise due to squeezed GWs as predicted in [3, 8, 10] while predicting further that squeezing – in particular squeezed-coherent states of GWs – can lead to an enhancement in the *signal* measurable by interferometric detectors.

If $\phi_0 = 0$ the signal becomes

$$\Delta\varphi = 4|h|q\, e^{-r}\, \sin\Omega t\,, \qquad (53)$$

and we find a suppression of the signal by a squeezing factor e^{-r}. We note that the case of $\phi_0 = 0$ displays $g_{GW}^2 \approx e^{-2r} < 1$ provided $|h|^2 \gg 8e^{4r}\sinh^2 r \cosh^2 r$. For $r = 1$, $g_{GW}^2 \approx$

$1/e^2 \approx 0.13$, and $\Delta\varphi/\Delta\varphi_0 = 1/e \approx 0.37$, indicating that a squeezed-coherent GW with suitable phase could display sub-Poissonian statistics – and hence quantum behavior – while only mildly attenuating the signal measured by LIGO.

It is instructive to compare the enhanced and suppressed phases due to squeezed-coherent waves with the corresponding phase due to a coherent state GW. Setting $r = 0$ in (51) the induced phase becomes [3],

$$\Delta\varphi_0 = 2q|h|\sin\Omega t\,, \qquad (54)$$

and note the same phase was found in the precession of the Bloch vector in (13). We can write the prefactor $q|h|$ in terms of experimentally accessible quantities. A GW has energy-density given by $E = (1/32\pi G)\Omega^2 f^2$ [53] where f denotes the GW strain for mode $\boldsymbol{k}$. A single graviton has energy-density $E_g = \Omega/V$, so a wave with energy density E has on average $\langle N \rangle = E/E_g$ gravitons. For coherent states the mean particle number is given by $|h| = \sqrt{\langle N \rangle}$, so the acquired phase becomes

$$\Delta\varphi_0 = \left(\frac{\omega}{4\Omega}\right)\, f \sin\Omega t\,, \qquad (55)$$

in accordance to the classical prediction that a GW induces an optical phase proportional to its strain and oscillating at the GW frequency [54].

The phase (55) allows us to connect our results with current gravitational wave detectors. Note the phase is proportional to the GW strain f, which originates from writing the prefactor $2q|h|$ in (54) in terms of experimentally accessible numbers. The same prefactor appears in the enhanced and suppressed phases (52), (53) due to squeezed-coherent states, and in the limit that $|h|^2 \gg \sinh^2 r$ we have $\langle N \rangle \approx |h|^2$. Therefore, if the squeezed-coherent GWs have mean numbers of gravitons comparable to the coherent GWs currently detected by LIGO – corresponding to strains of $f \approx 10^{-21}$ – and only a modest squeezing parameter, say $r \approx 1$, the signals due to these squeezed-coherent waves will be only slightly enhanced or suppressed with respect to signals currently detected.

Figure 2a) shows the relative phase imprinted by a squeezed-coherent GW state for different values of the squeezing angle, in comparison to the signal produced by a coherent state GW. As shown above, for the particular values $\phi_0 =$

$0, \pi/2$, the signal undergoes suppression and enhancement, respectively. For intermediate values of the squeezing angle (e.g. $\phi_0 = \pi/4$) the signal acquires a characteristic form with asymmetric amplitude, which can also be used to single out the squeezed-coherent state from other possible quantum states of the GW. Next, in Figure 2b) we show the maximum value of $\Delta\varphi$ normalized by the corresponding maximum phase attained through interaction with a coherent GW, $\max\left(\Delta\varphi_0\right) = \omega f/4\Omega$. As the squeezing parameter increases, the signal amplitude measured by the GW detector oscillates between enhancement and suppression according to the squeezing angle ϕ_0.

Detection of a signal as depicted in Figure 2 requires the coherent interaction between a GW and the optical modes on the order of a GW period. For GWs currently detected by LIGO, this corresponds to a photon lifetime of at least $\sim 300\,\mu$s, amounting to a cavity decay rate smaller than $\sim 3\,$kHz but note that optical state reconstruction as described in the previous section only requires time derivatives of the optical density matrix elements near the initial time of the experiment, relaxing the need for high-Q cavities.

We revisit the qubit example discussed in the previous section, now for the case of a squeezed-coherent GW. With (48) and (49) we can calculate the Bloch vector of the optical field associated to a squeezed-coherent GW finding – perhaps unsurprisingly at this point – a combination of the coherent and squeezed results,

$$\vec{r}_{|r,h\rangle}(t) = e^{-2e^{2r}q^2\sin^2(\Omega t)} \times$$
$$(\cos\left(\omega t + \Delta\varphi(t)\right), \sin\left(\omega t + \Delta\varphi(t)\right), 0) , \tag{56}$$

where we have assumed $\phi_0 = \pi/2$ and large r, for simplicity. We see the qubit inherits the shrinking factor associated to squeezing, plus an enhanced phase signal given in (52).

Similarly to the qubit model, an optical coherent state will acquire the phase signal manifest in the mean electric field $\langle\mathcal{E}(t)\rangle$, and also noise characteristics manifest in electromagnetic (EM) field correlation functions such as $g_{EM}^1(t) = \langle a^\dagger(0)a^\dagger(t)\rangle$, $g_{EM}^2 = \langle a^\dagger a^\dagger aa\rangle/\langle a^\dagger a\rangle^2$ and so on. Both signal and noise will provide information on the GW state. Calculation of the noise properties can be achieved through field theory methods, notably the Feynman-Vernon influence functional

formalism [8] or via the canonical quantization formalism. In certain situations Markovian approximations can be made; see for example [55] where the broadening of the power spectrum of a cavity due to inflationary squeezed states has been recently calculated.

As a final remark, we note similar effects are obtained if instead of a squeezed-coherent state we consider two-mode squeezed coherent states defined as

$$|\Psi\rangle_{12} = S_{12}(r, \phi(t))D_1(h_1(t))D_2(h_2(t))|0\rangle , \tag{57}$$

where $S_{12}(r, \phi(t))$ is the two-mode squeezing operator [56] acting on modes '1' and '2' defined by wave-vectors $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$, respectively, and $D_i(h_i(t))$ is a displacement operator acting on mode 'i' responsible for generating a seed coherent state; see Appendix E. Interestingly, two-mode squeezed(-coherent) states have been previously studied in cosmology [57] and could have been generated in the early universe [58].

7 Conclusion

In this work we have studied the optomechanical interaction of a GW in different quantum states – vacuum, coherent, squeezed and squeezed-coherent – with the quantized electromagnetic field in an optical cavity using the framework of effective field theory. We make no detailed assumption on the UV behavior of gravity, only requiring the existence of an energy range, well below the characteristic quantum gravity scale (assumed to be within a few order of magnitudes of the Planck scale) in which the use of effective theory is justified.

For the simplified situation in which the cavity state is given by a superposition of the vacuum and a single photon, referred to as an optical qubit, we have shown that trajectories of the associated Bloch vector are uniquely determined by the GW quantum state and are given by the combination of a shrinking factor associated to gravity-induced noise and precession motion resulting in a signal analogous to the one measured by conventional interferometric GW detectors.

Moreover, performing state tomography on an ensemble of electromagnetic quantum states which interacted with a single copy of a passing-by GW allows for the reconstruction of the GW

quantum state, provided the state is Gaussian. This offers a unique experimental perspective on the interplay between quantum mechanics and gravity.

For the case in which the GW is in a squeezed-coherent state, the gravitational induced fluctuations can exhibit an exponential enhancement, as originally pointed out in [3, 8, 10], and we show that the signal can also be exponentially enhanced or suppressed, depending on the squeezing phase. Squeezed-coherent GW states can also exhibit non-classical, sub-Poissonian graviton statistics. This opens the possibility of detecting quantum features associated to GWs in current or near-future detectors, provided possible sources of squeezed-coherent GWs emit with intensities comparable to the signals so far detected (strain $f \sim 10^{-21}$) and squeezing parameters of order one ($r \sim 1$).

In quantum optics, squeezed-coherent states are produced when nonlinear mode-mode couplings or parametric modulations are present, which occur in dielectric crystals when the polarizability of the material is probed beyond linear order [52]. In gravity, coupling between GW modes and between GWs and dynamical spacetime backgrounds naturally occurs due to the nonlinearity of General Relativity. Einstein's equations imply that the Riemann curvature tensor satisfies a nonlinear wave equation[1]. Using the so-called Brill-Hartle average [59], one can split the curvature into a background and a GW part. We can anticipate then at least two mechanisms for the parametric amplification of GWs:

I. The covariant derivative in the wave operator contains connection coefficients which are dependent on the propagating GW perturbations, thus producing higher-order terms in the GW field. This can be related to the graviton-graviton scattering processes in the effective field theory approach to gravity.

II. Even if quadratic couplings between the GWs are small, Einstein's equations contain terms coupling the curvature associated to GWs to the dynamical background curvature. If the radius of curvature of the background (given by the order of magnitude

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[1]The equation in components is $R_{\alpha\beta\gamma\delta;\mu}{}^{\mu} = 2R_{\alpha\beta\xi\mu}R_{\delta}{}^{\xi}{}_{\gamma}{}^{\mu} + 2R_{\alpha\xi\delta\mu}R_{\beta}{}^{\xi}{}_{\gamma}{}^{\mu} - 2R_{\alpha\xi\gamma\mu}R_{\beta}{}^{\xi}{}_{\delta}{}^{\mu}$.

of the components of the Riemann curvature tensor) is comparable to the wavelength of the waves, such curvature-coupling may lead to parametric amplification of the GWs, which in a quantum mechanical description amounts to squeezing of the seed coherent state. One example of this is the generation of squeezed GWs in cosmological perturbation theory.

Examples of mechanism I. include high intensity GW astrophysical events [60] and graviton-graviton scattering in field theory [61], while parametric processes of the type II. have been suggested in inflation [11–13, 58] and the end stages of black hole evaporation [14].

Noncoherent states of GWs are also expected to be produced via mechanism II. during the end stages of compact mergers [8, 10]. Numerical simulations indicate that perturbed black holes can generate outgoing gravitational radiation with characteristic nonlinear features, notably mode-mode coupling, second harmonic generation, stronger-than-linear output and parametric instabilities [62, 63], all phenomena associated to squeezing of quantum noise [43]. In Kerr black holes, mode coupling can occur even at linear order [64] and more recently, a novel mechanism for the nonlinear generation of GWs during the ringdown phase of a binary black hole merger has been proposed [65].

Finally, quantum enhanced GW detectors, in the sense of [66], might enable tests of the nonlinear features of gravity from the perspective of quantum mechanics.

Acknowledgments

References

[1] Alexander H Nitz, Collin D Capano, Sumit Kumar, Yi-Fan Wang, Shilpa Kastha, Marlin Schäfer, Rahul Dhurkunde, and Miriam Cabero. "3-ogc: Catalog of gravitational waves from compact-binary mergers". The Astrophysical Journal **922**, 76 (2021).

[2] Belinda Pang and Yanbei Chen. "Quantum interactions between a laser interferometer and gravitational waves". Phys. Rev. D **98**, 124006 (2018).

[3] Thiago Guerreiro. "Quantum effects in gravity waves". Classical and Quantum Gravity **37**, 155001 (2020).

[4] Luiz Davidovich. "Sub-poissonian processes in quantum optics". Rev. Mod. Phys. **68**, 127–173 (1996).

[5] Freeman Dyson. "Is a graviton detectable?". Int. J. Mod. Phys. A **28**, 1330041 (2013).

[6] A. I. Lvovsky. "Squeezed light". Chapter 5, pages 121–163. John Wiley & Sons, Ltd. (2015).

[7] Francesco Coradeschi, Antonia Micol Frassino, Thiago Guerreiro, Jennifer Rittenhouse West, and Enrico Junior Schioppa. "Can we detect the quantum nature of weak gravitational fields?". Universe**7** (2021).

[8] Maulik Parikh, Frank Wilczek, and George Zahariade. "Quantum mechanics of gravitational waves". Phys. Rev. Lett. **127**, 081602 (2021).

[9] Samarth Chawla and Maulik Parikh. "Quantum Gravity Corrections to the Fall of the Apple" (2021). arXiv:2112.14730.

[10] Maulik Parikh, Frank Wilczek, and George Zahariade. "Signatures of the quantization of gravity at gravitational wave detectors". Phys. Rev. D **104**, 046021 (2021).

[11] L. P. Grishchuk and Y. V. Sidorov. "Squeezed quantum states of relic gravitons and primordial density fluctuations". Phys. Rev. D **42**, 3413–3421 (1990).

[12] Andreas Albrecht, Pedro Ferreira, Michael Joyce, and Tomislav Prokopec. "Inflation and squeezed quantum states". Phys. Rev. D **50**, 4807–4820 (1994).

[13] Don Koks, Andrew Matacz, and B. L. Hu. "Entropy and uncertainty of squeezed quantum open systems". Phys. Rev. D **55**, 5917–5935 (1997).

[14] S. Hawking. "Black hole explosions?". Nature **248**, 30–31 (1974).

[15] Mark P. Hertzberg and Jacob A. Litterer. "Bound on Quantum Fluctuations in Gravitational Waves from LIGO" (2021). arXiv:2112.12159.

[16] W. Schleich and J. A. Wheeler. "Oscillations in photon distribution of squeezed states". J. Opt. Soc. Am. B **4**, 1715–1722 (1987).

[17] Charles W. Misner, K. S. Thorne, and J. A. Wheeler. "Gravitation". W. H. Freeman. San Francisco (1973).

[18] M. S. Safronova, D. Budker, D. DeMille, Derek F. Jackson Kimball, A. Derevianko, and Charles W. Clark. "Search for new physics with atoms and molecules". Rev. Mod. Phys. **90**, 025008 (2018).

[19] Fernando Monteiro, Gadi Afek, Daniel Carney, Gordan Krnjaic, Jiaxiang Wang, and David C. Moore. "Search for composite dark matter with optically levitated sensors". Phys. Rev. Lett. **125**, 181102 (2020).

[20] Charles P. Blakemore, Alexander Fieguth, Akio Kawasaki, Nadav Priel, Denzal Martin, Alexander D. Rider, Qidong Wang, and Giorgio Gratta. "Search for non-newtonian interactions at micrometer scale with a levitated test mass". Phys. Rev. D **104**, L061101 (2021).

[21] David C Moore and Andrew A Geraci. "Searching for new physics using optically levitated sensors". Quantum Science and Technology **6**, 014008 (2021).

[22] K. M. Backes et al. "A quantum enhanced search for dark matter axions". NaturePage 238 (2021).

[23] Deniz Aybas, Janos Adam, Emmy Blumenthal, Alexander V. Gramolin, Dorian Johnson, Annalies Kleyheeg, Samer Afach, John W. Blanchard, Gary P. Centers, Antoine Garcon, Martin Engler, Nataniel L. Figueroa, Marina Gil Sendra, Arne Wickenbrock, Matthew Lawson, Tao Wang, Teng Wu, Haosu Luo, Hamdi Mani, Philip Mauskopf, Peter W. Graham, Surjeet Rajendran, Derek F. Jackson Kimball, Dmitry Budker, and Alexander O. Sushkov. "Search

for axionlike dark matter using solid-state nuclear magnetic resonance". Phys. Rev. Lett. **126**, 141802 (2021).

[24] Peter W. Graham, David E. Kaplan, Jeremy Mardon, Surjeet Rajendran, William A. Terrano, Lutz Trahms, and Thomas Wilkason. "Spin precession experiments for light axionic dark matter". Phys. Rev. D **97**, 055006 (2018).

[25] K. Wurtz, B.M. Brubaker, Y. Jiang, E.P. Ruddy, D.A. Palken, and K.W. Lehnert. "Cavity entanglement and state swapping to accelerate the search for axion dark matter". PRX Quantum **2**, 040350 (2021).

[26] J. Estrada, R. Harnik, D. Rodrigues, and M. Senger. "Searching for dark particles with quantum optics". PRX Quantum **2**, 030340 (2021).

[27] D Carney, G Krnjaic, D C Moore, C A Regal, G Afek, S Bhave, B Brubaker, T Corbitt, J Cripe, N Crisosto, A Geraci, S Ghosh, J G E Harris, A Hook, E W Kolb, J Kunjummen, R F Lang, T Li, T Lin, Z Liu, J Lykken, L Magrini, J Manley, N Matsumoto, A Monte, F Monteiro, T Purdy, C J Riedel, R Singh, S Singh, K Sinha, J M Taylor, J Qin, D J Wilson, and Y Zhao. "Mechanical quantum sensing in the search for dark matter". Quantum Science and Technology **6**, 024002 (2021).

[28] Tanjung Krisnanda, Margherita Zuppardo, Mauro Paternostro, and Tomasz Paterek. "Revealing nonclassicality of inaccessible objects". Phys. Rev. Lett. **119**, 120402 (2017).

[29] Sougato Bose, Anupam Mazumdar, Gavin W. Morley, Hendrik Ulbricht, Marko Toroš, Mauro Paternostro, Andrew A. Geraci, Peter F. Barker, M. S. Kim, and Gerard Milburn. "Spin entanglement witness for quantum gravity". Phys. Rev. Lett. **119**, 240401 (2017).

[30] C. Marletto and V. Vedral. "Gravitationally induced entanglement between two massive particles is sufficient evidence of quantum effects in gravity". Phys. Rev. Lett. **119**, 240402 (2017).

[31] Teodora Oniga and Charles H.-T. Wang. "Quantum gravitational decoherence of light and matter". Phys. Rev. D **93**, 044027 (2016).

[32] Daniel Carney, Holger Müller, and Jacob M. Taylor. "Using an atom interferometer to infer gravitational entanglement generation". PRX Quantum **2**, 030330 (2021).

[33] Daniel Carney, Holger Müller, and Jacob M. Taylor. "Comment on using an atom interferometer to infer gravitational entanglement generation" (2021). arXiv:2111.04667.

[34] Kirill Streltsov, Julen Simon Pedernales, and Martin Bodo Plenio. "On the Significance of Interferometric Revivals for the Fundamental Description of Gravity". Universe **8**, 58 (2022). arXiv:2111.04570.

[35] Tobias Westphal, Hans Hepach, Jeremias Pfaff, and Markus Aspelmeyer. "Measurement of gravitational coupling between millimetre-sized masses". NaturePage 225 (2021).

[36] Markus Aspelmeyer. "When Zeh Meets Feynman: How to Avoid the Appearance of a Classical World in Gravity Experiments". Fundam. Theor. Phys. **204**, 85–95 (2022). arXiv:2203.05587.

[37] Rafal Demkowicz-Dobrzański, Marcin Jarzyna, and Jan Kołodyński. "Chapter four - quantum limits in optical interferometry". Volume 60 of Progress in Optics, pages 345–435. Elsevier. (2015).

[38] Marko Toroš, Anupam Mazumdar, and Sougato Bose. "Loss of coherence of matter-wave interferometer from fluctuating graviton bath" (2020). arXiv:2008.08609.

[39] Alessandra Buonanno and Yanbei Chen. "Scaling law in signal recycled laser-interferometer gravitational-wave detectors". Phys. Rev. D **67**, 062002 (2003).

[40] Marlan O. Scully and M. Suhail Zubairy. "Quantum optics". Cambridge University Press. (1997).

[41] Igor Brandão, Bruno Suassuna, Bruno Melo, and Thiago Guerreiro. "Entanglement dynamics in dispersive optomechanics: Nonclassicality and revival". Phys. Rev. Research **2**, 043421 (2020).

[42] M. P. Blencowe. "Effective field theory approach to gravitationally induced decoherence". Phys. Rev. Lett. **111**, 021302 (2013).

[43] A. A. Clerk, M. H. Devoret, S. M. Girvin, Florian Marquardt, and R. J. Schoelkopf. "Introduction to quantum noise, measurement, and amplification". Rev. Mod. Phys. **82**, 1155–1208 (2010).

[44] E. Oudot, P. Sekatski, F. Fröwis, N. Gisin, and N. Sangouard. "Two-mode squeezed states as schrödinger cat-like states". J. Opt. Soc. Am. B **32**, 2190–2197 (2015).

[45] Wojciech H. Zurek, Salman Habib, and Juan Pablo Paz. "Coherent states via decoherence". Phys. Rev. Lett. **70**, 1187–1190 (1993).

[46] Charles W Misner, Kip Thorne, and Wojciech Żurek. "John wheeler, relativity, and quantum information". Physics Today**62** (2009).

[47] DF Walls and GJ Milburn. "Quantum optics (springer, berlin" (1994).

[48] Edward B. Rockower. "Calculating the quantum characteristic function and the photon-number generating function in quantum optics". Phys. Rev. A **37**, 4309–4318 (1988).

[49] Christian Weedbrook, Stefano Pirandola, Raúl García-Patrón, Nicolas J. Cerf, Timothy C. Ralph, Jeffrey H. Shapiro, and Seth Lloyd. "Gaussian quantum information". Rev. Mod. Phys. **84**, 621–669 (2012).

[50] V. V. Dodonov, O. V. Man'ko, and V. I. Man'ko. "Multidimensional hermite polynomials and photon distribution for polymode mixed light". Phys. Rev. A **50**, 813–817 (1994).

[51] Michael Vanner, Igor Pikovski, and M. Kim. "Towards optomechanical quantum state reconstruction of mechanical motion". Annalen der Physik**527** (2015).

[52] Robert W. Boyd. "Nonlinear optics". Academic Press. (2008).

[53] L. D. Landau and E. M. Lifshitz. "The classical theory of fields course of theoretical physics". Butterworth-Heinemann. (1975).

[54] Benjamin P. Abbott et al. "The basic physics of the binary black hole merger GW150914". Annalen Phys. **529**, 1600209 (2017). arXiv:1608.01940.

[55] F. Shojaei Arani, M. Bagheri Harouni, B. Lamine, and A. Blanchard. "Imprints of the Squeezed Primordial Gravitational Waves on the Quantum Electromagnetic Field" (2021). arXiv:2110.10962.

[56] Bonny L. Schumaker and Carlton M. Caves. "New formalism for two-photon quantum optics. ii. mathematical foundation and compact notation". Phys. Rev. A **31**, 3093–3111 (1985).

[57] Andreas Albrecht, Pedro Ferreira, Michael Joyce, and Tomislav Prokopec. "Inflation and squeezed quantum states". Phys. Rev. D **50**, 4807–4820 (1994). arXiv:astro-ph/9303001.

[58] Sugumi Kanno and Jiro Soda. "Detecting nonclassical primordial gravitational waves with hanbury-brown–twiss interferometry". Phys. Rev. D **99**, 084010 (2019).

[59] Dieter R. Brill and James B. Hartle. "Method of the self-consistent field in general relativity and its application to the gravitational geon". Phys. Rev. **135**, B271–B278 (1964).

[60] R. F. Sawyer. "Quantum break in high intensity gravitational wave interactions". Phys. Rev. Lett. **124**, 101301 (2020).

[61] M. T. Grisaru, P. van Nieuwenhuizen, and C. C. Wu. "Gravitational born amplitudes and kinematical constraints". Phys. Rev. D **12**, 397–403 (1975).

[62] Yosef Zlochower, Roberto Gómez, Sascha Husa, Luis Lehner, and Jeffrey Winicour. "Mode coupling in the nonlinear response of black holes". Phys. Rev. D **68**, 084014 (2003).

[63] Aaron Zimmerman and Zachary Mark. "Damped and zero-damped quasinormal modes of charged, nearly extremal black holes". Phys. Rev. D **93**, 044033 (2016).

[64] Andrzej Rostworowski. "Towards a theory of nonlinear gravitational waves: A systematic approach to nonlinear gravitational perturbations in the vacuum". Phys. Rev. D **96**, 124026 (2017).

[65] Laura Sberna, Pablo Bosch, William E. East, Stephen R. Green, and Luis Lehner. "Nonlinear effects in the black hole ringdown: Absorption-induced mode excitation". Phys. Rev. D **105**, 064046 (2022).

[66] Hsin-Yuan Huang et al. "Quantum advantage in learning from experiments". Science **376**, abn7293 (2022). arXiv:2112.00778.

[67] Bruce Allen. "The Stochastic gravity wave background: Sources and detection" (1996). arXiv:gr-qc/9604033.

[68] G. Massimo Palma, Kalle-Antti Suominen, and Artur K. Ekert. "Quantum computers and dissipation". Proc. Roy. Soc. Lond. A **452**, 567–584 (1996). arXiv:quant-ph/9702001.

[69] V. Vedral. "Decoherence of massive superpositions induced by coupling to a quantized gravitational field" (2020). arXiv:2005.14596.

[70] Andreas Albrecht, Pedro Ferreira, Michael Joyce, and Tomislav Prokopec. "Inflation and squeezed quantum states". Phys. Rev. D **50**, 4807–4820 (1994).

A   Time evolution

We review the time evolution operator here. The discrete-mode Hamiltonian (eqns. (2) and (3)) in the main text can be exponentiated exactly. The result of exponentiating mode by mode is summarized in the unitary operator [See Eq. (2) in [41]]

$$U_{\boldsymbol{k}}(t) = e^{-i b_{\boldsymbol{k}}^\dagger b_{\boldsymbol{k}} \Omega t} e^{-i\omega a^\dagger a} e^{i B_k(t)(a^\dagger a)^2} e^{q_{\boldsymbol{k}} a^\dagger a (\gamma_k b_k - \gamma_k^* b_k^\dagger)} \tag{58}$$

where $\gamma_k = (1 - e^{-i\Omega_k t})$ and $B_k(t) = q_{\boldsymbol{k}}^2 (\Omega_k t - \sin \Omega_k t)$. From now on we will work on a single-mode and omit the mode subscript $\boldsymbol{k}$.

A useful formula is

$$e^{-\lambda a^\dagger a} a e^{\lambda a^\dagger a} = a e^\lambda \tag{59}$$

We can ignore the free electromagnetic evolution in (58) as it only generates a linear time-dependent phase on the electromagnetic annihilation operator. As in the main text, we also ignore the nonlinear $B_k(t)$ term, as it has a negligible effect scaling quadratically and higher in $q_{\boldsymbol{k}}$. The evolution of a GW state under (58) then reads

$$\begin{aligned} U_{\boldsymbol{k}}(t)|\Psi\rangle &= e^{-ib^\dagger b \Omega t} e^{q a^\dagger a (\gamma b - \gamma^* b^\dagger)} e^{ib^\dagger b \Omega t} e^{-ib^\dagger b \Omega t} |\Psi\rangle \\ &= e^{q a^\dagger a (\gamma e^{i\Omega t} b - \gamma^* e^{-i\Omega t} b^\dagger)} e^{-ib^\dagger b \Omega t} |\Psi\rangle \\ &= e^{q a^\dagger a (\gamma b^\dagger - \gamma^* b)} |\Psi(t)\rangle \end{aligned} \tag{60}$$

where we have defined the time-evolving state $|\Psi(t)\rangle = e^{-ib^\dagger b \Omega t}|\Psi\rangle$. We can regard (60) as an operator acting on the photon annihilation operator by conjugation, which results in

$$a(t) = \langle\Psi|U_{\boldsymbol{k}}(t)^\dagger a U_{\boldsymbol{k}}(t)|\Psi\rangle = a\langle\Psi(t)|D(q\gamma)|\Psi(t)\rangle \tag{61}$$

For the case of a squeezed-coherent state, for example,

$$|\Psi\rangle = S(r, \phi_0) D(|h|)|0\rangle \tag{62}$$

from which we can calculate

$$\begin{aligned} |\Psi(t)\rangle &= e^{-ib^\dagger b \Omega t}|\Psi\rangle \\ &= e^{-ib^\dagger b \Omega t} S(r, \phi_0) D(|h|) e^{ib^\dagger b \Omega t} e^{-ib^\dagger b \Omega t} |0\rangle \\ &= e^{-ib^\dagger b \Omega t} S(r, \phi_0) D(|h|) e^{ib^\dagger b \Omega t} |0\rangle \\ &= S(r, \phi) D(h)|0\rangle \end{aligned} \tag{63}$$

where $\phi = \phi_0 - \Omega t$ and $h = |h|e^{-i\Omega t}$, as considered in the main text.

B   Multi-mode corrections

In the main text, we consider the effects of a single-mode GW in a state with non-vanishing mean number of gravitons interacting with the detector and neglect the effects of the additional modes in the vacuum state. For completeness, we briefly review the results from [3] which justify this single-mode approximation.

The time evolution of the coupled optical-GW system is given by a product of the operators $U_k(t)$, defined in (5) in the main text. We now wish to estimate the effect of vacuum modes upon electromagnetic operators. As an example, consider the optical annihilation operator a. Its time evolution is then given by

$$
\begin{aligned}
a(t) &= \left(\prod_k U_k(t)\right)^\dagger a \left(\prod_k U_k(t)\right) \\
&= \prod_k e^{iB_k(t)a^\dagger a} e^{iB_k(t)/2} e^{q_k a^\dagger a\left(\gamma_k b^\dagger - \gamma_k^* b_k\right)} a \\
&= \prod_k e^{iB_k(t)a^\dagger a} e^{iB_k(t)/2} D(q_k\gamma_k)a
\end{aligned}
\tag{64}
$$

where $D(q_k\gamma_k)$ is the GW displacement operator. Consider now the GW field to be in the vacuum state

$$
|0\rangle = \prod_k |0_k\rangle
\tag{65}
$$

where $|0_k\rangle$ denotes the vacuum state in mode k. The annihilation operator of an optical mode interacting with such gravitational vacuum can then be written as

$$
a(t) = e^{i\mathcal{F}(t)a^\dagger a} e^{i\mathcal{F}(t)/2} \mathcal{G}(t)a
\tag{66}
$$

where

$$
\mathcal{F}(t) = \sum_k B_k(t)
\tag{67}
$$

and

$$
\mathcal{G}(t) = \prod_k \langle 0_k| D(q_k\gamma_k)|0_k\rangle = \exp\left[-\frac{1}{2}\sum_k q_k^2|\gamma_k|^2\right]
\tag{68}
$$

Physically, the expressions above must be cut-off at a maximum and minimum frequencies $\Omega_k = |k|$ to which the detector can be sensitive in principle [8]; for illustration purposes, we will exaggerate these infrared and ultraviolet cut-offs to the Hubble and Planck energies, E_{IR} and E_{pl}, respectively. Recovering the continuous limit yields, up to numerical factors of order one,

$$
\begin{aligned}
\mathcal{F}(t) &= \int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3}} 2\omega^2 \left(\frac{8\pi G}{k^3}\right)(\Omega_k t - \sin\Omega_k t) \\
&\approx \left(\frac{\omega}{E_{\mathrm{pl}}}\right)^2 \left(\int_{E_{\mathrm{IR}}}^{E_{\mathrm{pl}}} dk\right) t \approx \left(\frac{\omega}{E_{\mathrm{pl}}}\right)\omega t
\end{aligned}
\tag{69}
$$

where we have used $E_{\mathrm{pl}} \gg E_{\mathrm{IR}}$, considered large times $t \gg E_{\mathrm{IR}}^{-1}$ and neglected the bounded term $\sin\Omega_k t$. Corrections to the optical annihilation operator due to the phase $\mathcal{F}(t)$ are then on the order of (ω/E_{pl}) and only become relevant for optical energies close to the Planck energy. Similarly,

$$
\begin{aligned}
\mathcal{G}(t) &= \exp\left(-\frac{1}{2}\int \frac{d^3\boldsymbol{k}}{\sqrt{(2\pi)^3}}\omega^2\left(\frac{8\pi G}{k^3}\right)|\gamma_k|^2\right) \\
&\approx \exp\left(-2\left(\frac{\omega}{E_{\mathrm{pl}}}\right)^2 \int_{E_{\mathrm{IR}}}^{E_{\mathrm{pl}}} \frac{dk}{k}\right) \\
&= \exp\left(-2\left(\frac{\omega}{E_{\mathrm{pl}}}\right)^2 \ln\left(\frac{E_{\mathrm{pl}}}{E_{\mathrm{IR}}}\right)\right)
\end{aligned}
\tag{70}
$$

where we have considered the worst-case approximation $|\gamma_k|^2 = 2\left(1 - \cos\Omega_k t\right) \sim 4$. Notice that the ratio of ultraviolet to infrared cut-offs is $E_{\mathrm{pl}}/E_{\mathrm{IR}} \approx 10^{62}$, giving a correction to $a(t)$ approximately proportional to $(\omega/E_{\mathrm{pl}})^2$, a tremendously small number!

Instead of the vacuum, we could consider all modes to be populated by a thermal state at about $1\,\mathrm{K}$, the expected temperature for the cosmic GW background [67]. This would not alter $\mathcal{F}(t)$, which is state-independent, so the estimates in (69) remain. The $\mathcal{G}(t)$ term, however, would acquire a correction factor at most $e^{-q_{\mathrm{pl}}^2(2\bar{n}+1)}$, with $\bar{n} = 1/(e^{\hbar\Omega_k/k_B T} - 1)$ and q_{pl} the GW coupling strength at the Planck frequency. [3]. For a GW mode of $10\,\mathrm{Hz}$, peak of the expected cosmic GW background spectrum, this correction factor amounts to $\approx e^{-10^{-47}}$.

All in all, these estimates show that the effect of modes which are not populated by states with a very large mean number of gravitons upon optical observables is completely negligible. In other words, decoherence due to the gravitational vacuum, or even due to the thermal background of GWs is very weak, which is consistent with previous results [5, 42].

C Gravitational decoherence of cavity modes

Here we explore the idea that terms quadratic in the dimensionless coupling q appearing in the exponent of (10) are associated to GW-induced decoherence. Let us consider the electromagnetic-gravitational wave (EM-GW) system prepared at time $t = 0$ in the state

$$|\Psi\left(0\right)\rangle = \frac{|0\rangle_{\mathrm{EM}} + |N\rangle_{\mathrm{EM}}}{\sqrt{2}} \otimes |0\rangle_{\mathrm{GW}} \qquad (71)$$

This provides a simplified model of a qubit (defined by the span of the $\{|0\rangle, |N\rangle\}$ subspace) exhibiting features similar to the electromagnetic coherent states considered in the main, but with easily calculable coherence terms.

The initial state $|\Psi\left(0\right)\rangle$ can be evolved directly using operator (7) resulting in the state at arbitrary times,

$$|\Psi\left(t\right)\rangle = \frac{|0\rangle\,|0\rangle + |N\rangle\,|qN\gamma\rangle}{\sqrt{2}} \qquad (72)$$

where we omit the subscripts by intending that the left (right) kets refer to EM (GW), respectively; the order reverses when considering bras instead of kets. Notice that the time dependence is contained in the function $\gamma\left(t\right)$, and moreover $|qN\gamma\rangle$ is a coherent state. The total density matrix associated to $|\Psi\left(t\right)\rangle$ reads,

$$\rho\left(t\right) = \frac{1}{2}\left(|0\rangle\,|0\rangle\,\langle 0|\,\langle 0| + |0\rangle\,|0\rangle\,\langle qN\gamma|\,\langle N| + |N\rangle\,|qN\gamma\rangle\,\langle 0|\,\langle 0| + |N\rangle\,|qN\gamma\rangle\,\langle qN\gamma|\,\langle N|\right), \qquad (73)$$

from which we can trace out the GW subsystem to obtain the density matrix associated to the electromagnetic field,

$$\rho_{\mathrm{EM}}\left(t\right) = \begin{pmatrix} \frac{1}{2} & \rho_{01} \\ \rho_{01}^* & \frac{1}{2} \end{pmatrix} \qquad (74)$$

where the coherence term $\rho_{01} = \langle 0|qN\gamma\rangle = \exp\left\{-\frac{1}{2}q^2 N^2 |\gamma|^2\right\}$ oscillates with an amplitude proportional to q^2. This supports the idea that exponents proportional to the dimensionless coupling squared are associated to GW-induced decoherence.

To further explore this idea, let us now consider the GW field in a coherent state,

$$|\Psi\left(0\right)\rangle = \frac{|0\rangle + |N\rangle}{\sqrt{2}} \otimes |\alpha\rangle \qquad (75)$$

Time evolution yields,

$$|\Psi\left(t\right)\rangle = \frac{|0\rangle\,|\alpha\rangle + |N\rangle\,D\left(qN\gamma\right)|\alpha\rangle}{\sqrt{2}} \qquad (76)$$

Furthermore,

$$D\left(qN\gamma\right)\left|\alpha\right\rangle = D\left(qN\gamma\right)D\left(\alpha\right)\left|0\right\rangle \tag{77}$$

$$= e^{\frac{1}{2}Nq(\gamma\alpha^* - \gamma^*\alpha)}\left|qN\gamma + \alpha\right\rangle \tag{78}$$

and the density matrix now becomes,

$$
\begin{aligned}
\rho\left(t\right) = \ & \frac{1}{2}\left|0\right\rangle\left|\alpha\right\rangle\left\langle\alpha\right|\left\langle 0\right| \\
& + \frac{1}{2}e^{-\frac{1}{2}Nq(\gamma\alpha^* - \gamma^*\alpha)}\left|0\right\rangle\left|\alpha\right\rangle\left\langle qN\gamma + \alpha\right|\left\langle N\right| \\
& + \frac{1}{2}e^{\frac{1}{2}Nq(\gamma\alpha^* - \gamma^*\alpha)}\left|N\right\rangle\left|qN\gamma + \alpha\right\rangle\left\langle\alpha\right|\left\langle 0\right| \\
& + \frac{1}{2}\left|N\right\rangle\left|qN\gamma + \alpha\right\rangle\left\langle qN\gamma + \alpha\right|\left\langle N\right|
\end{aligned}
\tag{79}
$$

Tracing out the GW states as before yields the same form as (74) with ρ_{01} given by

$$\rho_{01} = e^{-\frac{1}{2}\left[q^2 N^2 |\gamma|^2 + qN(\gamma^*\alpha - \gamma\alpha^*)\right]} \tag{80}$$

We see the interaction between the EM field and a a quantized single-mode GW is given by the q^2 term present when the GW-field is in the vacuum state plus an oscillating factor linear in q and proportional to the GW amplitude.

Equation (80) naturally allows us to evaluate the expression of ρ_{01} for a single-mode GW in a thermal state with mean number of gravitons $\overline{n}$,

$$
\begin{aligned}
\rho_{01} &= e^{-\frac{1}{2}q^2 N^2 |\gamma|^2}\int\frac{d^2\alpha}{\pi\overline{n}}e^{-\frac{1}{2}\left[\frac{|\alpha|^2}{\overline{n}} + qN(\gamma^*\alpha - \gamma\alpha^*)\right]} \\
&= e^{-\frac{1}{2}q^2 N^2 |\gamma|^2(1+\overline{n})}
\end{aligned}
\tag{81}
$$

This expression is valid for a single-mode thermal state. We could generalize it to a continuum of GW modes in thermal states, with the initial state

$$\rho(0) = \left|\psi_0\right\rangle\langle\psi_0| \otimes \prod_{\boldsymbol{k}}\rho_{\boldsymbol{k}}^{\text{Th}} \tag{82}$$

where $\left|\psi_0\right\rangle = \frac{\left|0\right\rangle + \left|N\right\rangle}{\sqrt{2}}$ and $\rho_{\boldsymbol{k}}^{\text{Th}}$ is the thermal density matrix for mode $\boldsymbol{k}$. For such state, (81) generalizes to

$$\prod_{\boldsymbol{k}}e^{-\frac{1}{2}q_{\boldsymbol{k}}^2 N^2 |\gamma_{\boldsymbol{k}}|^2(1+\overline{n}_{\boldsymbol{k}})} = \exp\left(-\frac{1}{2}N^2\sum_{\boldsymbol{k}}q_{\boldsymbol{k}}^2\left|\gamma_{\boldsymbol{k}}\right|^2\left(1+\overline{n}_{\boldsymbol{k}}\right)\right) \tag{83}$$

If the GW thermal states are assumed to be highly populated $\overline{n}_{\boldsymbol{k}} \gg 1$ ($T \to \infty$), we can write $(1+\overline{n}_{\boldsymbol{k}}) \approx K_B T/\Omega_k$. Substituting $q_{\boldsymbol{k}} = (\omega/\Omega_k)\sqrt{8\pi G/\Omega_k V}/4$, $|\gamma_{\boldsymbol{k}}|^2 = 4\sin^2\left(\frac{\Omega_k t}{2}\right)$ and employing the density of modes for a bosonic field in a volume V [40] given by $\mathcal{D}(\Omega) \propto V\Omega^2 d\Omega$ we find, in the continuum limit and after summing over all modes,

$$\rho_{01} \approx \exp\left\{k_B T\left(\frac{N\omega}{E_P}\right)^2\zeta\int d\Omega\frac{\sin^2\left(\frac{\Omega t}{2}\right)}{\Omega^2}\right\} \tag{84}$$

where $E_{\text{pl}} = 1/\sqrt{G}$ is Planck's energy, ζ is a factor of order one and we note that the integral in the exponent is the same as Eq. (22) in [68] and Eq. (23) in [69]. From the scaling of the integral in (84) we see that

$$\rho_{01} \approx e^{-\Gamma t} \tag{85}$$

where

$$\Gamma \propto k_B T \left(\frac{\Delta E}{E_{\mathrm{pl}}} \right)^2 \tag{86}$$

up to a factor of order one, where we have defined the energy difference $\Delta E = N\omega$. With this we recover the scaling of the decoherence of a superposition with energy difference ΔE in the presence of a thermal GW background with temperature T as obtained in [42].

D   Reconstruction of Gaussian GW states from optical modes

As explained in the main text, the reduced density matrix elements of an optical state which interacted with a single mode quantum GW state are proportional to

$$\mathcal{I}_n(t) = \langle \Psi | D(-qn\gamma^*) | \Psi \rangle = \mathrm{Tr}\left(\sigma D(-nq\gamma^*) \right) = \mathrm{Tr}\left(\sigma e^{-qn\left(\gamma^* b^\dagger - \gamma b \right)} \right) \tag{87}$$

These correspond to the quantum characteristic function associated to the GW evaluated at certain contours in the complex plane, and as such contain information on the GW quantum state. By measuring different components of the optical reduced density matrix, we can then obtain such information and partially reconstruct the GW quantum state. As we now show, if the state is Gaussian reconstruction is perfect.

The idea behind the GW state reconstruction is to obtain relations between GW field correlators and time derivatives of $\mathcal{I}_n(t)$ evaluated at $t = 0$. Inverting these relations, we can obtain the first and second moments of the gravitational field quadratures. To see that, we Taylor expand $\mathcal{I}_n(t)$ in time using,

$$e^{nA(e^{it}-1)} = 1 + inAt - \frac{1}{2}\left(nA\left(nA + 1 \right) \right) t^2 +$$
$$- \frac{i}{6} nA\left(n^2 A^2 + 3nA + 1 \right) t^3 + \mathcal{O}(t^4) \tag{88}$$

$$e^{-nB(e^{-it}-1)} = 1 + inBt - \frac{1}{2}\left(nB\left(nB - 1 \right) \right) t^2 +$$
$$- \frac{i}{6} nB\left(n^2 B^2 - 3nB + 1 \right) t^3 + \mathcal{O}(t^4) \tag{89}$$

Observe also that

$$\mathcal{I}_n(t) = e^{-q^2 n^2 |\gamma|^2 / 2} C(-qn\gamma^*, -qn\gamma)$$
$$= \left(1 - \frac{1}{2} q^2 n^2 t^2 + \mathcal{O}(q^4) \right) C(-qn\gamma^*, -qn\gamma) \tag{90}$$

We now proceed order-by-order in t.

t-**terms:** The first order derivative of (90) gives us

$$\alpha_1(n) = inq \langle X \rangle \tag{91}$$

From which we obtain

$$\langle qX \rangle = -i\alpha_1(1) \tag{92}$$

t^2-**terms:** The second order derivative of (90) yields,

$$\alpha_2(n) = -n^2 u_1 - n u_2 - n^2 q^2 \tag{93}$$

where

$$u_1 = q^2 \langle b^{\dagger 2} + b^2 + 2b^\dagger b \rangle \tag{94}$$
$$u_2 = -q \langle b - b^\dagger \rangle \tag{95}$$

We can consider different values of n, say $n = 1, 2$ to obtain two independent relations which can be used to solve for u_1, u_2. We find,

$$u_1 = \alpha_2(1) - \frac{1}{2}\alpha_2(2) - q^2 \tag{96}$$

$$u_2 = -2\alpha_2(1) + \frac{1}{2}\alpha_2(2) \tag{97}$$

Now, observe that

$$u_1 = q^2 \langle b^{\dagger 2} + b^2 + 2b^\dagger b \rangle = \langle q^2 X^2 \rangle - \frac{i}{2}\langle [qX, qY] \rangle \tag{98}$$

We note $\left[b, b^\dagger \right] = \frac{i}{2} [X, Y] = \mathbb{1}$, but as we will see, it is sometimes convenient to keep expressions in terms of the commutator when taking the continuous limit.

We find,

$$\langle q^2 X^2 \rangle = \alpha_2(1) - \frac{1}{2}\alpha_2(2) \tag{99}$$

Similarly,

$$u_2 = -q \langle b - b^\dagger \rangle = i \langle qY \rangle = \frac{1}{2}\alpha_2(2) - 2\alpha_2(1) \tag{100}$$

t^3-terms: At third order we obtain

$$-\alpha_3(n) = 6n^3 v_1 + 6n^2 v_2 - n\alpha_1(1) \tag{101}$$

where

$$v_1 = \frac{iq^3}{6}\langle b^3 + 3b^\dagger b^2 + 3b^{\dagger 2}b + b^{\dagger 3} + 3\left(b + b^\dagger \right) \rangle \tag{102}$$

$$v_2 = \frac{iq^2}{2}\langle b^{\dagger 2} - b^2 \rangle \tag{103}$$

and once again, using different values of n we can recover v_1, v_2. For Gaussian states, it suffices to determine v_2. We have,

$$v_2 = -\frac{\alpha_3(1)}{3} + \frac{\alpha_3(2)}{24} - \frac{\alpha_1(1)}{4} \tag{104}$$

Using the relation

$$\langle b^{\dagger 2} - b^2 \rangle = i\langle XY \rangle - \frac{i}{2}\langle [X, Y] \rangle \tag{105}$$

we find,

$$\langle qXqY \rangle - \frac{1}{2}\langle [qX, qY] \rangle = \frac{2}{3}\alpha_3(1) - \frac{1}{12}\alpha_3(2) + \frac{1}{2}\alpha_1(1) \tag{106}$$

We can go on to higher orders, and determine more normal-ordered expectation values of graviton creation and annihilation operators, but for Gaussian quantum states, determining the moments $\langle X \rangle, \langle Y \rangle, \langle X^2 \rangle$ and $\langle XY \rangle$ are sufficient; we still need the expecation value $\langle Y^2 \rangle$, which can be obtained either by normalization of the Wigner function or by additional measurements of the mean number of gravitons $\langle N \rangle = \langle b^\dagger b \rangle$, since $\langle Y^2 \rangle = 2 + 4\langle b^\dagger b \rangle - \langle X^2 \rangle$. This leads to the first and second moments discussed in the main text.

E Quantum enhancement and suppression of GW signals

We provide further details on the calculations presented in the main text. We follow the conventions used in [56]. Useful formulae are:

$$\langle\gamma|D(\alpha)|\beta\rangle = e^{-|\alpha|^2/2}e^{-|\beta|^2/2}e^{-|\gamma|^2/2}e^{\alpha\gamma^* - \alpha^*\beta + \beta\gamma^*} \tag{107}$$

where $D(\alpha)$ is a displacement operator. Products of displacement operators satisfy:

$$D^\dagger(\beta)D(\alpha) = D(-\beta)D(\alpha) = e^{\frac{(\alpha\beta^* - \beta\alpha^*)}{2}}D(\alpha - \beta) \tag{108}$$

Moreover,

$$S(r, \phi) = \exp\left(\frac{1}{2}r\left(a^2 e^{-2i\phi} - a^{\dagger 2}e^{2i\phi}\right)\right) \tag{109}$$

is the single mode squeezing operator with squeezing parameter r and angle ϕ, and

$$e^{i\theta a^\dagger a}S(r, \phi)e^{-i\theta a^\dagger a} = S(r, \phi + \theta) \tag{110}$$

gives us the time evolution of the squeezing parameter in the Heisenberg picture. Another useful formula is

$$S^\dagger(r, \phi)D(\alpha)S(r, \phi) = D(\alpha\cosh r + \alpha^* e^{2i\phi}\sinh r) \tag{111}$$

Define two modes '1' and '2'. The above formulas generalize to the case of a two-mode squeezing operator,

$$S_{12}(r, \phi) = \exp\left(\frac{1}{2}r\left(b_1 b_2 e^{-2i\phi} - b_1^\dagger b_2^\dagger e^{2i\phi}\right)\right) \tag{112}$$

For instance, we have

$$S_{12}^\dagger \, b_1 \, S_{12} = b_1 \cosh r + b_2^\dagger e^{2i\phi}\sinh r \tag{113}$$

$$S_{12}^\dagger \, b_2 \, S_{12} = b_2 \cosh r + b_1^\dagger e^{2i\phi}\sinh r \tag{114}$$

which leads to the generalization of (111),

$$S_{12}^\dagger D_1(\alpha)D_2(\beta)S_{12} = D_1(\alpha\cosh r)D_1(-\beta^* e^{2i\phi}\sinh r)D_2(\alpha^* e^{2i\phi}\sinh r)D_2(\beta\cosh r) \tag{115}$$

We are interested in calculating the time-development of a single mode cavity electric field under the influence of a quantum GW. Here we focus on the case of a two-mode squeezed-coherent GW, since it generalizes the one mode calculation. Considering terms which generate only linear signals in the gravitational coupling q_k, the time evolution operator for two GW modes '1' and '2' reads

$$U(t) \approx e^{q_1 a^\dagger a(\gamma_1 b_1^\dagger - \gamma_1^* b_1)}e^{q_2 a^\dagger a(\gamma_1 b_2^\dagger - \gamma_1^* b_2)} \tag{116}$$

which for an uncorrelated electromagnetic-gravitational state leads to the electric field

$$\mathcal{E}(t) = \sqrt{\frac{\omega}{V_c}}\left(\frac{{}_{12}\langle\Psi(t)|D_1(q_1\gamma_1)D_2(q_2\gamma_2)|\Psi(t)\rangle_{12}a + h.c.}{\sqrt{2}}\right) \tag{117}$$

The two-mode GW induced phase originates from the term

$${}_{12}\langle\Psi(t)|D_1(q_1\gamma_1)D_2(q_2\gamma_2)|\Psi(t)\rangle_{12} \tag{118}$$

and thus we focus on calculating it explicitly.

We shall consider the two-mode squeezed-coherent state defined as in the main text,

$$|\Psi(t)\rangle_{12} = S_{12}(r, \phi(t))D_1(h(t))|0\rangle \tag{119}$$

where $\phi(t) = \phi_0 - \Omega t$ and $h(t) = |h|e^{-i\Omega t}$. In cosmology applications [70], we are interested in modes '1' and '2' defined by the wave-vectors $-\boldsymbol{k}$ and $\boldsymbol{k}$, respectively. Hence we have $q_1 = q_2 = q$ and $\gamma_1 = \gamma_2 = \gamma$. Therefore,

$$_{12}\langle\Psi(t)|D_1(q\gamma)D_2(q\gamma)|\Psi(t)\rangle_{12} = \tag{120}$$

$$= {}_2\langle 0|_1\langle h(t)|S_{12}^{\dagger}D_1(q\gamma)D_2(q\gamma)S_{12}|h(t)\rangle_1|0\rangle_2$$

$$= \underbrace{\langle h(t)|D_1(q\gamma\cosh r)D_1(-q\gamma^* e^{2i\phi}\sinh r)|h(t)\rangle}_{\text{(I)}}$$

$$\times \underbrace{\langle 0|D_2(q\gamma^* e^{2i\phi}\sinh r)D_2(q\gamma\cosh r)|0\rangle}_{\text{(II)}}$$

Defining the coherent state $|q\gamma\cosh r\rangle = D(q\gamma\cosh r)|0\rangle$ we can rewrite (II) as,

$$\langle 0|D(q\gamma^* e^{2i\phi}\sinh r)|q\gamma\cosh r\rangle \tag{121}$$

and using (107) we see that it only yields terms of order $\mathcal{O}(e^{-q^2})$. To leading order in q, then, (II) can be approximated as unity. As for the term (I) in (121) we write

$$\langle h|D(q\gamma\cosh r)D(-q\gamma^* e^{2i\phi}\sinh r)|h\rangle =$$

$$= \langle h|D^{\dagger}(-q\gamma\cosh r)D(-q\gamma^* e^{2i\phi}\sinh r)|h\rangle$$

$$= \mathcal{O}(e^{-q^2}) \times \langle h|D\big(q(\gamma\cosh r - \gamma^* e^{2i\phi}\sinh r)\big)|h\rangle$$

$$\approx \langle h|D\big(q(\gamma\cosh r - \gamma^* e^{2i\phi}\sinh r)\big)|h\rangle \tag{122}$$

This is equivalent, up to a phase, to Eq. (48) in the main text.