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Essays on Fiscal Policy
and Immigration

A dissertation submitted in partial satisfaction
of the requirements for the degree
Doctor of Philosophy in Economics

by

Jinwook Hur

2016

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ABSTRACT OF THE DISSERTATION

Essays on Fiscal Policy
and Immigration

by

Jinwook Hur

Doctor of Philosophy in Economics

University of California, Los Angeles, 2016

Professor Lee Ohanian, Chair

This dissertation studies how fiscal policy influences the macroeconomic variables in the long run under different economic environments and restrictions. Chapter 1 introduces a politico-economic model with a welfare state and immigration. In this model, policies on taxes and immigration are decided by the plurality voting system. While many studies on fiscal implications of immigration argue that relaxing immigration policies can substitute tax reforms in aging economies, I show that the democratic voting procedure can dampen the effect of relaxing immigration policies since desired policy reforms are not always implemented by the winning candidate of an election. This political economy results in social welfare losses through three different channels. Chapter 2 examines differences in hours worked between South Korea and Mexico during 1991-2008. Specifically, this paper assesses the extent to which this difference is explained by a neoclassical growth model with distortionary taxation of government. The model successfully captures the long-run trends of hours worked in Korea and Mexico. While gradual decline in hours worked of Korea is well explained by increasing tax wedges of the labor market, hours worked in Mexico exhibit a relatively flat trend since Mexico's labor wedges do not change dramatically during the sample period. This shows that long-term fiscal policy was significant in Korea economy around the 1997 crisis, while government fiscal policy played only a modest role in Mexico's labor market during the Peso crisis. Finally, Chapter 3 examines the large differences in long-run changes

in macroeconomic variables across 15 OECD countries. Specifically, this paper assesses the extent to which the neoclassical growth model equipped with government consumption, investment, and proportional taxes on expenditures and factor incomes explains the macroeconomic variables. According to the results, proportional taxes on income and expenditure plays a principal role in explanation of long-term trends in those variables. Changes in government investment influence the variables only modestly. This implies that it is not sizes of government expenditure, but distortions caused by tax wedges, that influence changes of macroeconomic variables in the long run.

The dissertation of Jinwook Hur is approved.

Mark J. Garmaise

Pablo David Fajgelbaum

Gary D. Hansen

Lee Ohanian, Committee Chair

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2016

To my family,

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CHAPTER 1

Political Economy of Immigration and Fiscal Sustainability

1.1 Introduction

In many developed economies, sustainability of the welfare state is an important issue. The aging of the baby boom generation and falling fertility rates make it harder to maintain the welfare programs without reforms in the fiscal policies. A tax reform is one option, but increasing tax burdens causes another controversy on its potential negative effects on productivity and social welfare.

In this sense, some economists argue that relaxing immigration policies can be an effective alternative to tax reforms. According to this literature, more liberal immigration policy on young and skilled workers can increase the tax revenue and resolve fiscal problems in many economies with a pay-as-you-go (PAYG) system of welfare programs. For example, Storesletten (2000) argues that admitting 1.6 million high-skilled immigrants at the age of 40-44 years old annually can replace tax reforms in resolving the fiscal problems that the U.S. economy experiences as its population is aging.

In most of the studies concentrating on fiscal implications of immigration policies the policy variables are assumed to be exogenous. However, in the actual decision-making process of immigration policies, there are many factors, especially political, that can keep desired policies from being implemented. For instance, young citizens that are of working age can worry that their wage would decrease because of competition with immigrants, which makes them vote for parties against generous immigration policies. Therefore, it is important to

what extent the desired reforms of immigration policies can be implemented through a democratic decision-making process.

In this paper, I analyze political equilibria in a simple overlapping-generation model with two skill levels in labor, proportional taxes on labor income, and a welfare state in form of simple lump-sum transfers. The native population, or the electorate, is divided into three different political parties depending on their age and skill level. Then the labor income tax rate, volume of immigration, and skill composition of immigrants are decided by the winning candidate of a plurality voting. As there exist three different political parties, the possibility of strategic voting and political coalitions is also considered.

Using this model economy, I demonstrate the pattern of different preferences on fiscal and immigration policies by different cohorts. The old retirees want to maximize tax revenue per capita because they do not have labor income, so they prefer Laffer rate of labor income tax and a balanced skill composition between skilled and unskilled workers to maximize production efficiency. On the other hand, the preference of the young workers depends on their skill level. Specifically, skilled workers want the immigrants to be unskilled, and unskilled workers want the immigrants to be skilled. Moreover, depending on the state of the economy, the young workers might not want to allow maximum amount of immigrants although their instantaneous utility is maximized by allowing as many immigrants as possible. If the young workers allow too many immigrants, the probability that they lose the election in the next period increases since the demographic composition of the electorate will be biased toward the young generation in the next period due to the high fertility of the immigrants, which might remove the current young cohort's opportunity to enjoy the benefit of the welfare state after they retire. So, the old retirees want maximum amount of immigration and welfare state as pure beneficiaries of the welfare program, while the skilled young workers want zero taxes with no welfare program in most of the parameter regions. The unskilled young workers are mostly in the middle of the two groups in the sense that they do not mind taxes as much as the skilled workers do. This emphasizes the role of the unskilled workers as a "swing" cohort in the election.

It is also shown that the skill composition of the economy exhibits “cycling” dynamics. That is, when the skill composition is biased toward unskilled labor in the beginning of a period, the unskilled workers are highly likely to win the election, thereby approving a high volume of skilled immigrants and balancing the skill composition. Similarly, when the skill composition leans too much toward skilled labor, the skilled workers are more likely to win the election, and the winning candidate will allow a high volume of unskilled immigrants. Since, in an economy with simple Cobb-Douglas technology, the optimal skill composition of workers is to have more skilled workers than unskilled workers, the democratic procedure of plurality voting causes welfare losses due to the gap between the optimal skill composition of immigrants and the realized skill composition of immigrants which tends to be lower than the optimal level.

I also show other factors causing welfare losses. First, the distortionary labor income tax in the model is a source of inefficiency. Since the old retirees implement Laffer rate whenever they win the election, this distortion is most severe when the old retirees become the decisive political party. Second, the volume of immigration might be smaller than the optimal level. Since the young workers worry that they might not regain the political power when they are old and lose their chances to obtain future social security benefits, the young workers have incentive to adjust the immigration volume to guarantee that they can win not only the election today but also in the future.

This paper is in line with the literature on macroeconomic implications of the political economy. Specifically, it builds on the researches on the political economy of immigration such as Benhabib (1996), Dolmas and Huffman (2004) and Ortega (2005), as well as the political economy of social security systems, such as Boldrin and Rustichini (2000) and Galasso and Profeta (2002). Specifically, the main goal of this paper is to examine whether the policy implication of immigration literature¹ is feasible in a positive theory setup.

This study technically follows the analyses of Ortega (2005) and Razin et al. (2014).

¹As explained above, many studies report that relaxing immigration policy can reduce fiscal problems in developed economies. Among others, see Auerbach and Oreopoulos (1999) and Storesletten (2000).

Ortega (2005) introduces a dynamic politico-economic model with immigration and skill upgrading. In his model, the voting decision of the electorate affects the skill composition of the economy, thereby influencing not only the skill complementarity but also the skill composition of the future electorate. Therefore, the voters confront the tradeoff between their skill premium and their future political power. On the other hand, Razin et al. (2014) abstracts from skill complementarity by assuming that the production function is linear in skilled and unskilled labor. Instead, their work concentrates on a more complicated political situation in which there exists a redistributive welfare state and three political parties. The three parties, formed according to skill levels and ages, make a voting decision in a plurality voting and, therefore, potentially vote strategically. This paper builds on the techniques in both papers. In a political equilibrium, each young worker considers not only the effect of the election result on his labor income but also how it will affect his political power when he becomes old and retires.

This paper proceeds as follows. The model economy is described in Section 1.2. The political procedure of policy implementation is explained in Section 1.3. Section 1.4 describes the political equilibria of the model. In Section 1.5, the dynamics of the model and welfare analysis are discussed. Section 1.6 concludes the paper.

1.2 Model Economy

The model builds on the dynamic overlapping generation model as in Razin et al. (2014) with application of democracy model with strategic voting behaviors in Besley and Coate (1997, 1998). In this model, there are two generations (the young and the old) and two skill levels (the skilled and the unskilled). Each individual works only in the first period of his life and retires in the second and last period. The government levies proportional labor income taxes from the young workers, and spends lump-sum transfers as a pay-as-you-go (PAYG) system. Furthermore, the government decides the tax rate, the volume of immigration, and the skill composition of immigrants by a democratic voting each period.

In this model, each period is divided into three sub-periods. In the first sub-period, the native individuals - both the young and the old - hold a plurality election, and the winner of the election decides the labor income tax rate (τ_t), the volume of immigration (μ_t), and the skill composition of immigrants (σ_t).² In the second sub-period, the immigrants enter the country as determined by the winning candidate. In the third and final sub-period, production, taxation, transfer, and consumption take place.

1.2.1 Preference

The lifetime utility function of a young worker with skill level $i \in \{s, u\}$ is assumed quasi-linear in consumption, such as:

$$U^y(c_t^{y,i}, l_t^i, c_{t+1}^{o,i}) = c_t^{y,i} - \frac{(l_t^i)^{1+1/\nu}}{1+1/\nu} + \beta c_{t+1}^{o,i}$$

where l_t^i is labor supply of skill level $i \in \{s, u\}$, β is the discount factor for the next period, and ν is the Frisch elasticity of labor supply.

The preference of an old retiree simply consists of the utility from today's consumption because he does not have a future continuation value. Thus, the utility function is given by:

$$U^o(c_t^{o,i}) = c_t^{o,i}$$

The budget constraints of the young and the old are given by:

$$\begin{aligned} c_t^{y,i} + d_t^i &\leq (1 - \tau_t)w_t^i l_t^i + b_t \\ c_t^{o,i} &\leq (1 + r_t)d_{t-1}^i + b_t \end{aligned}$$

where τ_t is the proportional labor income tax rate at period t , and d_t and b_t are savings and lump-sum transfers from the government, respectively.

Given that the utility function is quasi-linear in consumption, the Euler equation pins down the equilibrium interest rate at a constant, i.e.

$$r_t = \frac{1}{\beta} - 1 \quad \text{for all } t$$

²The process of voting will be demonstrated in detail in Section 1.3.

At this r_t , young workers are indifferent to the amount of savings as the marginal benefit of savings exactly offsets the marginal cost of savings, so any amount of savings can be an equilibrium solution. Throughout this paper, we will only consider the case of $d_t^i = 0$ for all $i \in \{s, u\}$ for simplicity. This makes the old retirees simply consume the lump-sum transfer regardless of their skill levels, so this setup reduces the two groups of old retirees (the skilled & the unskilled) into one single group.

The labor supply function of a young worker with skill level $i \in \{s, u\}$ is derived as:

$$l_t^i = [(1 - \tau_t)w_t^i]^\nu$$

Therefore, the indirect lifetime utility of each cohort is:

$$\begin{aligned} V_t^i &= c_t^{y,i} - \frac{(l_t^i)^{1+1/\nu}}{1 + 1/\nu} + \beta c_{t+1}^{o,i} \\ &= \frac{[(1 - \tau_t)w_t^i]^{1+\nu}}{1 + \nu} + b_t + \beta b_{t+1} \end{aligned} \tag{1.1}$$

$$V_t^o = b_t \tag{1.2}$$

1.2.2 Production Technology

This model is abstract from the capital accumulation, so the skilled and unskilled labor are the only factors of production.³ The output is produced according to the following Cobb-Douglas technology:

$$Y_t = (L_t^s)^\alpha (L_t^u)^{1-\alpha}$$

where L_t^i is the aggregate labor input of skill level $i \in \{s, u\}$, and α is assumed to be greater than 0.5. The labor demand equations for the two skill groups corresponding to this production function is thus given by:

$$\begin{aligned} w_t^s &= \alpha Y_t (L_t^s)^{-1} \\ w_t^u &= (1 - \alpha) Y_t (L_t^u)^{-1} \end{aligned}$$

³An alternative setup of this is to assume that capital stock per capita is fixed, and the immigrants bring the same amount of the capital stock when they are approved to immigrate.

1.2.3 Immigration and Demographics

Before production takes place in a period, the government decides the volume and the skill composition of immigrants. For simplicity, it is assumed that only young people are allowed to immigrate, thus all immigrants supply labor immediately at the period of their entrance into the economy. Then, the aggregate labor supply is characterized by following formulae:

$$\begin{aligned} L_t^s &= [s_t + \sigma_t \mu_t] N_t l_t^s \\ L_t^u &= [1 - s_t + (1 - \sigma_t) \mu_t] N_t l_t^u \end{aligned}$$

in which N_t is the total number of native-young individuals and $s_t \in [0, 1]$ is the fraction of native-born skilled workers in the labor force in the beginning of the period t . $\mu_t \in [0, \bar{\mu}]$ is the ratio of approved immigrants to the native-born young population, thereby governing the volume of immigration.⁴ Finally, $\sigma_t \in [0, 1]$ stands for the fraction of skilled immigrants in the group of new immigrants.

Also, there are some additional assumptions for convenience. First, the skill level of offspring follows his parent. Second, offspring of the immigrants are considered native, while the immigrants themselves are not considered as native.⁵ Then, the demographic dynamics can be described by the following equations:

$$\begin{aligned} N_{t+1} &= [1 + n + (1 + m) \mu_t] N_t && \text{(level of population)} \\ s_{t+1} N_{t+1} &= [(1 + n) s_t + (1 + m) \sigma_t \mu_t] N_t && \text{(skill composition)} \end{aligned}$$

⁴The volume of immigration has a finite upper bound because otherwise the choice of μ_t can be infinity in a number of political equilibria since the production function is constant-returns-to-scale. In an economy with a DRS technology, μ_t becomes a finite value even without an exogenous upper bound. In this DRS case, however, the state of economy depends not only on the relative population size of the voters but also the absolute number of young workers, thereby increasing the dimension of the state vector by one.

⁵In other words, the immigrants cannot vote for their life, while their descendants are legal voters.

where n and m are the population growth rate for the native and migrants, respectively.⁶ Combining these two equations, we can get the following law of motion for the skill composition of the natives:

$$s_{t+1} = \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+(1+m)\mu_t} \quad (1.3)$$

Equation (1.3) demonstrates the role of immigration policies. For instance, $\sigma_t < s_t$ implies that the fraction of skilled immigrants in the whole group of immigrants exceed the fraction of skilled workers in the native young workers. Then, the fraction of skilled workers in the native young workers will increase in the next period because of the assumption that the descendants succeed their parents' skill level. Also, if $\mu_t = 0$ (no immigrants accepted), then s_t obviously remains the same in the next period.⁷ As $\mu_t \rightarrow \infty$ (infinitely many immigrants are approved), $s_{t+1} \rightarrow \sigma_t$, implying that the skill composition of the natives will converge to the skill composition of accepted immigrants as more immigration is accepted.

1.2.4 Government and the Welfare State

The government levies proportional labor income taxes at the rate determined by voting. The tax revenue is transferred equally regardless of ages (young and old), skills (skilled and unskilled), and/or the residence status (natives and immigrants). The government is assumed to balance budget every period thus there is no government debt. Then the lump-sum transfer is given by:

$$b_t = \frac{\tau_t [(s_t + \sigma_t\mu_t) w_t^s N_t l_t^s + (1 - s_t + (1 - \sigma_t)\mu_t) w_t^u N_t l_t^u]}{(1 + \mu_t)N_t + (1 + \mu_{t-1})N_{t-1}}$$

⁶In many developed countries, it is consistently observed from the census data that the fertility rate is higher for immigrants than natives, implying $m > n$. One of the recent examples is the report of U.S. Census Bureau by Monte and Ellis (2014).

⁷In other words, immigration is the only method of changing aggregate skill composition in this model and there is no skill acquisition or loss. For studies on transition of aggregate skill level or task composition through immigration, refer to Hunt and Gauthier-Loiselle (2010) and Peri (2012) among others.

in which the numerator is total tax revenue, and the denominator stands for the total population including both natives and immigrants.

1.2.5 Equilibrium-Characterizing Equations

The general equilibrium of this economy is characterized by the following 9 equations.

$$l_t^s = [(1 - \tau_t)w_t^s]^\nu \quad (1.4)$$

$$l_t^u = [(1 - \tau_t)w_t^u]^\nu \quad (1.5)$$

$$Y_t = (L_t^s)^\alpha (L_t^u)^{1-\alpha} \quad (1.6)$$

$$w_t^s = \alpha Y_t (L_t^s)^{-1} \quad (1.7)$$

$$w_t^u = (1 - \alpha) Y_t (L_t^u)^{-1} \quad (1.8)$$

$$L_t^s = [s_t + \sigma_t \mu_t] N_t l_t^s \quad (1.9)$$

$$L_t^u = [1 - s_t + (1 - \sigma_t) \mu_t] N_t l_t^u \quad (1.10)$$

$$b_t = \frac{\tau_t \{ (s_t + \sigma_t \mu_t) w_t^s N_t l_t^s + [1 - s_t + (1 - \sigma_t) \mu_t] w_t^u N_t l_t^u \}}{(1 + \mu_t) N_t + (1 + \mu_{t-1}) N_{t-1}} \quad (1.11)$$

$$N_t = [1 + n + (1 + m) \mu_{t-1}] N_{t-1} \quad (1.12)$$

In this system of equations, there are:

- 9 Endogenous Variables: $l_t^s, l_t^u, w_t^s, w_t^u, Y_t, L_t^s, L_t^u, b_t, \frac{N_t}{N_{t-1}}$,
- 2 State Variables: μ_{t-1}, s_t , and,
- 3 Policy Variables: τ_t, μ_t, σ_t

Note that all nine endogenous variables are uniquely solvable given the state variables and the policy variables for a period t . It is obvious that the decision of output produced and labor input are purely static because there is no inter-temporal component in production, such as capital stock in a neoclassical model. However, the demographic structure of this economy is dynamic as the ratio of the native young to native old at time $t+1$ (N_{t+1}/N_t) and the skill composition of the natives at time $t+1$ depends on today's decision of immigration

policies (μ_t and σ_t). Therefore, the only dynamic component of this model will be the decision of voting, which will be discussed in detail in Section 1.3.

The equations are further normalized by dividing by N_t , which denotes the population of native young workers. Then, the equations above can be rewritten as follows:

$$\begin{aligned}
l_t^s &= [(1 - \tau_t)w_t^s]^\nu \\
l_t^u &= [(1 - \tau_t)w_t^u]^\nu \\
\tilde{Y}_t &= \left(\tilde{L}_t^s\right)^\alpha \left(\tilde{L}_t^u\right)^{1-\alpha} \\
w_t^s &= \alpha \tilde{Y}_t \left(\tilde{L}_t^s\right)^{-1} \\
w_t^u &= (1 - \alpha) \tilde{Y}_t \left(\tilde{L}_t^u\right)^{-1} \\
\tilde{L}_t^s &= [s_t + \sigma_t \mu_t] l_t^s \\
\tilde{L}_t^u &= [1 - s_t + (1 - \sigma_t) \mu_t] l_t^u \\
b_t &= \frac{\tau_t \left[w_t^s \tilde{L}_t^s + w_t^u \tilde{L}_t^u \right]}{(1 + \mu_t) + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} \\
\frac{N_t}{N_{t-1}} &= 1 + n + (1 + m) \mu_{t-1}
\end{aligned}$$

where the tilded variables stand for the original variables divided by N_t . For example, \tilde{L}_t^s is the aggregate input of skilled labor per native worker.

1.3 Political Decision

As discussed in Section 1.2, the policies on the income tax rate (τ_t), the volume of immigration (μ_t), and the skill composition of immigrants (σ_t) are decided at the beginning of each period by a plurality voting. In this section, the political decision-making process is explained in detail. This modeling of voting process is basically identical to the application of Besley and Coate (1997) by Razin et al. (2014), while there are slight differences in the electorate

composition.⁸ In modeling the implementation procedure of the winning candidate, the idea of Markov-perfect equilibria is applied.

1.3.1 Political Decision Process

The political decision-making process each period consists of the following three steps:

- **Step 1:** One individual is selected as a candidate in each of three political parties (the “Skilled Young”, the “Unskilled Young”, and the “Old Retirees”).
- **Step 2:** One and only one of the three candidates is elected by one single plurality voting.⁹
- **Step 3:** The winning candidate chooses the set of policy variables - $(\tau_t, \mu_t, \sigma_t)$

For convenience, these steps are described in the reverse order in the following sub-sections.

1.3.1.1 Policy Implementation of the Winning Candidate

Once one of the three candidates is elected by a plurality voting, the third and final step of the voting process is the decision making by the winning candidate. The winning candidate implements the policies on taxation and immigration as he prefers. As there is no mechanism

⁸For example, Razin et al. (2014) simplifies the political equilibria in two aspects. First, they assume that only the smallest party out of the three can choose to vote strategically, thereby closing the possibility that the smallest party wins the election. Second, they assume that the immigrants have the right to vote when they become old. This simplifies the equilibrium because the young-to-old ratio in the *electorate* is consistent with that of the whole pre-immigration population. Both simplifications are relaxed in this paper: the smallest party can win the election if the members of the second-largest party have incentive to vote strategically *and* the smallest party’s members do *not* have incentive to vote for the second-largest party. Also, immigrants do not have the right to vote for their life in this paper.

⁹In other words, there is no second voting such as, for example, the run-off primary in the U.S. presidential election.

to resolve commitment problems, all the voters recognize *ex ante* that the winning candidate will pursue his optimum after being elected and all other pledges are considered as cheap talk in any voting equilibrium. Let the optimal policy of the decisive candidate at the time t denoted by a triplet $(\tau_t^d, \mu_t^d, \sigma_t^d) \equiv \Phi_t^d(\mu_{t-1}, s_t)$ for the winning candidate's party $d \in \{s, u, o\}$. Then Φ_t^d solves the following maximization problem:

$$\Phi_t^d(\mu_{t-1}, s_t) = \underset{\tau_t, \mu_t, \sigma_t}{\operatorname{argmax}} V^d(\tau_t, \mu_t, \sigma_t, \Phi_{t+1})$$

where Φ_{t+1} is the perception of today's winning candidate on the policy variables in the next period. Obviously there can be multiple different rule of perception Φ_{t+1} that satisfies sub-game perfection, thus the complexity of this optimization problem depends on multiplicity of the expectation of the policy rule of the winning candidate of the next period. If there exist a multiple number of reasonable perceptions on future policies, then this period's winning candidate must consider multiple possibilities which can be caused by his decision today.

In order to refine this type of complexity and reduce the number of perceived future policy rules, this paper relies on Markov-perfect property.¹⁰ That is, the perception function Φ_{t+1} is assumed to be a function of the state variable for the period $t + 1$ *only*, without depending on anything else such as signals to eliminate any types of "sunspot" equilibria. With introduction of this Markov-perfect property, the winning candidate expects that next period's winning candidate will make a decision according to the identical rule with individuals living today. Formally, the winning candidate's problem is written as follows:

$$\begin{aligned} \Phi^d(\mu_{t-1}, s_t) &= \underset{\tau_t, \mu_t, \sigma_t}{\operatorname{argmax}} V^d(\tau_t, \mu_t, \sigma_t, \Phi^d(\mu_t, s_{t+1})) \\ \text{subject to } s_{t+1} &= \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+(1+m)\mu_t} \end{aligned}$$

if the winning candidate belongs to the cohort $d \in \{s, u\}$. Intuitively, this means that today's winning candidate decides policy variables with the belief that the winning candidate of the

¹⁰The Markov-perfection property in this paper is directly from Razin et al. (2014), which builds on the Markov-perfection setup used in previous works on short-term government and political economy such as Krusell and Rios-Rull (1996), Grossman and Helpman (1998), and Hassler et al. (2003).

next period will decide policy variables by exactly the same way as this period's candidates do, and, in any equilibrium, this belief is exactly realized.

If the winning candidate is an old retiree, his decision rule is simple since his utility consists only of the consumption of the lump-sum payment this period. Namely, the decision when winning the election is given by the solution of the following problem:

$$\Phi^o(\mu_{t-1}, s_t) = \operatorname{argmax}_{\tau_t, \mu_t, \sigma_t} V^o(\tau_t, \mu_t, \sigma_t) = b_t$$

where b_t is represented as a function only of the policy variables and the states by solving (1.4)~(1.12).

1.3.1.2 Voting Process

One step backward from the policy implementation procedure in the Section 1.3.1.1, now suppose that one candidate is selected from each of three political parties. The winning candidate will be decided by a single plurality voting, so the result of voting mainly depends on the size of the political parties. However, in analyzing the voting process, the possibility of strategic voting must be considered as there are three parties. That is, if a voter believes that the candidate from his cohort has no possibility of winning the election, he might have incentive to decide to vote for the next-best candidate in order to prevent the worst candidate from winning the election.

Also, in this model all the voters in the same cohort have identical preference, so voters in the same political party vote identically. This setup makes us concentrate on the behavior of a *representative* voter instead of analyzing complicated voting equilibrium within a cohort.

Formally, let $e_t^i = (p_t^{is}, p_t^{iu}, p_t^{io}) \in \Delta^2$ denote a voting decision of a representative voter in cohort $i \in \{s, u, o\}$, where p_t^{ij} stands for the probability that the voter in party i votes for the candidate from party j , and Δ^2 is a 2-simplex. Then, a voting profile $\mathbf{e}_t = (\mathbf{e}_t^s, \mathbf{e}_t^u, \mathbf{e}_t^o)$ is

a *voting equilibrium* if it solves the following problem:¹¹

$$\mathbf{e}_t^i = \operatorname{argmax}_{e_t^i} \left\{ \sum_{j \in \{s, u, o\}} \mathcal{P}^j(e_t^i, \mathbf{e}_t^{-i}) V^i(\Phi_t^j, \Phi_{t+1}, \mathbf{e}_{t+1}) \right\}$$

for all $i \in \{s, u, o\}$, where $\mathcal{P}^j(e_t)$ is the probability that the candidate from party j wins the election given the voting profile e_t . That is, in a voting equilibrium, each voter chooses whom to vote for so that the expected value of himself is maximized given the other voter's decision and the perceived future policy of the winning candidate next period.

In order to reduce the possibility of the multiple equilibria for a given set of state variables, I *only* consider the equilibria in which none of the voting decision is a weakly dominated strategy. In addition, I assume the following tie-breaking rules of the election:

- i) If there are two parties with the equally largest number of votes, then the candidate whose votes are from the least number of cohorts becomes the winner.
- ii) If two smaller groups are willing to form a coalition by voting for either candidate of the two, the coalition follows the larger party of the two. If their sizes are equal, the tie is broken with equal probability.
- iii) If all three parties have exactly the same size, the tie is broken with equal probability.

These tie-breaking assumptions, together with the assumption that equilibria with weakly dominated strategies are excluded, guarantees that there exists a unique pure-strategy voting equilibrium for each state vector of (μ_{t-1}, s_t) . In all of these equilibria, the members of the largest (not necessarily majority) cohort *always vote for the candidate from their own party* regardless of the formation of a coalition, thus a political coalition is potentially formed *only by the two smallest parties*.¹²

¹¹It is implicitly assumed that there is no additional cost for the electorate to participate in voting, and only the equilibria in which none of the voters abstains are considered.

¹²See Lemma 4 in Appendix 1.7.3 for a proof.

1.3.2 Size of Political Groups

Since the size of each of the political groups (a group of skilled young workers, unskilled young workers, and old retirees) is a critical component of the political decision,¹³ I start by looking at which group becomes the “largest” or the “majority” for all possible combinations of the state variables (μ_{t-1}, s_t) .

The group of native skilled young workers becomes the largest group under the following conditions:

(i) $s_t > \frac{1}{2}$, and,

(ii) $[1 + n + (1 + m)\mu_{t-1}] s_t > 1$

Namely, the group of skilled young workers becomes the largest if it is larger than the group of unskilled young workers *and* the group of native old retirees. It is straightforward to see that the first condition dominates if $n + (1 + m)\mu_{t-1} > 1$, and the second condition dominates otherwise.

Similarly, the group of unskilled young workers is the largest group under the following two conditions:

(i) $1 - s_t > \frac{1}{2}$, and,

(ii) $[1 + n + (1 + m)\mu_{t-1}] (1 - s_t) > 1$

and the first condition dominates if $n + (1 + m)\mu_{t-1} > 1$, and the second condition dominates otherwise.

The political party of the native old retirees becomes the largest under the following condition:

$$\max \{ [1 + n + (1 + m)\mu_{t-1}] s_t, [1 + n + (1 + m)\mu_{t-1}] (1 - s_t) \} < 1$$

¹³As emphasized before, note that the size of electorate is not the *only* component though, because whether a coalition is formed or not depends on not only the relative size of the two smallest groups but also their preference on the policy profiles of others parties.

Therefore, the relative sizes of the political groups depend on the combination of the state (μ_{t-1}, s_t) and the parameters m (fertility rate of the immigrants) and n (fertility rate of the natives). Specifically, the relative size of the electorate groups depends on the value of $n + (1 + m)\mu_{t-1}$, which governs the ratio between the native young and the native old. The following 4 cases categorizes the relative size of each political group at different skill composition s_t . The important fact is that the largest group does *not* always win because of the possibility that the two smaller parties form a coalition, so this section is *not* for describing who the winning candidate of the election is.

Case I: $\frac{N_t}{N_{t-1}} > 3$

s_t	0	$\frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{2} - \frac{1}{2} \frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{2}$	$\frac{1}{2} + \frac{1}{2} \frac{1}{1+n+(1+m)\mu_{t-1}}$	$1 - \frac{1}{1+n+(1+m)\mu_{t-1}}$	1
Largest	Unskilled*	Unskilled*	Unskilled	Skilled	Skilled*	Skilled*	
2nd Largest	Old	Skilled	Skilled	Unskilled	Unskilled	Old	
Smallest	Skilled	Old	Old	Old	Old	Unskilled	

*: More than 50%

Figure 1.1: Relative Sizes of Political Groups for Different Skill Composition - Case I

In this case, the size of the old retirees' party is less than one-third of the population of the native young workers, so the old group can never be the largest regardless of the skill composition of the young generation. Figure 1.1 displays the relative size of each political group depending on the value of $s_t \in [0, 1]$ in this case of $\frac{N_t}{N_{t-1}} > 3$. The old retirees' party constitutes the smallest unless the skill composition is extremely biased to either skilled- or unskilled labor, and either the skilled or the unskilled workers always become the largest group.

Case II: $2 < \frac{N_t}{N_{t-1}} \leq 3$

S_t	0	$\frac{1}{2} - \frac{1}{2} \frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{2}$	$1 - \frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{2} + \frac{1}{2} \frac{1}{1+n+(1+m)\mu_{t-1}}$	1
Largest	Unskilled*	Unskilled	Unskilled	Skilled	Skilled	Skilled*		
2nd Largest	Old	Skilled	Skilled	Unskilled	Unskilled	Old		
Smallest	Skilled	Old	Old	Old	Old	Unskilled		

*: More than 50%

Figure 1.2: Relative Sizes of Political Groups for Different Skill Composition - Case II

This case is similar to Case I in that the old retirees never have a chance to be the largest group since the population of the native old retirees is still less than a half the population of the native young workers.

Case III: $1 < \frac{N_t}{N_{t-1}} \leq 2$

S_t	0	$\frac{1}{2} - \frac{1}{2} \frac{1}{1+n+(1+m)\mu_{t-1}}$	$1 - \frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{2}$	$\frac{1}{1+n+(1+m)\mu_{t-1}}$	$\frac{1}{2} + \frac{1}{2} \frac{1}{1+n+(1+m)\mu_{t-1}}$	1
Largest	Unskilled*	Unskilled	Old	Old	Skilled	Skilled*	
2nd Largest	Old	Old	Unskilled	Skilled	Old	Old	
Smallest	Skilled	Skilled	Skilled	Unskilled	Unskilled	Unskilled	

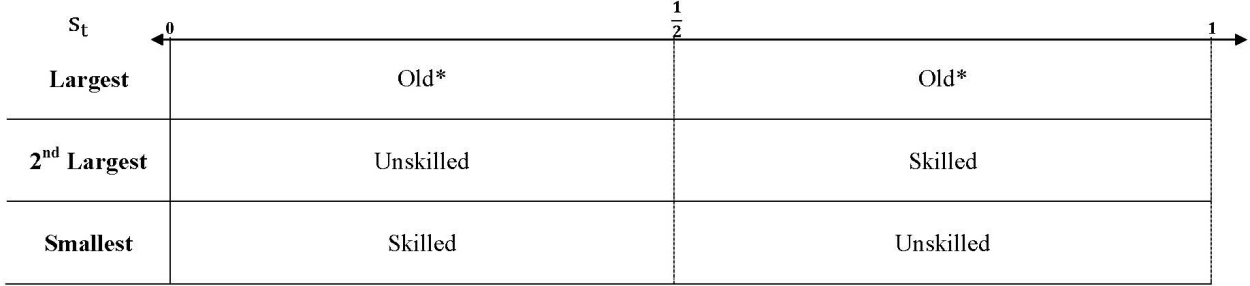
*: More than 50%

Figure 1.3: Relative Sizes of Political Groups for Different Skill Composition - Case III

This is probably the most interesting case. In this case, the native old population is still smaller than the population of the native young workers but larger than a half of it.

Therefore, the old retirees' group has a chance to be the largest group if the number of skilled young workers is close enough to the number of unskilled workers. However, since the population of the old retirees still less than a half of the total native population as a whole, the old retirees' group cannot constitute the majority for any value of s_t .

Case IV: $\frac{N_t}{N_{t-1}} \leq 1$



*: More than 50%

Figure 1.4: Relative Sizes of Political Groups for Different Skill Composition - Case IV

In this case, the population of the native old exceeds the population of the native young, so the old retirees' party always constitutes the majority of the electorate.

1.4 Political Equilibria

As mentioned in Section 1.2, while the political decision of the policy-makers is forward-looking, the equilibrium allocation and prices are purely static in this model because there are no inter-temporal components linking different periods such as bond market or capital accumulation. Therefore, the equilibrium allocation can be solved analytically with only a few simplifying conditions. For instance, the equilibrium allocation is solved in closed form if:

- (i) The marginal rate of substitution between consumption and leisure is multiplicative in labor input.
- (ii) The production function is multiplicative in both types of labor (skilled and unskilled).

Therefore, a Cobb-Douglas production function and our functional form of the utility function enable the equilibrium solvable in closed form for a given state variable (μ_{t-1}, s_t) and a set of policy variables $(\tau_t, \mu_t, \sigma_t)$.¹⁴ Specifically, the lump-sum transfer per capita and the wage of each type of labor are represented with respect only to (μ_{t-1}, s_t) and $(\tau_t, \mu_t, \sigma_t)$ as follows:

$$b_t = \frac{\tau_t (1 - \tau_t)^\nu \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t \mu_t)^\alpha [1 - s_t + (1 - \sigma_t) \mu_t]^{1-\alpha}}{1 + \mu_t + (1 + \mu_{t-1}) [1 + n + (1 + m) \mu_{t-1}]} \quad (1.13)$$

$$w_t^s = \alpha^{\frac{1+\alpha\nu}{1+\nu}} (1 - \alpha)^{\frac{(1-\alpha)\nu}{1+\nu}} (s_t + \sigma_t \mu_t)^{-\frac{1-\alpha}{1+\nu}} [1 - s_t + (1 - \sigma_t) \mu_t]^{\frac{1-\alpha}{1+\nu}} \quad (1.14)$$

$$w_t^u = \alpha^{\frac{\alpha\nu}{1+\nu}} (1 - \alpha)^{\frac{1+(1-\alpha)\nu}{1+\nu}} (s_t + \sigma_t \mu_t)^{\frac{\alpha}{1+\nu}} [1 - s_t + (1 - \sigma_t) \mu_t]^{-\frac{\alpha}{1+\nu}} \quad (1.15)$$

Therefore, the lifetime utility of individuals in each cohort is also represented only with the state variables and the policy variables by (1.1) and (1.2).

1.4.1 Equilibria without Forward-Looking

Before analyzing the equilibria of the full model, I start by demonstrating the equilibria without forward-looking behavior of the individuals, which is simply equivalent to assuming $\beta = 0$. This simplified version is considered because:

- i) all the equilibria in allocation and voting are solvable in closed form, so we can analyze all details of the equilibria, and,
- ii) more importantly, this is a good benchmark to analyze how the forward-looking motive affects the decision of the winning candidate, thus the voters' choice after all.

¹⁴For more detailed procedure of computing the closed-form equilibrium, see Appendix 1.7.1.

By assuming $\beta = 0$, this model becomes abstract from the inter-temporal components, and the winning candidate's policy decision becomes a simple optimization problem of maximizing his instantaneous utility. We first analyze the preference of each cohort on policy variables, then the political equilibrium after that.

1.4.1.1 Preference of Candidates on Policy Variables

After substituting the equilibrium allocation with closed-form solutions, the indirect utility function of an individual in each cohort can be represented by the following indirect utility functions with respect to policy variables and states:

$$\begin{aligned} V_t^i &= b_t + \cancel{\beta b_{t+1}} + \frac{[(1 - \tau_t) w_t^i]^{1+\nu}}{1 + \nu} \\ V_t^o &= b_t \end{aligned}$$

where b_t and w_t^i are of the form as in (1.13)~(1.15), for $i \in \{s, u\}$. Note that the continuation value is zero for the young workers as $\beta = 0$ is assumed. The ideal policies each political party, therefore, can be analyzed with these indirect utility functions. The following proposition displays the old retiree's preferred policies.

Proposition 1. *If an old retiree wins the election, his optimal policy is characterized by:*¹⁵

$$(\tau_t, \mu_t, \sigma_t) = \begin{cases} \left(\frac{1}{1+\nu}, \bar{\mu}, 1 \right) & \text{if } 0 \leq s_t < \alpha - (1 - \alpha)\bar{\mu} \\ \left(\frac{1}{1+\nu}, \bar{\mu}, \alpha + \frac{\alpha - s_t}{\bar{\mu}} \right) & \text{if } \alpha - (1 - \alpha)\bar{\mu} \leq s_t < \alpha + \alpha\bar{\mu} \\ \left(\frac{1}{1+\nu}, \bar{\mu}, 0 \right) & \text{if } \alpha + \alpha\bar{\mu} \leq s_t \leq 1 \end{cases}$$

Proof. See Appendix 1.7.2.1. □

The implication of this proposition is clear - as the old retirees do not have any labor endowment, they are *pure beneficiaries* of the welfare state. Therefore, if the candidate from

¹⁵Obviously, some of the ranges of s_t can be an empty set if $\bar{\mu}$ is large enough. For instance, if $\bar{\mu}$ is larger than $\alpha/(1 - \alpha)$, then the optimal σ_t always equals $\alpha + \frac{\alpha - s_t}{\bar{\mu}}$ since the first and the third intervals become empty sets.

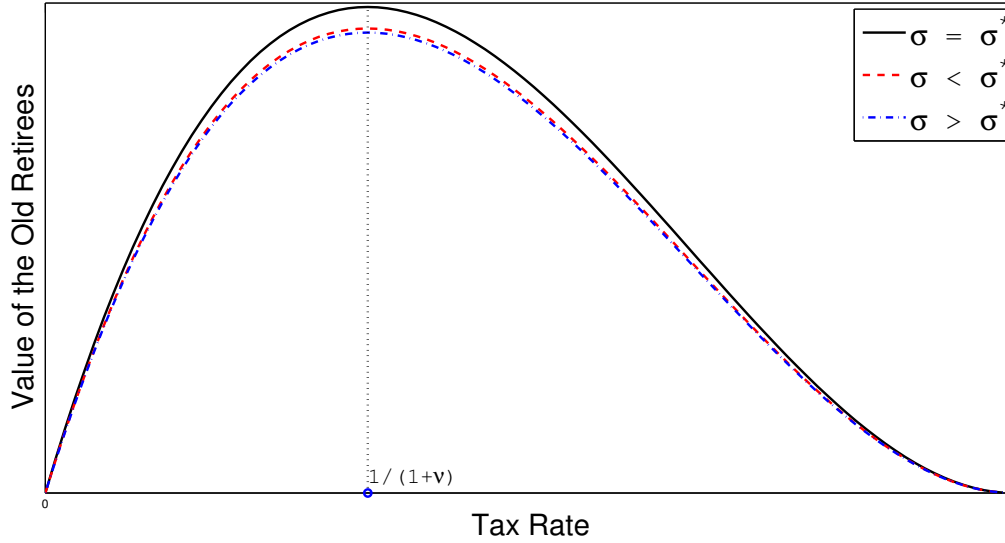


Figure 1.5: Tax Rates and the Indirect Utility of the Old Retirees

the old retirees wins the election, he will choose the policy variables so that the tax revenue of the government per capita is maximized. The Figure 1.5 displays the relationship between the tax rate and the lifetime value of the old retirees. The tax rate is decided at $1/(1 + \nu)$, which is the *Laffer rate* of the economy. Regarding the skill composition of immigrants (σ_t), the old retiree chooses $\sigma_t = 1$ if s_t is small enough and $\sigma_t = 0$ if s_t is large enough, because the production, thus the tax revenue, is maximized when the proportion of skilled workers in the labor force equals α , the exponent in the Cobb-Douglas production function. To better understand this implication, assume that $\bar{\mu} \rightarrow \infty$. Then, the decision of σ_t converges to α , which means that the old retirees want the skill composition of the economy's workers to be consistent with the Cobb-Douglas exponent and the wage rates of the skilled- and unskilled labor to be equalized. Figure 1.5 shows that the value of the old retirees is maximized when the σ_t is optimal as in Proposition 1. Finally, the immigration volume is decided at $\mu_t = \bar{\mu}$. This result relies on the assumption of constant-returns-to-scale technology. In other words, as long as the skill composition satisfies the optimality, the tax revenue is monotone increasing in the volume of immigration since the production efficiency is robust to the scale of factor inputs.

The next two propositions show the optimality of the young workers with each skill level:

Proposition 2. *If a skilled young worker wins the election, his optimal policy is characterized by:*

$$(\tau_t, \mu_t, \sigma_t) = \begin{cases} (0, \bar{\mu}, 0) & \text{if } 0 \leq s_t < \alpha \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] \\ (\tau_t^s, \bar{\mu}, 0) & \text{if } \alpha \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] < s_t \leq 1 \end{cases}$$

where

$$\tau_t^s = \frac{s_t - \alpha \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]}{s_t(1 + \nu) - \alpha \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]} \in \left(0, \frac{1}{1 + \nu} \right)$$

Proof. See Appendix at Section 1.7.2.2. □

Proposition 3. *If an unskilled young worker wins the election, his optimal policy is characterized by:*

$$(\tau_t, \mu_t, \sigma_t) = \begin{cases} (\tau_t^u, \bar{\mu}, 1) & \text{if } 0 \leq s_t \leq 1 - (1 - \alpha) \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] \\ (0, \bar{\mu}, 1) & \text{if } 1 - (1 - \alpha) \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] < s_t \leq 1 \end{cases}$$

where

$$\tau_t^u = \frac{1 - s_t - (1 - \alpha) \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]}{(1 - s_t)(1 + \nu) - (1 - \alpha) \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]} \in \left(0, \frac{1}{1 + \nu} \right)$$

Proof. The proof is omitted as it is symmetric to the proof of Proposition 2. It is simply proved by substituting s_t with $1 - s_t$, σ_t with $1 - \sigma_t$, and α with $1 - \alpha$ in Proposition 2. □

The main implication of the Proposition 2 and 3 is that the young workers consider not only the lump-sum transfer but also their labor income. For example, when s_t is sufficiently small thus there exist only a small number of skilled young workers in the economy, the wage rate of the skilled labor is so high that the skilled young become *net contributors* of the welfare state, thus they prefer *zero* tax rate. The upper panel of Figure 1.6 shows the utility of the skilled young workers for different tax rates and σ_t , the skill composition of immigrants, when their optimal tax rate equals zero. Given zero tax rate, the transfer also equals zero, thus the utility of the skilled young depends only on the labor income. Since

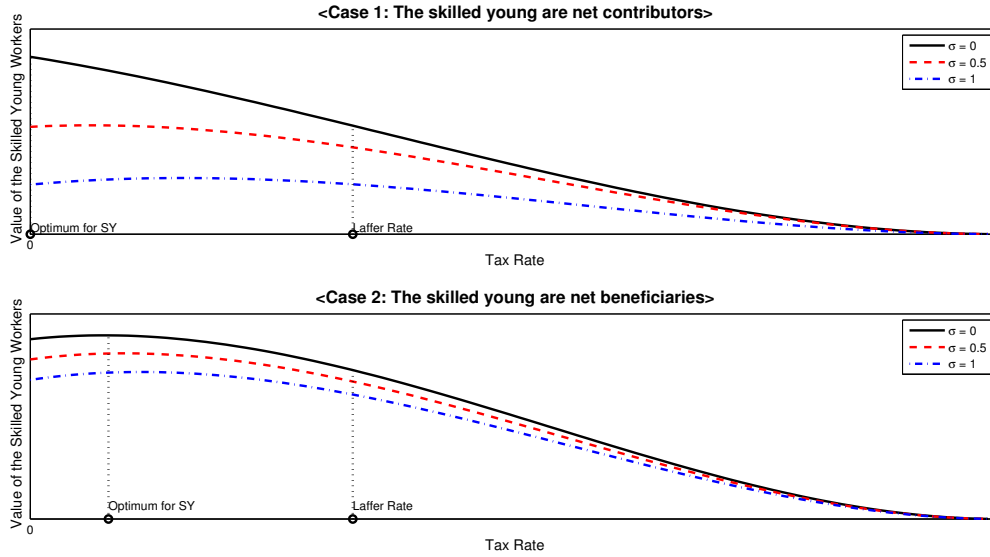


Figure 1.6: Tax Rates and the Indirect Utility of the Skilled Young

the wage rate of the skilled labor is increasing in the input of unskilled labor due to skill complementarity, the skilled workers prefer as many unskilled immigrants as possible.

The lower panel of Figure 1.6 displays the opposite case that s_t is large enough. In this case, the skilled workers now become *net beneficiaries* of the welfare state because there are already excessively many skilled workers so the equilibrium wage of the skilled labor is too small, so the skilled workers prefer a strictly positive tax rate τ_t^s . Nevertheless, their ideal tax rate is still smaller than the Laffer rate since they are anyway tax-payers while the old retirees are not. In this case, the skilled young workers' decision of σ_t depends on two factors: First, their wage rate increases as more unskilled labor enters the economy as in the previous case. Second, the direction of the changes in lump-sum transfer depends on the pre-immigration skill composition of the economy. Specifically, the transfer increases in σ_t if s_t is relatively small, and decreases if s_t is large. Regardless of the size of s_t , however, it can be shown that the wage effect dominates the transfer effect. Thus the skilled workers choose to approve the unskilled immigrants only.

The optimum of the unskilled young workers is symmetric to that of the skilled workers as in Figure 1.7. They set a zero tax rate when s_t is so large that the unskilled workers are

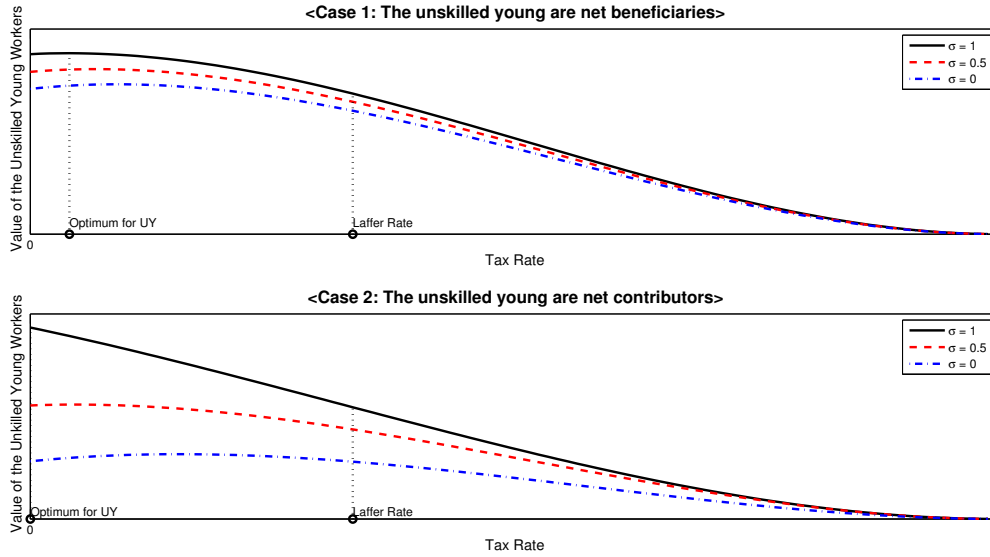


Figure 1.7: Tax Rates and the Indirect Utility of the Unskilled Young

net contributors of the welfare state, and they prefer a positive tax rate when s_t is small and they are net beneficiaries of the welfare state. In either case, the unskilled workers prefer the maximum amount of skilled immigrants.

In summary, the old retirees want to maximize the government's tax revenue by levying a high tax rate (at the Laffer rate) and balancing the skill composition of workers for efficient production. On the other hand, the young workers prefer a lower (even zero) tax rate and want all the immigrants to have the opposite skill to maximize their disposable labor income.

1.4.1.2 Political Equilibria

Basically the result of the election depends on the relative sizes of the three political parties as described in Section 1.3.2, while there is no guarantee that the largest party always wins the election since the two smallest parties potentially form a coalition when the largest party's preferred policy is believed to be the worst for either of the smaller groups. Therefore, not only the relative size of electorate but also the preference of each party on the preferred policy of one another determines the result of the election.

The pattern of the political equilibria depends on the relative values of parameters. For instance, a simple case is when $\frac{N_t}{N_{t-1}} < 1$ (the size of the native old exceeds the native young), in which the candidate of the old retirees' party always wins the election by majority. While the equilibria can be represented in closed form for this no forward-looking case, I will describe only one equilibrium with interesting implications rather than considering every possible case of parameter values and state. Specifically, the analysis concentrates on the case in which:

i) $1 < \frac{N_t}{N_{t-1}} \leq 2$

ii) α is sufficiently large and $\bar{\mu}$ is sufficiently small such that $\frac{1}{2} < \alpha - (1 - \alpha)\bar{\mu} < \alpha(1 + \bar{\mu})$

iii) μ_{t-1} is sufficiently large such that $\alpha - (1 - \alpha) \left[\bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] < \frac{1}{2}$.

In this case, the patterns of the political equilibrium can be described by the range of s_t as follows:

Case I: $0 \leq s_t \leq \frac{1}{2} - \frac{1}{2} \cdot \frac{N_{t-1}}{N_t}$

In this case, the population of the native unskilled young workers constitutes the majority, so the winning candidate's policy decision maximizes the lifetime value of the unskilled young. Since s_t is so small in this case that the unskilled young become the net beneficiary of the welfare state, the choice of the winner's policy variables is $\Phi^u(\mu_{t-1}, s_t) = (\tau_t^u, \bar{\mu}, 1)$ where τ_t^u is defined as in Proposition 3. Political coalition does not take place as it cannot even win the election.

Case II: $\frac{1}{2} - \frac{1}{2} \cdot \frac{N_{t-1}}{N_t} < s_t \leq 1 - \frac{N_{t-1}}{N_t}$

In this case, the unskilled young party is still the largest but does not form the majority. However, the smallest parties (the skilled young and the old retirees) do not have incentive to form a coalition because they perceive that the policy of each other is the worst. For the old retirees, the preferred policy of the skilled young includes a zero tax rate, which is the

worst to the old retirees as it leads to the zero welfare state. For the skilled young workers, the policies on σ_t of both the unskilled young and the old are equally the worst since both prefer $\sigma_t = 1$, while the old retirees even prefer a higher tax rate. Therefore, for each of the old retirees and the skilled young workers, it is the next-best choice to let the unskilled workers win the election, thereby forgoing the possibility of a political coalition. As a result, the unskilled young's party wins the election and it implements $(\tau_t, \mu_t, \sigma_t) = (\tau_t^u, \bar{\mu}, 1)$ by Proposition 3.

Case III: $1 - \frac{N_{t-1}}{N_t} < s_t \leq \alpha - (1 - \alpha) \left[\bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]$

In this case, the population of old retirees constitutes the largest, the unskilled workers are the second, and the skilled workers form the smallest group. Therefore, it is important whether the skilled- and unskilled workers have incentive to form a coalition with each other. It is clear that the skilled workers have incentive to strategically vote for the unskilled candidate. The preferred tax rate of the unskilled workers is $\tau^u \in (0, 1/(1 + \nu))$, which is advantageous to the skilled as it levies less taxes than the old retirees would do. Both old retirees and unskilled workers prefer maximum amount of skilled immigrants. Therefore, the skilled workers are willing to vote for the political party of the unskilled workers. Since the population of the unskilled workers exceeds that of the skilled workers, the coalition is formed in a way that the unskilled candidate represents the whole coalition. Therefore, the unskilled workers win the election.¹⁶ The policy implementation becomes $(\tau_t, \mu_t, \sigma_t) = (\tau^u, \bar{\mu}, 1)$ by Proposition 3.

¹⁶I do not consider whether the unskilled workers have incentive to strategically vote for the skilled candidate because the unskilled workers' party is a larger group than the skilled and the skilled group *does* have an incentive to vote for the unskilled candidate. In this case, regardless whether the unskilled party's second-best is the skilled or not, the coalition is formed such that the unskilled candidate represents the whole coalition by the equilibrium refinement procedure introduced in Section 1.3.1.2. Obviously, the case that the skilled party wins the election by forming a coalition with the unskilled workers can be a Nash equilibrium if the unskilled workers have incentive to do so.

Case IV: $\alpha - (1 - \alpha) \left[\bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] < s_t < \frac{1}{2}$

In this case the order of population size is still given by the old, the unskilled, and the skilled. The only difference from the previous case is that now the unskilled workers are net contributors so they prefer zero taxes as the skilled wants. Therefore, the coalition is formed so that the unskilled party wins the election, and the policy of $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 1)$ is implemented by Proposition 3.

Case V: $\frac{1}{2} \leq s_t < \frac{N_{t-1}}{N_t}$

In this case, the old retirees are the largest, the skilled are the second-largest, and the unskilled are the smallest in size. Also, since both skilled- and unskilled workers are net contributors to the welfare state, all of them prefer a zero tax rate, thus the old retirees equally dislike any young candidate to win. The skilled workers obviously prefer unskilled workers' policy profile to that of the old retirees. Therefore, the important problem is whether the unskilled prefer the skilled workers' ideal policy to the old retirees'. If the unskilled workers prefer skilled workers, then the skilled candidate would represent the coalition. Otherwise, the unskilled workers would have no reason to vote for the skilled workers, so the skilled workers strategically vote for the unskilled workers, and the unskilled candidate will represent the coalition.

For the unskilled workers, the preferred policies of the other two parties have the following trade-offs: Though the skilled workers' preferred policy, denoted by $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 0)$, is preferable since it levies zero taxes, it will allow many unskilled immigrants and make the competition in the unskilled labor market tighter. Though the old retirees' preferred policy, denoted by $(\tau_t, \mu_t, \sigma_t) = (1/(1+\nu), \bar{\mu}, 0)$, is beneficial because it will allow a maximum number of skilled immigrants, it levies high taxes to the labor income. Therefore, the problem is whether the unskilled workers can bear the high tax rate (at the Laffer rate) preferred by the old retirees in order to make as many skilled immigrants to enter as possible. Formally, this trade-off is characterized by the threshold tax rate, denoted by $\tilde{\tau}(\mu_{t-1}, s_t)$, which is

determined implicitly by solving for the following equation:

$$V^u(\mu_{t-1}, s_t; \tau_t = \tilde{\tau}, \mu_t = \bar{\mu}, \sigma_t = 1) = V^u(\mu_{t-1}, s_t; \tau_t = 0, \mu_t = \bar{\mu}, \sigma_t = 0)$$

where the left-hand side is the utility of the unskilled workers when the tax rate is at $\tilde{\tau}$ and the maximum number of skilled immigrants are approved, and the right-hand side is the utility of the unskilled workers when the skilled young win the election.¹⁷ In other words, $\tilde{\tau}$ is the maximum tax rate that the unskilled young workers can bear in return for approving maximum skilled immigrants. If $\tilde{\tau} < 1/(1 + \nu)$, then the tax rate which the old retirees would levy is excessively burdensome to the unskilled, so the unskilled will strategically vote for the skilled workers' party. Otherwise, if $\tilde{\tau} \geq 1/(1 + \nu)$, then the unskilled prefer the old retirees' policy at least as much as the skilled workers', thus coalition is formed in a way that the unskilled candidate represents the whole coalition and wins the election.

As a result, the skilled young win the election by the support of the unskilled party if $\tilde{\tau} < 1/(1 + \nu)$, which implements $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 0)$ by Proposition 2. If $\tilde{\tau} \geq 1/(1 + \nu)$, then the unskilled workers win the election by the support of the skilled workers and the policy implementation is $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 1)$ by Proposition 3.

Case VI: $\frac{N_{t-1}}{N_t} \leq s_t < \alpha - (1 - \alpha)\bar{\mu}$

In this case, the order of electorate size is given by the skilled, the old, and then the unskilled. As in the previous case, the old retirees do not have any incentive for a strategic voting because all the young workers prefer the zero tax rate. Therefore, the problem is whether the unskilled are willing to vote strategically for the old candidate or not, and this decision depends on the level of the unskilled workers' threshold tax rate $\tilde{\tau}$. If $\tilde{\tau} \leq 1/(1 + \nu)$, the unskilled do not participate in the coalition with the old retirees and the skilled party wins, thus the policy implementation will be $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 0)$ by Proposition 2. If $\tilde{\tau} > 1/(1 + \nu)$, then the old retirees win the election by the support of the unskilled workers, and the resulting policy implementation is give by $(\tau_t, \mu_t, \sigma_t) = (1/(1 + \nu), \bar{\mu}, 1)$ by Proposi-

¹⁷The existence of such $\tilde{\tau}$ is guaranteed by Intermediate Value Theorem

tion 1.

Case VII: $\alpha - (1 - \alpha)\bar{\mu} \leq s_t < \alpha + \alpha\bar{\mu}$

In this case, the order of electorate size is the same as in the previous case (the skilled, the old, then the unskilled). The difference is that now s_t is so large that the old retirees prefer “balanced” skill composition of immigrants at $\sigma_t = \alpha + \frac{\alpha - s_t}{\bar{\mu}}$ instead of allowing skilled immigrants only. This makes the unskilled workers less favorable to the old retirees, thereby *decreasing* the threshold tax rate for a strategic voting. Formally, the new threshold tax, denoted by $\hat{\tau}(\mu_{t-1}, s_t)$, is determined implicitly by the following equation:

$$V^u \left(\mu_{t-1}, s_t; \hat{\tau}, \bar{\mu}, \alpha + \frac{\alpha - s_t}{\bar{\mu}} \right) = V^u (\mu_{t-1}, s_t; 0, \bar{\mu}, 0)$$

If $\hat{\tau} \leq 1/(1 + \nu)$, the unskilled do not participate in the coalition with the old retirees and the skilled party wins, thus the policy implementation will be $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 0)$ by Proposition 2. If $\hat{\tau} > 1/(1 + \nu)$, then the old retirees win the election by the support of the unskilled workers, thus $(\tau_t, \mu_t, \sigma_t) = (1/(1 + \nu), \bar{\mu}, \alpha + \frac{\alpha - s_t}{\bar{\mu}})$ is implemented by Proposition 1.

Case VIII: $\alpha + \alpha\bar{\mu} < s_t < \frac{1}{2} + \frac{1}{2} \frac{N_{t-1}}{N_t}$

In this case, the electorate size is the same as in the previous case (the skilled, the old, then the unskilled). However, now s_t is so high that the old retirees want to implement $\sigma_t = 0$. Therefore, neither the old retirees nor the unskilled workers have incentive to support each other. As a result, no coalition is formed and the skilled candidate wins the election as the largest party. The policy of $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 0)$ is implemented by Proposition 2.

Case IX: $\frac{1}{2} + \frac{1}{2} \frac{N_{t-1}}{N_t} < s_t \leq 1$

In this case, the skilled young workers’ party constitutes the majority of the electorate. The skilled young candidate wins the election and implements $(\tau_t, \mu_t, \sigma_t) = (0, \bar{\mu}, 0)$ by Proposition 2.

The 9 cases above describes the pattern of political equilibria depending on the skill composition and the age composition of the economy. In short, a highly unbalanced skill composition makes the larger skill group more likely win the election, and the old retirees are likely to win when young-to-old ratio is small enough and skill composition of the young voters is relatively *balanced*. The forward-looking case will be shown to exhibit a similar pattern of equilibria in the next section.

1.4.2 Equilibria with Forward-Looking

In this section, now the young workers care not only the instantaneous utility but also the future value of themselves (alternatively, $\beta > 0$). This results in a trade-off for the young workers. For example, for an unskilled young worker, it is still advantageous to approve many skilled immigrants because the skill-complementarity will increase his wage this period as discussed in the previous section without a forward-looking behavior. However, the forward-looking behavior now causes a drawback to this strategy. If too many skilled young immigrants enter the economy this period, the descendants of those immigrants will be *native* skilled young workers in the next period (so they have the right to vote in the next period as members of the skilled young party). Therefore, it is highly likely that the skilled young workers' party, which is most hostile to the welfare state, will win the election next period. This will deprive the present unskilled workers of the future welfare state after they retire. The unskilled young workers of this period can expect this, so they might choose to approve only a small number of skilled workers so that the skilled young workers will not constitute a majority in the next period.

Formally, if a young candidate with skill level d wins the election, his policy rule is determined by the following optimization problem:

$$\begin{aligned} \Phi^d(\mu_{t-1}, s_t) = \operatorname{argmax}_{\tau_t, \mu_t, \sigma_t} & \frac{(1 - \tau_t) [w^i(\tau_t, \mu_t, \sigma_t; \mu_{t-1}, s_t)]^{1+\nu}}{1 + \nu} + b(\tau_t, \mu_t, \sigma_t; \mu_{t-1}, s_t) \\ & + \beta \cdot b(\Phi^d(\mu_t, s_{t+1})) \end{aligned} \quad (1.16)$$

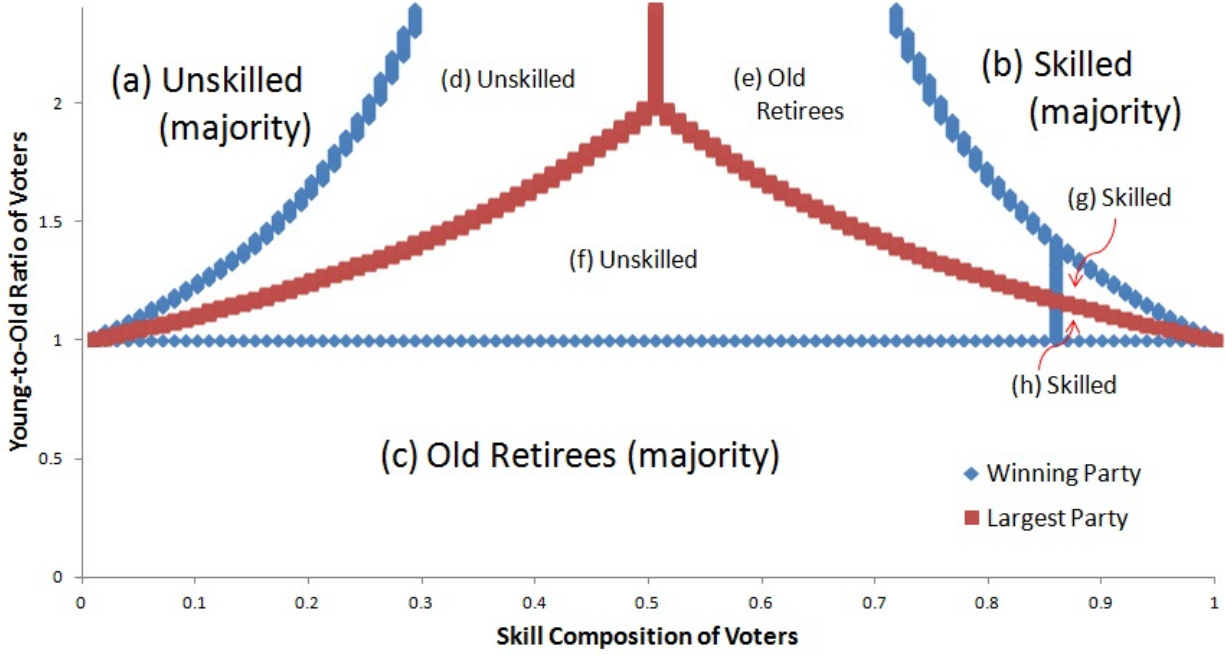


Figure 1.8: Winning Candidate for Each State ($\alpha = 0.8$)

subject to

$$s_{t+1} = \frac{(1+n)s_t + (1+m)\sigma_t\mu_t}{1+n+(1+m)\mu_t}$$

An old retiree's decision when he wins is identical to the previous section when $\beta = 0$ since his utility does not depend on future.

Due to complexity of this model, I rely on the numerical solution for this forward-looking case. Basically the solution numerically searches for the functional form of $\Phi^d(\mu_{t-1}, s_t)$ which solves the functional equation (1.16). In other words, the solution describes what the winning candidate's optimal choice of the policy profile is, given the state vector.

Figure 1.8 displays which cohort actually *wins* the election given the state vector (skill composition of the electorate in the horizontal axis, and the young-to-old ratio of the electorate in the vertical axis). First of all, the red line divides the state space into three regions according to which the *largest* party is. For instance, the regions (a) and (d) are where the unskilled workers' party forms the largest group, the regions (b), (e), and (g) are where the skilled workers' party is the largest, and the old retirees' party is the largest in

(c), (f), and (h). The blue line further divides the regions according to which group becomes the *winner* and how this group wins. See the following details:

- In region (a), (b), and (c), either one of the groups constitutes the majority (more than 50%) of the electorate. Therefore, the political party with majority wins the election.
- In region (d), the unskilled party is the largest. It wins the election since the two smaller parties (the skilled and the old) are expected to implement a policy profile which is the worst to each other thus no coalition is organized.
- In region (e), the old retirees' party wins while the skilled is the largest because the unskilled young workers prefer old retirees' policy profile to that of the skilled. In other words, the unskilled workers can bear the high tax rates that the old retirees are expected to levy as the old retirees will approve many skilled immigrants and increase the wage of the unskilled labor.
- In region (f), the unskilled workers' party wins the election though the old retirees are the largest because the skilled workers strategically vote for the unskilled candidate to keep the old retirees from winning the election.
- In region (g), the skilled workers' party wins the election as the largest party. The other two parties do not form a coalition.
- In region (h), the skilled workers' party wins the election by forming a coalition with the unskilled. The old retirees' party is the largest but fails to win since their preferred tax rate (Laffer rate) is excessively burdensome to the unskilled workers.

Therefore, political coalition is formed in (e), (f), and (h), while the smaller parties let the largest to win without formation of coalition in (d) and (g).

Figure 1.9 shows how the winning party and the equilibrium volume of immigration differ by the skill composition of voters for a fixed young-to-old ratio of the electorate. While the old retirees always approve the maximum amount of immigrants, the young workers can

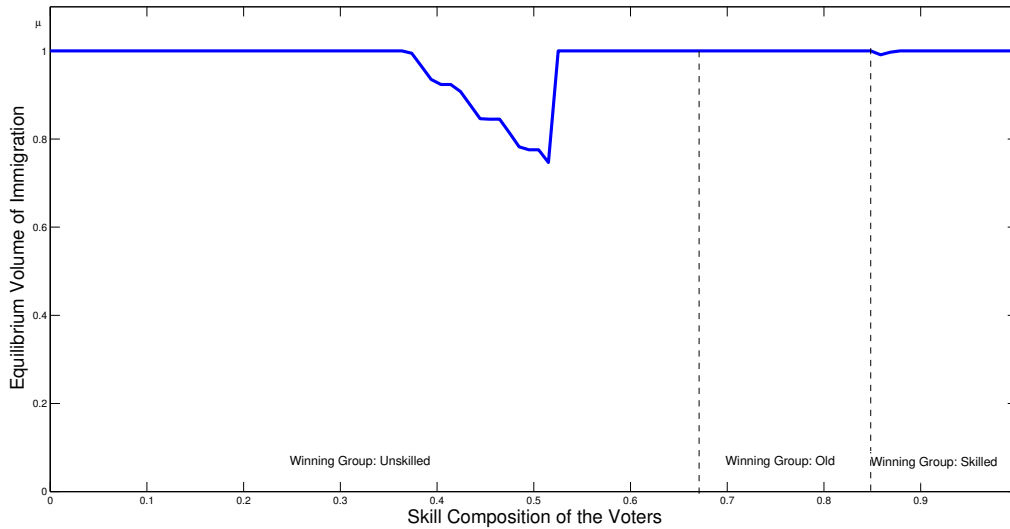


Figure 1.9: Decision of Immigration Volume by Skill Composition

choose a value smaller than the maximum for some values of s_t . Note that this happens solely by the forward-looking behavior, because the desired immigration volume is always the maximum level without the forward-looking component as shown in Proposition 1~3.

Figure 1.10~1.12 describe the reason why the unskilled workers prefer $\mu_t < \bar{\mu}$ at some states and $\mu_t = \bar{\mu}$ in the others. In Figure 1.10, the current state is given by the point A . Path (i) shows how the economy will move to a new state next period if the winning candidate decides to approve the maximum number of immigrants. As the diagram demonstrates, allowing the maximum number of skilled immigrants will make the skilled workers to be the majority group in the next period, thereby depriving the possibility to receive the transfer after they retire. To prevent this, an alternative option of the unskilled workers is to approve a slightly smaller number of immigrants instead (path ii), which will guarantee that today's unskilled young workers will win the election again in the next period (thus will receive the maximum lump-sum transfer), though it will reduce today's labor income of the unskilled workers relative to the path (i). Figure 1.11 shows the example in which it is too costly to reduce the immigration volume for winning in the next period if the economy is currently at the point B . In this case, the unskilled will abandon the possibility of winning in the

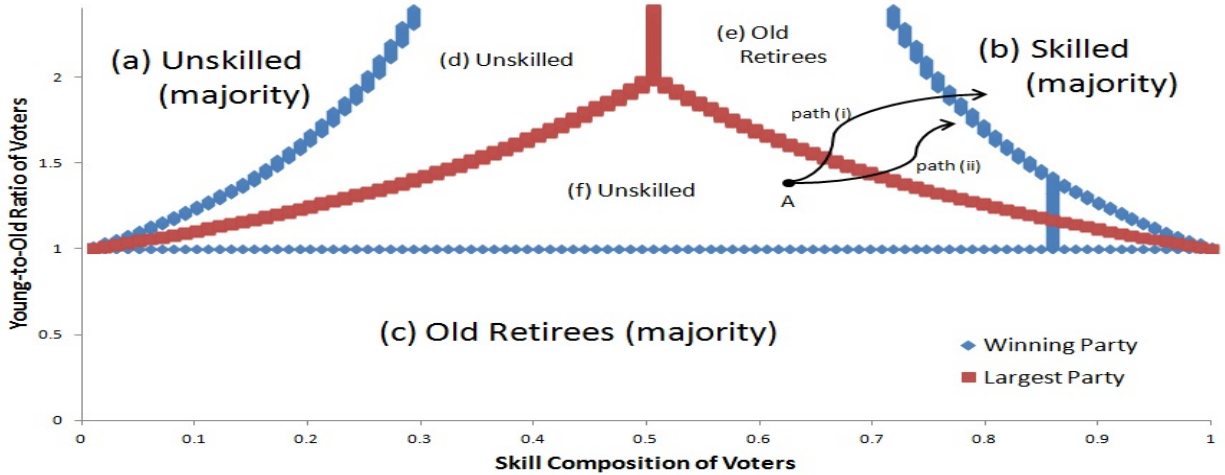


Figure 1.10: Case I - Less than Maximum Volume of Immigration is Optimal

next period and pursue to maximize today's utility by allowing the maximum number of immigrants, which is the reason why the immigration volume goes up to the maximum at around $s = 0.6$ in Figure 1.9.

Finally, Figure 1.12 shows the case where it is not feasible for the unskilled workers to regain the political power in the next period again by adjusting the immigration volume. If the skill composition is sufficiently biased toward unskilled labor, the unskilled workers cannot make the economy move toward the state for which the old retirees will win the election in the next period even if the maximum amount of skilled immigrants is accepted. In this case, the unskilled workers' optimal choice is simply to approve the maximum number of skilled immigrants to maximize the instantaneous piece of their utility.

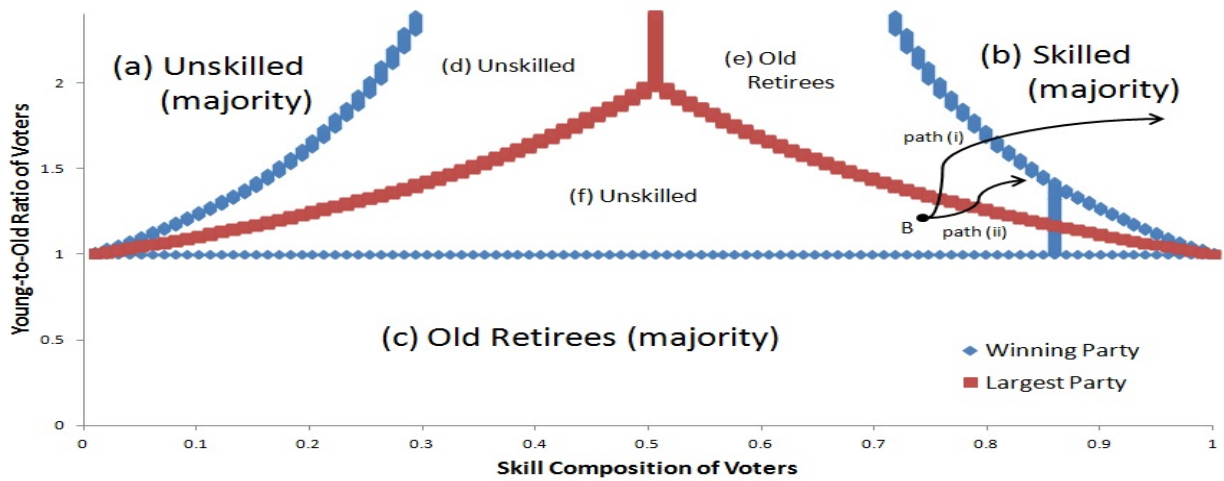


Figure 1.11: Case II - Less than Maximum Volume of Immigration is Too Costly

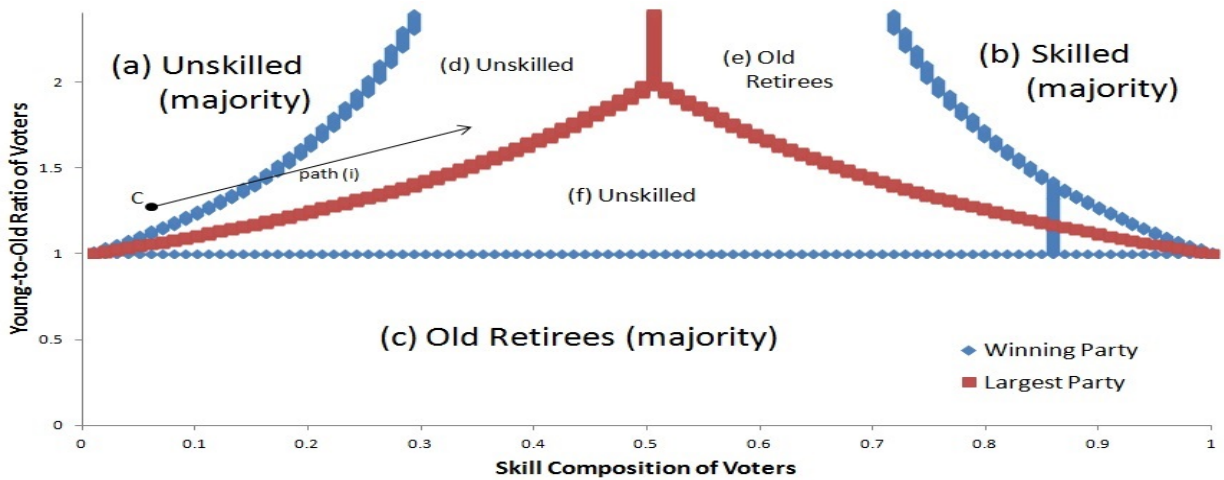


Figure 1.12: Case III - Less than Maximum Volume of Immigration has No Gaining

1.5 Dynamics and Social Welfare

In this section, the dynamics of key macroeconomic variables and their implications are analyzed. According to the analysis on political equilibria shown in the previous section, the skill composition of the economy is the key component which determines who wins the election and how the policies are implemented. Moreover, based on the results, it is expected that the skill composition exhibits cyclical movement - if the skill level of the young natives (i.e. the young voters) is biased toward unskilled labor, the skill composition will move toward the skilled labor in the next period because the unskilled workers' party is highly likely to win the election and approve the maximum number of skilled immigrants, whose descendants will be skilled workers in the next period. Similarly, if the proportion of the skilled workers is very large, the skill composition will move toward the unskilled labor in the next period by the same logic.

This cyclical behavior of the skill composition leads to welfare loss of the economy. Since the production function of this model is the Cobb-Douglas aggregation of the skilled and unskilled labor with exponent α for the skilled labor, the socially optimal skill composition is obviously in a way that α is the proportion of the skilled labor and $1 - \alpha$ is that of the unskilled labor. In this sense, the skill composition chosen by the old retirees is socially optimal, while the young workers' ideal decision might be suboptimal.

In this model, the immigration *per se* does not cause any inefficiency due to the constant-returns-to-scale technology. In other words, it is socially optimal to approve as many immigrants as possible as long as the post-immigration skill composition gets more balanced toward α . Therefore, the welfare loss of this economy is three-fold. First, the skill composition is not always balanced since the young workers prefer maximal skill complementarity for maximizing their own wage. Second, the proportional labor income tax is distortionary. Third, the young workers might not choose to allow the maximum number of immigrants for regaining their political power in the following period.

1.5.1 Welfare Loss by Skill Imbalance

In order to check that the skill composition exhibits suboptimal path, the dynamics of the skill composition is examined in this section. Figure 1.13 displays the dynamics of the *post-immigration* skill composition of workers when $\alpha = 0.8$ and the initial skill composition equals 0.5. As guessed in the previous paragraphs, the skill composition fluctuates over time. For instance, the skilled workers win the election at $t = 2$, decreasing the proportion of the skilled workers by allowing the maximum number of unskilled immigrants. This makes, at $t = 3$, the unskilled workers win the election and pushes the skill composition upward. The old retirees win the election at $t = 4$ since the unskilled workers adjust the immigration volume at $t = 3$ so that they can be the decisive party again when they retire. Since old retirees want the skill composition to be *balanced* at α , the skill composition of immigrants is adjusted such that the skill composition of workers is exactly at α after immigrants enter the economy. This makes the skilled workers win the election at $t = 5$, and this pattern is repeated later on.

It is notable that the value of s_t always stays below α . Since α is assumed to be larger

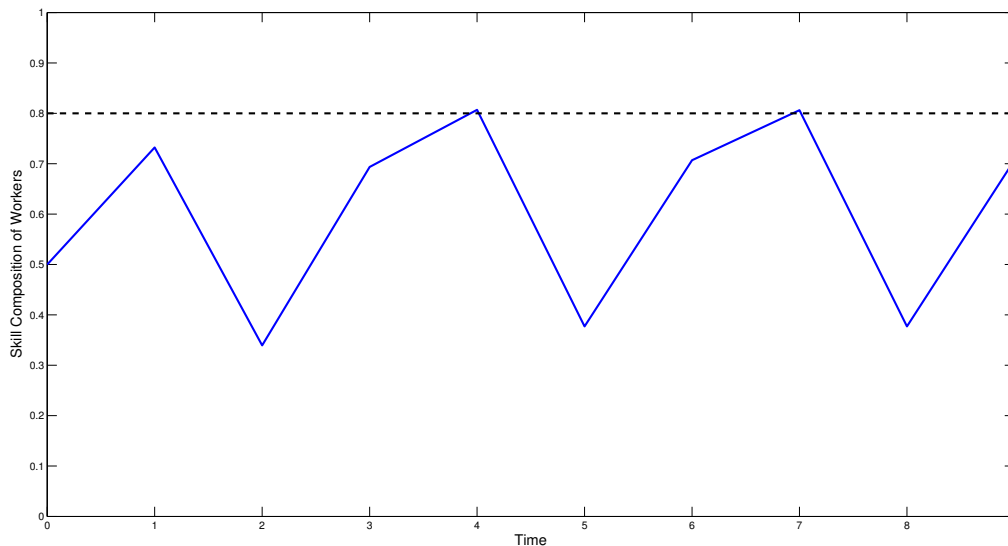


Figure 1.13: Dynamics of the Skill Composition of Workers ($\alpha = 0.8$)

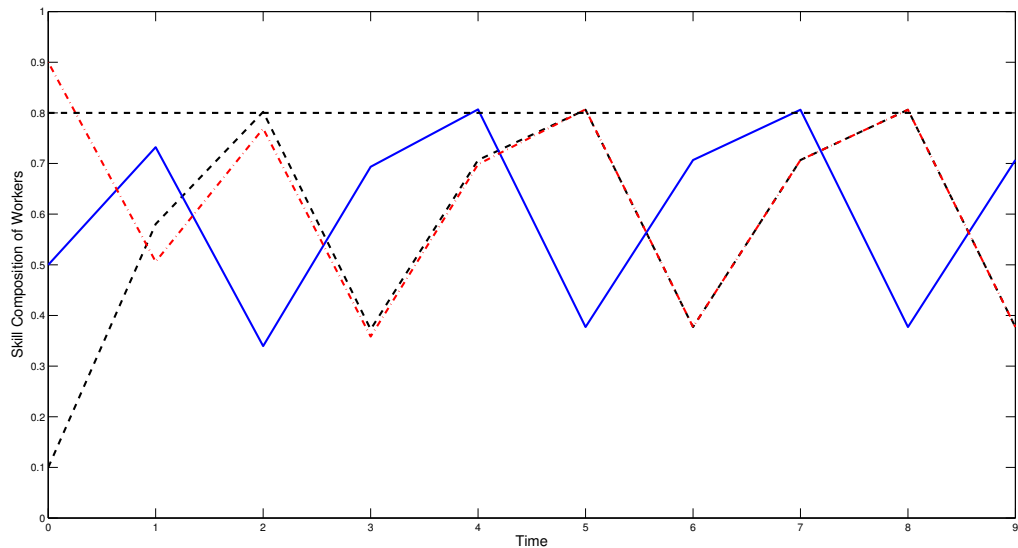


Figure 1.14: Dynamics of the Skill Composition of Workers for Different Initial s_t

than 0.5 for skill premium, the skilled workers' party is likely to win the election when s_t is close to α . Therefore, the welfare loss from skill imbalance is mainly caused by the skilled workers in this model. This skill imbalance not only causes welfare loss but also dampens the effect of immigration in sustaining the welfare state. Since the welfare state is most efficiently financed when $s_t = \alpha, \forall t$, the gap between α and s_t caused by the skilled workers' ideal policy makes the inefficiency of the welfare state deteriorate.

Furthermore, the fluctuation of the skill composition converges to the same pattern of cycles regardless of the initial skill composition. Figure 1.14 shows how different initial skill compositions end up exhibiting the same cyclical behavior. The skill composition converges to the identical cycles since the value of s_t is *pulled* into α whenever the old retirees win the election. In other words, whenever the old retirees' party wins the election, the economy is absorbed to the state of $s_t = \alpha$ in the next period.

Figure 1.15 displays the dynamics of wage rates for the skilled and unskilled labor, respectively. It is clear that the skilled workers implement a policy profile to maximize their own wage (e.g. $t = 1$) and the unskilled workers maximize their own wage (e.g. $t = 2$).

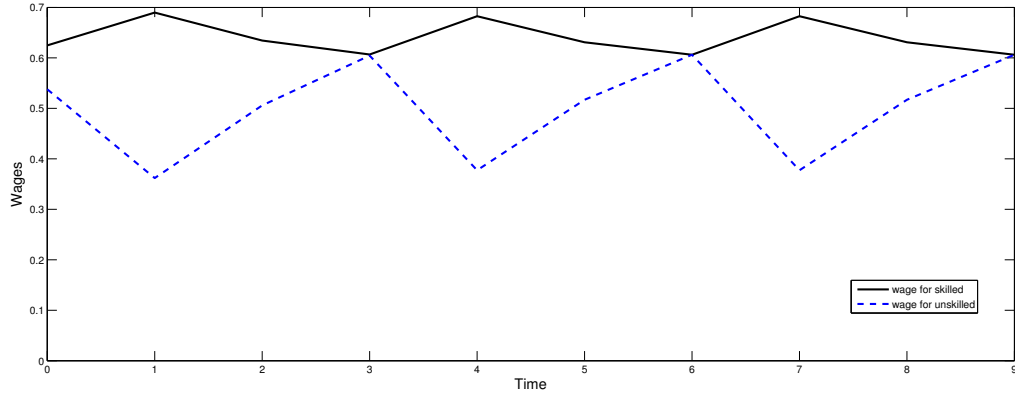


Figure 1.15: Dynamics of the Pre-Tax Wages

The wage rates are *equalized* at $t = 3$, $t = 6$, and $t = 9$, which are the period when the old retirees win the election. The implication is simple - As discussed above, the Cobb-Douglas exponent and the actual skill composition will be equalized by the old retirees. At the same time, the income share of the skilled workers is always α by the property of a Cobb-Douglas technology. Therefore, the wage rates for both skills are equalized when the old retirees win the election and implement the “skill-balancing” immigration policy.

1.5.2 Welfare Loss by Distortionary Taxation

It is obvious that the labor income tax in this model distorts the labor supply of both skills. This type of welfare loss is mainly caused by the old retirees because they prefer the Laffer rate to maximize the welfare state. As proved in Proposition 2 and Proposition 3, the tax distortion is either zero or is smaller than caused by the old retirees if any cohort of the young workers becomes the decisive party. Therefore, the principal contributors of the welfare loss by taxes are the opposite to the case of skill imbalance: The old retirees are the main contributor, and the skilled workers’ preferred tax policy is the least distortionary in terms of taxation.¹⁸

¹⁸Although the skilled workers potentially prefer a non-zero tax rate as proved in Proposition 2, it is mostly not realized because the skilled workers prefer a non-zero rate only if $\alpha \left[1 + \bar{\mu} + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] < s_t \leq 1$,

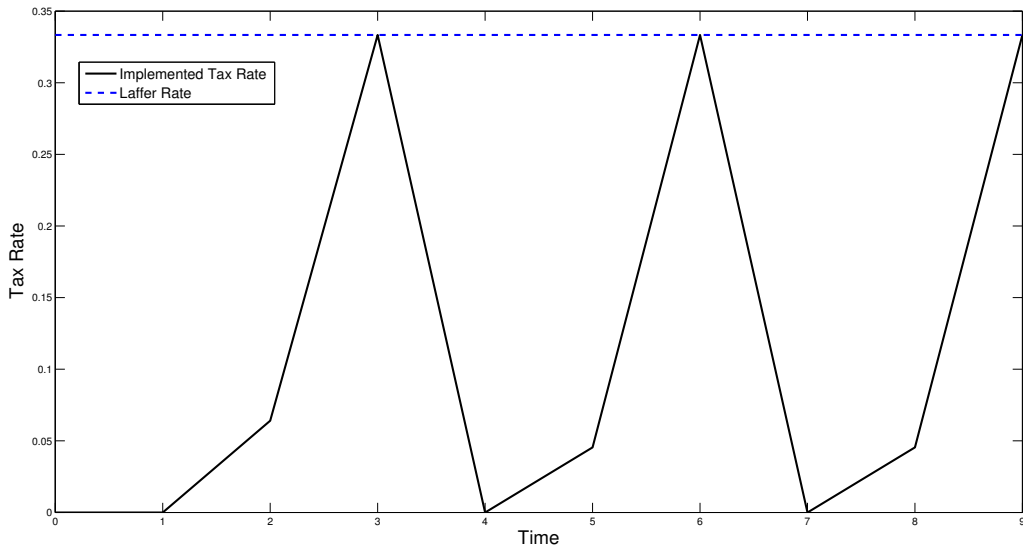


Figure 1.16: Dynamics of the Tax Rates

Figure 1.16 shows the implementation of the tax rate over time. It is straightforward to see that the old retirees levy $\tau_t = \frac{1}{1+\nu}$ at the Laffer rate, while the skilled workers levy zero taxes. The unskilled young workers implement a small non-zero tax rate when they win, as proved in Proposition 3.

1.5.3 Welfare Loss by Immigration Volume Adjustment

In this model, the only reason for the immigration volume to be smaller than $\bar{\mu}$ is the young workers' motive to regain political power after they retire, as discussed in Section 1.4.2. Figure 1.17 shows the implemented volume of immigration over time. Note that the volume is not the maximum at $t = 5$ and $t = 7$, both of which are the periods when the unskilled workers win the election. However, in this numerical example, the gap between $\bar{\mu}$ and the implemented μ_t is very small, and, moreover, such μ_t that is smaller than $\bar{\mu}$ is still very large (nearly 1, which means that the same number of immigrants with the existing native young

which is an empty set for most reasonable values of $\frac{N_{t-1}}{N_t}$, $\bar{\mu}$, and α . For instance, the interval becomes empty for the following conservative choice of parameters: $\alpha = 0.8$, $\bar{\mu} = 0$, $\frac{N_{t-1}}{N_t} = 0.25$.

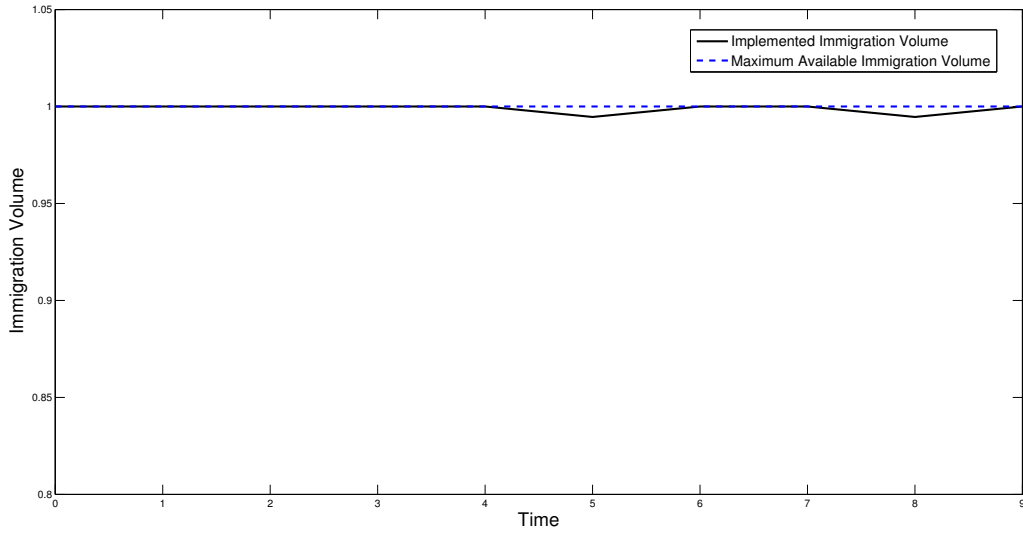


Figure 1.17: Dynamics of the Immigration Volume

workers enters the economy).

1.6 Concluding Remarks

This paper analyzes a politico-economic model of immigration and the welfare state to demonstrate the pattern of the political equilibria and the theoretical implication on the sustainability of the welfare state and the efficiency of the economy. In this 2-period overlapping-generation model, the size of the welfare state, the volume of immigration, and the skill composition of immigrants are determined by the winner of a plurality voting each period. Since there are more than 2 political parties, the possibility of strategic voting and consequent political coalitions among small political parties are also considered.

In the political processes there are two key conflicts among the cohorts: the old vs. the young (maximizing welfare state vs. small taxes) and the skilled workers vs. the unskilled workers (more unskilled immigrants vs. more skilled immigrants). In equilibrium, the skill composition of immigrants is biased toward unskilled labor relative to socially efficient level because, at the socially efficient level of skill composition, the skilled workers party is more

likely to win the election. Due to this inefficiency, the effect of immigration policy on sustainability of welfare state is weaker than expected by the literature.

Different sources of welfare loss are also analyzed. Not only does the imbalance of the skill composition but also distortionary taxation and the suboptimal volume of immigration also contribute to the welfare loss. Specifically, the ruling of the old retirees causes maximal tax distortion since the old retirees want to maximize the tax revenue of the government. If a young cohort becomes the ruling party, the volume of immigration can be determined at a lower level than optimal due to the young workers' incentive to regain political power in the future.

Finally, this study does not consider the possibility that the winning candidate's implementation of immigration policy is not feasible due to, for example, lack of foreign-born workers who want to migrate to the host country. Also, another interesting case which this paper does not study is the case in which there are a mass of refugees potentially entering several countries, and the refugees' country choice depends on the policy of each country as well as the economic state such as skill composition, aggregate productivity, and demographic structure.

1.7 Appendix

1.7.1 Full Equilibrium Solution of the Allocation and Prices

If the production function is a typical Cobb-Douglas function in skilled and unskilled labor, the equilibrium can be represented by the closed-form expressions.¹⁹ In other words, all the equilibrium allocation and prices are represented in closed-form functions of the policy variables (the labor-income tax rate, the volume of immigration, and the skill composition

¹⁹More rigorously, closed-form solutions are available when the marginal rate of substitution between the labor input and consumption is represented as a multiplicative function of the labor input, and the production function is multiplicative in both skills.

of immigrants), state variables (μ_{t-1} and s_t), and the parameters. In case of a Cobb-Douglas production function, the equilibrium is characterized by the following 9 equations:

$$l_t^s = [(1 - \tau_t)w_t^s]^\nu \quad (1.17)$$

$$l_t^u = [(1 - \tau_t)w_t^u]^\nu \quad (1.18)$$

$$\tilde{Y}_t = \left(\tilde{L}_t^s\right)^\alpha \left(\tilde{L}_t^u\right)^{1-\alpha} \quad (1.19)$$

$$w_t^s = \alpha \tilde{Y}_t \left(\tilde{L}_t^s\right)^{-1} \quad (1.20)$$

$$w_t^u = (1 - \alpha) \tilde{Y}_t \left(\tilde{L}_t^u\right)^{-1} \quad (1.21)$$

$$\tilde{L}_t^s = [s_t + \sigma_t \mu_t] l_t^s \quad (1.22)$$

$$\tilde{L}_t^u = [1 - s_t + (1 - \sigma_t) \mu_t] l_t^u \quad (1.23)$$

$$b_t = \frac{\tau_t \left[w_t^s \tilde{L}_t^s + w_t^u \tilde{L}_t^u \right]}{(1 + \mu_t) + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} \quad (1.24)$$

$$\frac{N_t}{N_{t-1}} = 1 + n + (1 + m) \mu_{t-1} \quad (1.25)$$

with 9 unknowns (l_t^s , l_t^u , \tilde{Y}_t , \tilde{L}_t^s , \tilde{L}_t^u , b_t , N_t/N_{t-1} , w_t^s , w_t^u), each of which is defined as in Section 1.2.

Plugging (1.20) and (1.21) into (1.17) and (1.18), respectively,

$$\begin{aligned} (l_t^s)^\frac{1}{\nu} &= (1 - \tau_t) \alpha \left(\tilde{L}_t^s\right)^{\alpha-1} \left(\tilde{L}_t^u\right)^{1-\alpha} \\ (l_t^u)^\frac{1}{\nu} &= (1 - \tau_t) (1 - \alpha) \left(\tilde{L}_t^s\right)^\alpha \left(\tilde{L}_t^u\right)^{-\alpha} \end{aligned}$$

Since now the equations are expressed in terms of variables for labor input only, this becomes a system of 2 equations and 2 unknowns (l_t^s and l_t^u) by substituting \tilde{L}_t^s and \tilde{L}_t^u with (1.22) and (1.23). As both equations are multiplicative in l_t^s and l_t^u , there exists a unique closed-form solution. After some algebra, labor input per skilled- and unskilled worker is represented only by policy variables and parameters as follows:

$$\begin{aligned} l_t^s &= (1 - \tau_t)^\nu \alpha^\frac{(1+\alpha\nu)\nu}{1+\nu} (1 - \alpha)^\frac{(1-\alpha)\nu^2}{1+\nu} (s_t + \sigma_t \mu_t)^\frac{(\alpha-1)\nu}{1+\nu} [1 - s_t + (1 - \sigma_t) \mu_t]^\frac{(1-\alpha)\nu}{1+\nu} \\ l_t^u &= (1 - \tau_t)^\nu \alpha^\frac{\alpha\nu^2}{1+\nu} (1 - \alpha)^\frac{[1+(1-\alpha)\nu]\nu}{1+\nu} (s_t + \sigma_t \mu_t)^\frac{\alpha\nu}{1+\nu} [1 - s_t + (1 - \sigma_t) \mu_t]^\frac{-\alpha\nu}{1+\nu} \end{aligned}$$

By plugging these back to (1.22) and (1.23) we get the following closed-form expressions of total skilled- and unskilled labor divided by native young population (\tilde{L}_t^s and \tilde{L}_t^u , respectively):

$$\begin{aligned}\tilde{L}_t^s &= (1 - \tau_t)^\nu \alpha^{\frac{(1+\alpha\nu)\nu}{1+\nu}} (1 - \alpha)^{\frac{(1-\alpha)\nu^2}{1+\nu}} (s_t + \sigma_t \mu_t)^{\frac{1+\alpha\nu}{1+\nu}} [1 - s_t + (1 - \sigma_t)\mu_t]^{\frac{(1-\alpha)\nu}{1+\nu}} \\ \tilde{L}_t^u &= (1 - \tau_t)^\nu \alpha^{\frac{\alpha\nu^2}{1+\nu}} (1 - \alpha)^{\frac{[1+(1-\alpha)\nu]\nu}{1+\nu}} (s_t + \sigma_t \mu_t)^{\frac{\alpha\nu}{1+\nu}} [1 - s_t + (1 - \sigma_t)\mu_t]^{\frac{1+(1-\alpha)\nu}{1+\nu}}\end{aligned}$$

By plugging these further back to (1.19), the output per native young worker (\tilde{Y}_t) is also represented by a closed form as follows:

$$\tilde{Y}_t = (1 - \tau_t)^\nu \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t \mu_t)^\alpha [1 - s_t + (1 - \sigma_t)\mu_t]^{1-\alpha}$$

Consequently the amount of lump-sum transfer (b_t) is represented in the function of policy variables and states by the results above:

$$b_t = \frac{\tau_t (1 - \tau_t)^\nu \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t \mu_t)^\alpha [1 - s_t + (1 - \sigma_t)\mu_t]^{1-\alpha}}{(1 + \mu_t) + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} \quad (1.26)$$

Finally, the wage rates of skilled- and unskilled labor are calculated as below:

$$\begin{aligned}w_t^s &= \alpha^{\frac{1+\alpha\nu}{1+\nu}} (1 - \alpha)^{\frac{(1-\alpha)\nu}{1+\nu}} (s_t + \sigma_t \mu_t)^{-\frac{1-\alpha}{1+\nu}} [1 - s_t + (1 - \sigma_t)\mu_t]^{\frac{1-\alpha}{1+\nu}} \\ w_t^u &= \alpha^{\frac{\alpha\nu}{1+\nu}} (1 - \alpha)^{\frac{1+(1-\alpha)\nu}{1+\nu}} (s_t + \sigma_t \mu_t)^{\frac{\alpha}{1+\nu}} [1 - s_t + (1 - \sigma_t)\mu_t]^{-\frac{\alpha}{1+\nu}}\end{aligned} \quad (1.27)$$

1.7.2 Proofs

1.7.2.1 Proof of Proposition 1

The old retiree's objective function at time t is simply the lump-sum transfer per capita:

$$V_t^o = b_t$$

thus we can differentiate it by each of τ_t , μ_t , and σ_t using (1.26).

First, the derivative with respect to the tax rate is:

$$\frac{\partial V_t^o}{\partial \tau_t} = \frac{\partial b_t}{\partial \tau_t} \quad (1.28)$$

$$= B_1 \cdot [(1 - \tau_t)^\nu - \tau_t \nu (1 - \tau_t)^{\nu-1}] \quad (1.29)$$

where

$$B_1 = \frac{\alpha^{\alpha\nu}(1-\alpha)^{(1-\alpha)\nu} (s_t + \sigma_t\mu_t)^\alpha [1 - s_t + (1 - \sigma_t)\mu_t]^{1-\alpha}}{1 + \mu_t + (1 + \mu_{t-1})\frac{N_{t-1}}{N_t}} > 0$$

Obviously $\frac{\partial V_t^o}{\partial \tau_t} = 0$ at either $\tau_t = 1/(1 + \nu)$ or $\tau_t = 1$. However, $\tau_t = 1$ implies no lump-sum payment by (1.26) thus zero utility for the old retirees, so $\tau_t = 1/(1 + \nu)$ is the optimal tax rate when the old retiree group wins the election.

Second, the derivative of V_t^o with respect to the skill composition of immigrants is:

$$\begin{aligned} \frac{\partial V_t^o}{\partial \sigma_t} &= \frac{\partial b_t}{\partial \sigma_t} \\ &= B_2 \cdot (\alpha + \alpha\mu_t - s_t - \sigma_t\mu_t) \end{aligned} \quad (1.30)$$

where

$$B_2 = \frac{\tau_t (1 - \tau_t)^\nu \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t\mu_t)^{\alpha-1} [1 - s_t + (1 - \sigma_t)\mu_t]^{-\alpha} \mu_t}{1 + \mu_t + (1 + \mu_{t-1})\frac{N_{t-1}}{N_t}} > 0 \quad (\because 0 < \tau_t^* = 1/(1 + \nu) < 1 \text{ as shown above})$$

Note that $\frac{\partial V_t^o}{\partial \sigma_t} < 0$ for any $\sigma_t \in (0, 1)$ if $s_t > \alpha + \alpha\mu_t$, and $\frac{\partial V_t^o}{\partial \sigma_t} > 0$ for any $\sigma_t \in (0, 1)$ if $s_t < \alpha + \alpha\mu_t - \mu_t$. Therefore, the optimal choice of the skill composition of immigrants when the old retirees party wins the election is:

$$\sigma_t = \begin{cases} 1 & \text{if } 0 \leq s_t < \alpha + \alpha\mu_t - \mu_t \\ \alpha + \frac{\alpha - s_t}{\mu_t} & \text{if } \alpha + \alpha\mu_t - \mu_t \leq s_t \leq \alpha + \alpha\mu_t \\ 0 & \text{if } \alpha + \alpha\mu_t < s_t \leq 1 \end{cases} \quad (1.31)$$

Finally, the derivative of V_t^o with respect to the volume of immigration is:

$$\begin{aligned} \frac{\partial V_t^o}{\partial \mu_t} &= \frac{\partial b_t}{\partial \mu_t} \\ &= B_3 \cdot \left[\left\{ 1 + \mu_t + (1 + \mu_{t-1})\frac{N_{t-1}}{N_t} \right\} \{ \alpha\sigma_t [1 - s_t + (1 - \sigma_t)\mu_t] \right. \\ &\quad \left. + (s_t + \sigma_t\mu_t)(1 - \alpha)(1 - \sigma_t) \right\} - (s_t + \sigma_t\mu_t)[1 - s_t + (1 - \sigma_t)\mu_t] \end{aligned} \quad (1.32)$$

where

$$B_3 = \frac{\tau_t (1 - \tau_t)^\nu \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t \mu_t)^{\alpha-1} [1 - s_t + (1 - \sigma_t) \mu_t]^{-\alpha}}{\left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right]^2}$$

$$> 0 \quad (\because 0 < \tau_t = 1/(1 + \nu) < 1 \text{ as shown above})$$

There can be three cases according to the optimal decision for σ_t .

- **Case I:** $s_t \in (\alpha + \alpha\mu_t, 1]$

In this case, $\sigma_t = 0$ as shown in (1.31). After substitution, (1.32) is rewritten as follows:

$$\begin{aligned} \frac{\partial V_t^o}{\partial \mu_t} &= B_3 \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right] (1 - \alpha) s_t - (1 + \mu_t - s_t) s_t \right\} \\ &= B_3 \cdot s_t \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right] (1 - \alpha) - (1 + \mu_t - s_t) \right\} \\ &> B_3 \cdot s_t \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right] (1 - \alpha) - (1 + \mu_t) (1 - \alpha) \right\} \\ &\quad (\because s_t > \alpha + \alpha\mu_t) \\ &> 0 \end{aligned}$$

Therefore, since $\frac{\partial V_t^o}{\partial \mu_t} > 0$ for any value of μ_t , it is optimal to choose $\mu_t = \bar{\mu}$.

- **Case II:** $s_t \in [0, \alpha + \alpha\mu_t - \mu_t)$

In this case, $\sigma_t = 1$ as shown in (1.31). After substitution, (1.32) is rewritten as follows:

$$\begin{aligned} \frac{\partial V_t^o}{\partial \mu_t} &= B_3 \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right] \alpha (1 - s_t) - (s_t + \mu_t) (1 - s_t) \right\} \\ &= B_3 \cdot (1 - s_t) \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right] \alpha - s_t - \mu_t \right\} \\ &> B_3 \cdot (1 - s_t) \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}\right] \alpha - (1 + \mu_t) \alpha \right\} \\ &\quad (\because s_t < \alpha + \alpha\mu_t - \mu_t) \\ &> 0 \end{aligned}$$

Therefore, since $\frac{\partial V_t^o}{\partial \mu_t} > 0$ for any value of μ_t , it is optimal to choose $\mu_t = \bar{\mu}$.

- **Case III:** $s_t \in [\alpha + \alpha\mu_t - \mu_t, \alpha + \alpha\mu_t]$

In this case, $\sigma_t = \alpha + \frac{\alpha - s_t}{\mu_t}$ as shown in (1.31). After reorganization, (1.32) is rewritten

as follows:

$$\begin{aligned} \frac{\partial V_t^o}{\partial \mu_t} &= B_3 \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] \sigma_t \{ \alpha [1 - s_t + (1 - \sigma_t) \mu_t] - (1 - \alpha)(s_t + \sigma_t \mu_t) \} \right. \\ &\quad + \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] (s_t + \sigma_t \mu_t)(1 - \alpha) \\ &\quad \left. - (s_t + \sigma_t \mu_t) [1 - s_t + (1 - \sigma_t) \mu_t] \right\} \end{aligned}$$

After substituting $\sigma_t = \alpha + \frac{\alpha - s_t}{\mu_t}$, the first term equals zero, and the whole expression is simplified into the following form:

$$\begin{aligned} \frac{\partial V_t^o}{\partial \mu_t} &= B_3 \cdot \left\{ \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] (1 - \alpha) - (1 + \mu_t)(1 - \alpha) \right\} \quad (1.33) \\ &> 0 \end{aligned}$$

Therefore the optimal choice is $\mu_t = \bar{\mu}$.

In conclusion, since $\frac{\partial V_t^o}{\partial \mu_t}$ is strictly positive, the optimal choice of the old retiree is to choose $\mu_t = \bar{\mu}$ regardless of the state of the economy, which ends the proof. \square

1.7.2.2 Proof of Proposition 2

The skilled young worker's objective function at time t consists of two terms: the utility from the lump-sum transfer, and the utility from labor income net of disutility of working:

$$V_t^s = b_t + \frac{[(1 - \tau_t) w_t^s]^{1+\nu}}{1 + \nu}$$

First, the derivative of V_t^s with respect to the tax rate is:

$$\begin{aligned} \frac{\partial V_t^s}{\partial \tau_t} &= \frac{\partial b_t}{\partial \tau_t} + [(1 - \tau_t) w_t^s]^\nu \left[(1 - \tau_t) \frac{\partial w_t^s}{\partial \tau_t} - w_t^s \right] \\ &= \frac{\partial b_t}{\partial \tau_t} - (1 - \tau_t)^\nu (w_t^s)^{1+\nu} \end{aligned}$$

since $\frac{\partial w_t^s}{\partial \tau_t} = 0$ by (1.27). Plugging (1.27) and (1.29) into this expression yields:

$$\begin{aligned} \frac{\partial V_t^s}{\partial \tau_t} &= B_4 \cdot \left[\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} [1 - (1 + \nu) \tau_t] - (1 - \tau_t) \alpha \right] \\ &= B_4 \cdot \left\{ \frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} - \alpha - \left[\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} (1 + \nu) - \alpha \right] \cdot \tau_t \right\} \end{aligned}$$

where

$$\begin{aligned} B_4 &= (1 - \tau_t)^{\nu-1} \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t \mu_t)^{\alpha-1} [1 - s_t + (1 - \sigma_t) \mu_t]^{1-\alpha} \\ &> 0 \end{aligned}$$

Therefore, the optimal tax rate for the skilled young worker depends on the state of the economy (s_t, μ_{t-1}) , and the following two cases are possible:

- **Case I:** $s_t < \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - \sigma_t \mu_t$

In this case we can observe the following two consequences:

- $\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} - \alpha < 0$, and,
- $\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} (1 + \nu) - \alpha$ is either a positive number, or a negative number smaller in absolute value than $\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} - \alpha < 0$

These two observations guarantees that $\frac{\partial V_t^s}{\partial \tau_t} < 0$. Therefore, the optimal tax rate for the skilled young worker in this case is given by $\tau_t = 0$.

- **Case II:** $s_t \geq \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - \sigma_t \mu_t$

In this case we have an interior solution at

$$\begin{aligned} \tau_t &= \frac{\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} - \alpha}{\frac{s_t + \sigma_t \mu_t}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} (1 + \nu) - \alpha} \\ &= \frac{s_t + \sigma_t \mu_t - \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]}{(s_t + \sigma_t \mu_t) (1 + \nu) - \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]} \\ &\in (0, 1) \end{aligned}$$

Therefore, these two cases show that the optimal decision of τ_t for the skilled young worker is given by:

$$\tau_t = \begin{cases} 0 & \text{if } s_t < \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - \sigma_t \mu_t \\ \frac{s_t + \sigma_t \mu_t - \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]}{(s_t + \sigma_t \mu_t) (1 + \nu) - \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]} & \text{if } s_t \geq \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - \sigma_t \mu_t \end{cases} \quad (1.34)$$

The derivative of V_t^s with respect to the skill composition of immigrants is:

$$\frac{\partial V_t^s}{\partial \sigma_t} = \frac{\partial b_t}{\partial \sigma_t} + (1 - \tau_t)^{1+\nu} (w_t^s)^\nu \frac{\partial w_t^s}{\partial \sigma_t}$$

Plugging (1.27) and (1.30) we get:

$$\frac{\partial V_t^s}{\partial \sigma_t} = B_5 \cdot \left\{ \frac{\tau_t (s_t + \sigma_t \mu_t)}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} (\alpha - s_t + \alpha \mu_t - \sigma_t \mu_t) - \frac{(1 - \tau_t)(1 + \mu_t)\alpha(1 - \alpha)}{1 + \nu} \right\} \quad (1.35)$$

where

$$B_5 = (1 - \tau_t)^\nu \mu_t \alpha^{\alpha\nu} (1 - \alpha)^{(1-\alpha)\nu} (s_t + \sigma_t \mu_t)^{\alpha-2} [1 - s_t + (1 - \sigma_t)\mu_t]^{-\alpha} > 0$$

Again, the optimal decision of σ_t can be separated into two cases as in (1.34).

- **Case I:** $s_t < \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - \sigma_t \mu_t$

In this case, the optimal τ_t is zero according to (1.34). After plugging $\tau_t = 0$ into (1.35), obviously $\frac{\partial V_t^s}{\partial \sigma_t} < 0$, implying that optimal σ_t equals zero.

- **Case II:** $s_t \geq \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - \sigma_t \mu_t$

This case is equivalent to $-s_t - \sigma_t \mu_t \leq -\alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]$. Plugging this inequality into (1.35),

$$\frac{\partial V_t^s}{\partial \sigma_t} \leq B_5 \cdot \left\{ \frac{\tau_t (s_t + \sigma_t \mu_t)}{1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t}} \left[\alpha (1 + \mu_t) - \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] \right] - \frac{(1 - \tau_t)(1 + \mu_t)\alpha(1 - \alpha)}{1 + \nu} \right\} < 0$$

Since $\frac{\partial V_t^s}{\partial \sigma_t} < 0$ for both cases, the optimal decision of σ_t is always zero for the skilled young worker.

Finally, the derivative of the skilled young worker's objective function with respect to the volume of immigration is give by:

$$\frac{\partial V_t^s}{\partial \mu_t} = \frac{\partial b_t}{\partial \mu_t} + (1 - \tau_t)^{1+\nu} (w_t^s)^\nu \frac{\partial w_t^s}{\partial \mu_t}$$

As it is already shown that $\sigma_t = 0$ in optimum, we can use (1.33) to derive the follows:

$$\left. \frac{\partial b_t}{\partial \mu_t} \right|_{\sigma_t=0} = B_3 \cdot s_t \cdot \left\{ (1 - \alpha) \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - (1 - s_t + \mu_t) \right\}$$

and there can be two cases depending on the value of s_t :

- **Case I:** $s_t < \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]$

In this case, $\tau_t = 0$ as shown above, therefore $b_t = 0$ and thus $\frac{\partial b_t}{\partial \mu_t} = 0$.

- **Case II:** $s_t \geq \alpha \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right]$

In this case:

$$\begin{aligned} \left. \frac{\partial b_t}{\partial \mu_t} \right|_{\sigma_t=0} &= B_3 \cdot s_t \cdot \left\{ (1 - \alpha) \left[1 + \mu_t + (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \right] - (1 - s_t + \mu_t) \right\} \\ &\geq B_3 \cdot s_t \cdot (1 + \mu_{t-1}) \frac{N_{t-1}}{N_t} \\ &\geq 0 \end{aligned}$$

Therefore, $\frac{\partial b_t}{\partial \mu_t}$ is non-negative in either case. Now, the derivative of the wage rate is written in detail as:

$$\begin{aligned} \frac{\partial w_t^s}{\partial \mu_t} &= \alpha^{\frac{1+\alpha\nu}{1+\nu}} (1 - \alpha)^{\frac{(1-\alpha)\nu}{1+\nu}} \left[-\frac{1 - \alpha}{1 + \nu} \sigma_t (s_t + \sigma_t \mu_t)^{-\frac{1-\alpha}{1+\nu}-1} [1 - s_t + (1 - \sigma_t) \mu_t]^{\frac{1-\alpha}{1+\nu}} \right. \\ &\quad \left. + \frac{1 - \alpha}{1 + \nu} (1 - \sigma_t) (s_t + \sigma_t \mu_t)^{-\frac{1-\alpha}{1+\nu}} [1 - s_t + (1 - \sigma_t) \mu_t]^{\frac{1-\alpha}{1+\nu}-1} \right] \end{aligned}$$

Substituting $\sigma_t = 0$, it is easy to see that $\frac{\partial w_t^s}{\partial \mu_t} > 0$, thereby implying that $\frac{\partial V_t^s}{\partial \mu_t} > 0$. Therefore, the optimal choice of the volume of immigration for the skilled young worker is $\bar{\mu}$.

Now go back to the intervals of s_t in (1.34). Since now we know that $\mu_t = \bar{\mu}$ and $\sigma_t = 0$ in optimality, we can substitute μ_t and σ_t with $\bar{\mu}$ and 0, respectively, ending the proof. \square

1.7.3 Political Coalition and Equilibria

In this section, I discuss details of the assumption on the determination process of the political equilibria discussed in Section 1.3.1.2. As in the model, there are three political parties, and all the individuals in one party have the identical preference. Throughout this section, let \mathcal{A} , \mathcal{B} , and \mathcal{C} denote the three political parties *in the order of the population sizes*, and $\#(\mathcal{A})$, $\#(\mathcal{B})$, and $\#(\mathcal{C})$ denote the relative size of the three parties, respectively. The sizes are normalized such that $\#(\mathcal{A}) + \#(\mathcal{B}) + \#(\mathcal{C}) = 1$, so the measure of the size of the whole electorate equals 1.²⁰ Assume that all the parties at least have one member, formally $\#(\mathcal{X}) > 0$ for all $\mathcal{X} \in \{\mathcal{A}, \mathcal{B}, \mathcal{C}\}$.

First of all, it will be proved that the largest party's members always vote for their own party regardless whether they constitute the majority or not. It is obvious that \mathcal{A} wins the election if $\#(\mathcal{A}) > 0.5$, which is a trivial case. So, assume that it does not exceed 50%. Then Lemma 4 follows:

Lemma 4. *A member of \mathcal{A} votes only for \mathcal{A} in any political equilibrium.*

This implies that the members of the largest political party does not vote strategically for a candidate from another party in any equilibria after repeatedly eliminating weakly dominated strategies. The proof is following:

Proof. It is obvious that the members of \mathcal{A} prefer \mathcal{A} 's policy profile most, so assume, without loss of generality, that members of \mathcal{A} prefers \mathcal{B} to \mathcal{C} . Then, it is obvious that \mathcal{C} is weakly dominated by the other two strategies. Therefore, let's assume that there is an equilibrium in which \mathcal{A} 's members vote for \mathcal{B} . Then there are the following possible cases:

- **Case I:** \mathcal{B} 's members vote for \mathcal{B} , and \mathcal{C} 's members vote for \mathcal{C}

In this case, for \mathcal{A} 's members, voting for \mathcal{B} gives a weakly smaller payoff than voting for \mathcal{A} because if \mathcal{A} 's members deviate to voting for \mathcal{A} , then \mathcal{A} wins the election.

²⁰Therefore, $\#(\mathcal{A}) \geq \#(\mathcal{B}) \geq \#(\mathcal{C})$ holds since \mathcal{A} , \mathcal{B} , and \mathcal{C} are labeled in the order of size.

- **Case II:** \mathcal{B} 's members vote for \mathcal{C} , and \mathcal{C} 's members vote for \mathcal{B}

Similar to Case I.

- **Case III:** \mathcal{B} 's members vote for \mathcal{B} , and \mathcal{C} 's members vote for \mathcal{B}

Then, for members of \mathcal{A} , voting for \mathcal{A} and \mathcal{B} result in the identical payoff because \mathcal{C} wins after all.

- **Case IV:** \mathcal{B} 's members vote for \mathcal{C} , and \mathcal{C} 's members vote for \mathcal{C}

Similar to Case III.

Therefore, in any of the cases above, voting for \mathcal{B} is at least weakly dominated by voting for \mathcal{A} , which contradicts the assumption that voting for \mathcal{B} occurs in equilibrium and ends the proof. \square

Therefore, it is clear that the members of the largest cohort always vote for the candidate of their own cohort. Together with this lemma, the following assumptions refine the political equilibrium so that there exists a unique pure-strategy equilibrium per state.

- (i) Weakly dominated strategies are repeatedly eliminated.
- (ii) If there are two parties with the equally largest number of votes, then the candidate whose votes are from the least number of cohorts becomes the winner.
- (iii) The voters strategically vote for the candidate of another cohort *only if* it is strictly preferred to not doing so.
- (iv) Suppose there are two parties. If *all* members of *both* parties prefer forming a coalition with each other, the candidate of the larger party out of the two becomes the candidate of the whole coalition. If the two parties have the exactly same sizes, the candidate is determined by equal probability.
- (v) Any remaining possibility of ties is broken with equal probability.

These assumptions, together with Lemma 4, implies the following results of the election.

Case 1: $\#(\mathcal{A}) \geq 0.5$

This is the simplest case to solve, in which \mathcal{A} 's candidate wins the election by majority. Note that $\#(\mathcal{A}) = 0.5$ also guarantees \mathcal{A} 's victory due to the assumption (ii) above.

Case 2: $0.5 > \#(\mathcal{A}) > \#(\mathcal{B}) \geq \#(\mathcal{C})$

This is the most interesting case. Since \mathcal{A} does not take the majority, it is important whether \mathcal{B} and \mathcal{C} form a political coalition or not, which depends on the preference of \mathcal{B} 's and \mathcal{C} 's members. The following exclusive and exhaustive cases are possible:

- **Case 2-1:** \mathcal{B} 's members at least weakly prefer \mathcal{A} to \mathcal{C} , and \mathcal{C} 's members at least weakly prefer \mathcal{A} to \mathcal{B} .

In this case, neither \mathcal{B} nor \mathcal{C} wants a coalition since each other's ideal policy is the worst. Therefore, \mathcal{A} wins the election by receiving the largest votes.

- **Case 2-2:** \mathcal{B} 's members at least weakly prefer \mathcal{A} to \mathcal{C} , and \mathcal{C} 's members strongly prefer \mathcal{B} to \mathcal{A}

In this case, while \mathcal{B} 's members does not have any incentive to vote for \mathcal{C} strategically, \mathcal{C} 's members have incentive to vote for \mathcal{B} 's candidate. Therefore, \mathcal{B} 's candidate represents the political coalition by \mathcal{B} and \mathcal{C} , and wins the election in the equilibrium.

- **Case 2-3:** \mathcal{B} 's members strongly prefer \mathcal{C} to \mathcal{A} , and \mathcal{C} 's members at least weakly prefer \mathcal{A} to \mathcal{B} .

In this case, \mathcal{C} 's candidate represents the coalition by \mathcal{B} and \mathcal{C} and wins the election by the same logic as in Case 2-2. Note that \mathcal{C} wins the election although it is the smallest cohort in the electorate.

- **Case 2-4:** \mathcal{B} 's members strongly prefer \mathcal{C} to \mathcal{A} , and \mathcal{C} 's members strongly prefer \mathcal{B} to \mathcal{A} .

In this case, both of the two smaller parties want to form a coalition. If $\#(\mathcal{B}) > \#(\mathcal{C})$,

then the candidate of \mathcal{B} represents the whole coalition by assumption (iv) and wins the election. If $\#(\mathcal{B}) = \#(\mathcal{C})$, then the candidate of the coalition is determined by a coin tossing, and the resulting candidate wins the election.

Case 3: $\#(\mathcal{A}) = \#(\mathcal{B}) > \#(\mathcal{C})$

In this case, \mathcal{C} plays a role of the swing cohort. If \mathcal{C} 's members strongly prefer \mathcal{A} to \mathcal{B} , then \mathcal{A} wins the election by forming a coalition with \mathcal{C} . If \mathcal{C} 's members strongly prefer \mathcal{B} to \mathcal{A} , \mathcal{B} wins the election similarly. If \mathcal{C} 's members are indifferent, then they do not vote strategically, so either \mathcal{A} or \mathcal{B} wins the election with equal probability (coin tossing).

Case 4: $\#(\mathcal{A}) = \#(\mathcal{B}) = \#(\mathcal{C})$

In this case, each party wins the election with probability $1/3$.

CHAPTER 2

Hours Worked and Tax Policy: Korea vs. Mexico, 1991-2008

2.1 Introduction

The Mexican Peso crisis in 1994 and the Asian crisis in 1997 played curious roles in the labor markets of South Korea and Mexico. Though both crises are called “financial” crisis and interpreted as caused by similar mechanisms, their effects on the labor market were seemingly different in those countries. Figure 2.1 displays the volume of employment, employment-to-population ratio, and the unemployment rate in these two countries for 1991-2008. It appears clear that both Mexican crisis in 1994 and Korean crisis in 1997 reduced the levels of employment and employment rates and caused severe unemployment in the short run. Considering hours worked instead of sizes of employment, however, the effect of crisis seems different in those countries. Figure 2.2 displays the annual hours worked per employee and annual hours worked per capita in both countries. Annual hours worked per employee in Korea exhibits a decreasing trend from 1991 to 2008, while hours worked in Mexico is roughly constant during the same period. In other words, Korea’s hours worked per capita exhibit a huge drop during Asian crisis in 1997 and hardly recover thereafter, while Mexico’s hours worked per capita decrease only modestly during Peso crisis in 1994.

This labor market phenomenon gives rise to the possibility that we might need a long-term structural theory rather than short-term crisis stories in order to better understand what happened in the labor markets of those economies. Specifically, one reasonable hypothesis is that the labor markets of Korea and Mexico experienced different structural

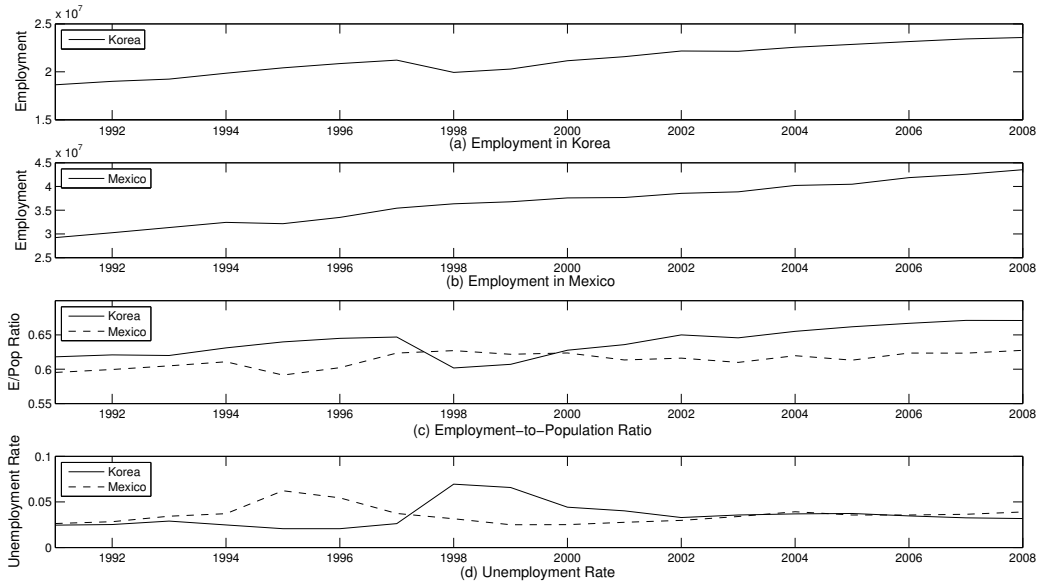


Figure 2.1: Employment Statistics, Korea vs. Mexico

changes around their crisis times, which led to different long-term changes in hours worked, though both countries experienced short-term drop in hours worked during their crises.

For this reason, in this paper it is examined whether a neoclassical growth model with government's tax policy can explain this difference. Since proportional taxes on labor income and consumption cause labor market wedges and thus long term changes of hours worked in a neoclassical model, many existing studies have examined whether this theoretical hypothesis is consistent with the actual data and shown that tax wedges play a significant role in explaining long-run changes in labor market variables. For example, Prescott (2004) uses a neoclassical growth model with proportional taxes to account for the difference in hours worked between the U.S. and European countries. While his analysis concentrates on comparison of hours worked between two particular points of time, Ohanian et al. (2008) extend his analysis so that a larger set of countries are included and all years in the data set are assessed. I follow the approach of Ohanian et al. (2008) in this paper.

This approach does not intend to argue that other labor market factors than taxes are unimportant in understanding patterns of labor supply. For instance, Cole and Ohanian (2002, 2004) show that other factors such as cartelization, bargaining, and unemployment

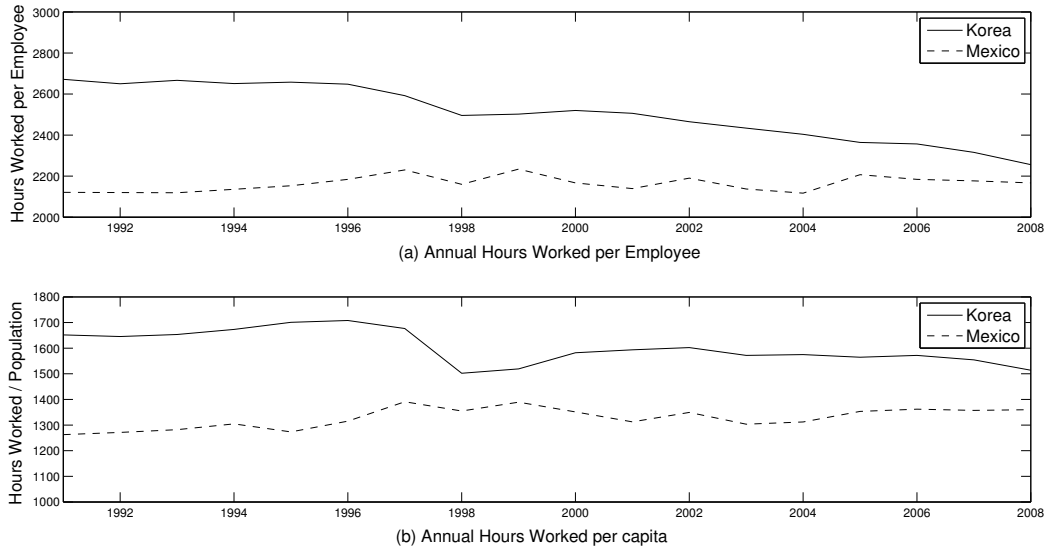


Figure 2.2: Hours Worked, Korea vs. Mexico

subsidies played key roles in the size and duration of the Great Depression in the U.S. and U.K.. Chari et al. (2007) also show that wedges in efficiency and investment were as important as labor wedges. Chang and Kim (2007) suggest a theory of labor wedge caused by failure of labor market aggregation in an economy with heterogeneous households and an incomplete market.

This paper proceeds as follows. Section 2.2 presents the model economy. Section 2.3 describes the computation strategy and data sources. Section 2.4 reports the results. Section 2.5 concludes the paper.

2.2 Model Economy

In order to evaluate the significance of tax wedges in the labor markets of South Korea and Mexico, I use the neoclassical labor tax wedge model introduced by Ohanian et al. (2008). The model economy consists of a representative household, a representative firm, and the government. The household's instantaneous utility is determined by private consumption (C_t), government consumption (G_t), and leisure ($\bar{H} - H_t$). The household maximizes the

lifetime utility given by

$$\sum_{t=0}^{\infty} \beta^t U(C_t, G_t, \bar{H} - H_t)$$

where $0 < \beta < 1$ is the time discount factor. The form of the utility function (U) can be a subtle issue since a family of utility functions that exhibit (asymptotic) balanced growth paths generates constant hours worked when there is no disturbance. It might be problematic to use this family of utility functions if the actual hours worked exhibits a clear trend. In this paper, I use a utility function consistent with asymptotic balanced growth and examine whether this function captures the actual series.¹ The form of the utility function is given as follows:

$$U(C_t + \lambda G_t, \bar{H} - H_t) = \alpha \log(C_t + \lambda G_t - \bar{C}) + (1 - \alpha) \frac{(\bar{H} - H_t)^{1-\gamma} - 1}{1 - \gamma}$$

where $\gamma \geq 0$, $0 \leq \alpha \leq 1$, $0 \leq \lambda \leq 1$, and $\bar{C} \geq 0$. γ governs the elasticity of substitution between consumption and leisure, λ measures the subjective value of the representative household on government consumption relative to private consumption, α measures the weight of consumption relative to leisure in the utility function, and \bar{C} is the subsistence consumption. With this setup, this utility function is consistent with (asymptotic²) balanced growth.

Production of the representative firm follows the technology represented by the following typical Cobb-Douglas production function:

$$Y_t = A_t F(K_t, H_t) = A_t K_t^\theta H_t^{1-\theta}$$

where Y_t is the output, A_t is the level of efficiency in technology, K_t is the capital input, and H_t is the labor input. The parameter θ stands for the capital income share.

¹In other words, I examine whether the trends of hours worked in both economies are properly replicated ONLY with tax wedges in the labor market without any help of specific functional forms causing a trend in hours worked.

²Precisely, the model exhibits asymptotic balanced growth paths when $\bar{C} > 0$. For further details, refer to the theory of *Generalized Balanced Growth (GBG)* by Kongsamut et al. (2001).

The government levies proportional taxes on labor income and consumption expenditure, and the tax rates for those items are denoted by $\tau_{ht} \in [0, 1]$ and $\tau_{ct} \in [0, 1]$, respectively. The government spends the whole tax revenue for either government consumption (G_t) or lump-sum transfer to the household so that the government's budget is balanced in each period.

The household determines its private consumption and leisure so that the marginal rate of substitution between leisure and consumption equals the after-tax real wage rate. Formally, the household's first-order condition is given by:

$$(1 + \tau_{ct}) \frac{U_h(C_t + \lambda G_t, \bar{H} - H_t)}{U_c(C_t + \lambda G_t, \bar{H} - H_t)} = (1 - \tau_{ht}) W_t \quad (2.1)$$

The firm's optimality condition implies that the wage rate equals the marginal product of labor as follows:

$$W_t = A_t F_h(K_t, H_t) \quad (2.2)$$

Combining (2.1) and (2.2), we obtain the following labor market equilibrium condition:

$$\frac{U_h(C_t + \lambda G_t, \bar{H} - H_t)}{U_c(C_t + \lambda G_t, \bar{H} - H_t)} = \frac{1 - \tau_{ht}}{1 + \tau_{ct}} A_t F_h(K_t, H_t) \quad (2.3)$$

Let τ_t be defined by $1 - \tau_t \equiv (1 - \tau_{ht}) / (1 + \tau_{ct})$. Then τ_t measures the size of labor wedge in this model economy. Plugging the specified forms of the utility function and the production function, (2.3) can be written as,

$$\frac{H_t}{(\bar{H} - H_t)^\gamma} = (1 - \tau_t) \frac{\alpha(1 - \theta)}{1 - \alpha} \frac{Y_t}{C_t + \lambda G_t - \bar{C}} \quad (2.4)$$

Hence, the hours worked in each period is represented as a function of right-hand side variables and parameters.

2.3 Computation and Data

2.3.1 Computation Strategy

From the equation (2.4), we can see that the sequence of H_t can be computed if the values of right-hand side variables (Y_t, C_t, G_t, τ_t) and parameters ($\alpha, \theta, \gamma, \lambda, \bar{C}, \bar{H}$) are given.

Therefore, the strategy of this paper is to compute the model-generated series of H_t using the data of those variables and calibration of parameters and compare them with the actual data of hours worked.

One of the main challenges in this exercise is to obtain the average income and expenditure tax rates of South Korea and Mexico. McDaniel (2007) proposes a method³ to calculate the average tax rates and provides a long computed series of average income and expenditure tax rates for 15 OECD countries. She calculates those tax rates based on national accounts statistics and detailed revenue statistics. Since tax rates of South Korea are unavailable in her table and tax rates of Mexico are available only from 2003, I replicate her computation methods to calculate the average tax rates of Korea and Mexico.⁴

2.3.2 Data and Calibration

First of all, the value of \bar{H} , which measures the endowment of annual hours, is chosen to $14 \times 365 = 5100$ for every period in both countries. For the benchmark case, the values of γ (inverse of Frisch elasticity) and λ (weight on government consumption in the utility function) are assumed $\gamma = 1$ (log utility in leisure) and $\lambda = 1$ (government consumption is a perfect substitute for private consumption). The subsistence level of consumption (\bar{C}) is assumed to equal 5% of Korea's consumption in 1990, with population measure and currency unit adjusted for calibrating Mexico's counterpart. In Section 2.6.1, it is also examined whether the results are robust when different values of γ , λ , and \bar{C} are used instead. The parameters α and θ are not calibrated individually. Instead, the value of $\alpha(1 - \theta)/(1 - \alpha)$ in equation (2.4) is calibrated as a whole so that the value of H_0 (actual hours worked in the initial year

³The approach of her paper basically extends the methods used in Mendoza et al. (1994) and Carey and Rabesona (2004).

⁴My computation includes the tax rates of Korea for 1990-2008, and Mexico for 1993-2008. The computed series is available in Section 2.6.2.

of the data set⁵) matches the actual value of the initial sample period.

Regarding the sources of data used in this exercise, the data on the private consumption (C_t), government consumption (G_t), and GDP (Y_t) are taken from the Penn World Table 7.0 (Heston et al., 2011). Data on employment and populations are taken from *Economic Outlook* published by OECD. The series of hours worked are taken from The Conference Board and The Groningen Growth and Development Centre (2012). The working-age population includes individuals with age 15-64. The actual series of hours worked is computed by the following formula:

$$H_t^{actual} = \frac{(\text{Annual Hours Worked per Employee})_t \times (\text{Employment})_t}{(\text{Working-age Population})_t}$$

The average tax rates on consumption expenditure and labor income are constructed by the author's calculation in which the brief computation strategy was introduced in Section 2.3.1. Further detailed calculation procedure is introduced in Section 2.6.2.

2.4 Results

Figure 2.3 displays the actual vs. model-generated series of hours worked in Korea and Mexico. Each of the upper and lower panel represents Korea and Mexico, respectively. One panel shows series of three variables: the actual hours worked from data (solid line), the hours worked generated by a model *without* tax wedges⁶ (dashed line), and the hours worked generated by the full model (dash-dot line).

As shown in the upper panel of Figure 2.3, tax wedges play a significant role in explanation of long-term reduction in hours worked for South Korea as we observe that the model-generated series without taxes is clearly distinguished from the model-generated series with taxes. While the model without tax wedges fails to replicate the decline of Korea's hours worked for 1990-2008, the model with tax wedges successfully mimics the long-run

⁵1990 for South Korea and 1993 for Mexico

⁶That is, it displays series of hours worked generated by a model in which $\tau_t = 0$ for all t .

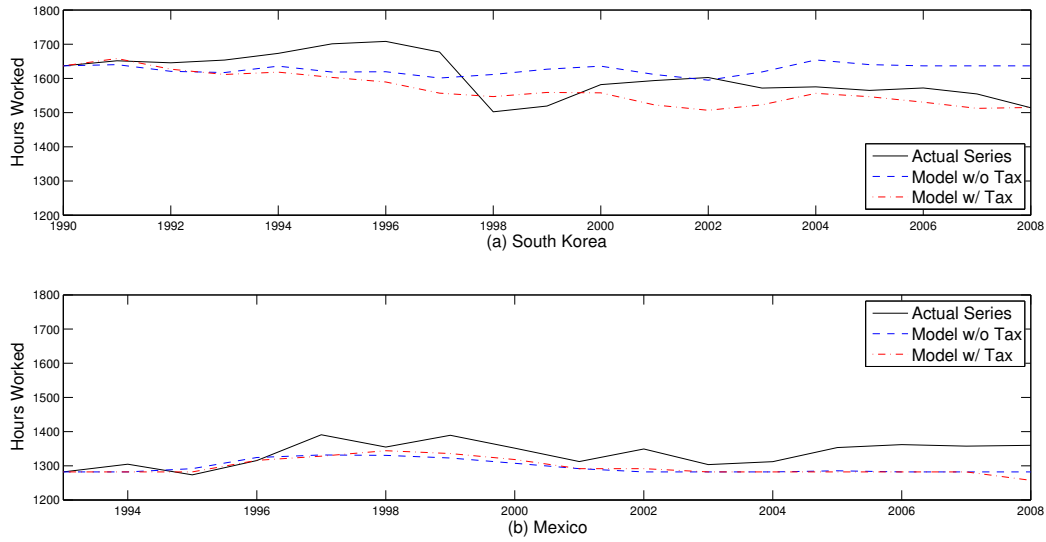


Figure 2.3: Actual vs. Model-generated Hours Worked

decline of hours worked. This supports the idea that government's long-term fiscal policy significantly influenced hours worked during the sample period in Korea and the 1997 crisis merely expedited such decline in hours worked instead of causing long-term structural influences. However, the existence of labor tax wedges does not explain the reason why hours worked drop rapidly during the crisis period in Korea, which implies that we need short-run crisis theories as well in order to better understand the labor market of Korea during that period.

On the other hand, the result for Mexico shows that the role of tax wedges in Mexico was only modest. As the lower panel of Figure 2.3 displays, the series of hours worked generated by the model with and without taxes exhibit virtually the same pattern, which implies that there were no big changes in tax wedges during the sample period. For instance, the actual hours worked of Mexico exhibit huge increase between 1995 and 1997, but the result shows that government's fiscal policy was hardly important in this increase.

This difference appears to be originated from the difference in tax policies of those countries. Figure 2.4 shows the average tax rates in those countries. The upper panel displays the labor tax rates (τ_{ht}), the middle panel displays the consumption tax rates (τ_{ct}), and the

lower panel displays the combined tax wedge (τ_t). Overall, the Korean government levies higher tax rates on labor and consumption than the Mexican government. Also, Korea's labor tax wedge increased from 1995 to 1997, and then exhibits a stabilized pattern from 1997 crisis. On the other hand, the Mexican tax rates and tax wedges do not have a specific pattern, and seem to fluctuate around a constant number. That is the reason why Mexican model-generated hours worked have similar patterns regardless of the existence of taxes in the model, while there are big differences in Korean series. This shows that the long-run differences in hours worked for those two countries partially rely on difference in fiscal policy of the governments. While Korean government effectively increased labor and consumption taxes around the 1997 crisis, Mexican government did not show such an action, which explains a big portion of the different reaction in long-run hours worked.

One by-product of this model exercise is that, considering the difference in macroeconomic variables such as the output, consumption, and the tax rates, Mexican workers do not seem to work less than Korean workers, though hours worked per capita are much larger in Korea. In Figure 2.3, the gap between actual and model-generated hours worked is relatively small, which means that the macroeconomic variables in the model explain the long-run changes of hours worked quite successfully. In other words, considering the output and consumption, it is hard to say that Korean workers work excessively much and vice versa for Mexican workers.

2.5 Concluding Remarks

This paper examines whether the standard neoclassical growth model with tax wedges can replicate different patterns of actual hours worked in Korea and Mexico. I construct a model with proportional labor income taxes and consumption taxes, and compare the hours worked generated by the model with the actual hours worked from the data around the financial crises in both economies. I show that the model explains the data well in the low frequency. Especially for the hours worked in Korea, the diminishing trend for 1990-2008 is matched by

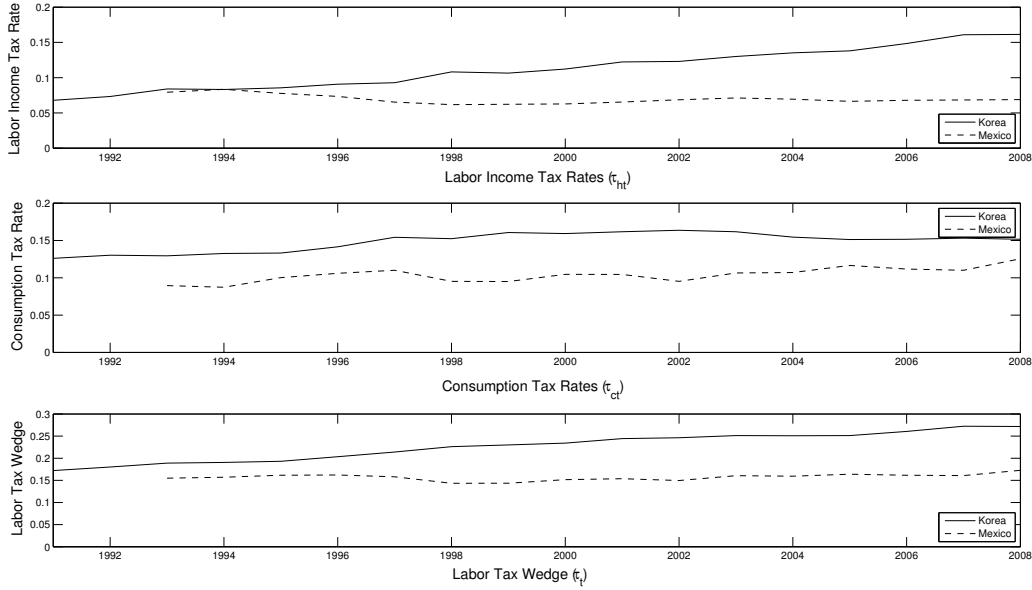


Figure 2.4: Tax Rates in Korea and Mexico

the tax wedges, implying that reduction in hours worked is mainly caused by the increasing taxes rather than the 1997 crisis *per se*. On the other hand, the tax rates in Mexico exhibit no big changes after the Peso crisis, suggesting that the role of fiscal policy is relatively modest during the 1990s and early 2000s in Mexico. The model is not very successful in explaining short-term fluctuations in hours worked in those countries, implying that other factors than fiscal variables might be important in short-term explanations.

Then a naturally following question is – what caused such different reactions of the two governments, and the different effect of the different policies toward a financial crisis. For Mexico, a future research needs to explain the existing gap between the model and the actual hours. Also, an explanation on why Korean workers work more than Mexican workers, despite higher average taxes in the labor market, can be a succeeding research question.

2.6 Appendix

2.6.1 Parameter Specifications and Robustness

As described in Section 2.3.2, the parameter values of γ (inverse of Frisch elasticity), λ (weight on government consumption in the household preference), and \bar{C} (subsistence level of private consumption) might influence the results. Therefore, it is examined whether the results are robust to the specifications of the values of these variables in this section.

Figure 2.5 displays how sensitively the model-generated series changes in the value of γ .⁷ The upper panel shows the result for Korea, and the lower panel for Mexico. As seen in the figure, the model-generated hours worked in Korea significantly increase in the value of γ , while the change is only modest for Mexico.⁸ Even for Korea's case, however, the diminishing trend itself is well explained for any choice of γ , implying that the tax wedges in Korea was sufficiently significant that decline in hours occurs even for very small Frisch elasticity.

Figure 2.6 shows how sensitively the results change in λ .⁹ It seems that the results hardly change in the choice of λ . This happens because the effect of the volume of government consumption toward hours worked is not significant in either country. This is actually very obvious because the model assumes that the whole government consumption is financed either by proportional income and expenditure taxes or *lump-sum* taxes. The size of distortionary taxes, rather than the size of government expenditure *per se*, matters in the model.

⁷Be reminded that the benchmark value is $\gamma = 1$.

⁸Theoretically, γ governs the inverse of Frisch elasticity of labor supply. For Korean economy, a higher value of γ makes workers less sensitive to increase in taxes (and subsequent decrease of real wages), thereby increasing the hours worked. For Mexico, such difference is hardly shown because the extent of tax wedges is relatively very small.

⁹The benchmark value is $\lambda = 1$, meaning that private and government consumption perfect substitute each other.

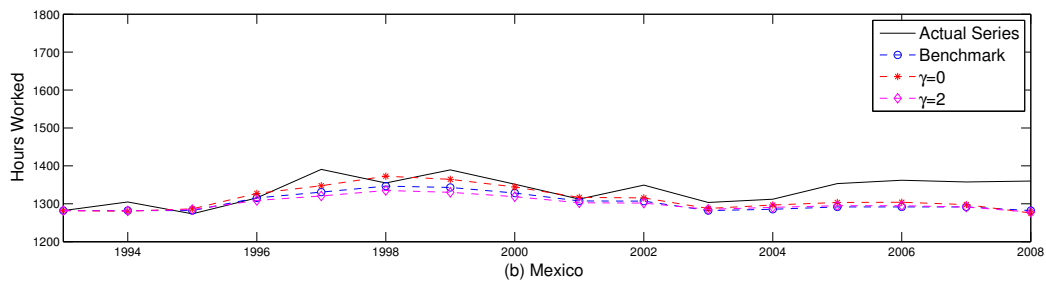
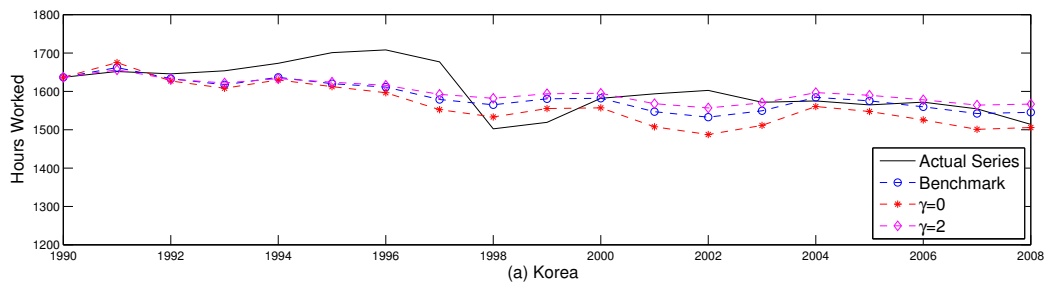


Figure 2.5: Effect of Elasticity of Substitution between Leisure and Consumption (γ)

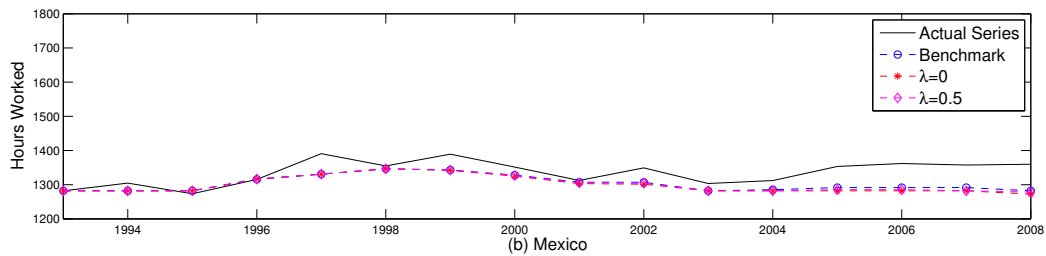
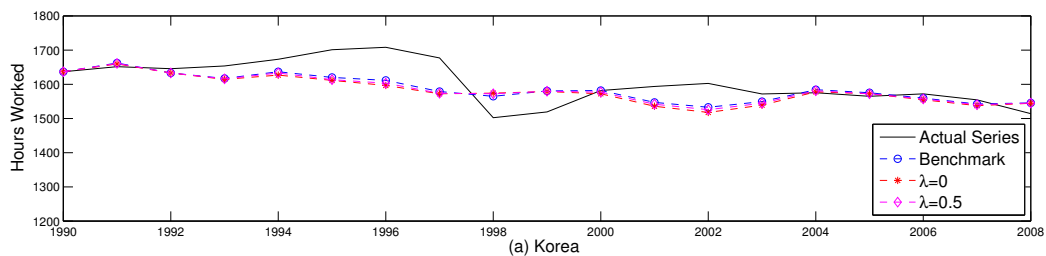


Figure 2.6: Effect of Government Consumption (λ)

Finally, Figure 2.7 demonstrates how the results differ by the value of subsistence level of private consumption (\bar{C}).¹⁰ The changes are still bigger for Korean economy, but the direction of the trend itself is unaffected by the value of \bar{C} .

¹⁰The benchmark value of \bar{C} is 5% of the private consumption in Korea at 1990.

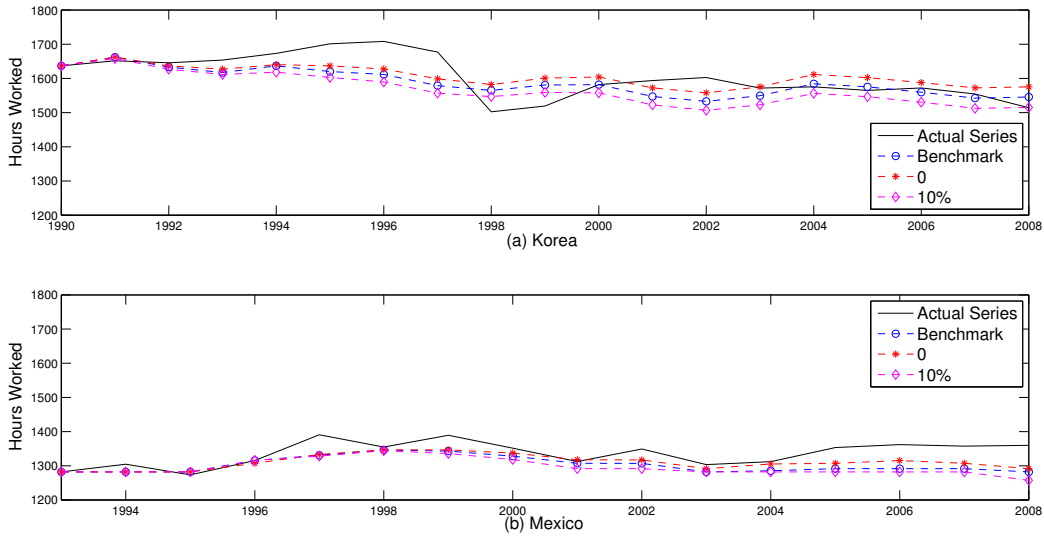


Figure 2.7: Effect of Subsistence Consumption (\bar{C})

2.6.2 Average Tax Rates

To compute the average tax rates, I directly follow the procedure of McDaniel (2007). The basic strategy is to categorize the government’s tax revenue by its sources - consumption expenditure, labor income, capital income, and investment expenditure. Dividing these revenues by corresponding income or expenditure yields the average tax rates of a country in a given period. See McDaniel (2007) for the detailed procedure. In this section, only the sources of data and the country-specific procedures are introduced below.

2.6.2.1 Korea

The national accounts data for Korea is obtained from OECD (2003) for 1990-1997, and from OECD (2010) for 1997-2008. The reason why 1997 is overlapping is that the data after 1997 is once modified by the Korean government and thus there is inconsistency between OECD (2003) and OECD (2010). Therefore, I compute the tax rates for 1990-1997 and for 1997-2008 separately, and re-scale the predating tax rates from 1997 so that the 1997 tax rates are the same in both periods. The volume of each item of the government revenue is

obtained from OECD (2011).

2.6.2.2 Mexico

Mexican data is obtained from OECD (2003) for 1993-1995, OECD (2008) for 1995-2003, and OECD (2010) for 2003-2008. By the same reason as Korea, I use three different sources, between each pair of which one year overlaps, and re-scale the pre-dating tax series so that the series is continuous. The volume of each item of the government revenue is obtained from OECD (2011).

	Labor Income Tax	Consumption Tax	Investment Tax	Capital Income Tax
1990	0.070	0.136	0.091	0.117
1991	0.068	0.126	0.084	0.117
1992	0.073	0.130	0.088	0.120
1993	0.084	0.129	0.088	0.109
1994	0.083	0.133	0.090	0.114
1995	0.086	0.133	0.090	0.120
1996	0.091	0.141	0.096	0.126
1997	0.093	0.154	0.105	0.115
1998	0.108	0.152	0.108	0.130
1999	0.106	0.161	0.112	0.109
2000	0.112	0.159	0.111	0.147
2001	0.122	0.162	0.113	0.133
2002	0.123	0.164	0.114	0.129
2003	0.130	0.162	0.113	0.152
2004	0.135	0.154	0.108	0.139
2005	0.138	0.151	0.106	0.159
2006	0.148	0.152	0.106	0.161
2007	0.161	0.153	0.107	0.176
2008	0.161	0.152	0.106	0.180

Table 2.1: Average Tax Rates of Korea

	Labor Income Tax	Consumption Tax	Investment Tax	Capital Income Tax
1990				
1991				
1992				
1993	0.079	0.089	0.063	0.095
1994	0.083	0.087	0.062	0.092
1995	0.078	0.100	0.071	0.070
1996	0.073	0.106	0.074	0.067
1997	0.065	0.110	0.076	0.073
1998	0.062	0.095	0.066	0.077
1999	0.062	0.095	0.066	0.080
2000	0.063	0.105	0.073	0.081
2001	0.065	0.104	0.073	0.087
2002	0.069	0.095	0.067	0.087
2003	0.071	0.106	0.075	0.083
2004	0.069	0.107	0.075	0.079
2005	0.066	0.116	0.082	0.089
2006	0.068	0.112	0.078	0.091
2007	0.068	0.110	0.077	0.085
2008	0.069	0.125	0.087	0.098

Table 2.2: Average Tax Rates of Mexico

CHAPTER 3

Fiscal Policy and Long-run Changes in Macroeconomy: 15 OECD Countries, 1960-2001

3.1 Introduction

Current developed economies have exhibited different patterns of economic growth since the end of WWII. For example, average hours worked in the U.S. and Canada have slightly increased since 1950s, while average hours worked has been gradually diminishing in most Western European countries during the same period. This well known empirical fact has been studied by many economists with neoclassical perspectives, including Prescott (2004) and Ohanian et al. (2008). Prescott (2004) examines whether a neoclassical model with labor wedged by taxes can explain different patterns of long-term changes in hours worked between the U.S. economy and developed economies in Europe. Ohanian et al. (2008) extends his methods including a larger set of countries and longer time periods.

However, other macroeconomic variables exhibit seemingly different patterns of changes in the long run as well. Figure 3.1 displays the long-run changes in average hours worked (northwest panel), consumption-output ratio (northeast panel), capital-output ratio (southwest panel), and investment-output ratio (southeast panel) of four countries - the U.S., Canada, Germany, and Italy - during 1960-2001.¹ As mentioned above, hours worked of the U.S. and Canada are slightly increasing since 1960. On the other hand, hours worked exhibit gradual decline in Germany and Italy for the same sample period. Regarding consumption-

¹The data is obtained at OECD and KIEL Institute for the World Economy (2006). For detailed explanation of the data sources, see Section 3.3.

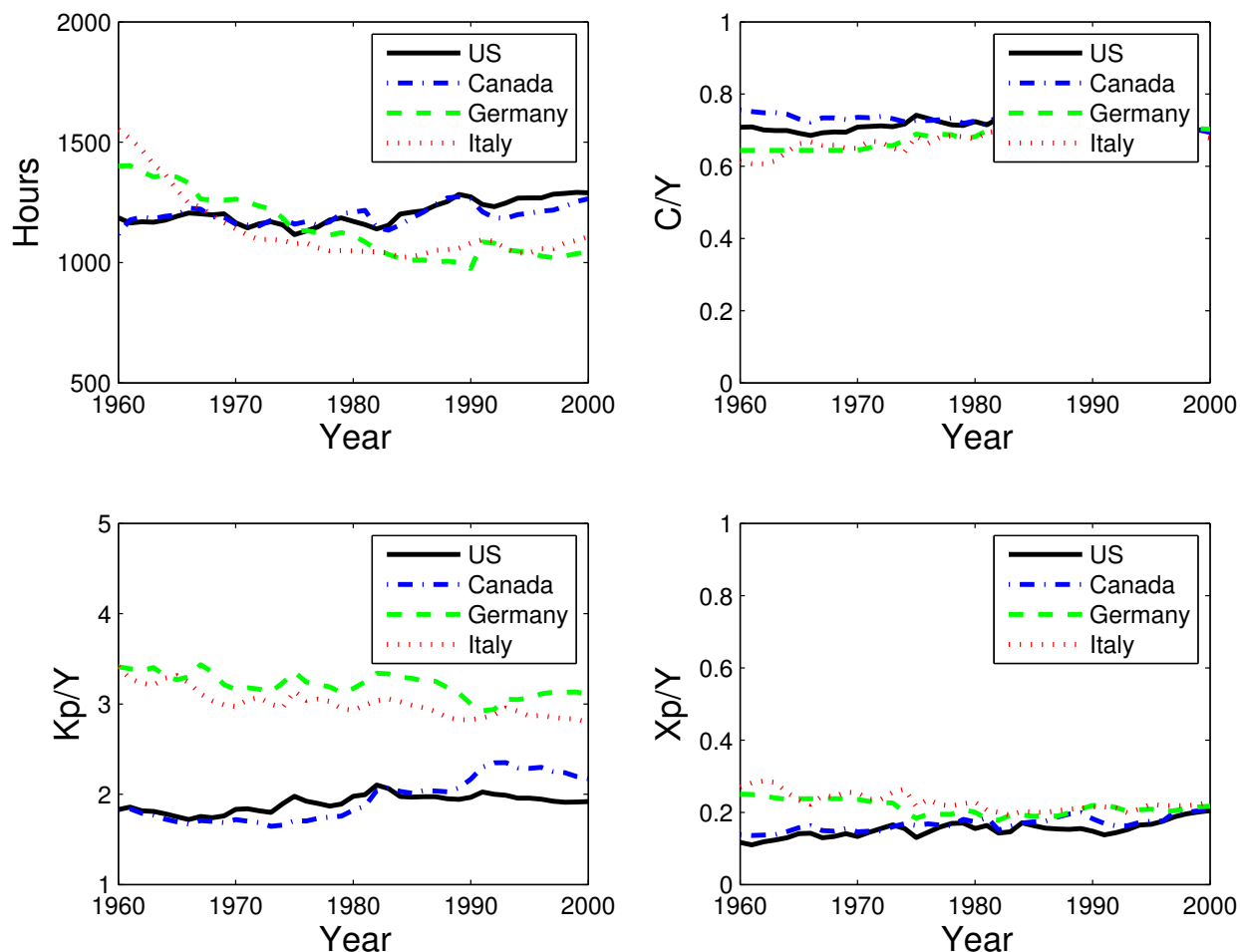


Figure 3.1: Long-run Changes in Key Macroeconomic Variables, 1960-2001

output ratio, the U.S. and Canada have relatively stable values, while the ratio in Germany and Italy seems to slowly increase. Moreover, the capital accumulation and investment also exhibit different patterns among these countries. While Germany and Italy own large amounts of capital stocks relative to their GDP, the capital-output ratios of the U.S. and Canada are much smaller in the data. Similarly, investment-output ratio also differs among these countries.

These differences in capital accumulation path and consumption as well as hours worked suggest that there might be other factors than labor wedge² that significantly influence the

²In Ohanian et al. (2008), labor wedge is generated by proportional taxes on wage and consumption

long-term patterns of those key macroeconomic variables. Therefore, in this paper, an extended version of neoclassical model is tested. Not only tax wedges in the labor market but also the wedges in capital accumulation and investment procedure are considered. Also, it is examined whether the size of government expenditure *per se* is significant in long-term changes of macroeconomic variables.

Though this paper extends studies of Prescott (2004) and Ohanian et al. (2008) into the capital accumulation process, I mainly follow the methodology of deterministic simulation used in Cole and Ohanian (1999), Kehoe and Prescott (2002), and Chen et al. (2006). 15 OECD countries³ are selected based on data availability and reliability. The data on government capital stock and government investment is taken from OECD and KIEL Institute for the World Economy (2006), and the average tax rates are taken from McDaniel (2007). All remaining macroeconomic data is taken from Heston et al. (2011).

This paper proceeds as follows. Section 3.2 describes how the benchmark model is set. Section 3.3 presents detailed data sources and calibration procedure. Section 3.4 demonstrates results of model solutions and their interpretations. Section 3.5 concludes the paper.

3.2 Model Economy

The model is a variation of a typical neoclassical model in several aspects. As in typical neoclassical models, there is a representative household and a representative firm. The household supplies labor and private capital, and the firm conducts production using these factors of production. However, there are a couple of differences. First of all, output is produced by not only labor and private capital but also public capital, which is formed by the government's investment. Second, there exists a government, which levies proportional taxes on different types of expenditure and income and spends its tax revenue for public

expenditure.

³The 15 countries are: Australia, Austria, Belgium, Canada, Finland, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, Switzerland, the U.K., and the U.S..

consumption and investment for public capital formation.⁴

Therefore, the long-run trends of the macroeconomic variables depend on the long-term changes of the tax rates on expenditure and income as well as the government spending (government consumption plus government investment). The following subsections demonstrate the details of the model economy.

3.2.1 Production Technology

There is a representative firm that follows the technology given by the following production function:

$$Y_t = A_t F(K_{pt}, K_{gt}, H_t)$$

where A_t is the total factor productivity (TFP, henceforth), K_{pt} is the aggregate private capital input, K_{gt} is the aggregate public (government) capital input, and H_t is the aggregate labor input.

The functional form of $F(K_{pt}, K_{gt}, H_t)$ is important. As long as the author's knowledge, there is little consensus on the mathematical form of the contribution of public capital in the neoclassical production function, and the results potentially highly depend on how we assume the functional form of the production function.⁵ This paper, however, is abstract from this discussion. Instead, the following production function is used throughout this paper:

$$Y_t = A_t K_{pt}^\theta H_t^{1-\theta} K_{gt}^\psi$$

where $0 < \theta < 1$ and $\psi > 0$. This production function is used in Baxter and King (1993) and Leeper et al. (2010). It is a homogeneous function with degree one with respect to private factor inputs (private capital and labor), while it exhibits increasing-returns-to-scale in all

⁴This type of variation of the neoclassical growth model has been used in several studies. For example, see Baxter and King (1993) and Leeper et al. (2010) among others.

⁵For instance, if private and public capital are mutually substitutable, then the functional form must be $F(K_{pt} + \chi K_{gt}, H_t)$ for some constant χ .

factors including the government capital.

The growth factor of the TFP is defined by:

$$\gamma_t \equiv \left(\frac{A_{t+1}}{A_t} \right)^{\frac{1}{1-\theta}}$$

which is allowed to be time-varying.

The firm determines its demand for labor and private capital so that the profit is maximized, and the demand equation for each factor input is given by the following typical first-order conditions of the profit-maximization problem:

$$\begin{aligned} w_t &= (1 - \theta) \frac{Y_t}{H_t} = (1 - \theta) \frac{y_t}{h_t} \\ r_t &= \theta \frac{Y_t}{K_{pt}} = \theta \frac{y_t}{k_{pt}} \end{aligned}$$

where y_t and k_{pt} mean output per capita and private capital stock per capita, respectively.

3.2.2 Preference

The representative household consists of N_t members at time t , and maximizes the following lifetime utility function:

$$\sum_{t=0}^{\infty} \beta^t N_t \left\{ \log(c_t + \lambda g_{ct}) + \alpha \frac{(\bar{H} - h_t)^{1-\sigma} - 1}{1-\sigma} \right\}$$

subject to:

$$(1 + \tau_{ct})C_t + (1 + \tau_{xt})X_t = (1 - \tau_{ht})w_t H_t + (1 - \tau_{kt})r_t K_{pt} - \Pi_t$$

$$K_{pt+1} = (1 - \delta)K_{pt} + X_t$$

$$K_0 \text{ given}$$

$$n_t \equiv \frac{N_{t+1}}{N_t}$$

in which c_t , h_t , and g_{ct} are the consumption per capita, labor input per capita, and government consumption per capita at time t , respectively. C_t , X_t , and Π_t are the aggregate consumption, aggregate private investment, and the aggregate lump-sum tax, respectively.

There are parameters β , λ , \bar{H} , σ , and δ , which stand for the discount factor, the measure of substitutability between private and government consumption, the total hours endowed, the measure of labor supply elasticity, and the depreciation rate of the private capital, respectively. w_t and r_t are the wage rate and the rental rate of the private capital. The population growth factor between period t and $t + 1$ is denoted by n_t . Finally, τ_{ct} , τ_{xt} , τ_{ht} , and τ_{kt} are the proportional tax rates on the private consumption, private investment, labor income, and capital income, respectively.

3.2.3 Government

The government levies proportional taxes on private consumption, private investment, labor income, and capital income, and spends the tax revenue for government consumption and government investment. The government is assumed to balance its budget every period. Formally:

$$\begin{aligned}\tau_{ct}C_t + \tau_{xt}X_t + \tau_{ht}w_tH_t + \tau_{kt}r_tK_{pt} + \Pi_t &= G_{ct} + G_{kt} \\ K_{gt+1} &= (1 - \delta_{gt})K_{gt} + G_{kt}\end{aligned}$$

in which G_{ct} and G_{kt} are the government consumption and the government investment.

3.2.4 Equilibrium

After some algebra, the general equilibrium of this economy is characterized by the following system of difference equations for $t = 0, 1, 2, \dots$:

$$\alpha(\bar{H} - h_t)^{-\sigma} = \frac{1 - \tau_{ht}}{1 + \tau_{ct}}(1 - \theta) \frac{1}{h_t} \cdot \frac{1}{\frac{\tilde{c}_t}{\tilde{y}_t} + \lambda \frac{\tilde{g}_{ct}}{\tilde{y}_t}} \quad (3.1)$$

$$\frac{1 + \tau_{xt}}{1 + \tau_{ct}} \cdot \frac{1}{\tilde{c}_t + \lambda \frac{\tilde{g}_{ct}}{\tilde{y}_t}} \gamma_t = \beta \frac{1}{1 + \tau_{ct+1}} \cdot \frac{1}{\tilde{c}_{t+1} + \lambda \frac{\tilde{g}_{ct+1}}{\tilde{y}_{t+1}}} \left[(1 - \tau_{kt+1}) \theta \frac{\tilde{y}_{t+1}}{\tilde{k}_{pt+1}} + (1 + \tau_{xt+1})(1 - \delta) \right] \quad (3.2)$$

$$\tilde{y}_t^{1-\psi} = \tilde{k}_{pt}^\theta h_t^{1-\theta} \left(\frac{\tilde{k}_{gt}}{\tilde{y}_t} \right)^\psi \quad (3.3)$$

$$\frac{\tilde{c}_t}{\tilde{y}_t} + n_t \gamma_t \frac{\tilde{k}_{pt+1}}{\tilde{y}_t} - (1 - \delta) \frac{\tilde{k}_{pt}}{\tilde{y}_t} + \frac{\tilde{g}_{ct}}{\tilde{y}_t} + \frac{\tilde{g}_{kt}}{\tilde{y}_t} = 1 \quad (3.4)$$

where variables with a tilde stand for the detrended values of corresponding variables.⁶ The equation (3.1) is the labor market equilibrium condition, (3.2) is the Euler equation for the private investment, (3.3) is the production function, and (3.4) is the feasibility condition of the economy.

The steady state, where all the exogenous sequences⁷ are assumed to be constants, is

⁶For example, \tilde{c}_t stands for the detrended value of the consumption per capita, which is formally defined by $\tilde{c}_t \equiv c_t/A_t^{\frac{1}{1-\theta}}$

⁷such as the proportional tax rates (τ 's), government spending (government consumption and investment), the population growth (n_t), and the growth rate of TFP (γ_t).

characterized by the following system of equations:

$$\begin{aligned}\alpha(\bar{H} - h)^{-\sigma} &= \frac{1 - \tau_h}{1 + \tau_c} (1 - \theta) \frac{1}{h} \cdot \frac{1}{\frac{\tilde{c}}{\tilde{y}} + \lambda \frac{\tilde{g}_c}{\tilde{y}}} \\ (1 + \tau_x)\gamma &= \beta \left[(1 - \tau_k)\theta \frac{\tilde{y}}{\tilde{k}_p} + (1 + \tau_x)(1 - \delta) \right] \\ \tilde{y}^{1-\psi} &= \tilde{k}_p^\theta h^{1-\theta} \left(\frac{\tilde{k}_g}{\tilde{y}} \right)^\psi \\ \frac{\tilde{c}}{\tilde{y}} + n\gamma \frac{\tilde{k}_p}{\tilde{y}} - (1 - \delta) \frac{\tilde{k}_p}{\tilde{y}} + \frac{\tilde{g}_c}{\tilde{y}} + \frac{\tilde{g}_k}{\tilde{y}} &= 1\end{aligned}$$

3.3 Data and Computation

As there is no uncertainty in the model economy, the equilibrium values of all the endogenous variables are determined solely by the movement of exogenous variables. Therefore, my computation strategy is to compute the model-generated series of hours worked (h_t), consumption-output ratio (c_t/y_t), private investment-output ratio (x_t/y_t), and private capital-output ratio (k_{pt}/y_t) *given* the actual series of average tax rates, public investment, population growth rate, and the growth factor of TFP.

Therefore, a deterministic simulation is conducted in my computation. First, we are given the actual data of tax rates, government spending, population growth, and TFP growth for 1960-2001. After this period, I assume there are T more years during which all the exogenous variables are constant at their 1960-2001 average. T must be large enough for the economy to approach its steady state sufficiently closely. Given the initial value of private capital (k_{p0}) and the terminal value at the steady state, all the intermediate series of the endogenous variables can be computed as a solution of a system of equations. This type of computation technique has been used in many existing studies, including Cole and Ohanian (1999) and Chen et al. (2006) among others.

3.3.1 Calibration

First of all, \bar{H} , the total endowment of hours, is calculated by $14 \times 365 = 5,110$. The capital share and depreciation rate also follow the literature as $\theta = 0.36$ and $\delta = 0.1$. The calibration of λ (measure of substitutability between private consumption and government consumption) and σ (measure of labor supply elasticity) is not clear, so I use $\lambda = 1$ and $\sigma = 1$ in the benchmark case and change the values to observe the potential significance of them.⁸ There is no consensus on the value of ψ (exponent of government capital in the production function) either⁹, so I use $\psi = 0.1$ and check the significance of it by changing its values.¹⁰ The discount factor (β) is determined so that the private capital-output ratio matches the data in the steady state Euler equation, and the weight of disutility from working in the utility function (α) is determined so that the average hours worked match the data in the steady state labor-leisure condition.

3.3.2 Source of Data

The data series of the average tax rates on consumption, private investment, labor income, and capital income are from McDaniel's (2007) computation. She computes the average tax rates on each source of expenditure and income by the following procedure - first, she categorizes the government's tax revenue by the source of tax. After that, the amount of each source is divided by the total amount of each item, which yields to the average tax rate for each item of tax sources. A similar strategy has been conducted in Mendoza et al. (1994) and Carey and Rabesona (2004).

The government consumption-output ratio (g_{ct}/y_t) is obtained from Penn World Table 7.1 by Heston et al. (2011). The government capital-output ratio (k_{gt}/y_t) and the govern-

⁸See Section 3.6.1 for robustness checks of these variables.

⁹Baxter and King (1993) choose $\psi = 0.05$ as their benchmark.

¹⁰See Section 3.6.1 for robustness checks of ϕ .

ment investment-output ratio (g_{kt}/y_t) are from OECD and KIEL Institute for the World Economy.¹¹ Population data is from OECD, and the series of TFP growth factor (γ_t) is obtained by the author’s computation.¹²

3.4 Results

As mentioned in Section 3.1, the main purpose of this model is to match the patterns of long-run changes of key macroeconomic variables. Since the model has four key endogenous variables (hours, private consumption-output ratio, private capital-output ratio, and private investment-output ratio), this section will examine validity of this model for each variable. First, a two-point comparison as in Prescott (2004) is introduced in Section 3.4.1, and country-by-country results are briefly demonstrated in Section 3.4.2.

3.4.1 Two-Point Comparison

3.4.1.1 Hours Worked

Many existing studies show that there is a large amount of cross-country difference in hours worked among OECD countries in the long run¹³ and a large portion of these differences can be explained by tax wedges in the labor market only. Therefore, in this section we examine how importantly tax wedges can explain the long-term changes in hours worked.

Table 3.1 shows the ratio between hours worked in 2001 and 1960 for 15 OECD countries.¹⁴ The second column, labeled with “Data”, shows the actual data of the ratios. As

¹¹This data set provides measurement of government capital and investment of OECD countries. See Kamps (2006) for details

¹²The series of γ_t is computed by $\gamma_t \equiv \left(\frac{A_{t+1}}{A_t}\right)^{\frac{1}{1-\theta}}$, where $A_t \equiv \frac{Y_t}{K_{pt}^\theta H_t^{1-\theta} K_{gt}^\psi}$.

¹³See Prescott (2004) and Ohanian et al. (2008) among others.

¹⁴Formally, it means $\frac{H_{2001}}{H_{1960}}$ using model variables.

Ohanian et al. (2008) demonstrate, most European countries experienced large decline in hours worked, roughly 5 to 35 %, while the U.S., Canada, and Australia experienced no such decline in hours worked (actually some of them experienced even a slight increase in hours worked).

Right 3 columns of Table 3.1 show the ratio of each country in three different versions of the model - a model with no government at all¹⁵ (*No-Gov't Model*, henceforth), a model with tax wedges but no government investment nor government capital (*Tax-Only Model*, henceforth), and the full model introduced in Section 3.2 (*Benchmark Model*, henceforth), respectively. The best-fit model is labeled with an asterisk (*). As we can see, for most European countries the Tax Only model work well. However, for the countries whose data ratio is close to unity (Australia, Canada, and the U.S.), the Tax-Only model does not sufficiently explain why hours worked do not decline much in these countries. The hours worked solved in the Benchmark model are only slightly different from the Tax Only model, implying that the volume of government investment is not significantly large that it does not affect hours worked much in the data.

¹⁵Therefore, only TFP and population growth affects the solution of the model.

Country	Data	No-Gov't	Tax-Only	Benchmark
Australia	0.9901	1.1352*	0.9284	0.9302
Austria	0.7444	1.1168	0.7956	0.7831*
Belgium	0.6897	1.2037	0.8041*	0.8211
Canada	1.1317	1.0213*	0.8184	0.8192
Finland	0.6917	1.3759	0.8656*	0.9073
France	0.6601	1.1165	0.8259*	0.8367
Germany	0.7451	1.2455	0.8717*	0.8898
Italy	0.7136	1.3308	0.8883*	0.8998
Japan	0.8015	1.0600	0.8978*	0.9247
Netherlands	0.6872	1.1729	0.8917	0.8883*
Spain	0.8673	1.1589	0.8504*	0.8725
Sweden	0.9537	1.1494*	0.6708	0.6869
Switzerland	0.8632	1.1229	0.8620*	0.9094
UK	0.7768	0.9814	0.8089	0.7973*
US	1.0877	1.0358*	0.9339	0.9326

Table 3.1: Long-run Changes in Hours Worked - Model Comparison

3.4.1.2 Consumption

Table 3.2 displays the data and model-generated values of relative consumption-output ratio. As in Table 3.1, each number is calculated by dividing private consumption-output ratio in 2001 by the ratio in 1960. Therefore, considering the volume of GDP, a country consumes more in 2001 than 1996 if this ratio exceeds 1, and vice versa.

Comparing the actual data and the No-Gov't model, we can see that the No-Gov't model is relatively unsuccessful in explaining the ratio of consumption-output that is close to 1. That is, while the measure of TFP and population growth forecast that most countries consume less relative to GDP in 2001, the actual data shows that most countries consume a higher portion of their GDP. The existence of labor wedge by itself cannot explain this

Country	Data	No-Gov't	Tax-Only	Benchmark
Australia	1.1791	0.8446	0.9901*	0.9819
Austria	1.0034	0.8902	1.0272*	1.0394
Belgium	0.9397	0.7604	0.8741*	0.8479
Canada	0.9182	0.9718*	1.0659	1.0573
Finland	1.0483	0.6319	0.7732*	0.7262
France	1.0184	0.8711	0.9869*	0.9648
Germany	1.0899	0.7489	0.9345*	0.9044
Italy	1.1046	0.6853	0.8077*	0.7918
Japan	0.9072	0.9378*	0.9961	0.9550
Netherlands	1.0129	0.8102	0.9325*	0.9323
Spain	0.8923	0.8139	0.9319	0.8972*
Sweden	1.0174	0.7947	0.9267*	0.9035
Switzerland	1.0053	0.8445	1.0062*	0.9289
UK	1.0217	1.0849*	1.2032	1.2111
US	0.9888	1.0556*	1.1269	1.1157

Table 3.2: Long-run Changes in Consumption-Output Ratio - Model Comparison

high consumption, thus tax wedges on investment and capital income play roles in this case. Since capital income and investment expenditure are taxed, the post-tax real interest rate decreases, thereby avoiding investment and consuming more.

As in this hypothesis, in most countries, existence of tax wedges explains non-decreasing consumption-output ratio significantly better. Especially, European countries, in most of which relatively higher investment and capital income taxes are observed, experience virtually no decline in consumption-output ratio, while countries with small taxes, such as the U.S. and Japan, experience decline in consumption-output ratio.

Country	Data	No-Gov't	Tax-Only	Benchmark
Australia	0.9092	0.6671	0.7975*	0.6874
Austria	1.1843	0.7582	0.9043*	0.8163
Belgium	0.9405	0.6421	0.7333*	0.6517
Canada	1.1925	0.9490	1.1635*	0.9843
Finland	0.6810	0.4085	0.4275*	0.3822
France	1.0897	0.7579	0.8913*	0.7816
Germany	0.9095	0.5164	0.6421*	0.5718
Italy	0.8189	0.5203	0.6560*	0.5765
Japan	1.4544	1.0173	1.3916*	1.1687
Netherlands	0.9375	0.6308	0.7213*	0.6360
Spain	1.0784	0.7869	1.1544*	0.9994
Sweden	0.9718	0.6968	0.8151*	0.7077
Switzerland	1.1671	0.7439	1.1001*	0.9288
UK	1.3386	1.0596	1.2011*	1.0707
US	1.0487	0.9414	1.2276	1.0494*

Table 3.3: Long-run Changes in Private Capital-Output Ratio - Model Comparison

3.4.1.3 Capital Stock

Table 3.3 demonstrates the relative increase of private capital stock-to-output ratio. While No-Gov't model is very unsuccessful in explaining the actual increase of capital-output ratio, the tax wedges work significantly better in representing the actual increase. The Benchmark model with government investment does not improve Tax-Only model except for the U.S. economy, for which the full model derives quite accurate value of increase in private capital-output ratio.

3.4.1.4 Investment

Table 3.4 displays the relative increase of private investment-to-output ratio. Since the level of investment can be very small (or even negative) in the model, some model ratio has an extremely large or negative number, which is labeled “NaN” in the table.

As we see in the table, the Tax-Only model performs best as well for explaining long-term changes in private investment-output ratio. However, it is unsuccessful in showing why the private investment-output ratio exhibits large increases in the U.S. during the sample period. While the investment-output ratio increases by more than 75%¹⁶, none of the models tested in this paper successfully explain this large increase. For some countries, such as Finland and Italy, the models forecast too high increase in investment-output ratio relative to the actual data, implying that we might have to suspect other factors than tax wedges and government spending restraining private investment in these economies.

¹⁶Note that it is NOT 75% points, but 75% increase of the ratio.

Country	Data	No-Gov't	Tax-Only	Benchmark
Australia	0.7544	4.8199	1.0687*	1.3442
Austria	1.3333	2.2057	1.0711	1.2488*
Belgium	1.1218	9.0350	1.5478*	2.1615
Canada	1.5255	1.2370	0.8631*	0.8974
Finland	0.8166	NaN	3.1351*	134.3114
France	1.0643	2.3466	1.1285*	1.3768
Germany	0.8674	NaN	1.3455*	2.1122
Italy	0.8492	NaN	4.5042*	NaN
Japan	1.4514	1.1622	1.0804	1.1945*
Netherlands	1.1371	28.1852	1.2961*	1.5581
Spain	1.4281	2.5045	1.3055*	1.6082
Sweden	0.8379	2.6945	1.1687*	1.4611
Switzerland	0.9167	2.4720	0.9589*	1.2080
UK	1.3261	0.9586*	0.7792	0.7721
US	1.7551	1.1809*	0.9948	1.0526

Table 3.4: Long-run Changes in Private Investment-Output Ratio - Model Comparison

3.4.1.5 Remarks on the Two-point Comparison

Overall, the Tax-Only model obviously performs the best, meaning that the role of government investment is questionable, at least in the long-run sense, in most of the sample countries. However, though the Benchmark model severely fails in explaining the actual data for most countries, it performs relatively well in some of the sample countries, including the U.S., Australia, Canada, and Japan. These countries experience relatively flat hours worked over 1960-2001 while the other countries experience huge decline in hours. Therefore, there is still some room to consider the role of government investment in the long-run macroeconomic variables.

3.4.2 Country-by-Country Analysis

In this section, full results of model performances are shown for some selected countries. Especially, this section focuses on comparing the countries for which models perform well with those for which models are unsuccessful. For results of all sample countries, refer to Section 3.6.2.

3.4.2.1 The U.S., Canada, and Japan

As mentioned in this paper, the U.S. and Canada are important in this literature because in these economies hours worked do not exhibit a long-run decline since 1960s while most other developed countries exhibit declines in hours worked for the same period. For these countries, the Tax-Only model and Benchmark model relatively success in explaining the trends of four key macroeconomic variables.

Figure 3.2 shows the long-term changes of four key variables in the U.S.. The full model (red dotted line) performs better than Tax-Only model (green dashed line) in most variables. However, models with government are on average outperformed by the No-Gov't model, meaning that introduction of government taxation and expenditure makes a worse explanation on the actual data.

Figure 3.3 demonstrates the case of Canada. The Benchmark model performs well in explaining the long-run changes. However, it is hard to say that it significantly outperforms the Tax-Only model. Overall, all models well mimic the actual data, which means that the model works well with the TFP and population growth, being improved by introduction of taxes and government expenditures.

In case of Japan (Figure 3.4), both models with government perform well for explaining the trends for consumption and private capital accumulation. However, the model does not represent the level of hours worked very closely. While Tax-Only model and Benchmark model exhibit similar percentage decreases in hours worked between 1960 and 2011, the level of hours is expected to be lower than the actual data, implying that the reason why Japanese

workers work for such many hours is not well explained by the class of models used in this paper.

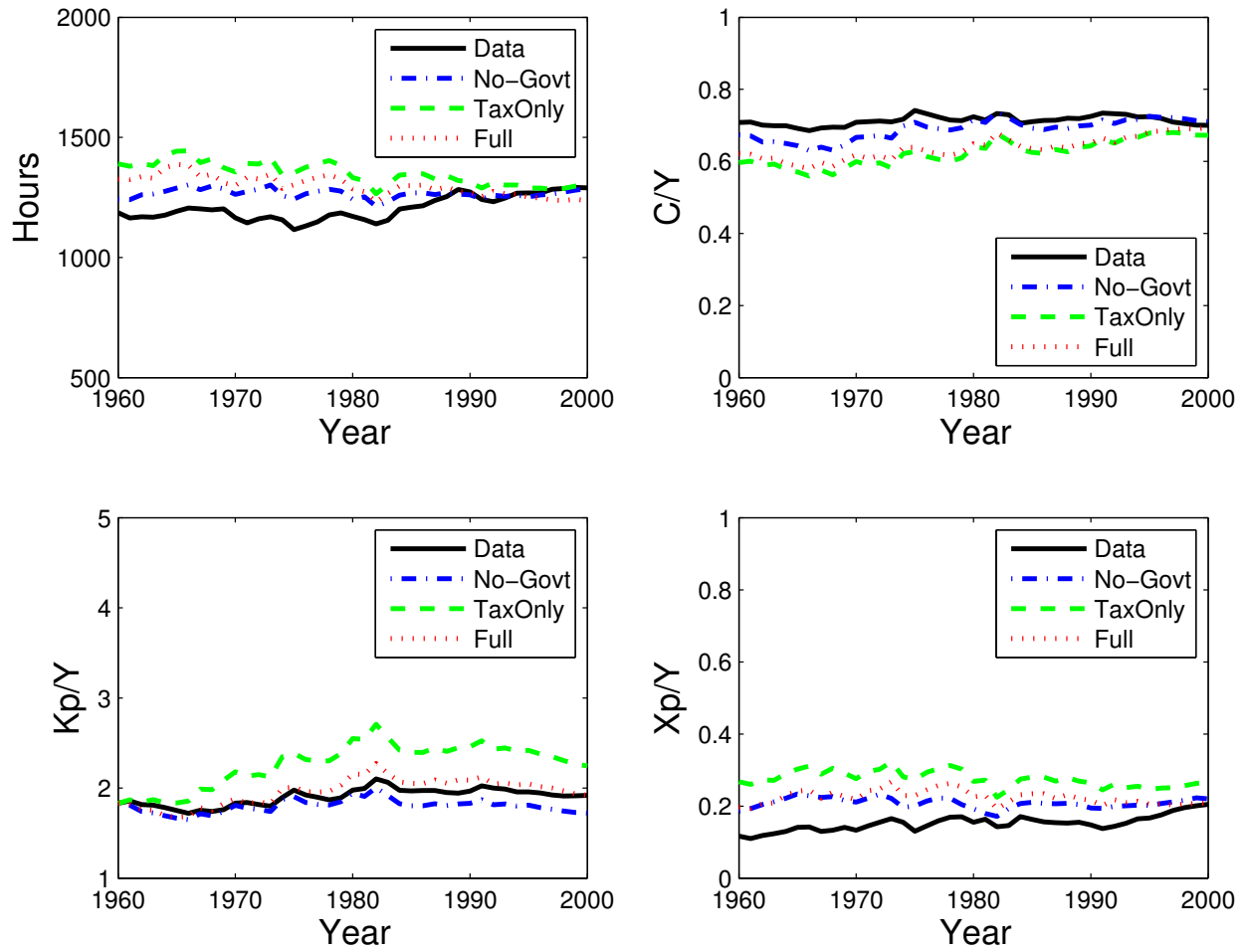


Figure 3.2: U.S. - Model Comparison

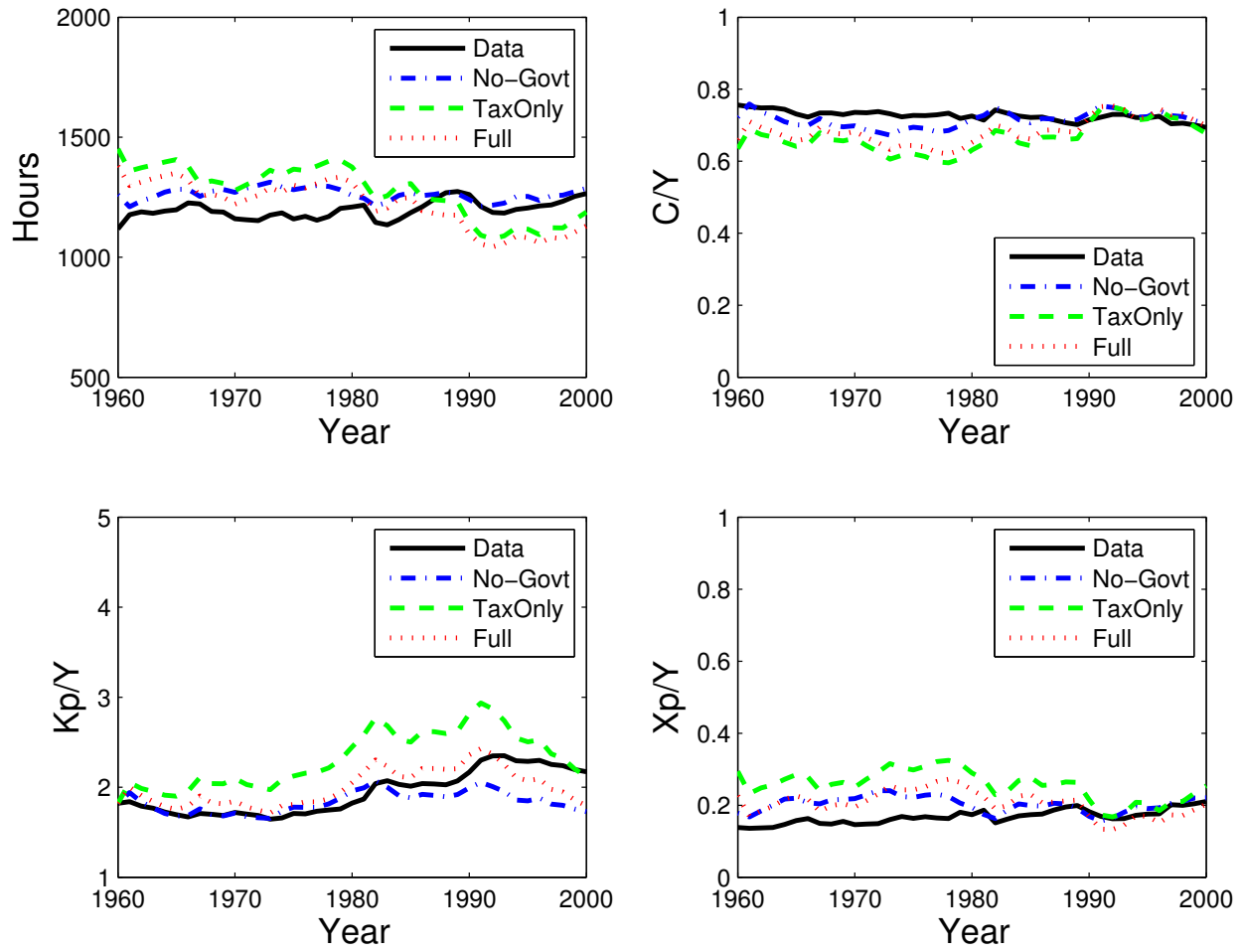


Figure 3.3: Canada - Model Comparison

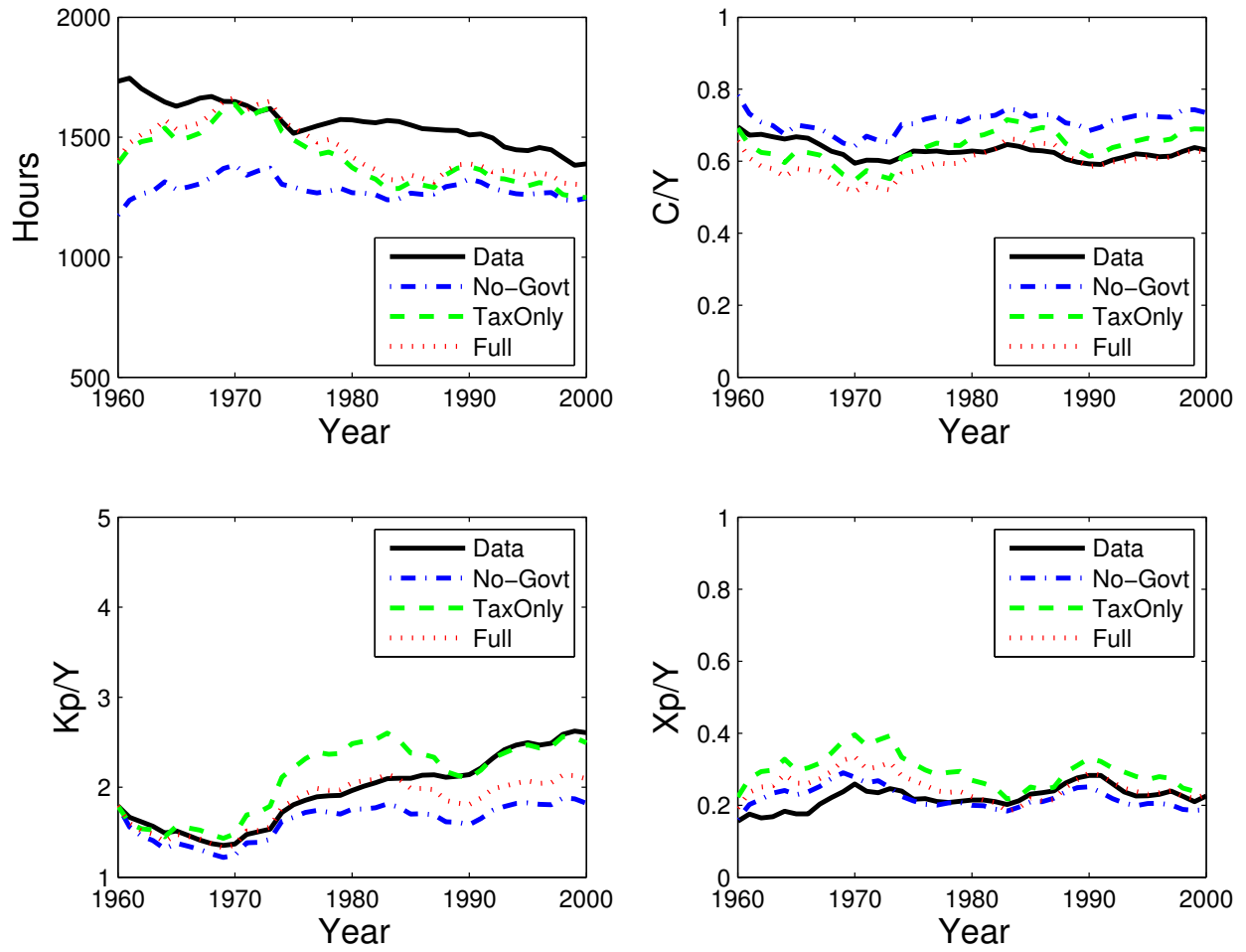


Figure 3.4: Japan - Model Comparison

3.4.2.2 Germany, Italy, and Japan

Figure 3.5 and 3.6 describe the data and model-generated series of four key variables for Germany and Italy. From these figures we can see that long-term changes in those variables are not well represented by the neoclassical growth models used in this paper. For instance, large decreases in hours of these two economies are not well mimicked by the models. The model does not explain slight increases in consumption-output ratio for Germany and Italy either: The model suggests that Germany and Italy must have consumed higher portion in 1960s considering TFPs and population growth rates, which is not true in the actual data though. Similarly, the models expect reduction in private capital-output ratio for 1960-2001, which is not the case in the actual data.

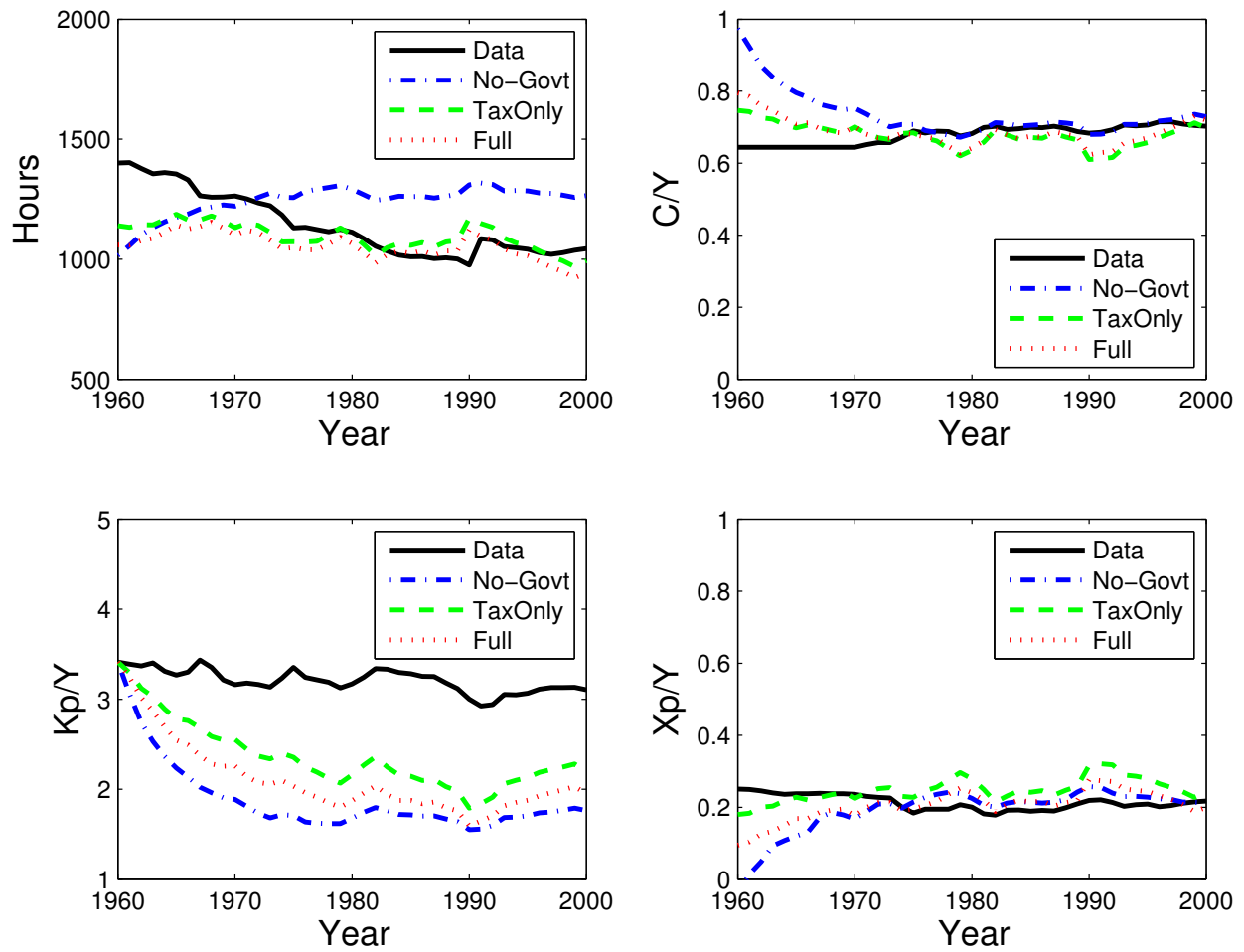


Figure 3.5: Germany - Model Comparison

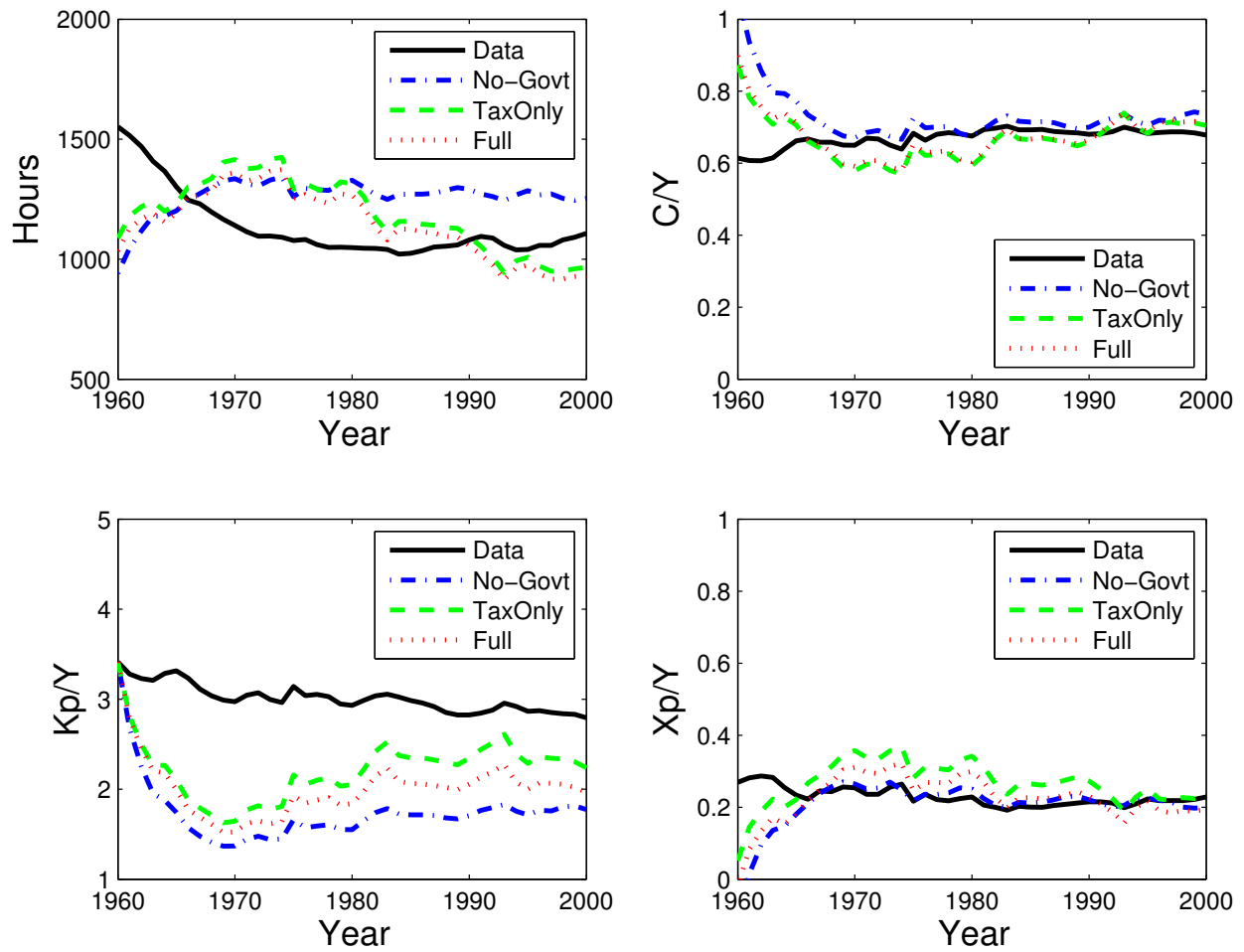


Figure 3.6: Italy - Model Comparison

3.5 Concluding Remarks

This paper examines the performance of neoclassical growth models with proportional taxes, government consumption, and government investment. As discussed so far, the model with tax wedges performs best for most of countries and most of the model variables. Considering government investment (the Benchmark model) does not significantly help the model's performance except for only a few cases, one of which is the U.S. private capital accumulation path. Therefore, the exercises on different versions of neoclassical growth models appear to show that the tax wedges in labor market, capital market, and investment procedure play the most significant role in explaining long-term variations of key macroeconomic variables.

For many economies¹⁷, even the model with tax wedges performs poor in explaining the long-run path of private investment-output ratio. This might mean that the assumptions on investment procedure do not reflect the data-generation process well, so additional assumptions on investment might be necessary, such as adjustment costs or time-to-build procedure¹⁸.

This paper does not intend to argue that government investment is not important in explanation of the economic growth though. Since the model in this paper assumes only a simple form of production function with government investment, further researches must be necessary in order to better understand the role of government-driven capital accumulation.

3.6 Appendix

3.6.1 Parameter Specifications and Robustness

This section examines the effect of parameter specification. Specifically, it aims to show how robust the model-generated series are to the parameter values of ϕ (exponent of government

¹⁷For example, Germany and Finland.

¹⁸Kydland and Prescott (1982)

capital in the production function), λ (household's weight on government consumption relative to private consumption), and σ (inverse of Frisch elasticity). Instead of displaying the results of this exercise for all sample countries, six countries¹⁹ are selected to be shown in this version of paper.

3.6.1.1 Robustness on ϕ

As we can see Figures 3.7-3.12, the model-generated series in the Benchmark model do not change significantly in the value of ϕ . This supports the conjecture in Section 3.5 that the role of government investment is only modest in this framework.

¹⁹The U.S., Canada, the U.K., France, Germany, and Italy are selected. Note that those countries are NOT chosen because the model perform well for those countries. This sample is chosen so that this section can represent different extents of model performances.

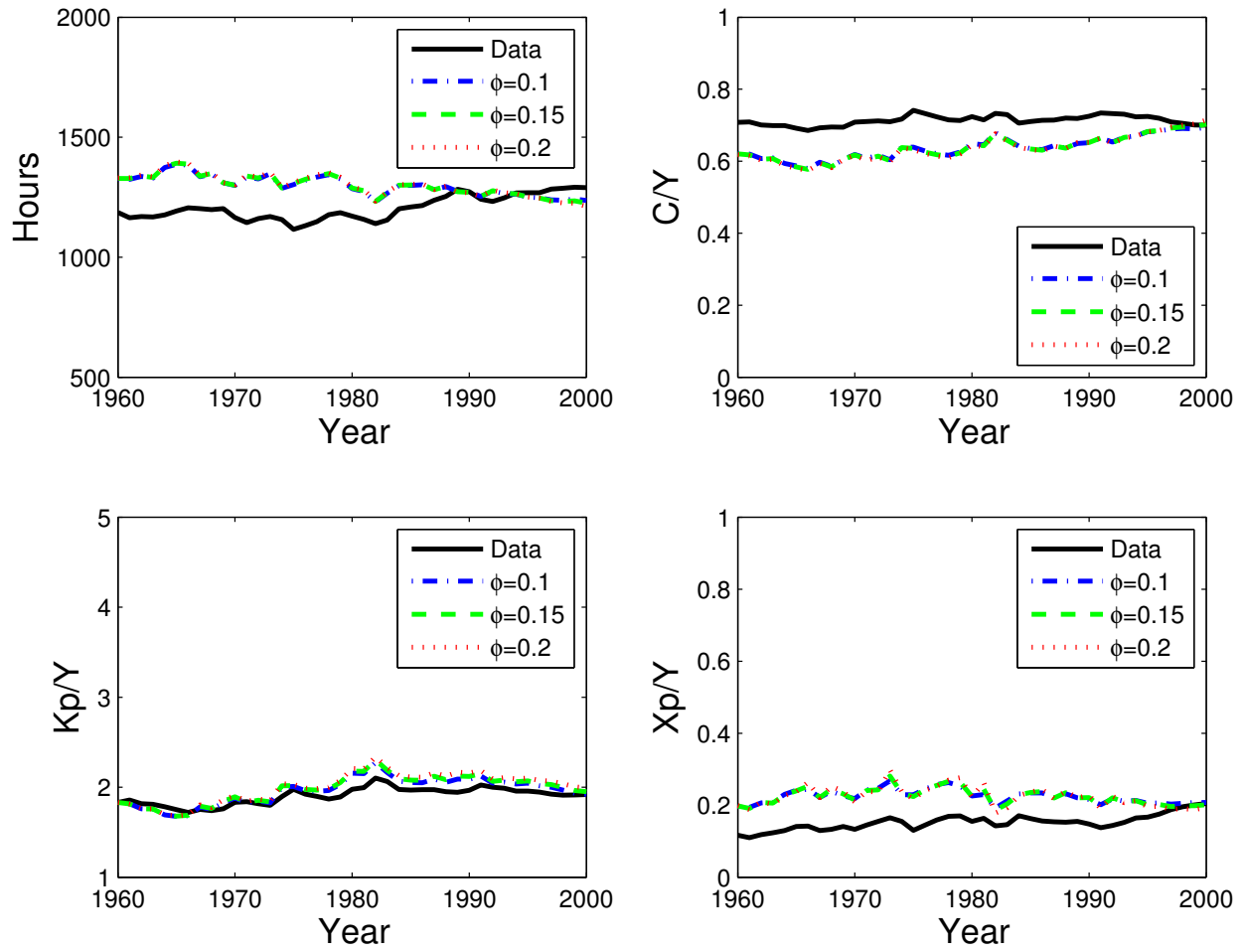


Figure 3.7: U.S. - Values of ϕ

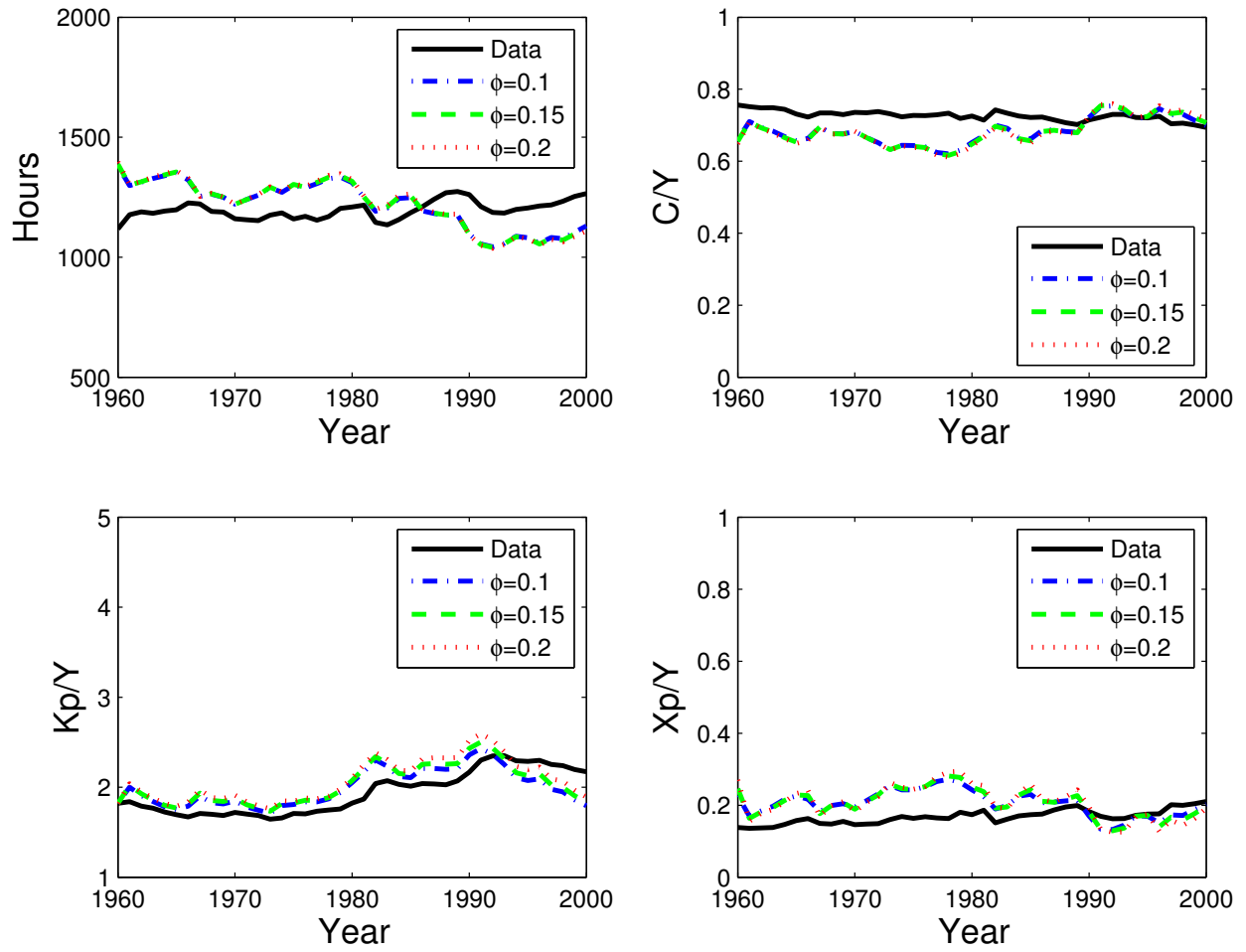


Figure 3.8: Canada - Values of ϕ

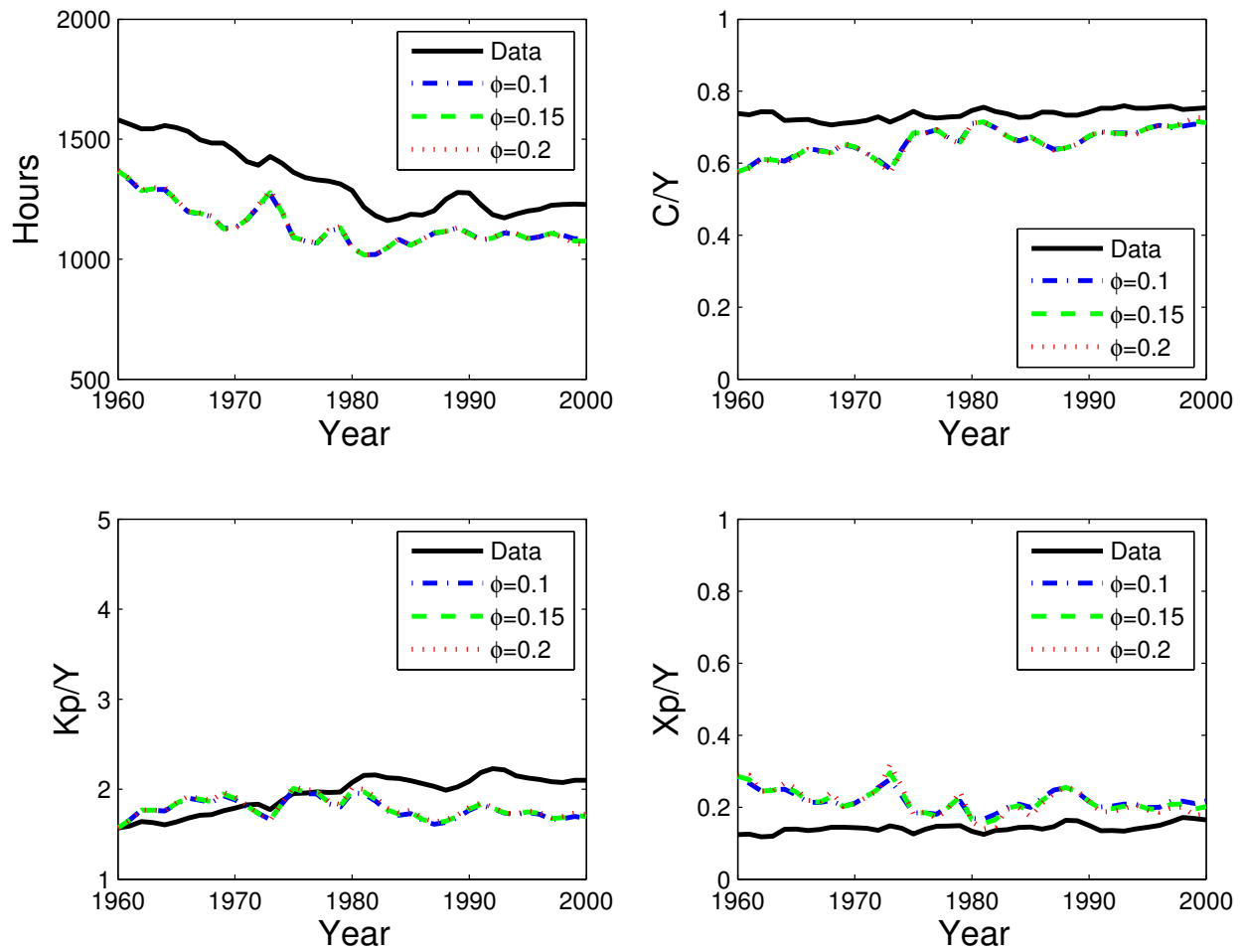


Figure 3.9: U.K. - Values of ϕ

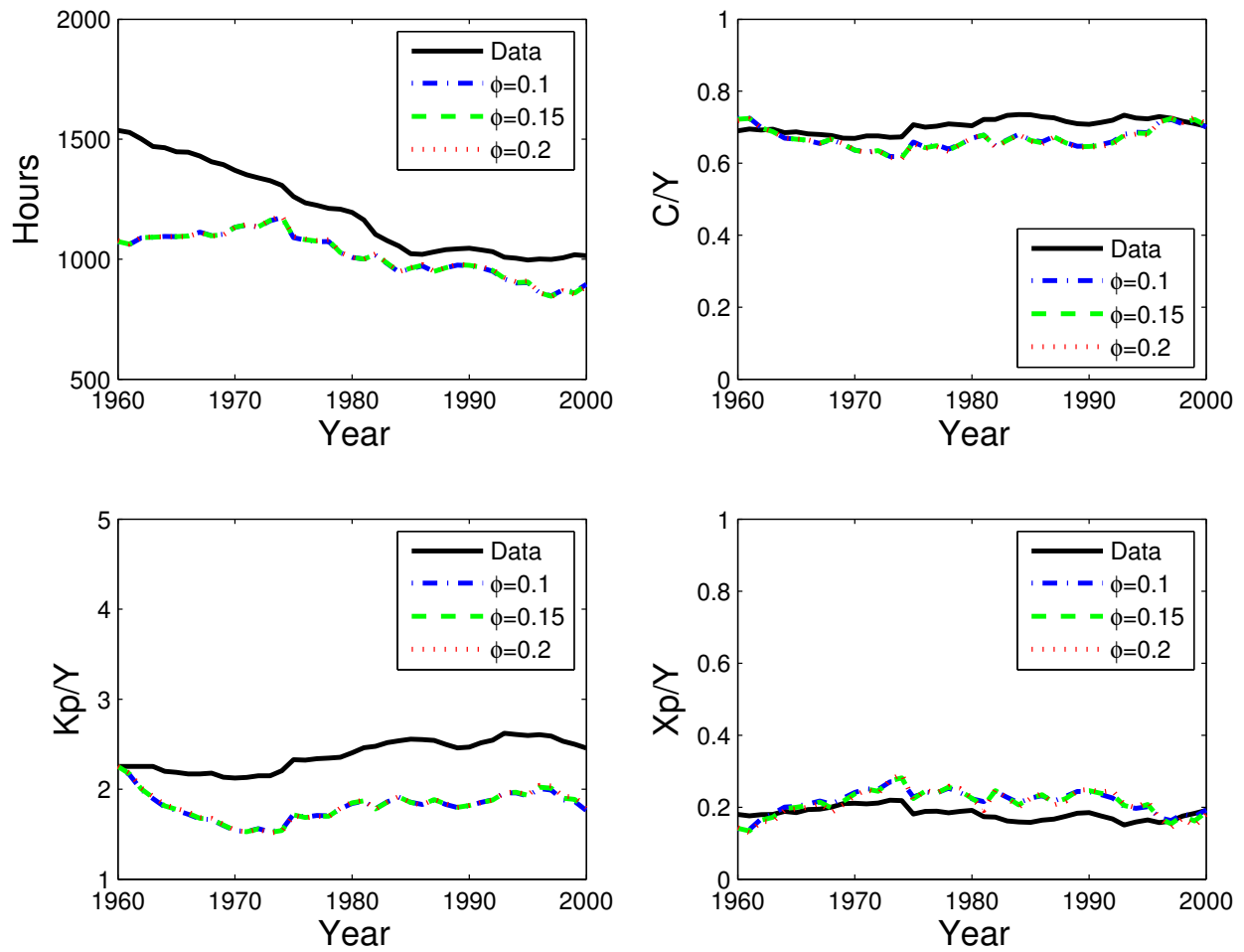


Figure 3.10: France - Values of ϕ

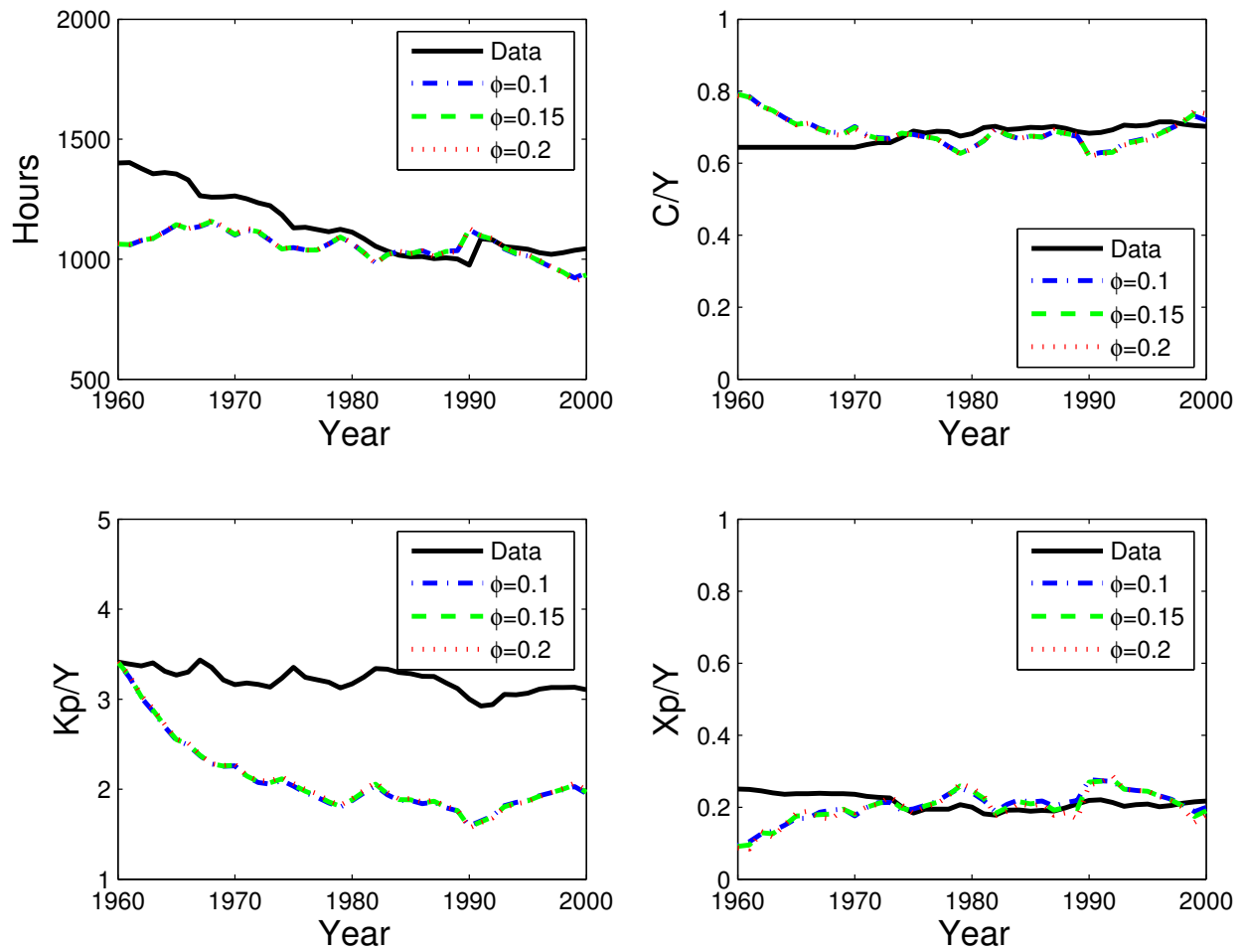


Figure 3.11: Germany - Values of ϕ

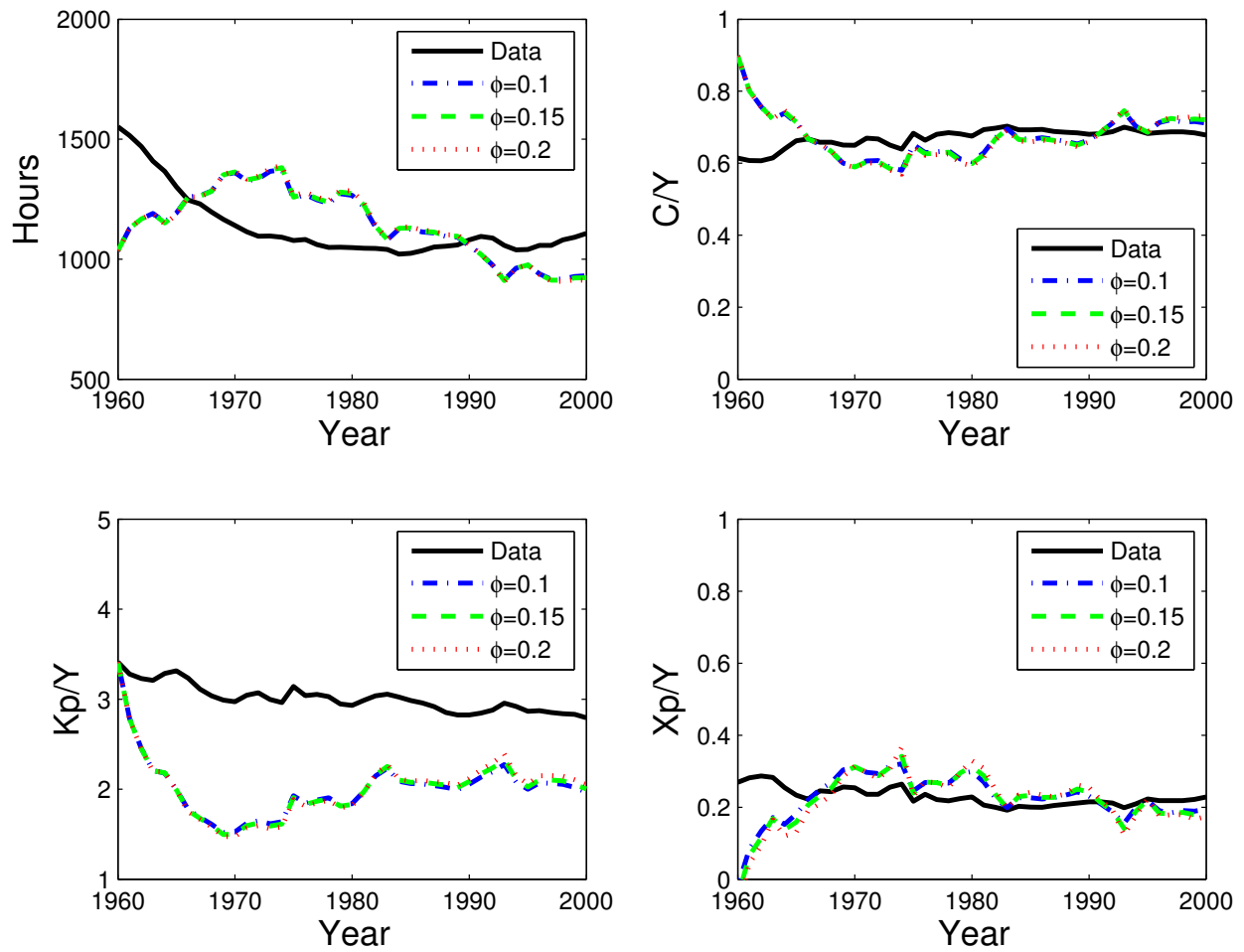


Figure 3.12: Italy - Values of ϕ

3.6.1.2 Robustness on λ

Figures 3.13-3.18 demonstrate the effect of λ . The model-generated series in the Benchmark model do not change significantly in the value of λ as in the previous case of ϕ . Again, this supports the conjecture that the role of government consumption is only modest in the class of models used in this paper.

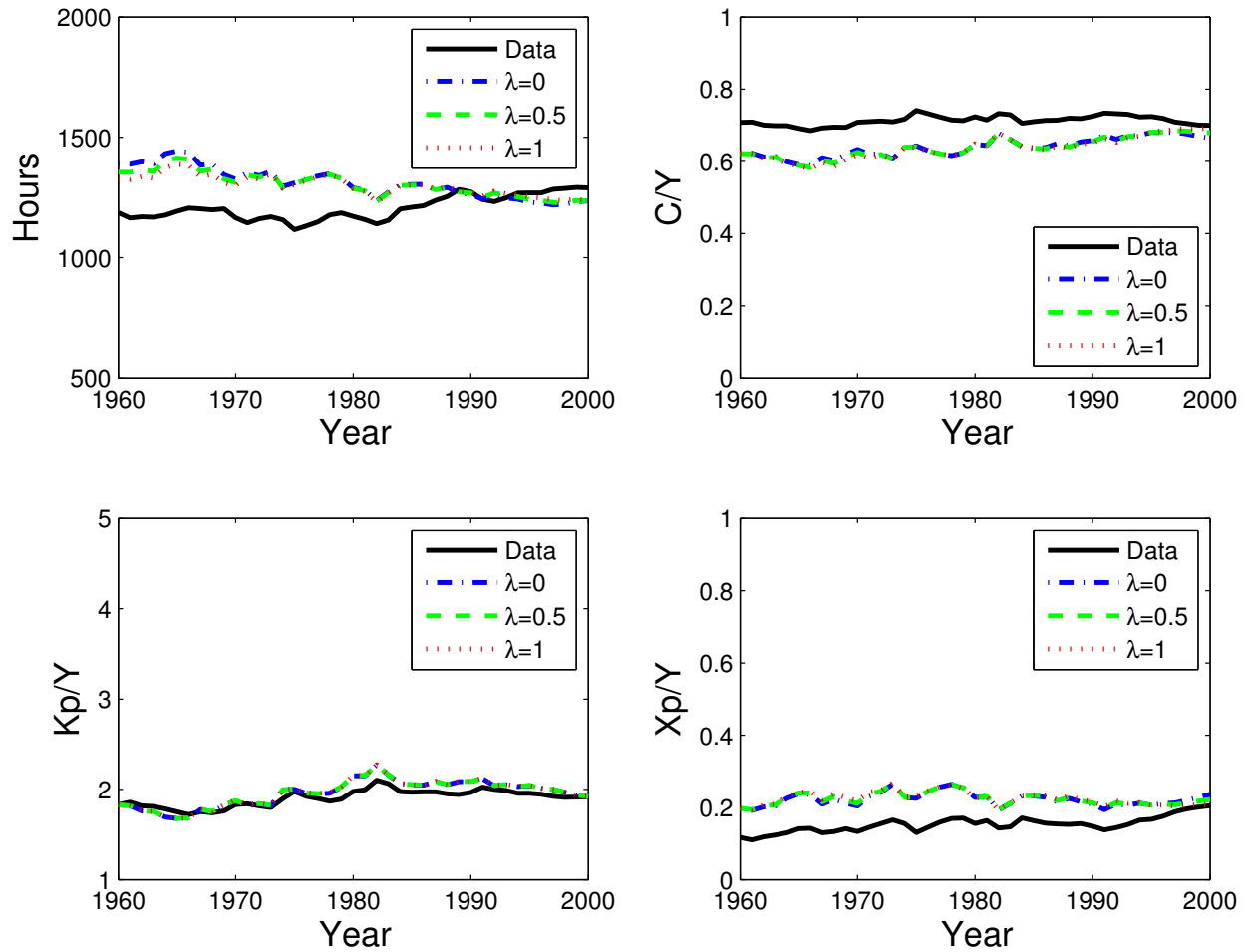


Figure 3.13: U.S. - Values of λ

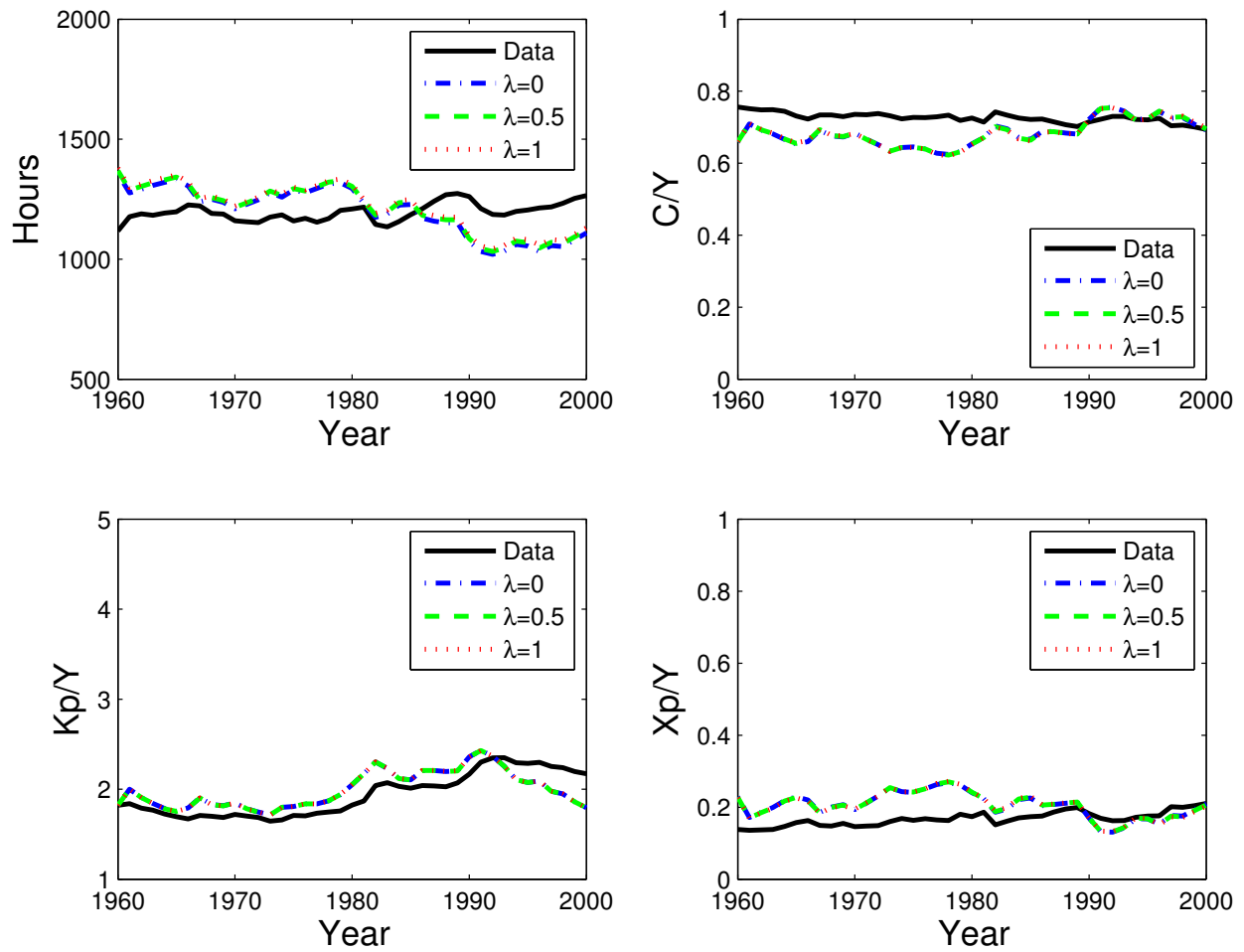


Figure 3.14: Canada - Values of λ

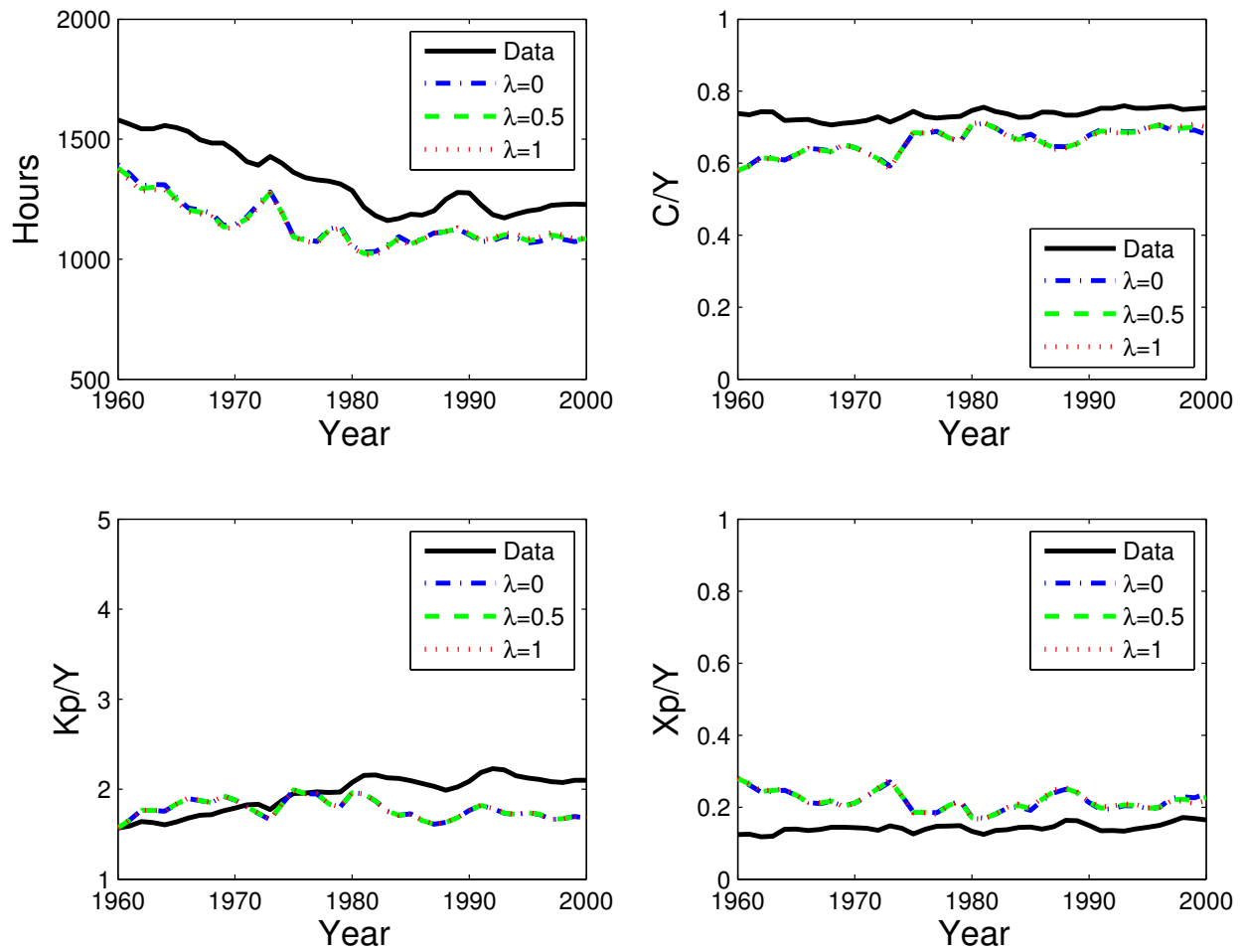


Figure 3.15: U.K. - Values of λ

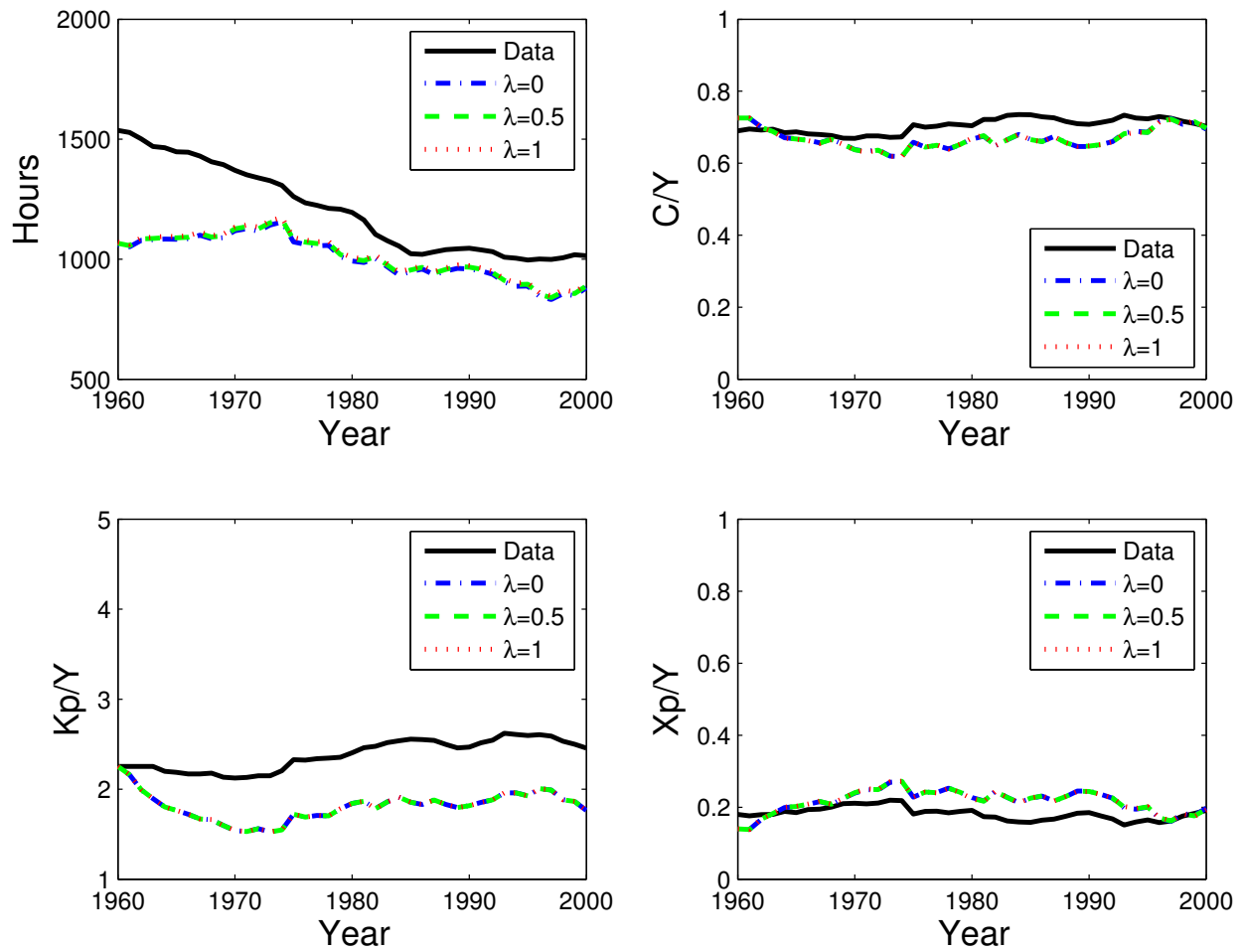


Figure 3.16: France - Values of λ

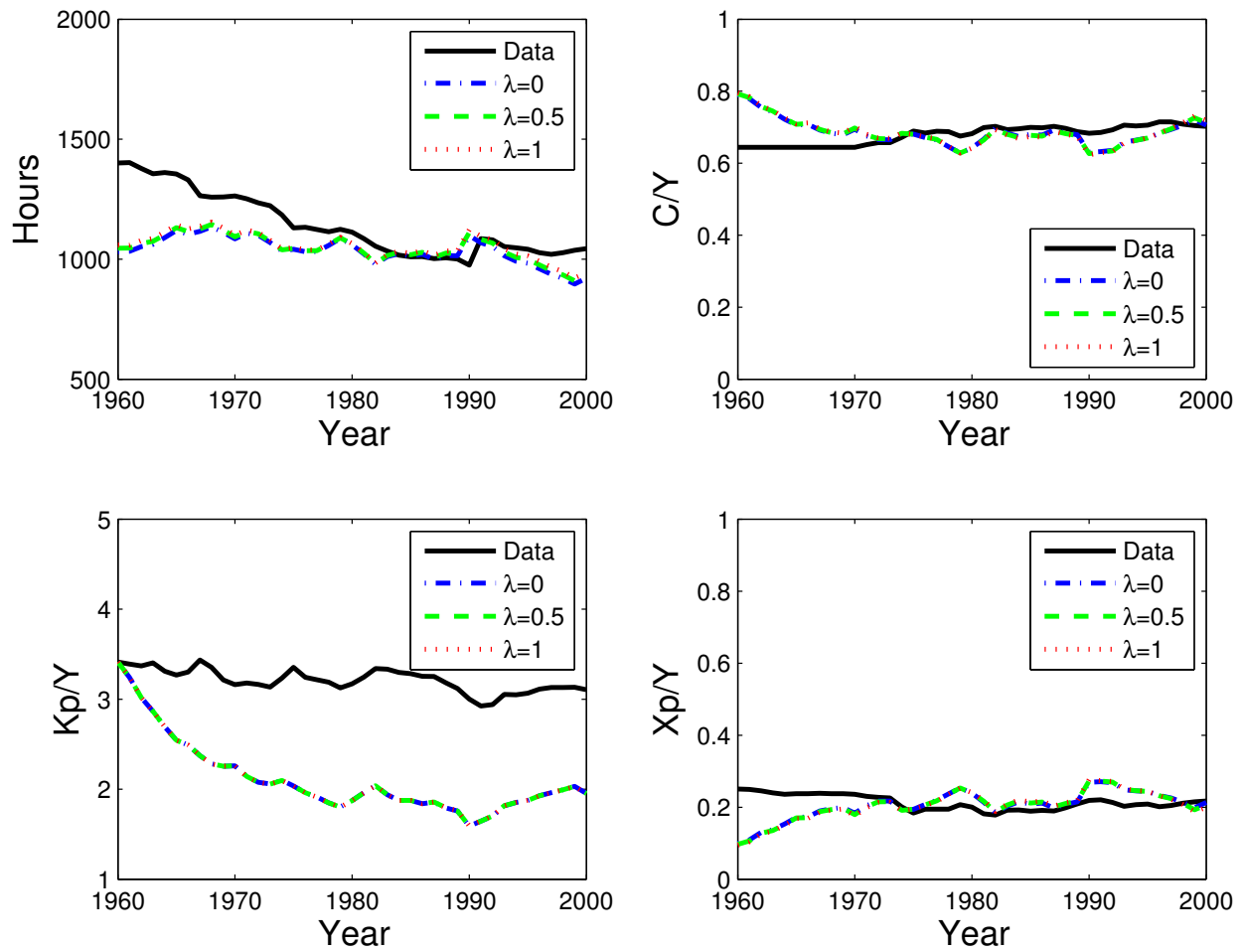


Figure 3.17: Germany - Values of λ

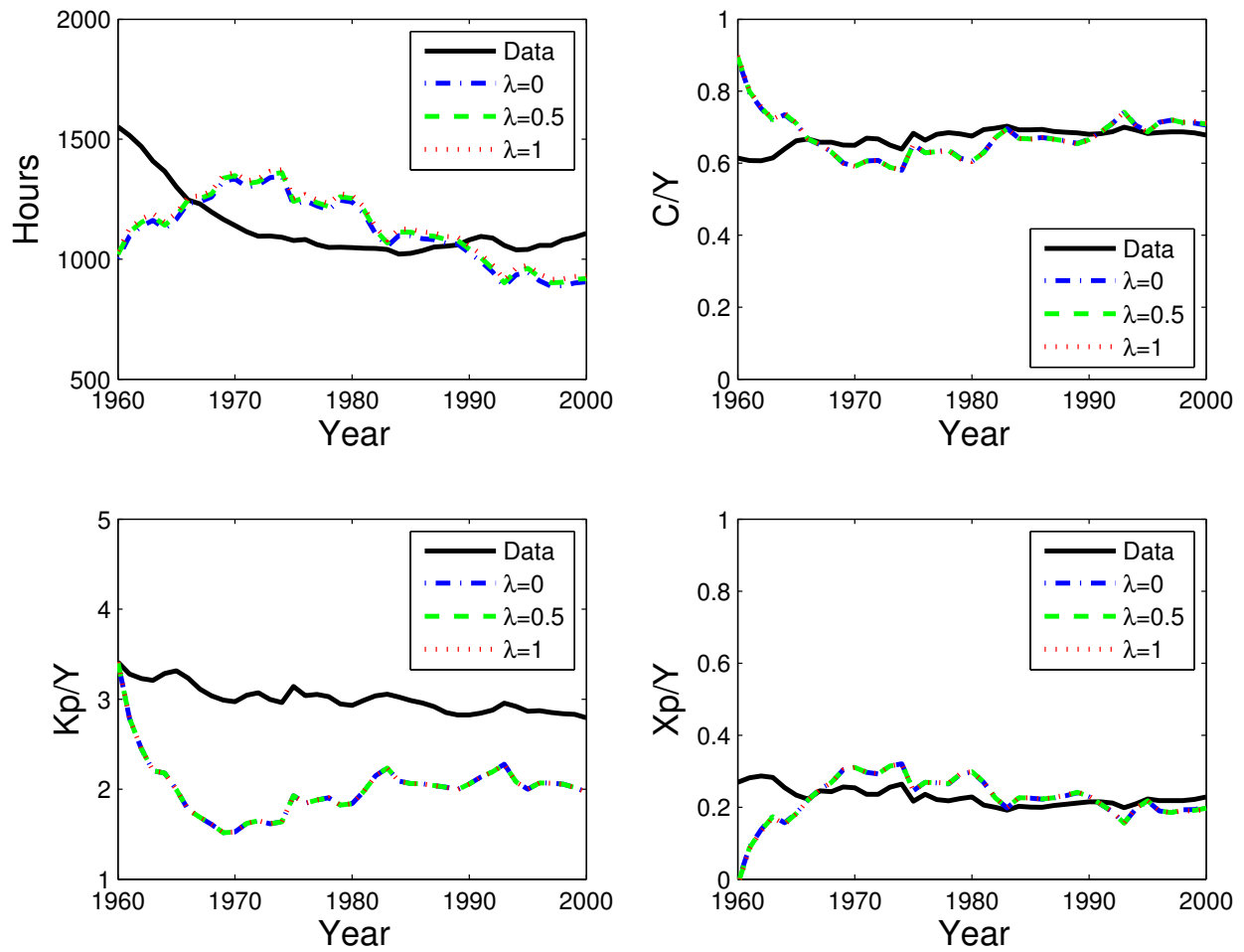


Figure 3.18: Italy - Values of λ

3.6.1.3 Robustness on σ

Figures 3.19-3.24 shows the effect of how we specify σ . The model-generated series of hours worked change significantly by different values of σ , while the changes in consumption, capital stock, and investment are only modest. Theoretically, a higher value of σ means that the workers react to the changes in real wages less elastically, so the case of $\sigma = 2$ exhibits a flatter path of hours, while the series fluctuate more when $\sigma = 0$ ²⁰

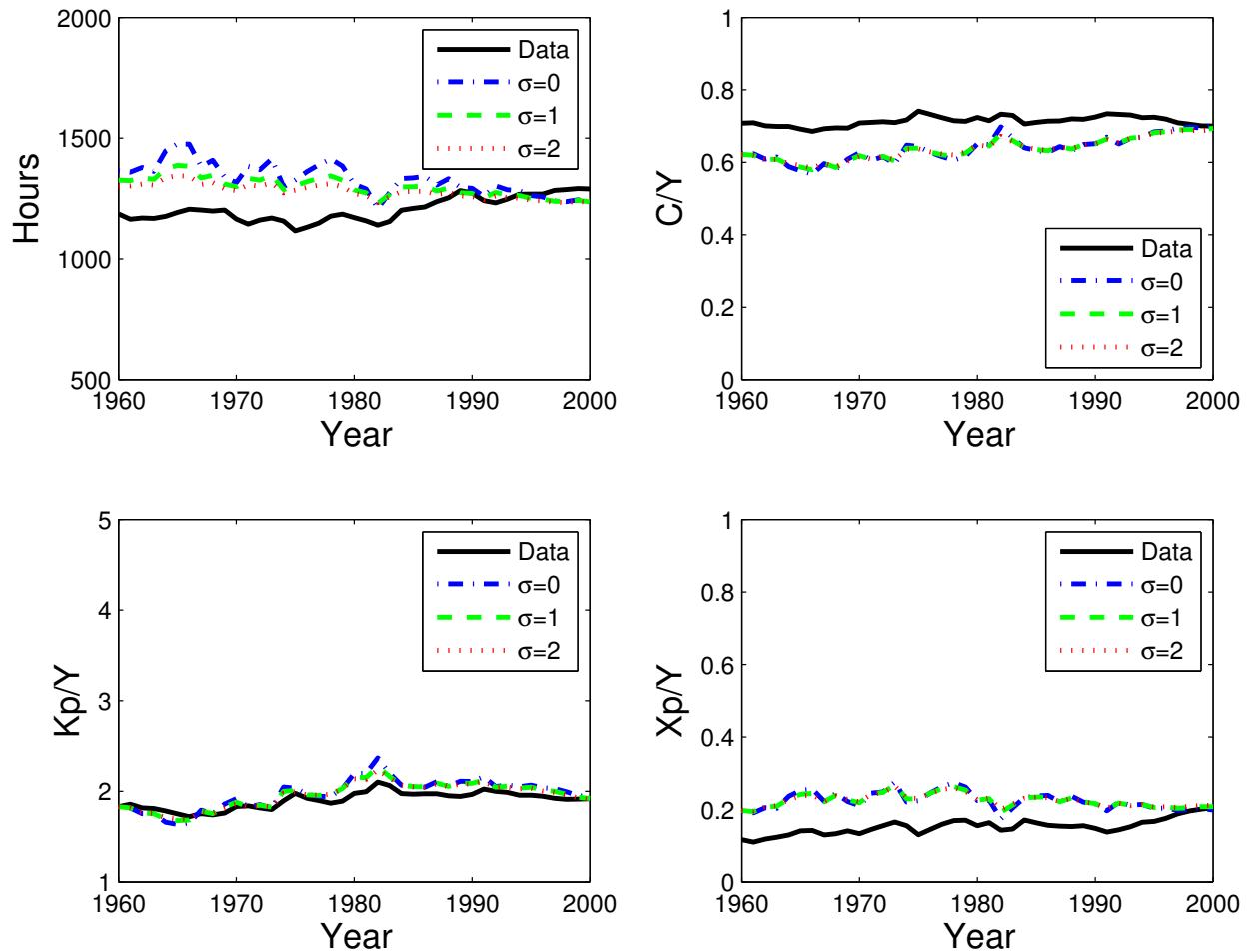


Figure 3.19: U.S. - Values of σ

²⁰This means the utility function is linear in hours and thus the labor supply is perfectly elastic. See Hansen (1985).

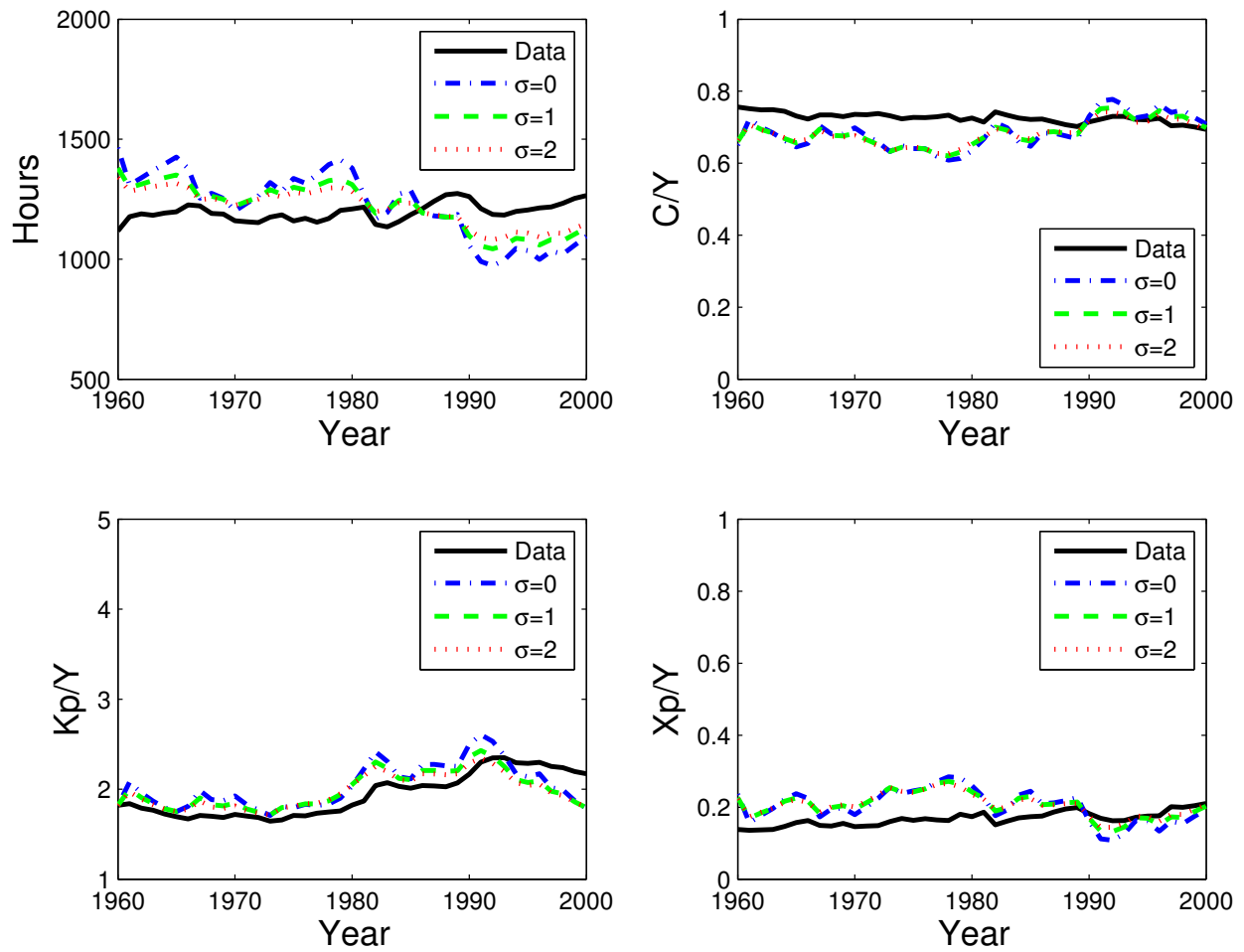


Figure 3.20: Canada - Values of σ

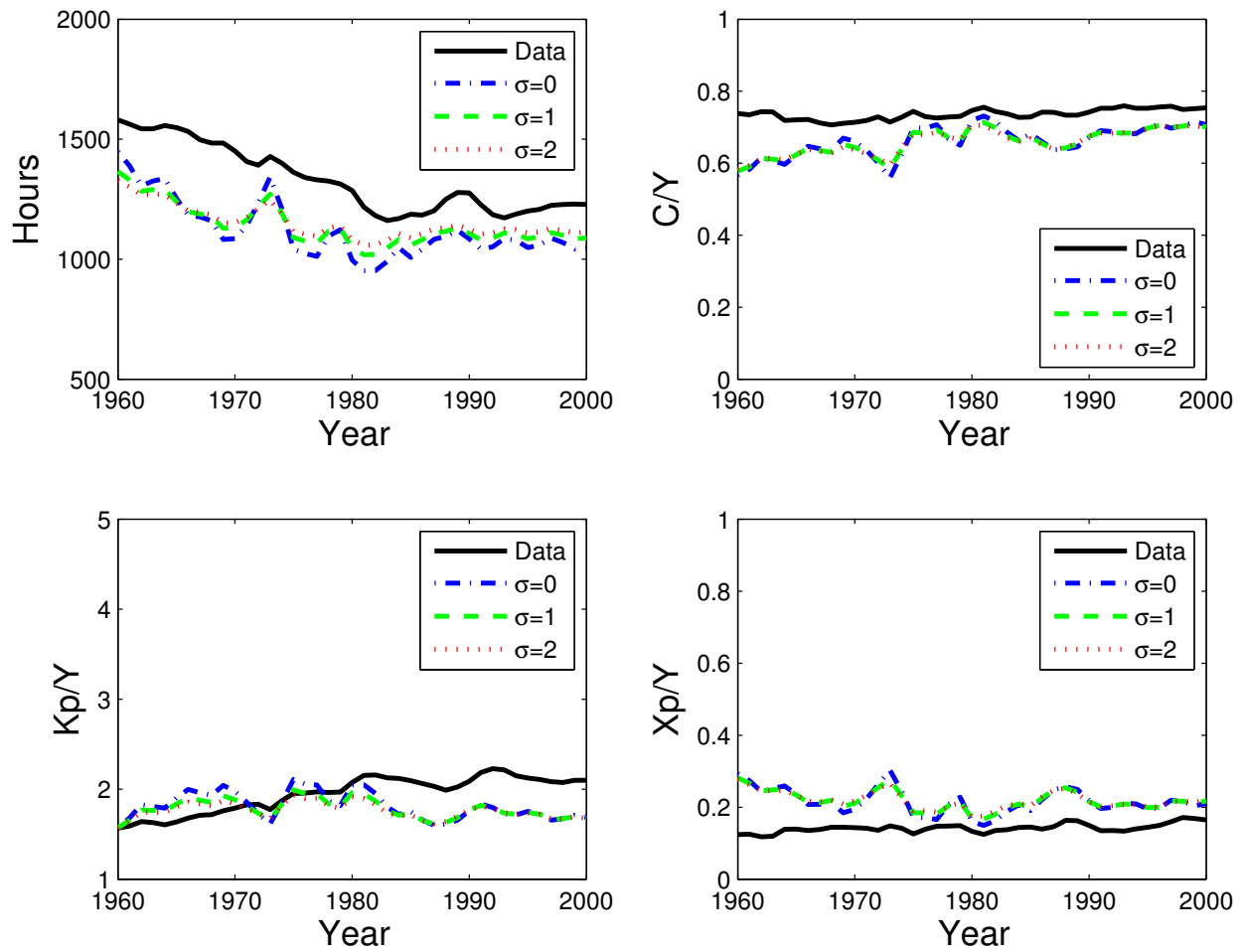


Figure 3.21: U.K. - Values of σ

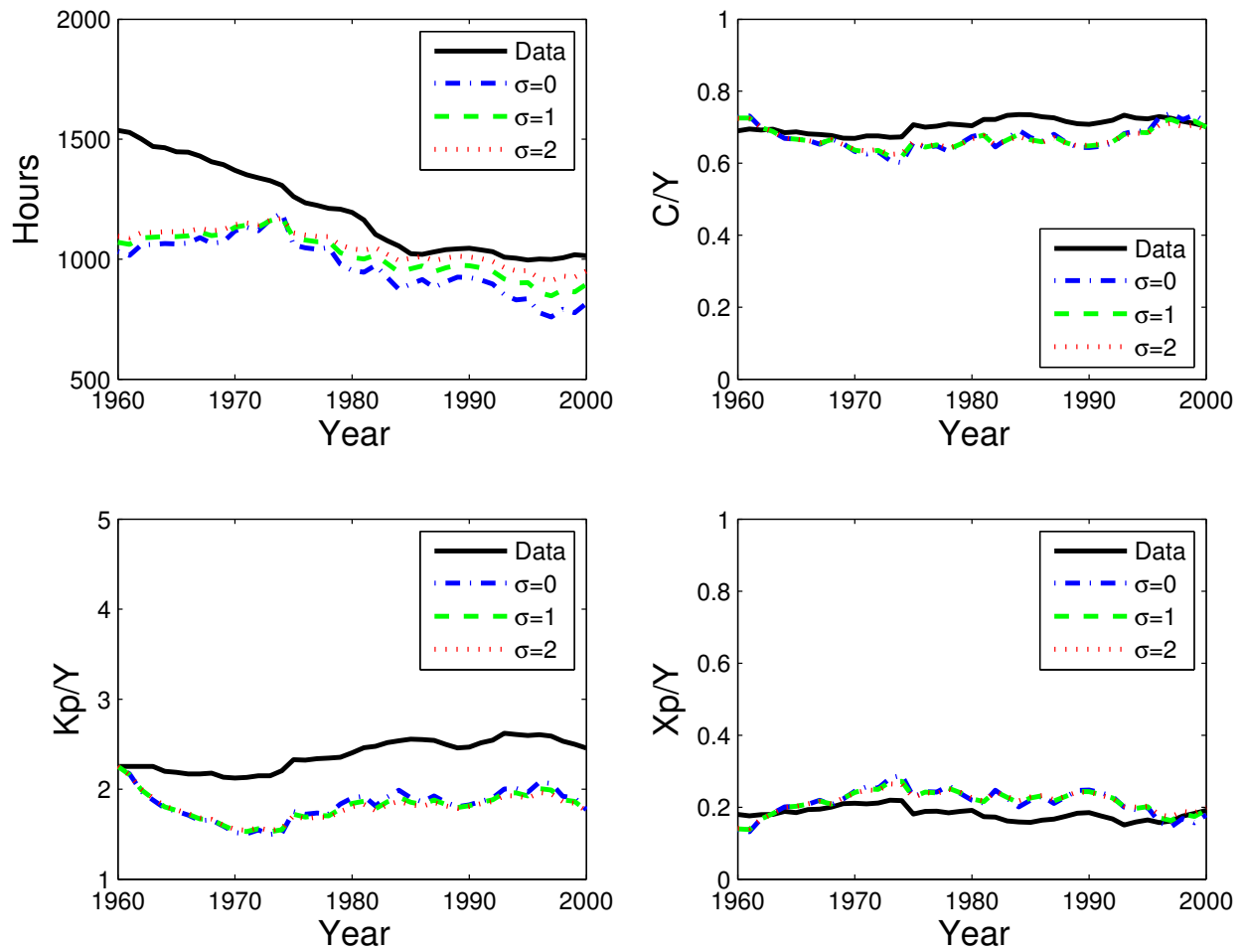


Figure 3.22: France - Values of σ

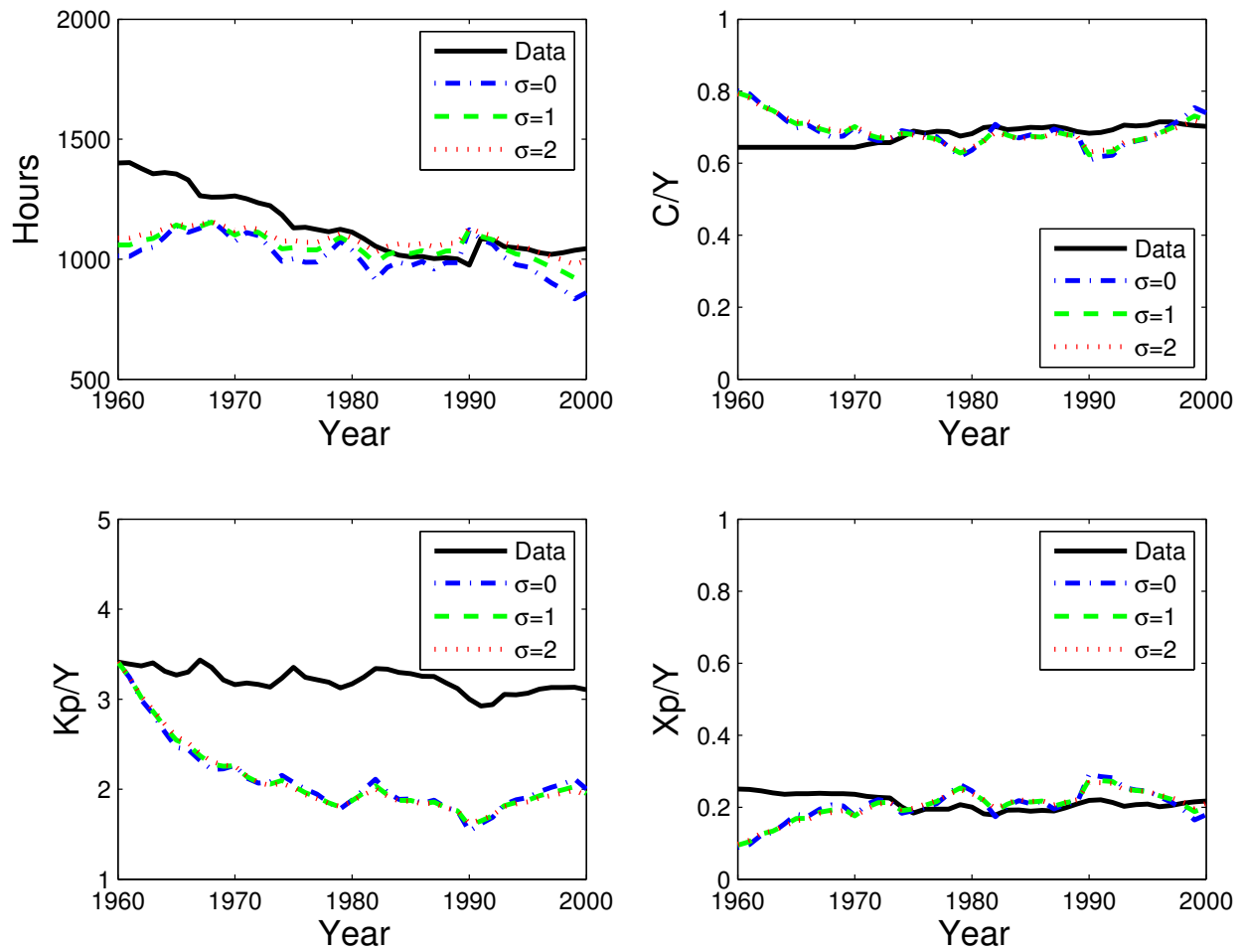


Figure 3.23: Germany - Values of σ

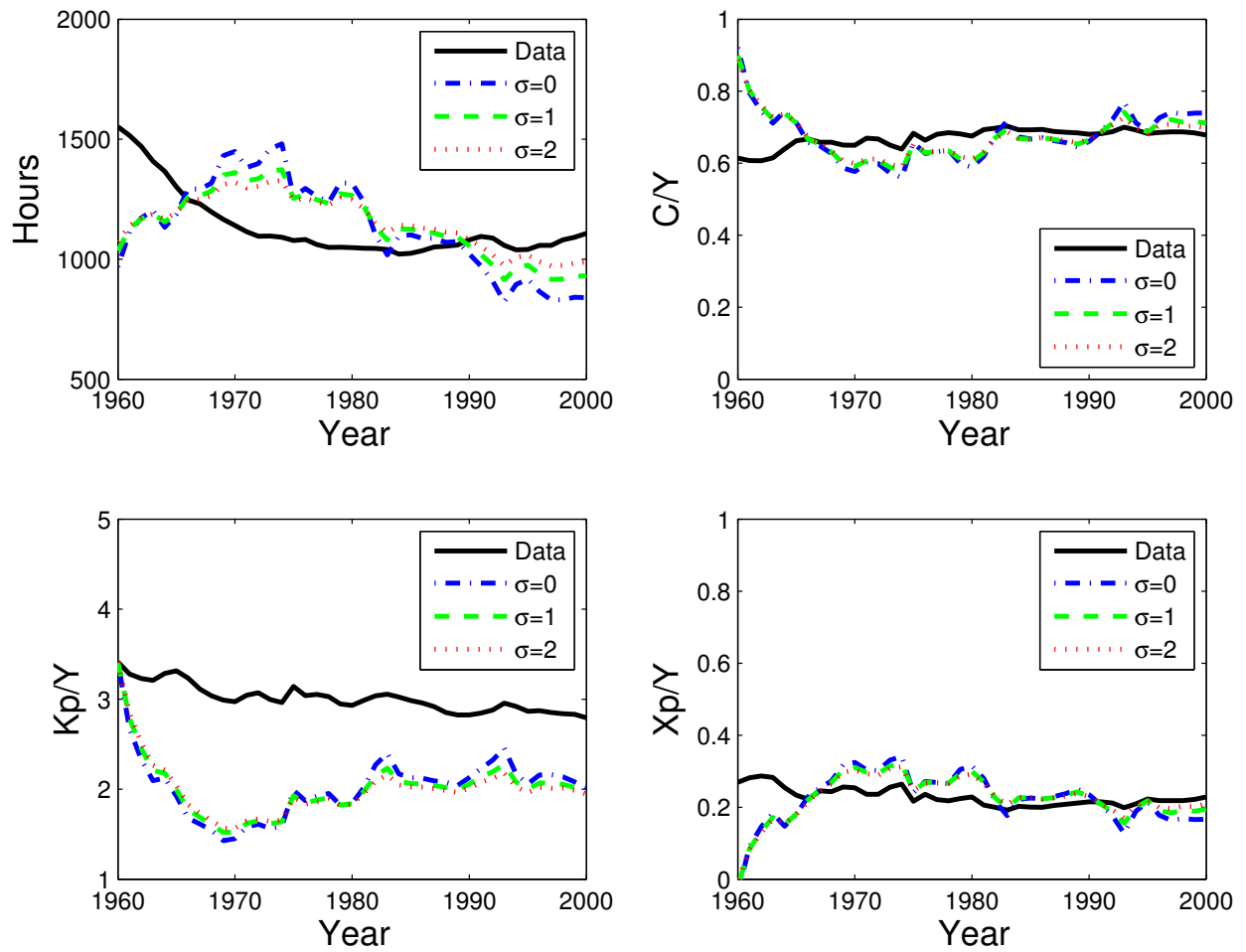


Figure 3.24: Italy - Values of σ

3.6.2 Full Results of all Sample Countries

This section displays the results of different models for the countries that are not shown in Section 3.4.2.

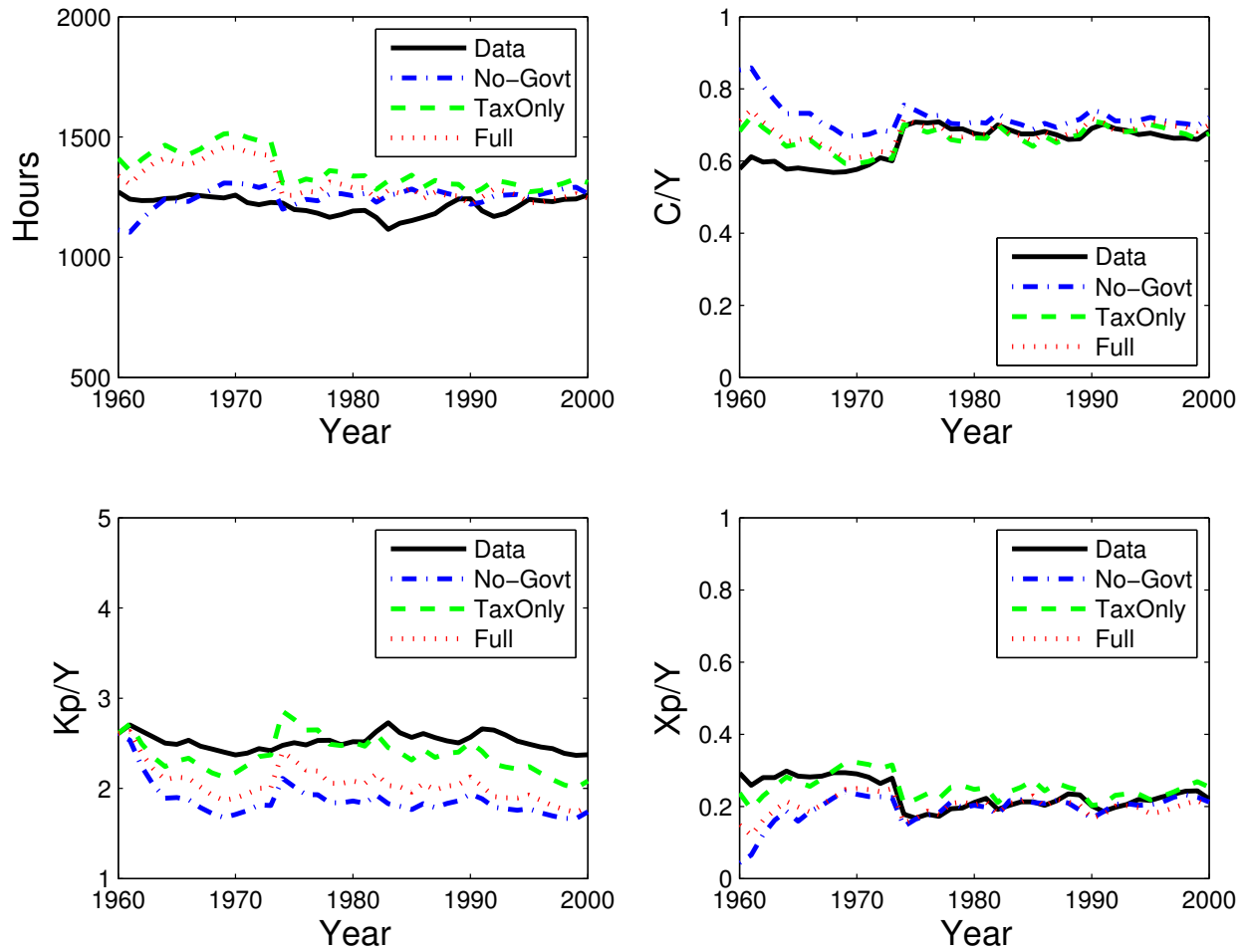


Figure 3.25: Australia - Model Comparison

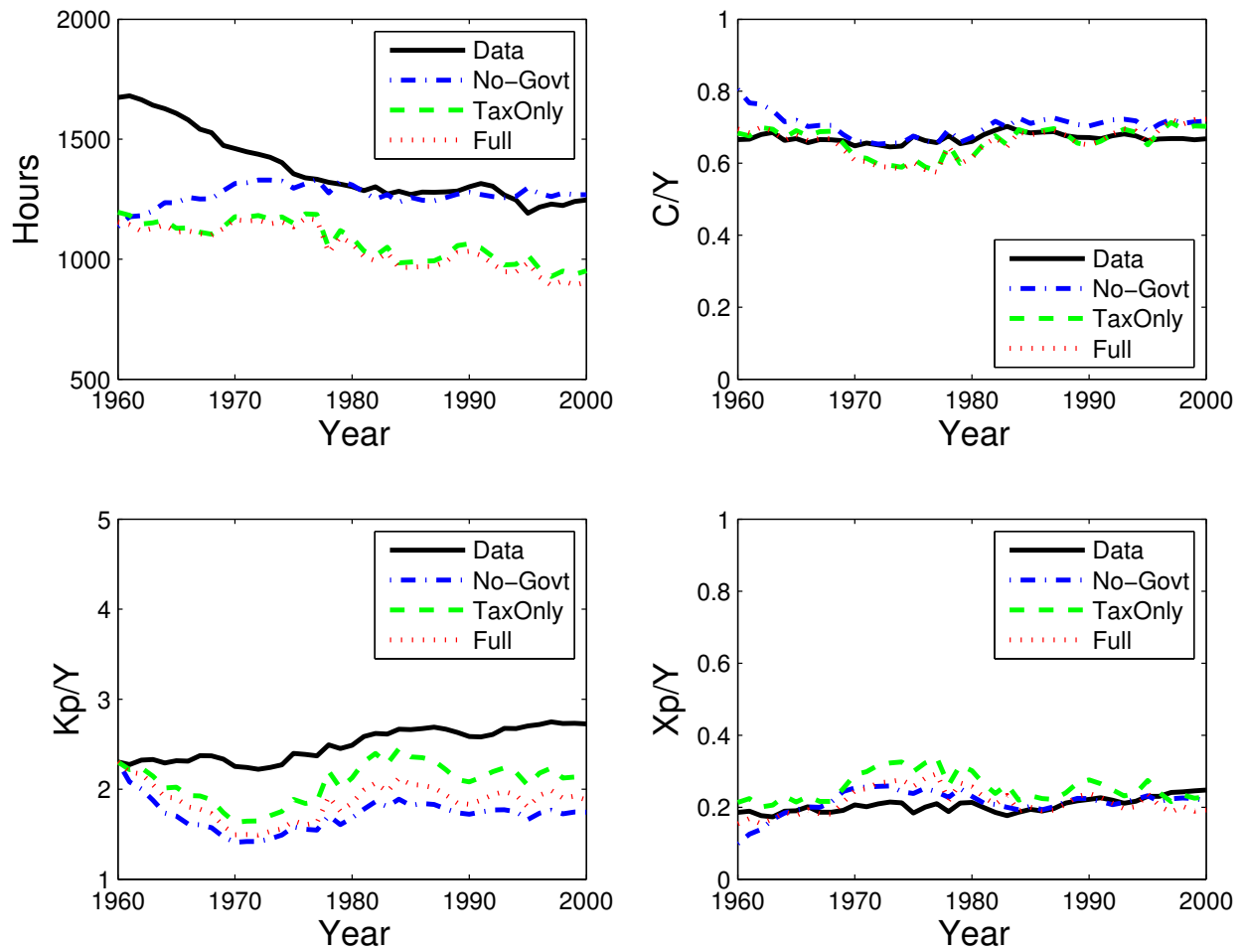


Figure 3.26: Austria - Model Comparison

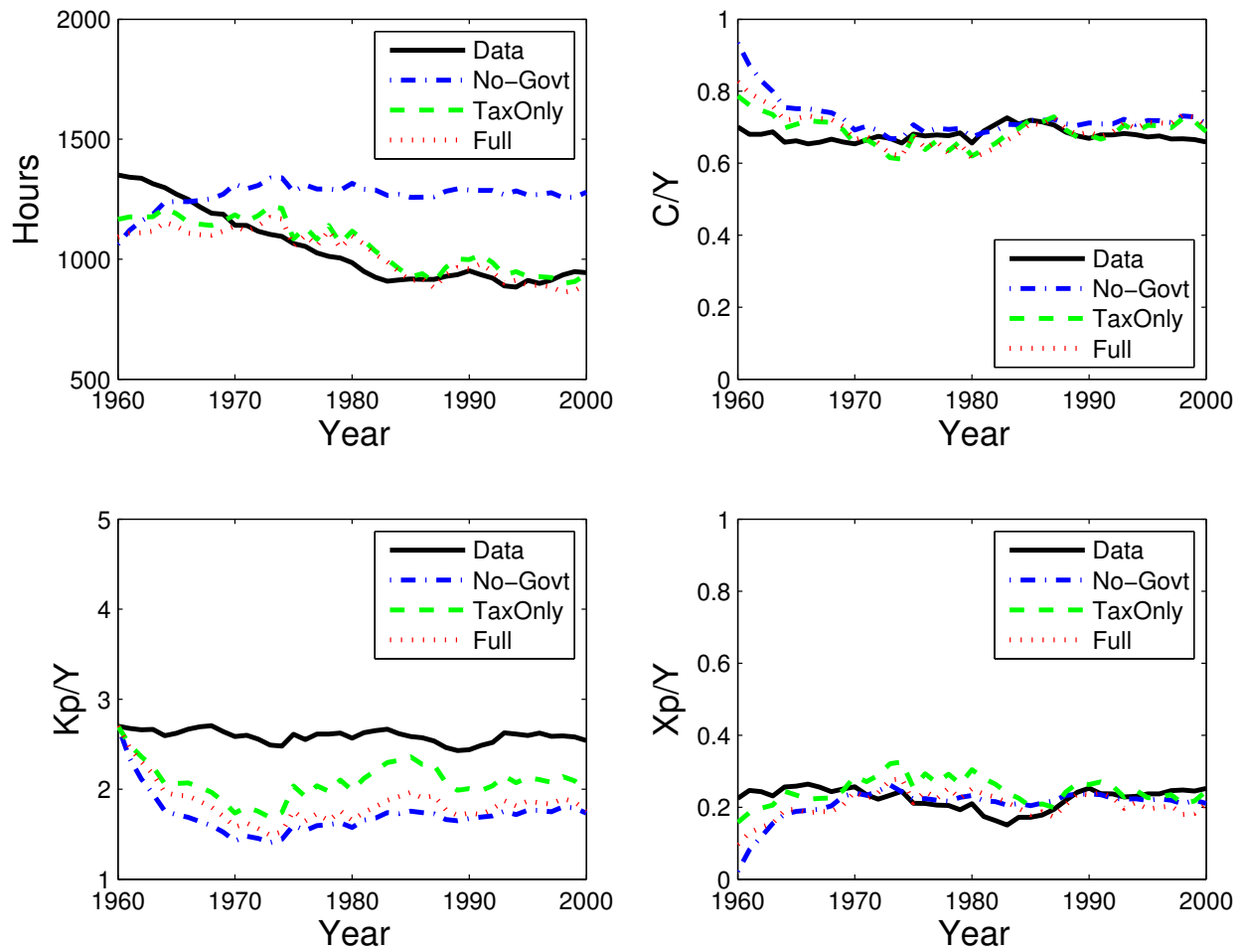


Figure 3.27: Belgium - Model Comparison

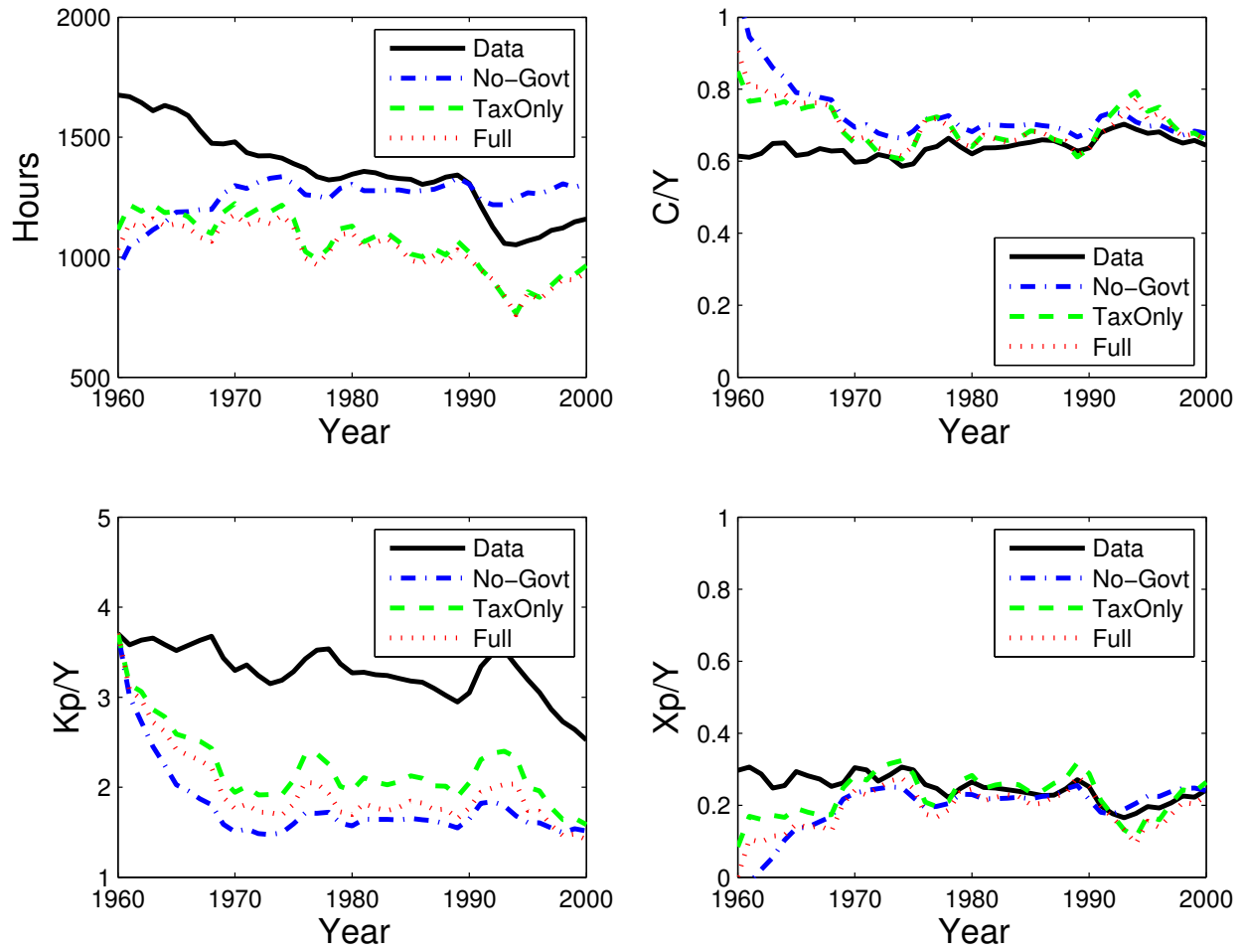


Figure 3.28: Finland - Model Comparison

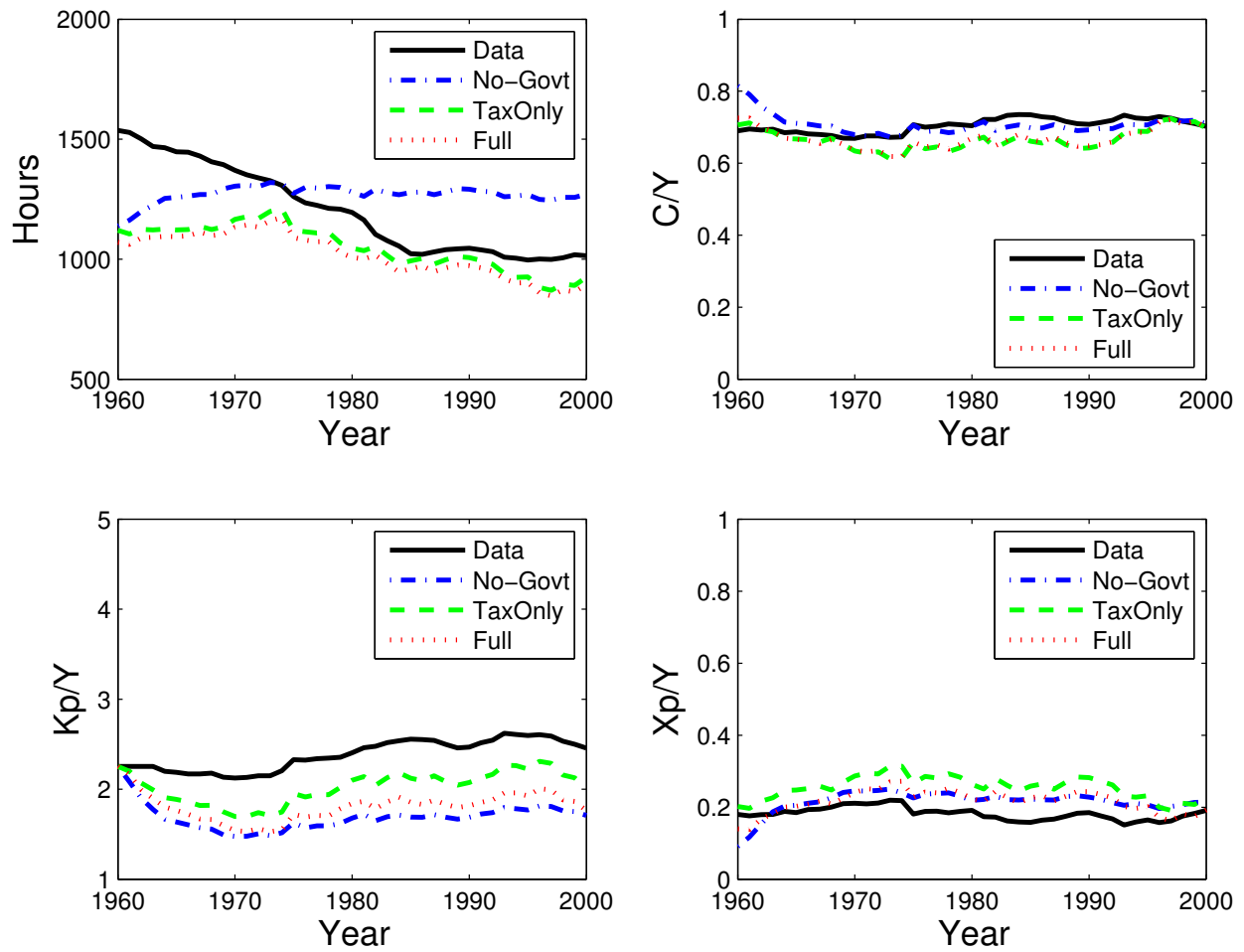


Figure 3.29: France - Model Comparison

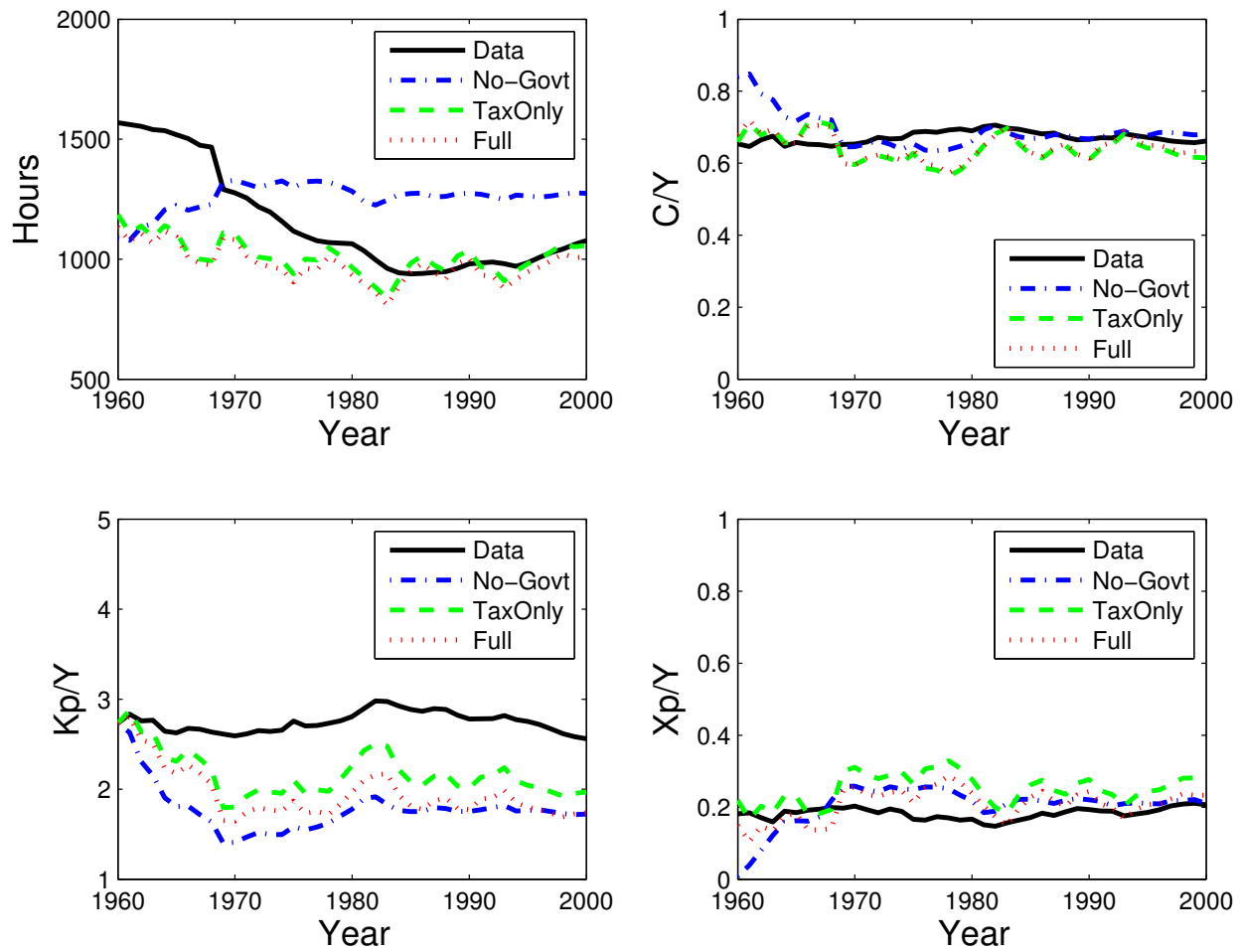


Figure 3.30: Netherlands - Model Comparison

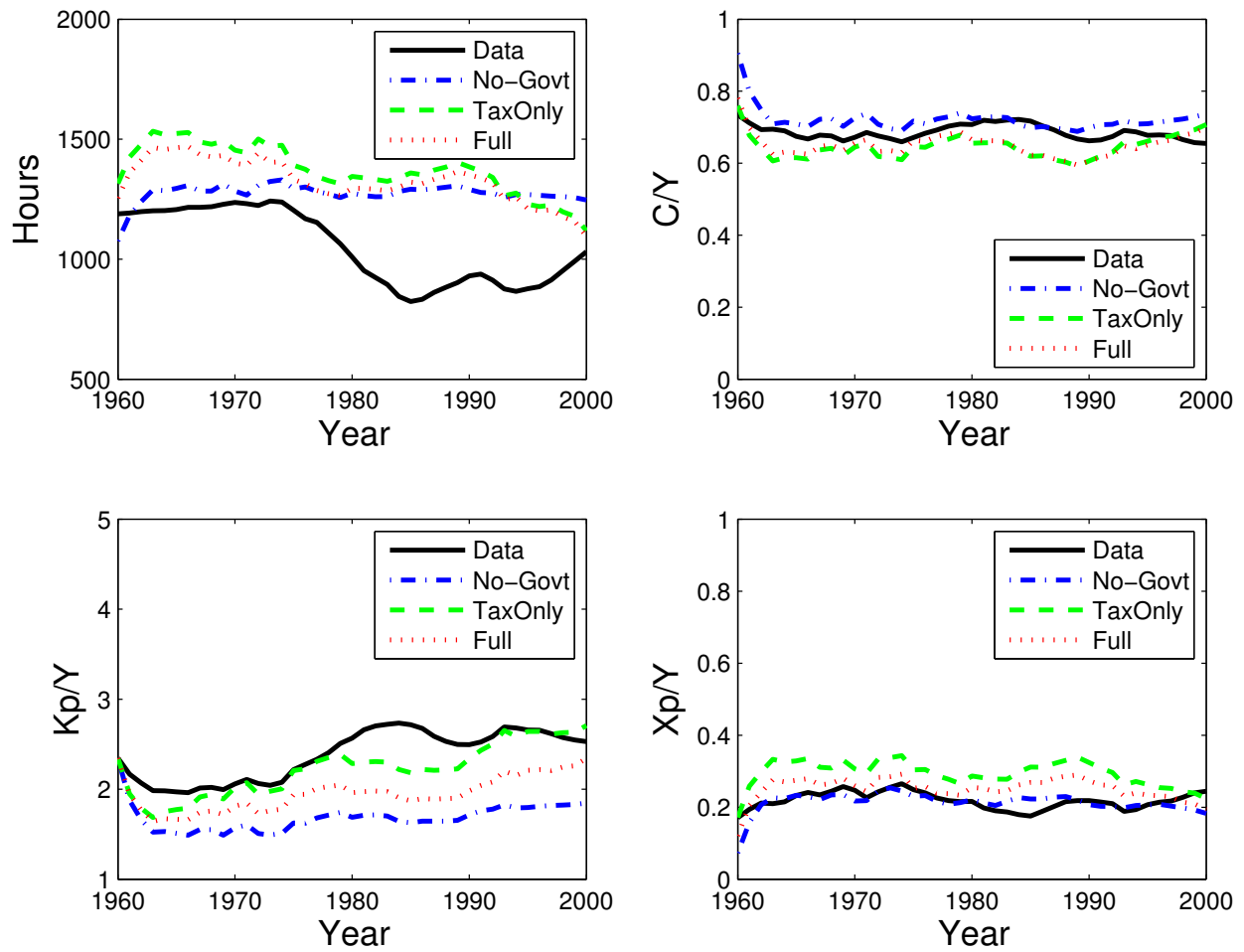


Figure 3.31: Spain - Model Comparison

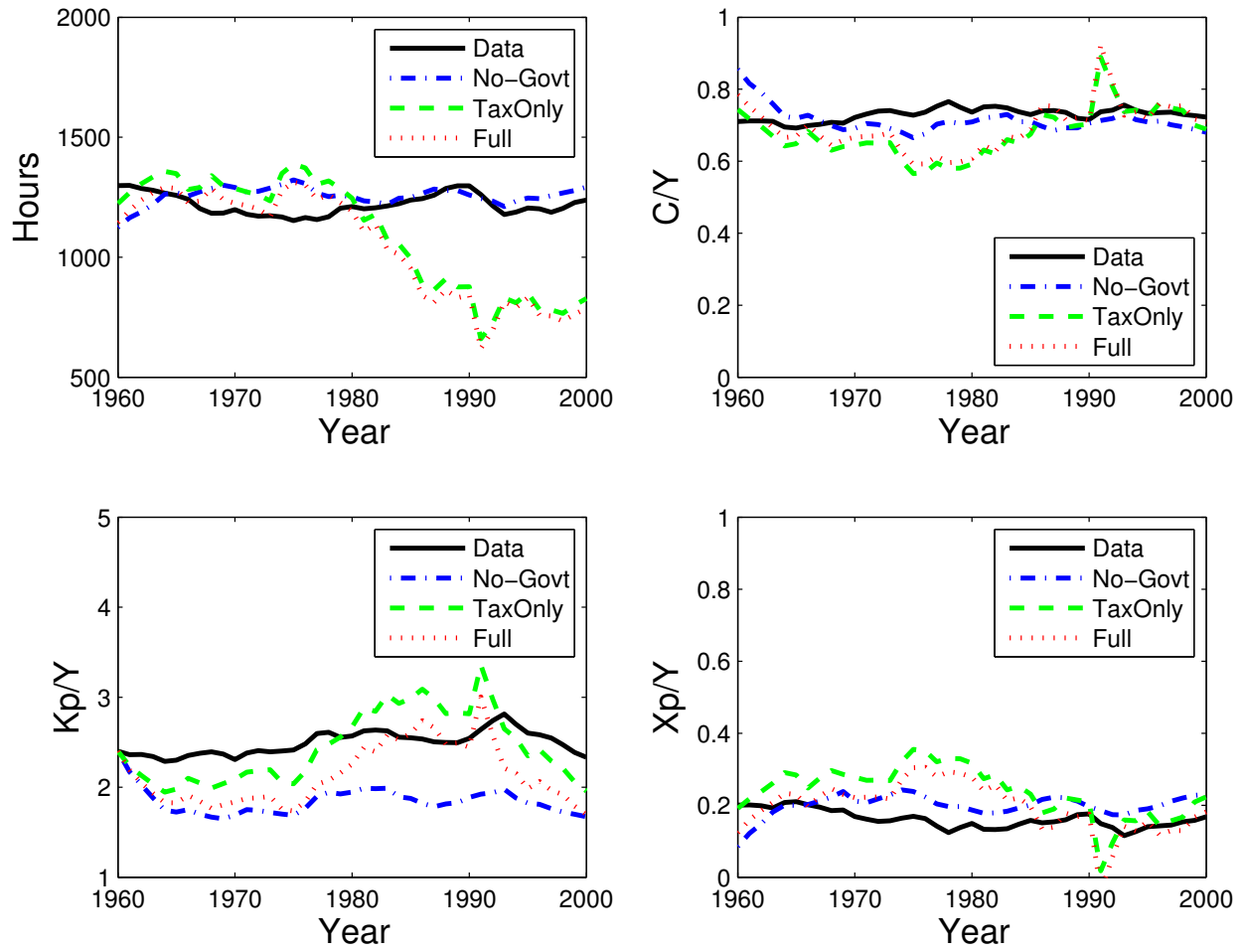


Figure 3.32: Sweden - Model Comparison

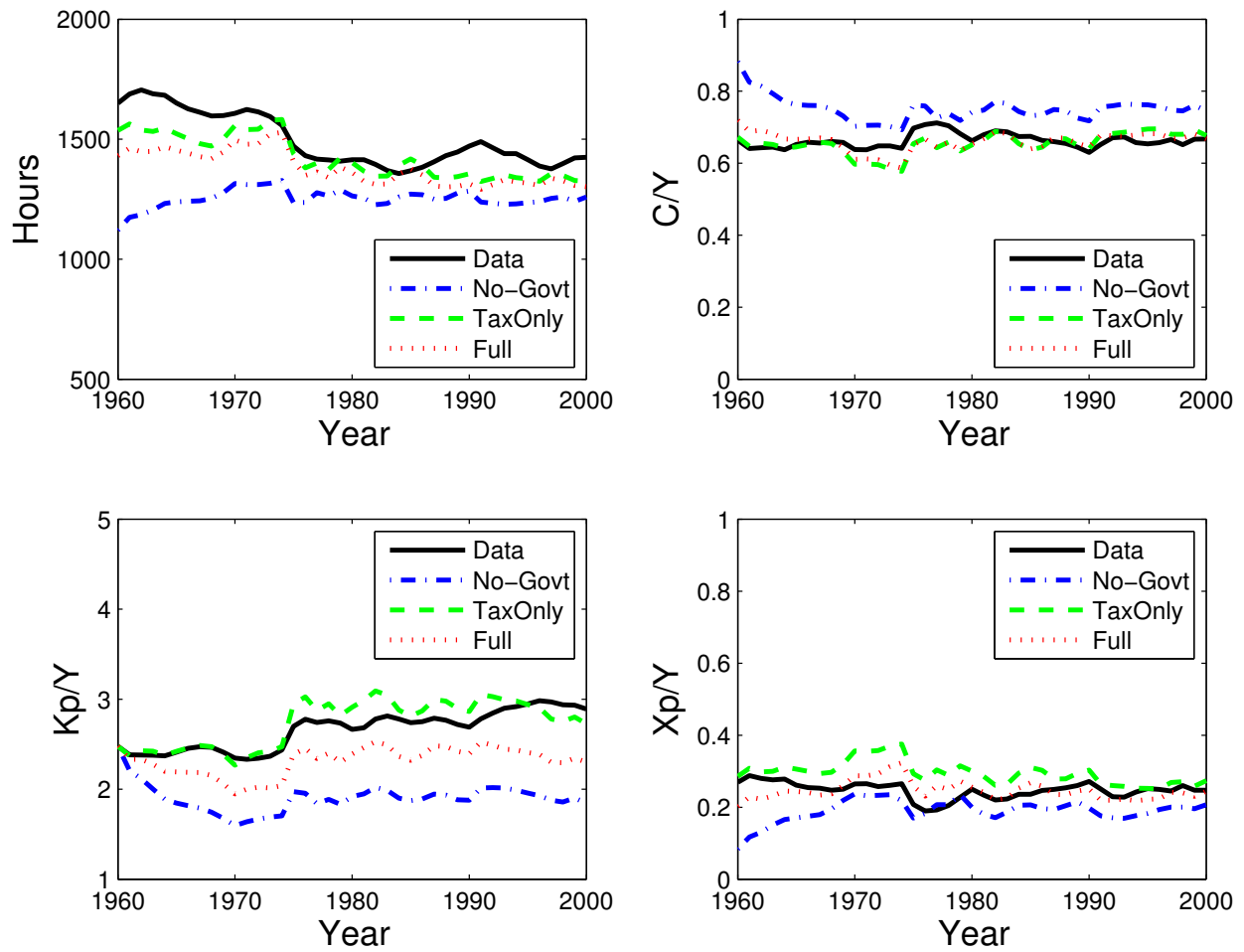


Figure 3.33: Switzerland - Model Comparison

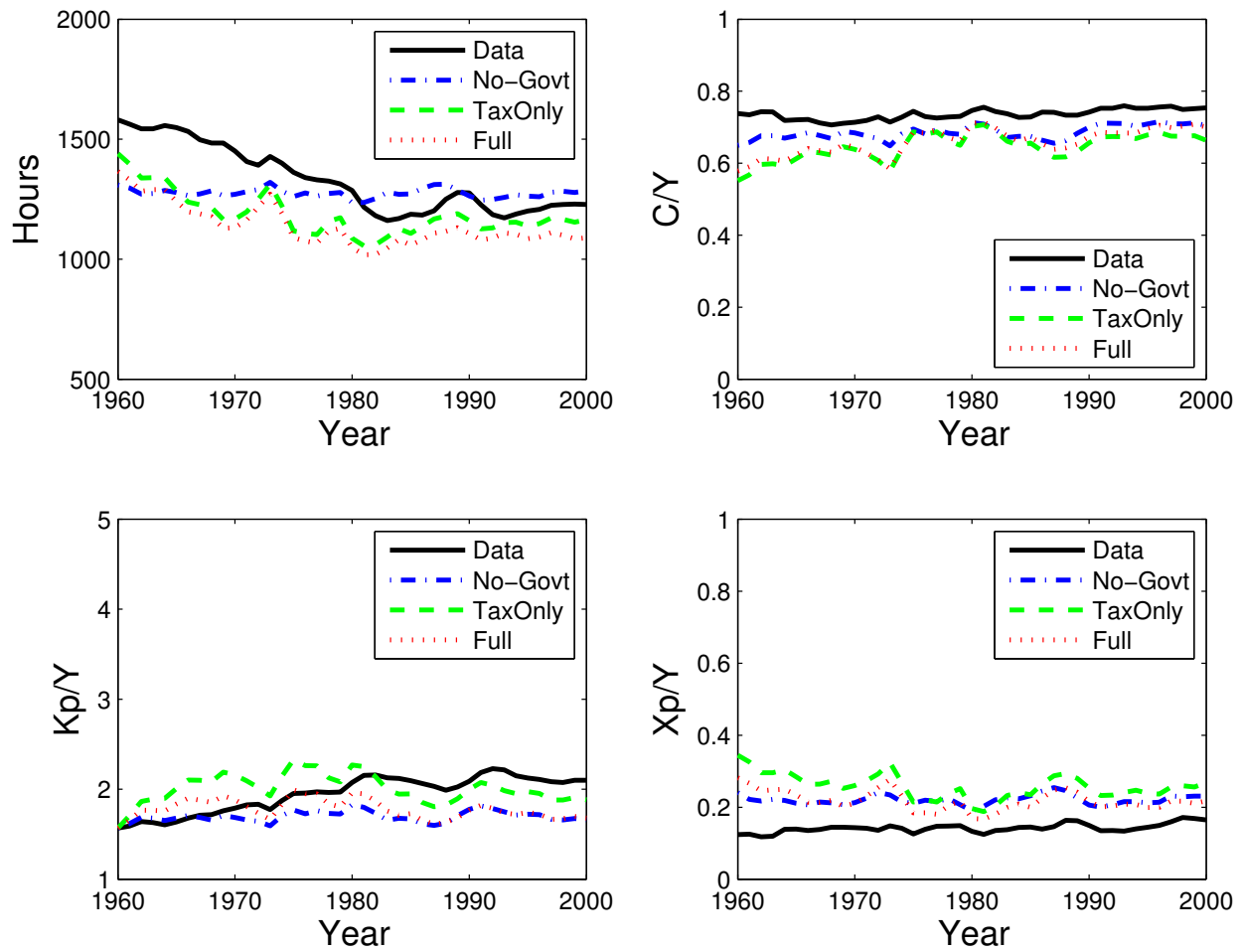


Figure 3.34: U.K. - Model Comparison

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