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Left-Right SU(4) Vector Leptoquark Model for Flavor Anomalies

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Building on our recent proposal to explain the experimental hints of new physics in B meson decays within the framework of Pati-Salam quark-lepton unification, through the interactions of the $(3,1)_{2/3}$ vector leptoquark, we construct a realistic model of this type based on the gauge group $\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{U}(1)'$ and consistent with all experimental constraints. The key feature of the model is that $\mathrm{SU}(4)_R$ is broken at a high scale, which suppresses right-handed lepton flavor changing currents at the low scale and evades the stringent bounds from searches for lepton flavor violation. The mass of the leptoquark can be as low as 10 TeV without the need to introduce mixing of quarks or leptons with new vector-like fermions. We provide a comprehensive list of model-independent bounds from low energy processes on the couplings in the effective Hamiltonian that arises from generic leptoquark interactions, and then apply these to the model presented here. We discuss various meson decay channels that can be used to probe the model and we investigate the prospects for discovering the new gauge boson at future colliders.

I. INTRODUCTION

The Standard Model (SM) provides a remarkably successful description of nature at the elementary particle level and, so far, there are only a handful of experimental indications of deviations from its predictions. Perhaps the most significant direct hint of physics beyond the SM are the recently observed anomalies in B meson decays [1, 2], which suggest that lepton universality might be violated. Assuming that those anomalies are not a result of experimental systematics, they are best accounted for by the vector leptoquark $(3,1)_{2/3}$ or $(3,3)_{2/3}$ [3]. However, building viable UV complete models involving those particles is challenging, especially in light of very stringent constraints on lepton flavor violation (LFV) from various experimental searches.

The first attempt to construct a vector leptoquark model for the $R_{K^{(*)}}$ anomalies was made in [4], where we proposed that the vector leptoquark $(3,1)_{2/3}$ explaining the anomalies might be the gauge boson of a theory with Pati-Salam unification. The conclusion was that the minimal model based on $\mathrm{SU}(4) \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ is not capable of this because of strict bounds on kaon and B meson rare decays [5–10]. The underlying problem in that model arises from the interference between left-handed (LH) and right-handed (RH) lepton flavor changing currents. We outlined a possible solution to this: extending the gauge group to $\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{U}(1)'$ and breaking $\mathrm{SU}(4)_R$ at a high scale, such that the RH lepton flavor changing currents are suppressed.

A viable realization of this idea is the subject of this paper. We demonstrate that a Pati-Salam gauge leptoquark as light as 10 TeV can explain the $R_{K^{(*)}}$ anomalies and remain consistent with all experimental bounds without introducing any mixing of quarks and leptons with new fermions. We discuss in detail the constraints arising from LFV searches and show that the absence of RH lepton flavor changing currents relaxes the bounds considerably. The model is expected to have clean signatures at future colliders, which we investigate in the case of the prospective 100 TeV machine.

Several other models for the flavor anomalies based on Pati-Salam unification have been proposed, some appearing almost immediately after our initial work [11–17]. Those models overcome the experimental constraints by mixing all or a subset of SM quarks and leptons with new vector-like fermions. Other approaches to account for the B meson decay anomalies involving scalar leptoquarks or Z' rather than vector leptoquarks have been also proposed (see, e.g. [18–24]).

In App. C we provide a model-independent analysis of the low energy consequences of a $(3,1)_{2/3}$ vector leptoquark that interacts with both LH and RH fields. We present an extensive list of bounds from flavor physics on generic coupling constants in this model-independent approach; App. C is thus a resource in its own right, of use to researchers interested in any specific model of this type. Appendix D is one such example, where we apply the results of App. C to the specific Pati-Salam model constructed in this work. The calculations in App. C update and extend previous results [5–10]. For instance, for B decays we use the most recent lattice results for the form factors [25], which weaken the bounds considerably compared to assuming the nonphysical values $f_+ = f_0 = 1$ adopted previously in the literature.

II. THE MODEL

The theory we propose is based on the gauge group

$$\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R \times \mathrm{SU}(2)_L \times \mathrm{U}(1)' \,. \tag{1}$$

The crucial feature of the model is that the subgroup $\mathrm{SU}(4)_R$ is broken at a much higher scale than $\mathrm{SU}(4)_L$, leading to a suppression of RH lepton flavor changing currents.

Fermion particle content

The matter fields in the model, along with their decomposition into $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ multiplets, are

$$\begin{aligned}
\hat{\Psi}_L &= (4,1,2,0) \;=\; (3,2)_{\frac{1}{6}} \oplus (1,2)_{-\frac{1}{2}} \,, \\
\hat{\Psi}^u_R &= (1,4,1,\tfrac{1}{2}) \;=\; (3,1)_{\frac{2}{3}} \oplus (1,1)_0 \,, \\
\hat{\Psi}^d_R &= (1,4,1,-\tfrac{1}{2}) \;=\; (3,1)_{-\frac{1}{3}} \oplus (1,1)_{-1} \,,
\end{aligned} \tag{2}$$

$$\hat{\chi}_L = (\bar{4},1,2,0) \;=\; (\bar{3},2)_{-\frac{1}{6}} \oplus (1,2)_{\frac{1}{2}}\,,$$
$$\hat{\chi}_R = (1,\bar{4},2,0) \;=\; (\bar{3},2)_{-\frac{1}{6}} \oplus (1,2)_{\frac{1}{2}}$$

for each generation, where $\hat{\Psi}_L$, $\hat{\Psi}^u_R$, $\hat{\Psi}^d_R$ contain the SM fields Q_L, L_L, u_R, d_R, e_R and a RH neutrino ν_R, whereas $\hat{\chi}_L$, $\hat{\chi}_R$ assure gauge anomaly cancellation and result in two vector-like pairs of fields Q'_L, Q'_R and L'_L, L'_R that are heavy and do not mix with SM fermions. This is the minimal fermion content for a consistent theory based on the gauge group (1).

Scalar sector and symmetry breaking

The Higgs sector contains the scalar representations

$$\hat{\Sigma}_L = (4,1,1,\tfrac{1}{2})\,,\quad \hat{\Sigma}_R = (1,4,1,\tfrac{1}{2})\,,\quad \hat{\Sigma} = (\bar{4},4,1,0)\,,$$
$$\hat{H}_d = (4,\bar{4},2,\tfrac{1}{2})\,,\quad \hat{H}_u = (4,\bar{4},2,-\tfrac{1}{2})\,. \tag{3}$$

The scalar potential is given in App. A. The parameters can be chosen such that the fields $\hat{\Sigma}_L$, $\hat{\Sigma}_R$ and $\hat{\Sigma}$ develop the vacuum expectation values (vevs),

$$\langle\hat{\Sigma}_L\rangle = \frac{v_L}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix},\quad \langle\hat{\Sigma}_R\rangle = \frac{v_R}{\sqrt{2}}\begin{pmatrix}0\\0\\0\\1\end{pmatrix},$$
$$\langle\hat{\Sigma}\rangle = \frac{v_\Sigma}{\sqrt{2}}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&z\end{pmatrix}, \tag{4}$$

where $z>0$. This results in the symmetry breaking pattern

$$\begin{aligned}&\mathrm{SU}(4)_L\times\mathrm{SU}(4)_R\times\mathrm{SU}(2)_L\times\mathrm{U}(1)'\\ &\quad\to\;\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\,.\end{aligned} \tag{5}$$

The relation between the SM hypercharge Y and the $\mathrm{U}(1)'$ charge Y' is given by

$$Y = Y' + \sqrt{\frac{2}{3}}\left(T^{15}_L + T^{15}_R\right)\,, \tag{6}$$

where

$$T^{15}_L = T^{15}_R = \frac{1}{2\sqrt{6}}\,\mathrm{diag}(1,1,1,-3)\,. \tag{7}$$

The scalar representations decompose into SM fields as

$$\begin{aligned}\hat{\Sigma}_L &= (3,1)_{\frac{2}{3}} \oplus (1,1)_0\,,\;\; \hat{\Sigma}_R = (3,1)_{\frac{2}{3}} \oplus (1,1)_0\,,\\ \hat{\Sigma} &= (8,1)_0 \oplus (3,1)_{\frac{2}{3}} \oplus (\bar{3},1)_{-\frac{2}{3}} \oplus 2\,(1,1)_0\,,\\ \hat{H}_d &= (8,2)_{\frac{1}{2}} \oplus (3,2)_{\frac{7}{6}} \oplus (\bar{3},2)_{-\frac{1}{6}} \oplus 2\,(1,2)_{\frac{1}{2}}\\ &\equiv O_1 \oplus T_1 \oplus T_2^\dagger \oplus S_1 \oplus S_2\,,\\ \hat{H}_u &= (8,2)_{-\frac{1}{2}} \oplus (3,2)_{\frac{1}{6}} \oplus (\bar{3},2)_{-\frac{7}{6}} \oplus 2\,(1,2)_{-\frac{1}{2}}\\ &\equiv O_2 \oplus T_3 \oplus T_4^\dagger \oplus S_3^* \oplus S_4^*\,.\end{aligned} \tag{8}$$

Under the symmetry breaking pattern (5) the $\hat{H}_d$, $\hat{H}_u$ fields have $(\bar{4},4)\to(\bar{3}\oplus1)\otimes(3\oplus1)$; S_1, S_3 stand for the singlet in $1\otimes1$, while S_2, S_4 are the singlets in $\bar{3}\otimes3$. The components of $\hat{\Sigma}_R$, $\hat{\Sigma}_L$, $\hat{\Sigma}$ have masses on the order of the $\mathrm{SU}(4)_R$ and $\mathrm{SU}(4)_L$ breaking scales. This is also the natural mass scale for the components of $\hat{H}_d$ and $\hat{H}_u$. However, as shown in App. B, it is possible to fine-tune the parameters of the potential such that only one linear combination of the fields $S_{1,2,3,4}$ is light. In particular, there exists a choice of parameters for which the light state is given by

$$H = -\,c_e\,S_1 - c_d\,S_2 + c_\nu\,S_3 + c_u\,S_4\,, \tag{9}$$

where $c_u \approx 1 \gg c_d \gg c_\nu$ and $1 \gg c_e \gg c_\nu$, with the ratio $c_d : c_e \approx m_b : m_\tau$. This reduces the scalar sector of the model to that of the SM at low energies.

Gauge sector

The gauge and kinetic terms are

$$\begin{aligned}\mathcal{L}_{\mathrm{g+k}} = &-\tfrac{1}{4}G^A_{L\mu\nu}G^{A\mu\nu}_L - \tfrac{1}{4}G^A_{R\mu\nu}G^{A\mu\nu}_R - \tfrac{1}{4}W^a_{\mu\nu}W^{a\mu\nu}\\ &-\tfrac{1}{4}Y'_{\mu\nu}Y'^{\mu\nu} + |D_\mu\hat{\Sigma}_L|^2 + |D_\mu\hat{\Sigma}_R|^2\\ &+|D_\mu\hat{\Sigma}|^2 + |D_\mu\hat{H}_d|^2 + |D_\mu\hat{H}_u|^2\\ &+\overline{\hat{\Psi}}_L i\not{D}\,\hat{\Psi}_L + \overline{\hat{\Psi}}^u_R\, i\not{D}\,\hat{\Psi}^u_R + \overline{\hat{\Psi}}^d_R\, i\not{D}\,\hat{\Psi}^d_R\,,\end{aligned} \tag{10}$$

with $A = 1,...,15$ and $a = 1,2,3$. The gauge covariant derivative takes the form

$$\begin{aligned}D_\mu = &\;\partial_\mu + ig_L\,G^A_{L\mu}T^A_L + ig_R\,G^A_{R\mu}T^A_R\\ &+ ig_2\,W^a_\mu\,t^a + ig'_1\,Y'_\mu\,Y'\,,\end{aligned} \tag{11}$$

where T^A_L, T^A_R, t^a, Y' are the $\mathrm{SU}(4)_L$, $\mathrm{SU}(4)_R$, $\mathrm{SU}(2)_L$, $\mathrm{U}(1)'$ generators. The gauge couplings at the low scale are related to the SM strong and hypercharge couplings via

$$g_s = \frac{g_L\,g_R}{\sqrt{g_L^2+g_R^2}}\,,\quad g_1 = \frac{g'_1 g_L g_R}{\sqrt{\frac{2}{3}g_1'^2(g_R^2+g_L^2)+g_L^2\,g_R^2}}\,. \tag{12}$$

The new gauge bosons are

$$X_L = (3,1)_{\frac{2}{3}}\,,\quad X_R = (3,1)_{\frac{2}{3}}\,,\quad G' = (8,1)_0\,,$$
$$Z'_L = (1,1)_0\,,\quad Z'_R = (1,1)_0\,. \tag{13}$$

The mass of G' is $M_{G'} = \frac{1}{\sqrt{2}}\sqrt{g_L^2+g_R^2}\,v_\Sigma$. The squared mass matrix for the gauge leptoquarks X_L, X_R is

$$\mathcal{M}^2_X = \tfrac{1}{4}\begin{pmatrix} g_L^2\left[v_L^2+v_\Sigma^2(1+z^2)\right] & -\,2\,g_L g_R v_\Sigma^2 z\\ -\,2\,g_L g_R v_\Sigma^2 z & g_R^2\left[v_R^2+v_\Sigma^2(1+z^2)\right]\end{pmatrix}. \tag{14}$$

The leptoquark mass eigenstates can be written as

$$\begin{pmatrix}X_1\\X_2\end{pmatrix} = \begin{pmatrix}\cos\theta_4 & \sin\theta_4\\ -\sin\theta_4 & \cos\theta_4\end{pmatrix}\begin{pmatrix}X_L\\X_R\end{pmatrix}, \tag{15}$$

where the mixing angle θ_4 depends on the parameters in Eq. (14). In the limit $v_R \gg v_L$ and $v_R \gg v_\Sigma$ the mixing vanishes, $\sin\theta_4 = 0$, and the leptoquark masses become

$$\begin{aligned}M_{X_1} &= \tfrac{1}{2}\,g_L\sqrt{v_L^2+v_\Sigma^2(1+z^2)}\,,\\ M_{X_2} &= \tfrac{1}{2}\,g_R\,v_R\,.\end{aligned} \tag{16}$$

The Z'_L and Z'_R squared masses are given by the two nonzero eigenvalues of the matrix

$$\mathcal{M}^2_{Z'} = \tfrac{3}{8} \times \tag{17}$$
$$\begin{pmatrix} g_L^2\left[v_L^2+v_\Sigma^2(\frac{1}{3}+z^2)\right] & -g_L g_R v_\Sigma^2(\frac{1}{3}+z^2) & -\frac{\sqrt{2}}{\sqrt{3}}g_1' g_L v_L^2 \\ -g_L g_R v_\Sigma^2(\frac{1}{3}+z^2) & g_R^2\left[v_R^2+v_\Sigma^2(\frac{1}{3}+z^2)\right] & -\frac{\sqrt{2}}{\sqrt{3}}g_1' g_R v_R^2 \\ -\frac{\sqrt{2}}{\sqrt{3}}g_1' g_L v_L^2 & -\frac{\sqrt{2}}{\sqrt{3}}g_1' g_R v_R^2 & \frac{2}{3}g_1'^2(v_L^2+v_R^2) \end{pmatrix}.$$

Taking the limit $v_R \gg v_L$ and $v_R \gg v_\Sigma$ yields

$$M_{Z'_L} = \sqrt{\frac{g_1'^2(g_L^2+g_R^2)+\frac{3}{2}g_L^2 g_R^2}{8(g_1'^2+\frac{3}{2}g_R^2)}}\sqrt{3v_L^2+v_\Sigma^2(1+3z^2)}\,,$$
$$M_{Z'_R} = \frac{1}{2}\sqrt{g_1'^2+\tfrac{3}{2}g_R^2}\,v_R\,. \tag{18}$$

Fermion masses

The Yukawa interactions are

$$\begin{aligned}\mathcal{L}_Y &= y^d_{ij}\,\overline{\hat{\Psi}^i_L}\hat{H}_d\hat{\Psi}^{dj}_R + y^u_{ij}\,\overline{\hat{\Psi}^i_L}\hat{H}_u\hat{\Psi}^{uj}_R + Y_{ij}\,\overline{\hat{\chi}^i_L}\,\hat{\Sigma}\,\hat{\chi}^j_R + \text{h.c.}\\ &\supset y^d_{ij}\,\overline{L^i_L}S_1 e^j_R + y^d_{ij}\,\overline{Q^i_L}S_2 d^j_R + y^u_{ij}\,\overline{L^i_L}S_3^* \nu^j_R\\ &\quad + y^u_{ij}\,\overline{Q^i_L}S_4^* u^j_R + \tfrac{1}{\sqrt{2}}Y_{ij}v_\Sigma\big(\overline{Q'^i_L}\,Q'^j_R + z\,\overline{L'^i_L}\,L'^j_R\big) + \text{h.c.}\\ &\supset -c_e\,y^d_{ij}\,\overline{L^i_L}He^j_R - c_d\,y^d_{ij}\,\overline{Q^i_L}Hd^j_R + c_\nu\,y^u_{ij}\,\overline{L^i_L}\tilde{H}\nu^j_R\\ &\quad + c_u\,y^u_{ij}\,\overline{Q^i_L}\tilde{H}u^j_R + \tfrac{1}{\sqrt{2}}Y_{ij}v_\Sigma\big(\overline{Q'^i_L}\,Q'^j_R + z\,\overline{L'^i_L}\,L'^j_R\big)\\ &\quad + \text{h.c.}\,,\end{aligned} \tag{19}$$

where $i,j=1,2,3$ are family indices and the coefficients "c" are those in Eq. (9). Typically, in theories with quark-lepton unification, the up-type quark and neutrino masses of a given generation are the same at the unification scale, and similarly the down-type quark and charged lepton masses. In our model this is not the case, but since there are only two Yukawa matrices y^u and y^d, without additional mass contributions the hierarchy of the up-type quark masses is, a priori, the same as for the neutrinos, and the down-type quark mass hierarchy the same as for the charged leptons at the unification scale.

Regarding the up-type quarks and neutrinos, for which the experimentally determined mass hierarchies differ considerably, this is solved by introducing a new scalar representation $\hat{\Phi}_{10} = (1,\overline{10},1,-1)$. If the SM singlet component of $\hat{\Phi}_{10}$ develops a vev v_{10} at a high scale, this provides a seesaw mechanism for the neutrinos via the interaction

$$y^{u\prime}_{ij}\,\overline{(\hat{\Psi}^{ui}_R)^c}\,\hat{\Phi}_{10}\hat{\Psi}^{uj}_R\,. \tag{20}$$

The contribution to the up-type quarks vanishes. Therefore, the up-type quark masses are $m_u \sim y^u v$, whereas the neutrino masses are $m_\nu \sim (c_\nu\,y^u v)^2/(y^{u\prime}v_{10})$.

The relative mass hierarchies of the down-type quarks versus charged leptons are not in vast disagreement with experiment. The running of the masses will largely account for m_b/m_τ. One can also introduce the scalar representation $\hat{\Phi}_{15} = (15,1,1,0)$ into the model, with the SM singlet component developing the vev $v_{15}\,\text{diag}(1,1,1,-3)$. New mass contributions to the down-type quarks and charged leptons would then result from loop processes, parameterized via the effective dimension five interaction $y^{d\prime}_{ij}\,\overline{\hat{\Psi}^i_L}\hat{H}_d\hat{\Psi}^{dj}_R\hat{\Phi}_{15}/\Lambda$, and mediated, e.g., by heavy vector-like fermions, leading to additional mass splitting.

Flavor structure

In terms of SM fermion mass eigenstates, the interactions of the vector leptoquarks with quarks and leptons are given by

$$\begin{aligned}\mathcal{L} &\supset \frac{g_L}{\sqrt{2}}\,X_{L\mu}\Big[L^u_{ij}\,(\bar{u}^i\gamma^\mu P_L\,\nu^j) + L^d_{ij}\,(\bar{d}^i\gamma^\mu P_L\,e^j)\Big]\\ &\quad + \frac{g_R}{\sqrt{2}}\,X_{R\mu}\Big[R^u_{ij}\,(\bar{u}^i\gamma^\mu P_R\,\nu^j) + R^d_{ij}\,(\bar{d}^i\gamma^\mu P_R\,e^j)\Big] + \text{h.c.}\,,\end{aligned} \tag{21}$$

where $L^{u/d}$, $R^{u/d}$ are unitary mixing matrices. They are related via $L^u = VL^dU$ and $R^u = VR^dU$, where V is the Cabibbo-Kobayashi-Maskawa matrix and U is the Pontecorvo-Maki-Nakagawa-Sakata matrix.

Proton stability

The vector boson $(3,1)_{2/3}$ does not mediate proton decay [4], neither does any of the scalars in our model. In particular, for the scalar $(3,2)_{1/6}$, which by itself would be problematic [26], gauge invariance forbids tree-level proton decay. In broader terms, the Lagrangian in Eq. (19) is invariant under the global symmetries $\mathrm{U}(1)'_B$ and $\mathrm{U}(1)'_L$, with the matter fields Ψ_L, Ψ^d_R and Ψ^u_R carrying charges $B' = L' = 1/4$ and all scalar fields being neutral. After symmetry breaking the charges under the remaining global $\mathrm{U}(1)_B$ and $\mathrm{U}(1)_L$ are

$$\begin{aligned}B &= B' + \tfrac{1}{\sqrt{6}}\left(T^{15}_L + T^{15}_R\right),\\ L &= L' - \tfrac{\sqrt{6}}{2}\left(T^{15}_L + T^{15}_R\right),\end{aligned} \tag{22}$$

which are simply the SM baryon and lepton number. Proton decay is thus forbidden at all orders in perturbation theory.

III. FLAVOR ANOMALIES

In this section we discuss how the vector leptoquark of $\mathrm{SU}(4)_L$ can explain the recent hints of physics beyond the SM in B meson decays, i.e., the deficit in the ratios

$$\begin{aligned}R_K &= \frac{\mathrm{Br}\big(B^+\to K^+\mu^+\mu^-\big)}{\mathrm{Br}\big(B^+\to K^+e^+e^-\big)}\,,\\ R_{K^*} &= \frac{\mathrm{Br}\big(B^0\to K^{*0}\mu^+\mu^-\big)}{\mathrm{Br}\big(B^0\to K^{*0}\,e^+e^-\big)}\end{aligned} \tag{23}$$

with respect to SM predictions [1, 2]. For an analysis of the anomalies at the effective operator level see [27–32].

To describe the decays in Eq. (23) quantitatively, it is convenient to start out from the effective Lagrangian for flavor changing neutral current processes with a $b\to s$ transition. Up to four-quark operators, it can be written as

$$\begin{aligned}\mathcal{L} = \frac{4G_F}{\sqrt{2}}V_{tb}V^*_{ts}\sum_{i,j}\Big[\sum_{k=7}^{10}C^{ij}_k\mathcal{O}^{ij(\prime)}_k + C^{ij}_\nu\mathcal{O}^{ij}_\nu\\ + C^{ij(\prime)}_S\mathcal{O}^{ij(\prime)}_S + C^{ij(\prime)}_P\mathcal{O}^{ij(\prime)}_P\Big]\,.\end{aligned} \tag{24}$$

The operators $\mathcal{O}_7^{ij}$ and $\mathcal{O}_8^{ij}$ correspond to electromagnetic and chromomagnetic moment transitions, the $\mathcal{O}_9^{ij(\prime)}$, $\mathcal{O}_{10}^{ij(\prime)}$, $\mathcal{O}_\nu^{kl}$ are the semileptonic operators

$$\begin{aligned}
\mathcal{O}_{9(10)}^{ij} &= \frac{e^2}{16\pi^2}\,(\bar{s}\,\gamma_\mu P_L\,b)\,\big[\,\overline{l^i}\,\gamma^\mu(\gamma_5)\,l^j\,\big]\,,\\
\mathcal{O}_{9(10)}^{ij\,\prime} &= \frac{e^2}{16\pi^2}\,(\bar{s}\,\gamma_\mu P_R\,b)\,\big[\,\overline{l^i}\,\gamma^\mu(\gamma_5)\,l^j\,\big]\,,\\
\mathcal{O}_\nu^{ij} &= \frac{e^2}{8\pi^2}\,(\bar{s}\,\gamma_\mu P_L\,b)\,\big(\bar{\nu}^i\gamma^\mu P_L\nu^j\big)\,,
\end{aligned} \tag{25}$$

and $\mathcal{O}_S^{ij(\prime)}$, $\mathcal{O}_P^{ij(\prime)}$ are the scalar operators

$$\begin{aligned}
\mathcal{O}_S^{ij(\prime)} &= \frac{e^2}{16\pi^2}\big[\bar{s}\,P_{R(L)}b\big]\big(\,\overline{l^i}\,l^j\big),\\
\mathcal{O}_P^{ij(\prime)} &= \frac{e^2}{16\pi^2}\big[\bar{s}\,P_{R(L)}b\big]\big(\,\overline{l^i}\gamma_5\,l^j\big).
\end{aligned} \tag{26}$$

Tensor operators were neglected since they cannot arise from short distance new physics with SM linearly realized [33].

The $R_{K^{(*)}}$ anomalies are best fit by [27]

$$\mathrm{Re}\,(\Delta C_9^{\mu\mu} - \Delta C_9^{ee}) = -\mathrm{Re}\,(\Delta C_{10}^{\mu\mu} - \Delta C_{10}^{ee}) \approx -0.6 \tag{27}$$

and with the contributions to other Wilson coefficients being small. In our model, the vector leptoquarks X_1, X_2 modify the coefficients by

$$\begin{aligned}
\Delta C_9^{ij} &= -\Delta C_{10}^{ij} = -\frac{\sqrt{2}\,\pi^2 g_L^2\,L_{2i}^d L_{3j}^{d*}}{G_F\,e^2\,V_{tb}V_{ts}^*}\left[\frac{\cos^2\theta_4}{M_{X_1}^2} + \frac{\sin^2\theta_4}{M_{X_2}^2}\right],\\
\Delta C_9^{ij\prime} &= \Delta C_{10}^{ij\prime} = -\frac{\sqrt{2}\,\pi^2 g_R^2 R_{2i}^d R_{3j}^{d*}}{G_F\,e^2\,V_{tb}V_{ts}^*}\left[\frac{\sin^2\theta_4}{M_{X_1}^2} + \frac{\cos^2\theta_4}{M_{X_2}^2}\right],\\
\Delta C_S^{ij} &= -\Delta C_P^{ij}\\
&= -\frac{\sqrt{2}\,\pi^2 g_L g_R R_{2i}^d L_{3j}^{d*}\sin 2\theta_4}{G_F\,e^2\,V_{tb}V_{ts}^*}\left[\frac{1}{M_{X_1}^2} - \frac{1}{M_{X_2}^2}\right],\\
\Delta C_S^{ij\prime} &= \Delta C_P^{ij\prime}\\
&= -\frac{\sqrt{2}\,\pi^2 g_L g_R L_{2i}^d R_{3j}^{d*}\sin 2\theta_4}{G_F\,e^2\,V_{tb}V_{ts}^*}\left[\frac{1}{M_{X_1}^2} - \frac{1}{M_{X_2}^2}\right],\\
\Delta C_\nu^{ij} &= 0\,.
\end{aligned} \tag{28}$$

Guided by the tightness of the bounds from LFV searches (discussed in Sec. IV and App. C), we assume that $\mathrm{SU}(4)_R$ is broken at a much higher scale than $\mathrm{SU}(4)_L$, i.e.

$$v_R \gg v_L \;\text{ and }\; v_R \gg v_\Sigma\,. \tag{29}$$

This suppresses RH lepton flavor changing currents and results in the contributions to the Wilson coefficients other than $\Delta C_{9,10}^{ij}$ being small. The condition in Eq. (27) becomes

$$\frac{M_{X_L}}{g_L\sqrt{\mathrm{Re}\,\big(L_{22}^d L_{32}^{d*} - L_{21}^d L_{31}^{d*}\big)}} \approx 23\ \mathrm{TeV}\,. \tag{30}$$

IV. EXPERIMENTAL CONSTRAINTS

The leptoquark masses and the mixing matrices are subject to experimental constraints from a number of null searches for LFV, with the most stringent bounds coming from rare decays of pions [34–36], kaons [37–42], B mesons [43–50], τ leptons [51–53] and $\mu - e$ conversion [54].

Implications of those constraints for Pati-Salam unification have been considered in the literature [5–10], but focused on models in which the vector leptoquark $(3,1)_{2/3}$ couples to both LH and RH fermion fields with similar strength. The conclusion of those analyzes, updated with the most recent experimental bounds [48–50], is that the leptoquark mass has to be $\gtrsim 90$ TeV [10]. In addition, constraints from searches for $\mu \to e\,\gamma$ when both LH and RH leptoquark interactions are present can push this limit much higher due to the bottom quark mass enhancement of the one-loop diagram (see App. C and also [55] for a discussion of a similar effect in scalar leptoquark models). Such a heavy leptoquark would not explain the $R_{K^{(*)}}$ anomalies, since the required relation, analogous to the one in Eq. (30), could not be satisfied for a perturbative gauge coupling and unitary mixing matrices.

In our model, for a sufficiently high scale of $\mathrm{SU}(4)_R$ breaking, the constraints arising from the presence of leptoquark RH couplings to fermions are eliminated and the remaining bounds on LH interactions can be satisfied for a significantly lower leptoquark mass. The tightest limits are listed in the appendix, for arbitrary LH and RH leptoquark interactions in App. C and for the case of just LH interactions in App. D.

If the mixing matrix entries L_{11}^d, L_{12}^d are $\mathcal{O}(1)$, the limits from searches for $K_L^0 \to e^\pm\mu^\mp$ and $\mu - e$ conversion a priori push the leptoquark mass up to hundreds of TeV in our model (thousands of TeV for models in which both LH and RH leptoquark interactions are present, due to the enhancement of the scalar current contribution, see App. C). The bounds, however, are satisfied for a much lighter leptoquark provided $L_{11}^d, L_{12}^d \ll 1$. Unitarity then implies that $L_{13}^d \approx 1$ and $L_{23}^d, L_{33}^d \ll 1$, therefore L^d takes the form

$$L^d \approx e^{i\phi}\begin{pmatrix} \delta_1 & \delta_2 & 1\\ e^{i\phi_1}\cos\theta & e^{i\phi_2}\sin\theta & \delta_3\\ -e^{-i\phi_2}\sin\theta & e^{-i\phi_1}\cos\theta & \delta_4\end{pmatrix}, \tag{31}$$

where $|\delta_i| \ll 1$. Note that the suppression of RH flavor changing currents in our model implies that there are no significant bounds from $\pi^0 \to \nu\bar{\nu}$ or $K_L^0 \to \nu\bar{\nu}$.

The remaining entries of L^d are subject to further constraints, mainly from B meson and τ decays. If both LH and RH leptoquark interactions were present, the $B^0 \to \mu^+\mu^-$ decay would provide the most stringent bound. However, with only LH interactions the tightest limits arise from searches for $B^+ \to K^+e^\pm\mu^\mp$. We calculated the corresponding branching fractions (see App. C) using the most recent lattice results for the form factors [25] based on the Bourrely-Caprini-Lellouch parameterization [56], which relaxes the bounds considerably compared to taking the nonphysical values $f_+ = f_0 = 1$ [9].

The resulting bound on M_{X_L} is minimized for $\theta \approx \pi/4$ and requires merely $M_{X_L}/g_L \gtrsim 9.2$ TeV. Given the relation between the gauge couplings in Eq. (12) and assuming $g_R \approx \sqrt{3\pi}$ (close to the perturbative limit) implies $g_L \approx 1.06\, g_s$, where $g_s \approx 0.96$ is the strong coupling constant at 10 TeV. This leads to the constraint

$$M_{X_L} \gtrsim 10 \text{ TeV} . \tag{32}$$

(If one chose instead $g_L = g_R = \sqrt{2}\, g_s$, this would result in the constraint $M_{X_L} \gtrsim 14$ TeV.) Saturating the bound in Eq. (32), the condition in Eq. (30) for explaining the $R_{K^{(*)}}$ anomalies is fulfilled if $\cos(\phi_1 + \phi_2) \approx 0.18$. We also note that for $M_{X_L} \approx 10$ TeV one could have $|\delta_i| \sim 0.02$, so the matrix L^d in Eq. (31) does not need to be highly tuned.

Finally, let us note that all loop-level constraints, including $K-\overline{K}$, $B-\overline{B}$, $B_s-\overline{B}_s$ mixing, radiative decays $\mu \to e\,\gamma$ (see App. C), $\tau \to e\,\gamma$, anomalous magnetic and electric moments of leptons, $Z \to b\,\bar{b}$ and others [57] are satisfied due to the unitarity of L^d and the leptoquark mass being $\gtrsim 10$ TeV.

V. COLLIDER PHENOMENOLOGY

The aim of this limited phenomenological analysis is to simply demonstrate that the leptoquark X_L in our model accounting for the flavor anomalies can be searched for at the next generation collider. Focussing on the proposed 100 TeV Future Circular Collider (FCC), we find that one of the best signature to look for is provided by the single leptoquark production process

$$p\,p \to X_L\, j\, \mu^- \to j\, j\, \mu^+ \mu^- . \tag{33}$$

In an in-depth analysis one could also investigate final states involving other leptons, which for the case of neutrinos would lead to missing energy signatures. Pair production of 10 TeV leptoquarks is suppressed even at a 100 TeV collider.

To simulate the SM background and the leptoquark signal for the process (33) we used MadGraph 5 [58] (version 2.6.3) with the default cuts apart from the lower cut on the transverse momentum of jets and leptons, which was set to 300 GeV. The leptoquark model file for MadGraph was implemented using FeynRules [59] (version 2.3.32).

Figure 1 plots the number of background (B) and signal (S) events for a leptoquark mass 10, 12 and 14 TeV expected within the first year of FCC running (estimated to be 250 fb^{-1} of data [60]) as a function of the invariant mass of the highest transverse momentum jet j and μ^+. Implementing the invariant mass cut $|\, M_{j\mu^+} - M_{X_L}| < \Gamma_X$, where Γ_X is the width of the leptoquark, the significance of the signal, $S/\sqrt{B}$, is very high: 19 σ for $M_{X_L} = 10$ TeV, 6.7 σ for 12 TeV and 4.5 σ for 14 TeV. More sophisticated cuts may make the search more efficient. A detailed analysis of the X_L vector leptoquark collider phenomenology is beyond the scope of this paper.

Were the B decay anomalies in R_K and R_{K^*} confirmed and established, inspection of Eq. (30) indicates this model can be ruled out at a future 100 TeV high luminosity hadron collider. Not only does the right-hand side of Eq. (30) provide an upper bound on the mass of the vector leptoquark, but Eq. (12) shows the strength of the coupling constant g_L is bounded from below, and therefore the height of the resonant signal in Fig. 1 is bounded from below.

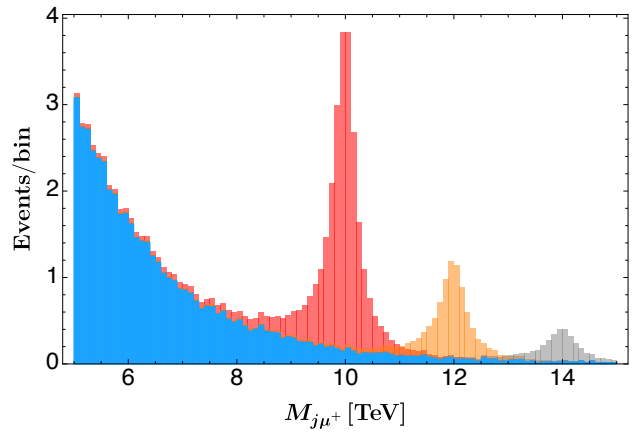

FIG. 1. Expected number of events in 250 fb^{-1} of data collected by a 100 TeV pp collider for the SM background $p\,p \to j\,j\,\mu^+\mu^-$ (blue) and the leptoquark signal $p\,p \to X_L\, j\,\mu^- \to j\,j\,\mu^+\mu^-$ for masses $M_{X_L} = 10, 12, 14$ TeV (red, orange, gray) as a function of the invariant mass of the highest p_T jet and μ^+. The values of parameters discussed in Sec. IV were used.

VI. CONCLUSIONS

We have constructed a new model to account for the recently observed anomalies in B meson decays set within the framework of Pati-Salam unification. The theory avoids all experimental bounds without introducing any vector-like fields mixing with the Standard Model fermions. This was achieved by suppressing the leptoquark right-handed interactions by associating them with a symmetry broken at a high scale, which eliminates the most stringent constraints arising from the simultaneous presence of left- and right-handed lepton flavor changing currents. In some regions of parameter space the mass of the leptoquark can be as low as 10 TeV while remaining consistent with all experimental data.

The tightest constraints on the model come from the experimental limits on rare kaon, B meson and τ decays, as well as $\mu-e$ conversion. In the appendix we presented general model-independent formulae for the various decay rates and listed the corresponding bounds. Those results can be used to read off the constraints on any model with one or more $(3,1)_{2/3}$ vector leptoquarks with arbitrary left- and right-handed interactions with Standard Model quarks and leptons.

In our analysis we chose parameters to explain the $R_{K^{(*)}}$ flavor anomalies. Although the vector leptoquark $(3,1)_{2/3}$ in our model is too heavy to account also for the $R_{D^{(*)}}$ anomalies, it has been shown [61] that the scalar leptoquark $(3,2)_{1/6}$ might be a good candidate for that. This leptoquark appears in the scalar sector of our model and can be made sufficiently light. It would be interesting to investigate this in more detail.

Currently, there exist many models that account for the hints of lepton universality violation in B meson decays. If these anomalies are established, new physics must emerge at a scale similar to that of the mass of the "left-handed" leptoquark in our model. We have demonstrated that simple kinematic cuts can isolate clearly observable signals with 250 fb^{-1} of accumulated data at a 100 TeV pp collider. Further analysis is badly required to determine whether such apparatus could distinguished among the many proposed models.

Acknowledgments

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APPENDIX

Appendix A: $\mathbf{SU(4)}_L \times \mathbf{SU(4)}_R$ symmetry breaking

The scalar potential of the model is given by

$$\begin{aligned} V = &- \mu_1^2\,|\hat\Sigma_L|^2 + \lambda_1|\hat\Sigma_L|^4 - \mu_2^2\,|\hat\Sigma_R|^2 + \lambda_2|\hat\Sigma_R|^4 - \mu_3^2\,|\hat\Sigma|^2 \\ &+ \lambda_3(\hat\Sigma\hat\Sigma^\dagger)^2 + \lambda_3'|\hat\Sigma\hat\Sigma^\dagger|^2 - \mu_4^2\,|\hat H_d|^2 + \lambda_4\big(\hat H_d\hat H_d^\dagger\big)^2 \\ &+ \lambda_4'|\hat H_d\hat H_d^\dagger|^2 - \mu_5^2\,|\hat H_u|^2 + \lambda_5\big(\hat H_u\hat H_u^\dagger\big)^2 + \lambda_5'|\hat H_u\hat H_u^\dagger|^2 \\ &+ \lambda_{12}|\hat\Sigma_L|^2|\hat\Sigma_R|^2 + \lambda_{13}|\hat\Sigma_L|^2|\hat\Sigma|^2 + \lambda_{14}|\hat\Sigma_L|^2|\hat H_d|^2 \\ &+ \lambda_{15}|\hat\Sigma_L|^2|\hat H_u|^2 + \lambda_{23}|\hat\Sigma_R|^2|\hat\Sigma|^2 + \lambda_{24}|\hat\Sigma_R|^2|\hat H_d|^2 \\ &+ \lambda_{25}|\hat\Sigma_R|^2|\hat H_u|^2 + \lambda_{34}|\hat\Sigma|^2|\hat H_d|^2 + \lambda_{35}|\hat\Sigma|^2|\hat H_u|^2 \\ &+ \lambda_{45}|\hat H_d|^2|\hat H_u|^2 + \lambda_{13}'|\hat\Sigma_L\hat\Sigma|^2 + \lambda_{14}'|\hat\Sigma_L^\dagger\hat H_d|^2 \\ &+ \lambda_{15}'|\hat\Sigma_L^\dagger\hat H_u|^2 + \lambda_{23}'|\hat\Sigma_R^\dagger\hat\Sigma|^2 + \lambda_{24}'|\hat\Sigma_R\hat H_d|^2 \\ &+ \lambda_{25}'|\hat\Sigma_R\hat H_u|^2 + \lambda_{34}'|\hat\Sigma\hat H_d|^2 + \lambda_{35}'|\hat\Sigma\hat H_u|^2 \\ &+ \lambda_{45}'|\hat H_d^\dagger\hat H_u|^2 + \lambda_{34}''\,\mathrm{Tr}(\hat\Sigma\hat H_d\hat H_d^\dagger\hat\Sigma^\dagger) \\ &+ \lambda_{35}''\,\mathrm{Tr}(\hat\Sigma\hat H_u\hat H_u^\dagger\hat\Sigma^\dagger) + \lambda_{45}''\,\mathrm{Tr}(\hat H_d^\dagger\hat H_u\hat H_u^\dagger\hat H_d) \\ &+ \big[\lambda_{345}'(\hat\Sigma\hat H_d)(\hat\Sigma\hat H_u) + \lambda_{345}''\mathrm{Tr}(\hat\Sigma\hat H_d\hat\Sigma\hat H_u) \\ &+ \lambda_{345}'''\mathrm{Tr}(\hat H_d\hat\Sigma\hat\Sigma\hat H_u) + \kappa\,\hat\Sigma_L\hat\Sigma\,\hat\Sigma_R^\dagger + \text{h.c.}\big]\,, \end{aligned} \tag{A1}$$

where we have adopted the notation:
$|\hat\Sigma_L|^2 \equiv (\hat\Sigma_L)_{\alpha_L}(\hat\Sigma_L^\dagger)^{\alpha_L}$, $|\hat\Sigma|^2 \equiv (\hat\Sigma)^{\alpha_L}_{\alpha_R}(\hat\Sigma^\dagger)^{\alpha_R}_{\alpha_L}$, $(\hat\Sigma\hat H_d) \equiv (\hat\Sigma)^{\alpha_L}_{\alpha_R}(\hat H_d)^{\alpha_R}_{\alpha_L}$, $|\hat\Sigma_L\hat\Sigma|^2 \equiv (\hat\Sigma_L)_{\alpha_L}(\hat\Sigma)^{\alpha_L}_{\alpha_R}(\hat\Sigma^\dagger)^{\alpha_R}_{\beta_L}(\hat\Sigma_L^\dagger)^{\beta_L}$, $|\hat\Sigma\hat H_d|^2 \equiv (\hat\Sigma)^{\alpha_L}_{\alpha_R}(\hat H_d)^{\alpha_R}_{\beta_L}(\hat H_d^\dagger)^{\beta_L}_{\beta_R}(\hat\Sigma^\dagger)^{\beta_R}_{\alpha_L}$, $\mathrm{Tr}(\hat\Sigma\hat H_d\hat\Sigma\hat H_u) \equiv (\hat\Sigma)^{\alpha_L}_{\alpha_R}(\hat H_d)^{\alpha_R}_{\beta_L}(\hat\Sigma)^{\beta_L}_{\beta_R}(\hat H_u)^{\beta_R}_{\alpha_L}$, etc.

Let us consider $\langle\hat\Sigma_L\rangle$, $\langle\hat\Sigma_R\rangle$ and $\langle\hat\Sigma\rangle$. Via a suitable $\mathrm{SU}(4)_L$ and $\mathrm{SU}(4)_R$ transformation, it is possible to bring $\langle\hat\Sigma_L\rangle$ and $\langle\hat\Sigma_R\rangle$ to the form

$$\langle\hat\Sigma_L\rangle = \frac{v_L}{\sqrt2}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \quad \langle\hat\Sigma_R\rangle = \frac{v_R}{\sqrt2}\begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \tag{A2}$$

where v_L and v_R are real and positive.

The remaining $\mathrm{SU}(3)$ invariance can be utilized to obtain

$$\langle\hat\Sigma\rangle = \begin{pmatrix} a_1 & 0 & 0 & b_1\\ 0 & a_2 & 0 & b_2\\ 0 & 0 & a_3 & b_3\\ c_1 & c_2 & c_3 & d\end{pmatrix}. \tag{A3}$$

To argue that $\langle\hat\Sigma\rangle$ can be brought to the diagonal form as in Eq. (4), it is sufficient to consider the potential terms $|\hat\Sigma|^2$, $(\hat\Sigma\hat\Sigma^\dagger)^2$, $|\hat\Sigma\hat\Sigma^\dagger|^2$, $|\hat\Sigma_L\hat\Sigma|^2$, $|\hat\Sigma_R^\dagger\hat\Sigma|^2$ and $\hat\Sigma_L\hat\Sigma\,\hat\Sigma_R^\dagger$. Since,

$$\begin{aligned} &\lambda_{13}'|\hat\Sigma_L\hat\Sigma|^2 = \tfrac12\lambda_{13}'v_L^2(c_1^2+c_2^2+c_3^2+d^2)\,,\\ &\lambda_{23}'|\hat\Sigma_R^\dagger\hat\Sigma|^2 = \tfrac12\lambda_{23}'v_R^2(b_1^2+b_2^2+b_3^2+d^2)\,,\\ &-\mu_3^2\,|\hat\Sigma|^2 + \lambda_3(\hat\Sigma\hat\Sigma^\dagger)^2 = \lambda_3\big(a_1^2+a_2^2+a_3^2+b_1^2+b_2^2+b_3^2\\ &\qquad\qquad + c_1^2+c_2^2+c_3^2+d^2-v_\Sigma^2\big)^2 - \lambda_3 v_\Sigma^4\,, \end{aligned} \tag{A4}$$

the potential is minimized for $\langle\hat\Sigma\rangle = \mathrm{diag}(a_1,a_2,a_3,d)$. In addition, the terms

$$\begin{aligned} &\lambda_3'|\hat\Sigma\hat\Sigma^\dagger|^2 = \lambda_3'\big(a_1^4+a_2^4+a_3^4+d^4\big)\,,\\ &\kappa\,\hat\Sigma_L\hat\Sigma\,\hat\Sigma_R^\dagger = \tfrac12\kappa\,v_Lv_R\,d \end{aligned} \tag{A5}$$

imply that the minimum occurs at $a_1=a_2=a_3$. Finally, we are free to choose κ to be real and negative, which through an appropriate redefinition of $\hat\Sigma$ leads to real $d>0$; therefore

$$\langle\hat\Sigma\rangle = \frac{v_\Sigma}{\sqrt2}\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&z\end{pmatrix} \tag{A6}$$

with z being real and positive. Note that only one of the parameters λ_{345}', λ_{345}'' and λ_{345}''' can be made real by a field redefinition. If any of the other two has a nonzero imaginary part, the scalar potential is CP-violating. A rigorous minimization procedure is beyond the scope of this work.

Appendix B: Scalar masses

To show that Eq. (9) can be satisfied, it is again sufficient to consider only a few terms in the scalar potential. In terms of hard masses, the relevant part of the Lagrangian is

$$\begin{aligned} \mathcal{L}_m \supset\; & M_d^2\,|\hat H_d|^2 + M_u^2\,|\hat H_u|^2 - \lambda_{24}'|\hat\Sigma_R\hat H_d|^2 - \lambda_{25}'|\hat\Sigma_R\hat H_u|^2\\ &- \lambda_{34}'|\hat\Sigma\hat H_d|^2 - \lambda_{35}'|\hat\Sigma\hat H_u|^2 - \lambda_{345}'(\hat\Sigma\hat H_d)(\hat\Sigma\hat H_u)\,. \end{aligned} \tag{B1}$$

This results in the masses for the color octets and triplets,

$$\begin{aligned} m_{O_1} = m_{T_1} = m_{T_2} \equiv M_d^2\,,\\ m_{O_2} = m_{T_3} = m_{T_4} \equiv M_u^2\,. \end{aligned} \tag{B2}$$

The mass squared matrix for the fields $S_{1,2,3,4}$ is

$$\mathcal{M}_S^2 = \begin{pmatrix} M_d^2+a_d & 0 & \frac12\lambda_{345}'z^2v_\Sigma^2 & \frac{\sqrt3}{2}\lambda_{345}'z\,v_\Sigma^2\\ 0 & M_d^2+b_d & \frac{\sqrt3}{2}\lambda_{345}'z\,v_\Sigma^2 & \frac32\lambda_{345}'v_\Sigma^2\\ \frac12\lambda_{345}'z^2v_\Sigma^2 & \frac{\sqrt3}{2}\lambda_{345}'z\,v_\Sigma^2 & M_u^2+a_u & 0\\ \frac{\sqrt3}{2}\lambda_{345}'z\,v_\Sigma^2 & \frac32\lambda_{345}'v_\Sigma^2 & 0 & M_u^2+b_u \end{pmatrix}, \tag{B3}$$

where

$$a_d = \tfrac{1}{2}\lambda'_{24} v_R^2 + \tfrac{1}{2}\lambda'_{34} z^2 v_\Sigma^2 \,, \quad b_d = \tfrac{3}{2}\lambda'_{34} v_\Sigma^2 \,,$$
$$a_u = \tfrac{1}{2}\lambda'_{25} v_R^2 + \tfrac{1}{2}\lambda'_{35} z^2 v_\Sigma^2 \,, \quad b_u = \tfrac{3}{2}\lambda'_{35} v_\Sigma^2 \,. \tag{B4}$$

We have verified that there exists a class of solutions with only one linear combination of the four scalars being light. To reproduce the SM fermion masses while keeping the Yukawas perturbative, it is sufficient to have the light mass eigenstate, identified with the SM Higgs, given by

$$H = -\, c_e\, S_1 - c_d\, S_2 + c_\nu\, S_3 + c_u\, S_4 \,, \tag{B5}$$

where $c_u \approx 1 \gg c_d \gg c_\nu$ and $1 \gg c_e \gg c_\nu$, with the ratio $c_d : c_e \approx m_b : m_\tau$.

Appendix C: Flavor constraints: Model-independent analysis

The general form of the Lagrangian describing interactions of vector leptoquarks $(3,1)_{2/3}$ with fermions is given by

$$\mathcal{L} \;\supset\; \sum_\alpha X_\mu^{(\alpha)} \Big[f^{Lu}_{ij(\alpha)} \left(\bar{u}^i \gamma^\mu P_L \nu^j\right) + f^{Ru}_{ij(\alpha)} \left(\bar{u}^i \gamma^\mu P_R \nu^j\right) + f^{Ld}_{ij(\alpha)} \left(\bar{d}^i \gamma^\mu P_L\, e^j\right) + f^{Rd}_{ij(\alpha)} \left(\bar{d}^i \gamma^\mu P_R\, e^j\right) \Big] \,, \tag{C1}$$

where the field $X_\mu^{(\alpha)}$ corresponds to a leptoquark with mass M_α. The resulting contributions to rare processes are listed below, along with the most severe experimental bounds.

The numerical values for particle masses and lifetimes were adopted from PDG [62]. The single-particle state normalization chosen is

$$\langle \vec{p} \,|\, \vec{p}\,' \rangle = 2E\, (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{p}\,') \tag{C2}$$

and the decay constant $f_\mathcal{M}$ for a meson consisting of quarks/antiquarks q_1, q_2 is defined via

$$\langle 0 | \bar{q}_1 \gamma^5 q_2 | \mathcal{M}(p) \rangle = -i f_\mathcal{M} \frac{m_\mathcal{M}^2}{m_{q_1} + m_{q_2}} \,, \qquad \langle 0 | \bar{q}_1 \gamma^\mu \gamma^5 q_2 | \mathcal{M}(p) \rangle = i f_\mathcal{M}\, p^\mu \,. \tag{C3}$$

Values of the meson decay constants, obtained from averaging the lattice results, were also taken from PDG,

$$f_{\pi^+} = 130 \text{ GeV} \,, \quad f_{K_L^0} = f_{K^+} = 156 \text{ GeV} \,, \quad f_{B^0} = 191 \text{ GeV} \,, \quad f_{B^+} = 187 \text{ GeV} \,, \quad f_{B_s^0} = 227 \text{ GeV} \,. \tag{C4}$$

(1) Neutral meson decays to two charged leptons

The leptoquark contribution to the decay of a meson $\mathcal{M}$ with mass $m_\mathcal{M}$ to two charged leptons, l_i^+ with mass m_i and l_j^- with mass m_j, is given by

$$\Gamma(\mathcal{M} \to l_i^+ l_j^-)_X = \frac{m_\mathcal{M} f_\mathcal{M}^2}{64\,\pi} \left[A_{ij} \left(1 - \frac{m_i^2 + m_j^2}{m_\mathcal{M}^2} \right) + B_{ij} \frac{4\, m_i m_j}{m_\mathcal{M}^2} \right] \sqrt{\left[1 - \frac{(m_i + m_j)^2}{m_\mathcal{M}^2} \right] \left[1 - \frac{(m_i - m_j)^2}{m_\mathcal{M}^2} \right]} \,, \tag{C5}$$

where

$$A_{ij} \equiv \left| \sum_\alpha \frac{a^{LR}_{ij(\alpha)}}{M_\alpha^2} \right|^2 + \left| \sum_\alpha \frac{a^{RL}_{ij(\alpha)}}{M_\alpha^2} \right|^2 \,,$$
$$B_{ij} \equiv \sum_{\alpha,\beta} \frac{\mathrm{Re}\left[a^{LR}_{ij(\alpha)} a^{RL\,*}_{ij(\beta)} \right]}{M_\alpha^2 M_\beta^2} \,,$$
$$a^{LR\,(K_L^0)}_{ij(\alpha)} \equiv \frac{1}{\sqrt{2}} \Big[m_i\, f^{Rd}_{1i(\alpha)} f^{Rd\,*}_{2j(\alpha)} + m_j\, f^{Ld}_{1i(\alpha)} f^{Ld\,*}_{2j(\alpha)} + \frac{2\, m_{K^0}^2 Q}{m_s + m_d} f^{Ld}_{1i(\alpha)} f^{Rd\,*}_{2j(\alpha)} \Big] + (1 \leftrightarrow 2) \,,$$
$$a^{LR\,(B^0)}_{ij(\alpha)} \equiv m_i\, f^{Rd\,*}_{1i(\alpha)} f^{Rd}_{3j(\alpha)} + m_j\, f^{Ld\,*}_{1i(\alpha)} f^{Ld}_{3j(\alpha)} + \frac{2\, m_{B^0}^2 Q}{m_b + m_d} f^{Ld\,*}_{1i(\alpha)} f^{Rd}_{3j(\alpha)} \,,$$
$$a^{LR\,(B_s^0)}_{ij(\alpha)} \equiv m_i\, f^{Rd\,*}_{2i(\alpha)} f^{Rd}_{3j(\alpha)} + m_j\, f^{Ld\,*}_{2i(\alpha)} f^{Ld}_{3j(\alpha)} + \frac{2\, m_{B_s^0}^2 Q}{m_b + m_s} f^{Ld\,*}_{2i(\alpha)} f^{Rd}_{3j(\alpha)} \,,$$
$$a^{RL}_{ij(\alpha)} \equiv a^{LR}_{ij(\alpha)} \, (L \leftrightarrow R) \,. \tag{C6}$$

In Eq. (C6) the quark masses m_q and the factor Q depend on the energy scale, $m_q = m_q(\mu)$ and $Q = Q(\mu)$, with $Q(\mu)$ given by the formula

$$Q(\mu) = \left[\frac{\alpha^{(6)}(m_t)}{\alpha^{(6)}(M_{X_L})} \right]^{\frac{4}{7}} \left[\frac{\alpha^{(5)}(m_b)}{\alpha^{(5)}(m_t)} \right]^{\frac{12}{23}} \left[\frac{\alpha^{(4)}(\mu)}{\alpha^{(4)}(m_b)} \right]^{\frac{12}{25}} , \tag{C7}$$

applicable for $m_b > \mu > m_c$. The coupling constant α is calculated from

$$\alpha^{(N_f)}(\mu, \Lambda) = \frac{4\,\pi}{(11 - 2N_f/3) \log(\mu^2/\Lambda^2)} \,, \tag{C8}$$

where N_f is the number of quark flavors at a given scale, by matching

$$\alpha^{(6)}(m_t) \equiv \alpha^{(6)}(m_t, \Lambda_6) = \alpha^{(5)}(m_t, \Lambda_5) \equiv \alpha^{(5)}(m_t), \qquad \alpha^{(5)}(m_b) \equiv \alpha^{(5)}(m_b, \Lambda_5) = \alpha^{(4)}(m_b, \Lambda_4) \equiv \alpha^{(4)}(m_b). \tag{C9}$$

The ratio $Q(\mu)/m_q(\mu)$ is a renormalization group invariant. Adopting the PDG values for the quark masses at $\mu = 2$ GeV and for the strong coupling constant at $\mu = M_Z$ [62], the value of Q depends only on the leptoquark mass scale through

$$Q(2 \text{ GeV}) = \frac{0.45}{[\alpha^{(6)}(M_{X_L})]^{4/7}} \,. \tag{C10}$$

As evident from Eq. (C6), the constraints on the leptoquark contribution to the branching fraction of kaon and B meson decays are much weaker when the leptoquarks have only LH or only RH interactions with SM fermions, as opposed to models with both LH and RH interactions. The bounds on the branching fraction are milder by a factor of

$$\sqrt{2\, m_\mathcal{M}^2 Q/(m_l\, m_q)} \,,$$

which is reflected by the much weaker constraints on the leptoquark mass in our model compared to generic leptoquark models (see App. D).

For the majority of decays considered here only the upper bound on the rate was experimentally established. However, in the four cases: $K_L^0 \to e^+e^-$, $K_L^0 \to \mu^+\mu^-$, $B^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-$ nonzero rates have been measured. For those particular decays not only the pure leptoquark contribution is relevant, but also the interference effects with the SM short-distance (SD) contribution. This can be taken into account by making the following substitution in the expressions for A_{ij} and B_{ij} in Eq. (C6),

$$\sum_\alpha \frac{a^{LR}_{ij(\alpha)}}{M_\alpha^2} \longrightarrow \sqrt{\frac{64\,\pi\,\Gamma(\mathcal{M} \to l_i^+ l_j^-)^{\rm SD}_{\rm SM}}{m_{\mathcal{M}} f_{\mathcal{M}}^2}}\,\delta_{ij} \pm \sum_\alpha \frac{a^{LR}_{ij(\alpha)}}{M_\alpha^2}\,, \tag{C11}$$

where the $+/-$ depends on the decay considered and corresponds to the SM short-distance amplitude for $\mathcal{M} \to l_i^+ l_j^-$ being negative/positive. The leptoquark-induced contribution is then obtained by subtracting off the pure SM part.

(a) Neutral kaon decays

The decays $K_L^0 \to e^\pm\mu^\mp$ are absent in the SM and the constraint on the leptoquark mass is derived directly from the experimental bound on the branching fraction, $\mathrm{Br}_X \lesssim \Delta\mathrm{Br}$. The rates for $K_L^0 \to e^+e^-$, $\mu^+\mu^-$ were measured [37, 62]. They are dominated by long-distance SM effects [63, 64]. For $K_L^0 \to e^+e^-$ the experimental branching fraction $\left(8.7^{+5.7}_{-4.1}\right) \times 10^{-12}$ [37] agrees well with the SM long-distance estimate of $(9.0 \pm 0.5) \times 10^{-12}$ [63]. In that case we use the experimental uncertainty for the measured branching fraction as the upper bound for the leptoquark contribution. For $K_L^0 \to \mu^+\mu^-$ the measured branching fraction is $(6.84 \pm 0.11) \times 10^{-9}$ [39], but it was shown that the short-distance SM contribution is only 0.9×10^{-9} [63], whereas the upper bound on the total short-distance contribution is 2.5×10^{-9} [64].

The constraints below reflect the most conservative bound on the leptoquark mass obtained using Eq. (C5). The branching fractions were left in explicitly for easier use of the formulae given future experimental improvements.

- $$\frac{A_{11}^{(K_L^0)}}{m_{K^0}^2} \lesssim \left[\frac{\mathrm{Br}(K_L^0 \to e^+e^-)}{5.7 \times 10^{-12}}\right] (672\ \mathrm{TeV})^{-4}\,, \quad [37] \tag{C12}$$

- $$\frac{A_{12}^{(K_L^0)} + A_{21}^{(K_L^0)}}{m_{K^0}^2} \lesssim \left[\frac{\mathrm{Br}(K_L^0 \to e^\pm\mu^\mp)}{4.7 \times 10^{-12}}\right] \times (689\ \mathrm{TeV})^{-4}\,, \quad [38] \tag{C13}$$

- $$\frac{A_{22}'^{(K_L^0)} + 0.2\,B_{22}'^{(K_L^0)}}{m_{K^0}^2} \lesssim \left[\frac{\mathrm{Br}(K_L^0 \to \mu^+\mu^-)}{2.5 \times 10^{-9}}\right] \times (140\ \mathrm{TeV})^{-4}\,, \quad [64] \tag{C14}$$

where $A_{22}'^{(K_L^0)}$ is given by $A_{22}^{(K_L^0)}$ with the substitution (C11) with $\Gamma(K_L^0 \to \mu^+\mu^-)^{\rm SD}_{\rm SM} = 0.9 \times 10^{-9}$; similarly for $B_{22}'^{(K_L^0)}$.

(b) Neutral B meson decays

For most of the B^0 and B_s^0 decays only the limit on the branching fraction is determined, therefore the bounds on leptoquark parameters are derived using $\mathrm{Br}_X \lesssim \Delta\mathrm{Br}$. In the case of $B^0 \to \mu^+\mu^-$ and $B_s^0 \to \mu^+\mu^-$ the branching fractions were actually measured, $\mathrm{Br}(B^0 \to \mu^+\mu^-) = \left(1.6^{+1.6}_{-1.4}\right) \times 10^{-10}$ [62] and $\mathrm{Br}(B_s^0 \to \mu^+\mu^-) = \left(3.0 \pm 0.6^{+0.3}_{-0.2}\right) \times 10^{-9}$ [49], and are dominated by short-distance SM effects. We arrive at the following set of constraints,

- $$\frac{A_{11}^{(B^0)}}{m_{B^0}^2} \lesssim \left[\frac{\mathrm{Br}(B^0 \to e^+e^-)}{8.3 \times 10^{-8}}\right] (29.4\ \mathrm{TeV})^{-4}\,, \quad [47] \tag{C15}$$

- $$\frac{A_{12}^{(B^0)} + A_{21}^{(B^0)}}{m_{B^0}^2} \lesssim \left[\frac{\mathrm{Br}(B^0 \to e^\pm\mu^\mp)}{1.0 \times 10^{-9}}\right] \times (88.6\ \mathrm{TeV})^{-4}\,, \quad [48] \tag{C16}$$

- $$\frac{A_{22}'^{(B^0)}}{m_{B^0}^2} \lesssim \left[\frac{\mathrm{Br}(B^0 \to \mu^+\mu^-)}{1.6 \times 10^{-10}}\right] (140\ \mathrm{TeV})^{-4}\,, \quad [62] \tag{C17}$$

- $$\frac{A_{13}^{(B^0)} + A_{31}^{(B^0)}}{m_{B^0}^2} \lesssim \left[\frac{\mathrm{Br}(B^0 \to e^\pm\tau^\mp)}{2.8 \times 10^{-5}}\right] \times (6.4\ \mathrm{TeV})^{-4}\,, \quad [46] \tag{C18}$$

- $$\frac{A_{23}^{(B^0)} + A_{32}^{(B^0)}}{m_{B^0}^2} \lesssim \left[\frac{\mathrm{Br}(B^0 \to \mu^\pm\tau^\mp)}{2.2 \times 10^{-5}}\right] \times (6.8\ \mathrm{TeV})^{-4}\,, \quad [46] \tag{C19}$$

- $$\frac{A_{33}^{(B^0)} + 0.59\,B_{33}^{(B^0)}}{m_{B^0}^2} \lesssim \left[\frac{\mathrm{Br}(B^0 \to \tau^+\tau^-)}{2.1 \times 10^{-3}}\right] \times (2.0\ \mathrm{TeV})^{-4}\,, \quad [50] \tag{C20}$$

- $$\frac{A_{11}^{(B_s^0)}}{m_{B_s^0}^2} \lesssim \left[\frac{\mathrm{Br}(B_s^0 \to e^+e^-)}{2.8 \times 10^{-7}}\right] (23.9\ \mathrm{TeV})^{-4}\,, \quad [47] \tag{C21}$$

- $$\frac{A_{12}^{(B_s^0)} + A_{21}^{(B_s^0)}}{m_{B_s^0}^2} \lesssim \left[\frac{\mathrm{Br}(B_s^0 \to e^\pm\mu^\mp)}{5.4 \times 10^{-9}}\right] \times (64.1\ \mathrm{TeV})^{-4}\,, \quad [48] \tag{C22}$$

- $$\frac{A_{22}'^{(B_s^0)}}{m_{B_s^0}^2} \lesssim \left[\frac{\mathrm{Br}(B_s^0 \to \mu^+\mu^-)}{0.7 \times 10^{-9}}\right] (107\ \mathrm{TeV})^{-4}\,, \quad [49] \tag{C23}$$

- $$\frac{A_{33}^{(B_s^0)} + 0.56\,B_{33}^{(B_s^0)}}{m_{B_s^0}^2} \lesssim \left[\frac{\mathrm{Br}(B_s^0 \to \tau^+\tau^-)}{6.8 \times 10^{-3}}\right] \times (1.7\ \mathrm{TeV})^{-4}\,, \quad [50] \tag{C24}$$

where $A_{22}'^{(B^0)}$ is given by $A_{22}^{(B^0)}$ with the substitution (C11) with $\Gamma(B^0 \to \mu^+\mu^-)^{\rm SD}_{\rm SM} = 1.6 \times 10^{-10}$, and similarly for $A_{22}'^{(B_s^0)}$ with $\Gamma(B_s^0 \to \mu^+\mu^-)^{\rm SD}_{\rm SM} = 3.0 \times 10^{-9}$. We listed the

constraint on the leptoquark contribution to $\mathrm{Br}(B_s^0 \to \mu^+\mu^-)$ for completeness, but this branching fraction is actually determined by the fit that yields ΔC_9 and ΔC_{10} in Eq. (27).

(2) Charged meson decays to a charged lepton and neutrino

Decays of mesons to a charged lepton and a neutrino exist in the SM. The leading order leptoquark contribution comes from interference effects. The theoretical uncertainty in the SM calculation is reduced by taking ratios of decay rates,

$$\frac{\Gamma(\mathcal{M} \to l_i^+ \nu)}{\Gamma(\mathcal{M} \to l_{i'}^+ \nu)} = \frac{\Gamma(\mathcal{M} \to l_i^+ \nu)}{\Gamma(\mathcal{M} \to l_{i'}^+ \nu)}\bigg|_{\mathrm{SM}} \left(1 + \frac{D_{i'} - D_i}{\sqrt{2}\, G_F}\right), \tag{C25}$$

where

$$D_i \equiv \sum_{\alpha;\, j} \frac{1}{M_\alpha^2}\, \mathrm{Re}\Big[d_{ij(\alpha)}^{LR} + d_{ij(\alpha)}^{RL}\Big]\,. \tag{C26}$$

For the case of Dirac neutrinos,

$$\begin{aligned} d_{ij(\alpha)}^{LR\,(\pi^+)} &\equiv \frac{U_{ij}}{V_{ud}} \\ &\quad\times \left[f_{1i(\alpha)}^{Rd} f_{1j(\alpha)}^{Ru\,*} + \frac{2\, m_{\pi^+}^2 Q}{m_i(m_d + m_u)} f_{1i(\alpha)}^{Ld} f_{1j(\alpha)}^{Ru\,*}\right], \\ d_{ij(\alpha)}^{LR\,(K^+)} &\equiv \frac{U_{ij}}{V_{us}} \\ &\quad\times \left[f_{1i(\alpha)}^{Rd} f_{2j(\alpha)}^{Ru\,*} + \frac{2\, m_{K^+}^2 Q}{m_i(m_s + m_u)} f_{1i(\alpha)}^{Ld} f_{2j(\alpha)}^{Ru\,*}\right], \\ d_{ij(\alpha)}^{RL} &\equiv d_{ij(\alpha)}^{LR}\,(L \leftrightarrow R)\,, \end{aligned} \tag{C27}$$

whereas for Majorana neutrinos the only nonzero terms are,

$$d_{ij(\alpha)}^{RL\,(\pi^+)} \equiv \frac{U_{ij}}{V_{ud}} f_{1i(\alpha)}^{Ld} f_{1j(\alpha)}^{Lu\,*}\,, \quad d_{ij(\alpha)}^{RL\,(K^+)} \equiv \frac{U_{ij}}{V_{sd}} f_{1i(\alpha)}^{Ld} f_{2j(\alpha)}^{Lu\,*}\,. \tag{C28}$$

The tightest bounds of this type originate from measurements of the branching fraction ratios

$$R(\pi^+) \equiv \frac{\Gamma(\pi^+ \to e^+\nu)}{\Gamma(\pi^+ \to \mu^+\nu)}\,, \quad R(K^+) \equiv \frac{\Gamma(K^+ \to e^+\nu)}{\Gamma(K^+ \to \mu^+\nu)}\,.$$

(a) Charged pion decays

The experimental measurement and the SM prediction yield

$$\begin{aligned} R(\pi^+) &= (1.2327 \pm 0.0023) \times 10^{-4} \quad [62]\,, \\ R(\pi^+)_{\mathrm{SM}} &= (1.2352 \pm 0.0001) \times 10^{-4} \quad [65]\,, \end{aligned}$$

which, given Eq. (C25), leads to

$$\big|D_1^{(\pi^+)} - D_2^{(\pi^+)}\big| \lesssim (3.9\ \mathrm{TeV})^{-2}\,. \tag{C29}$$

(b) Charged kaon decays

In this case,

$$\begin{aligned} R(K^+) &= (2.493 \pm 0.031) \times 10^{-5} \quad [42]\,, \\ R(K^+)_{\mathrm{SM}} &= (2.477 \pm 0.001) \times 10^{-5} \quad [65]\,, \end{aligned}$$

which results in

$$\big|D_1^{(K^+)} - D_2^{(K^+)}\big| \lesssim (3.1\ \mathrm{TeV})^{-2}\,. \tag{C30}$$

(3) Charged meson three-body decays to a meson and charged leptons

When the leptoquark has both LH and RH interactions with SM fermions, the three-body meson decays are less restrictive than the two-body decays. However, in the case of our model, with predominantly LH interactions, the bounds arising from $B^+ \to K^+ e^\pm \mu^\mp$ impose the most severe constraints on the leptoquark mass. The corresponding decay rate is expressed in terms of the form factors $f_+(q^2)$ and $f_0(q^2)$ defined via

$$\begin{aligned} &\langle \mathcal{M}'(p')|\bar{q}_1 \gamma^\mu q_2|\overline{\mathcal{M}}(p)\rangle \\ &= f_+(q^2)\left[p^\mu + p'^\mu - \frac{\Delta M^2}{q^2}\, q^\mu\right] + f_0(q^2)\frac{\Delta M^2}{q^2} q^\mu\,, \\ &\langle \mathcal{M}'(p')|\bar{q}_1 q_2|\overline{\mathcal{M}}(p)\rangle = f_0(q^2)\frac{\Delta M^2}{m_{q_1} - m_{q_2}}\,, \end{aligned} \tag{C31}$$

where the four-momentum transfer $q = p' - p$ and the meson squared mass difference $\Delta M^2 = m_{\mathcal{M}}^2 - m_{\mathcal{M}'}^2$. The contribution to the decay rate mediated by leptoquarks is

$$\begin{aligned} \Gamma(\mathcal{M} \to \mathcal{M}'\, l_i^+ l_j^-)_X &= \frac{1}{2048\,\pi^3}\frac{1}{m_{\mathcal{M}}^3} \\ &\times \int_{(m_i+m_j)^2}^{(m_{\mathcal{M}}-m_{\mathcal{M}'})^2} \frac{dq^2}{q^4}\sqrt{\lambda(q^2, m_{\mathcal{M}}^2, m_{\mathcal{M}'}^2)\,\lambda(q^2, m_i^2, m_j^2)} \\ &\times \Bigg\{\Bigg[\frac{1}{3} N_{ij}^+ \left(2\,q^2 - m_i^2 - m_j^2 - \frac{(m_i^2 - m_j^2)^2}{q^2}\right) \\ &\qquad + 2\, N_{ij}^-\, m_i\, m_j\Bigg]\, \lambda(q^2, m_{\mathcal{M}}^2, m_{\mathcal{M}'}^2)\, |f_+(q^2)|^2 \\ &+ \Bigg[N_{ij}^+ \left(m_i^2 + m_j^2 - \frac{(m_i^2 - m_j^2)^2}{q^2}\right) - 2\, N_{ij}^-\, m_i\, m_j \\ &\quad + 4\, P_{ij}^+\, q^2(q^2 - m_i^2 - m_j^2) - 8\, P_{ij}^-\, q^2\, m_i\, m_j \\ &\quad - 4\left(R_{ij}^+ + R_{ij}^-\right) m_i \left(q^2 - m_i^2 + m_j^2\right) \\ &\quad + 4\left(R_{ij}^+ - R_{ij}^-\right) m_j \left(q^2 + m_i^2 - m_j^2\right)\Bigg] \\ &\times \left(m_{\mathcal{M}}^2 - m_{\mathcal{M}'}^2\right)^2 |f_0(q^2)|^2\Bigg\}\,, \end{aligned} \tag{C32}$$

where

$$\begin{aligned} \lambda(x, y, z) &\equiv (x - y - z)^2 - 4\, y\, z\,, \\ N_{ij}^\pm &\equiv \left|\sum_\alpha \frac{n_{ij(\alpha)}^{LL} + n_{ij(\alpha)}^{RR}}{M_\alpha^2}\right|^2 \pm \left|\sum_\alpha \frac{n_{ij(\alpha)}^{LL} - n_{ij(\alpha)}^{RR}}{M_\alpha^2}\right|^2, \\ P_{ij}^\pm &\equiv \left|\sum_\alpha \frac{p_{ij(\alpha)}^{LR} + p_{ij(\alpha)}^{RL}}{M_\alpha^2}\right|^2 \pm \left|\sum_\alpha \frac{p_{ij(\alpha)}^{LR} - p_{ij(\alpha)}^{RL}}{M_\alpha^2}\right|^2, \\ R_{ij}^\pm &\equiv \sum_{\alpha,\beta} \frac{\mathrm{Re}\Big[\Big(n_{ij(\alpha)}^{LL} \pm n_{ij(\alpha)}^{RR}\Big)\Big(p_{ij(\beta)}^{LR\,*} \pm p_{ij(\beta)}^{RL\,*}\Big)\Big]}{M_\alpha^2\, M_\beta^2}\,, \end{aligned}$$

$$n_{ij(\alpha)}^{LL\,(K^+,\pi^+)} = f_{1i(\alpha)}^{Ld} f_{2j(\alpha)}^{Ld\,*}\,, \quad n_{ij(\alpha)}^{LL(\,B^+,\pi^+)} = f_{1i(\alpha)}^{Ld} f_{3j(\alpha)}^{Ld\,*}\,,$$
$$n_{ij(\alpha)}^{LL\,(B^+,K^+)} = f_{2i(\alpha)}^{Ld} f_{3j(\alpha)}^{Ld\,*}\,, \quad n_{ij(\alpha)}^{RR} = n_{ij(\alpha)}^{LL}\,(L \leftrightarrow R)\,,$$
$$p_{ij(\alpha)}^{LR\,(K^+,\pi^+)} = \frac{f_{1i(\alpha)}^{Ld} f_{2j(\alpha)}^{Rd\,*} Q}{m_s - m_d}, \; p_{ij(\alpha)}^{LR\,(B^+,\pi^+)} = \frac{f_{1i(\alpha)}^{Ld} f_{3j(\alpha)}^{Rd\,*} Q}{m_b - m_d},$$
$$p_{ij(\alpha)}^{LR\,(B^+,K^+)} = \frac{f_{2i(\alpha)}^{Ld} f_{3j(\alpha)}^{Rd\,*} Q}{m_b - m_s}\,, \quad p_{ij(\alpha)}^{RL} = p_{ij(\alpha)}^{LR}\,(L \leftrightarrow R)\,. \tag{C33}$$

The form factors $f_+(q^2)$ and $f_0(q^2)$ are calculated using lattice methods. For the $K \to \pi$ form factor we use the linear fit given in [66]. For the $B \to \pi$ and $B \to K$ form factors we adopt the results of [67] and [25], respectively, where the interpolating functions were obtained using the Bourrely-Caprini-Lellouch parameterization [56].

The $K \to \pi$ form factor is [66]

$$f_{\{+,0\}}(q^2)_{K\pi} = f_+(0)_{K\pi}\left[1 + \lambda'_{\{+,0\}}\frac{q^2}{m_{\pi^+}^2}\right]\,, \tag{C34}$$

with $f_+(0)_{K\pi} = 0.9636$, $\lambda'_+ = 0.0308$ and $\lambda'_0 = 0.0198$.

The $B \to \pi$ and $B \to K$ form factors are given by

$$f_+(q^2) = \frac{1}{P_+(q^2)}\sum_{n=0}^{N_+-1} b_+^{(n)}\left[z^n - (-1)^{n-N_+}\frac{n}{N_+}\,z^{N_+}\right],$$
$$f_0(q^2) = \sum_{n=0}^{N_0} b_0^{(n)} z^n\,, \tag{C35}$$

where

$$z \equiv z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}\,. \tag{C36}$$

In the $B \to \pi$ case [67]:

$t_+ = (m_{B^+} + m_{\pi^+})^2$, $t_- = (m_{B^+} - m_{\pi^+})^2$,
$t_0 = (m_{B^+} + m_{\pi^+})(\sqrt{m_{B^+}} - \sqrt{m_{\pi^+}})^2$,
$b_+^{(0)} = 0.42$, $b_+^{(1)} = -1.46\,b_+^{(0)}$, $b_+^{(2)} = -4.7\,b_+^{(0)}$,
$b_0^{(0)} = 0.516$, $b_0^{(1)} = -3.94\,b_0^{(0)}$, $b_0^{(2)} = 0.7\,b_0^{(0)}$,
$P_+(q^2) = 1 - q^2/m_{B^*}^2$, $m_{B^*} = 5.325$ GeV,

whereas for $B \to K$ [25]:

$t_+ = (m_{B^+} + m_{K^+})^2$, $t_- = (m_{B^+} - m_{K^+})^2$,
$t_0 = (m_{B^+} + m_{K^+})(\sqrt{m_{B^+}} - \sqrt{m_{K^+}})^2$,
$b_+^{(0)} = 0.432$, $b_+^{(1)} = -0.65$, $b_+^{(2)} = -0.97$,
$b_0^{(0)} = 0.550$, $b_0^{(1)} = -1.89$, $b_0^{(2)} = 1.98$, $b_0^{(3)} = -0.02$,
$P_+(q^2) = 1 - q^2/(m_{B^+} + \Delta_+^*)^2$, $\Delta_+^* = 0.04578$ GeV.

The resulting constraints on B^+ decays are much weaker than the corresponding bounds presented in [9]. This is due to the fact that the calculation in [9] assumed $f_+(q^2) = f_0(q^2) = 1$. This assumption for the $B \to \pi$ and $B \to K$ form factors is quite far from their actual shape.

(a) Charged kaon decays

Experimental constraints from searches for the processes $K^+ \to \pi^+ e^\pm \mu^\mp$ yield,

- $$N_{12}^{+\,(K^+,\pi^+)} + (0.54\ \mathrm{GeV}^2)\,P_{12}^{+\,(K^+,\pi^+)} + (0.83\ \mathrm{GeV})\left(R_{12}^{+\,(K^+,\pi^+)} - R_{12}^{-\,(K^+,\pi^+)}\right) \lesssim \left[\frac{\mathrm{Br}(K^+ \to \pi^+ e^+ \mu^-)}{5.2\times 10^{-10}}\right](32.1\ \mathrm{TeV})^{-4}\,, \tag{C37}$$ [40]

- $$N_{21}^{+\,(K^+,\pi^+)} + (0.54\ \mathrm{GeV}^2)\,P_{21}^{+\,(K^+,\pi^+)} - (0.83\ \mathrm{GeV})\left(R_{21}^{+\,(K^+,\pi^+)} + R_{21}^{-\,(K^+,\pi^+)}\right) \lesssim \left[\frac{\mathrm{Br}(K^+ \to \pi^+ e^- \mu^+)}{1.3\times 10^{-11}}\right](80.6\ \mathrm{TeV})^{-4}\,. \tag{C38}$$ [41]

(b) Charged B meson decays

The experimental bounds on the decays $B^+ \to \pi^+ e^\pm \mu^\mp$, $B^+ \to K^+ e^\pm \mu^\mp$ and $B^+ \to K^+ \mu^\pm \tau^\mp$ give,

- $$N_{12}^{+\,(B^+,\pi^+)} + (138\ \mathrm{GeV}^2)\,P_{12}^{+\,(B^+,\pi^+)} + (0.76\ \mathrm{GeV})\left(R_{12}^{+\,(B^+,\pi^+)} - R_{12}^{-\,(B^+,\pi^+)}\right) \lesssim \left[\frac{\mathrm{Br}(B^+ \to \pi^+ e^+ \mu^-)}{9.2\times 10^{-8}}\right](13.5\ \mathrm{TeV})^{-4}\,, \tag{C39}$$ [44]

- $$N_{21}^{+\,(B^+,\pi^+)} + (138\ \mathrm{GeV}^2)\,P_{21}^{+\,(B^+,\pi^+)} - (0.76\ \mathrm{GeV})\left(R_{21}^{+\,(B^+,\pi^+)} + R_{21}^{-\,(B^+,\pi^+)}\right) \lesssim \left[\frac{\mathrm{Br}(B^+ \to \pi^+ e^- \mu^+)}{9.2\times 10^{-8}}\right](13.5\ \mathrm{TeV})^{-4}\,, \tag{C40}$$ [44]

- $$N_{12}^{+\,(B^+,K^+)} + (109\ \mathrm{GeV}^2)\,P_{12}^{+\,(B^+,K^+)} + (1.0\ \mathrm{GeV})\left(R_{12}^{+\,(B^+,K^+)} - R_{12}^{-\,(B^+,K^+)}\right) \lesssim \left[\frac{\mathrm{Br}(B^+ \to K^+ e^+ \mu^-)}{9.1\times 10^{-8}}\right](16.2\ \mathrm{TeV})^{-4}\,, \tag{C41}$$ [43]

- $$N_{21}^{+\,(B^+,K^+)} + (109\ \mathrm{GeV}^2)\,P_{21}^{+\,(B^+,K^+)} - (1.0\ \mathrm{GeV})\left(R_{21}^{+\,(B^+,K^+)} + R_{21}^{-\,(B^+,K^+)}\right) \lesssim \left[\frac{\mathrm{Br}(B^+ \to K^+ e^- \mu^+)}{1.3\times 10^{-7}}\right](14.9\ \mathrm{TeV})^{-4}\,, \tag{C42}$$ [43]

- $$N_{23}^{+\,(B^+,K^+)} + (96\ \mathrm{GeV}^2)\,P_{23}^{+\,(B^+,K^+)} + (10.3\ \mathrm{GeV})\left(R_{23}^{+\,(B^+,K^+)} - 1.2\,R_{23}^{-\,(B^+,K^+)}\right) \lesssim \left[\frac{\mathrm{Br}(B^+ \to K^+ \mu^+ \tau^-)}{7.7\times 10^{-5}}\right](2.7\ \mathrm{TeV})^{-4}\,, \tag{C43}$$ [45]

- $$N_{32}^{+\,(B^+,K^+)} + (96\ \mathrm{GeV}^2)\,P_{32}^{+\,(B^+,K^+)} - (10.3\ \mathrm{GeV})\left(R_{23}^{+\,(B^+,K^+)} + 1.2\,R_{23}^{-\,(B^+,K^+)}\right) \lesssim \left[\frac{\mathrm{Br}(B^+ \to K^+ \mu^- \tau^+)}{7.7\times 10^{-5}}\right](2.7\ \mathrm{TeV})^{-4}\,. \tag{C44}$$ [45]

(4) Tau decays

The leptoquark contribution to τ decay rates, neglecting the mass of the lepton in the final state, is

$$\Gamma(\tau^- \to \mathcal{M}' \, l_i^-)_X \;=\; \frac{m_\tau^3 f_{\mathcal{M}'}^2}{128\,\pi}\, T_i \left[1 - \frac{m_{\mathcal{M}'}^2}{m_\tau^2}\right]^2 , \quad \text{(C45)}$$

where

$$T_i \;\equiv\; \left|\sum_\alpha \frac{t^L_{i(\alpha)}}{M_\alpha^2}\right|^2 + \left|\sum_\alpha \frac{t^R_{i(\alpha)}}{M_\alpha^2}\right|^2 ,$$

$$t^{L\,(\pi^0)}_{i(\alpha)} \;\equiv\; f^{Rd}_{13(\alpha)} f^{Rd\,*}_{1i(\alpha)} + \frac{2 m_{\pi^0}^2 Q}{m_\tau (m_d + m_u)} f^{Ld}_{13(\alpha)} f^{Rd\,*}_{1i(\alpha)} \;,$$

$$t^{L\,(K_S^0)}_{i(\alpha)} \;\equiv\; \frac{1}{\sqrt{2}}\left[f^{Rd}_{13(\alpha)} f^{Rd\,*}_{2i(\alpha)} + \frac{2 m_{K_S^0}^2 Q}{m_\tau (m_s + m_d)} f^{Ld}_{13(\alpha)} f^{Rd\,*}_{2i(\alpha)}\right] - (1 \leftrightarrow 2) \;,$$

$$t^R_{i(\alpha)} \;\equiv\; t^L_{i(\alpha)}\,(L \leftrightarrow R) \quad \text{(C46)}$$

and $f_{\pi^0} = f_{\pi^+}/\sqrt{2}$. The bounds are

- $T_1^{(\pi^0)} \;\lesssim\; \left[\dfrac{\mathrm{Br}(\tau^- \to \pi^0\, e^-)}{8.0 \times 10^{-8}}\right] (5.0\ \mathrm{TeV})^{-4} \,, \qquad [52]$ (C47)
- $T_2^{(\pi^0)} \;\lesssim\; \left[\dfrac{\mathrm{Br}(\tau^- \to \pi^0\, \mu^-)}{1.1 \times 10^{-7}}\right] (4.7\ \mathrm{TeV})^{-4} \,, \qquad [51]$ (C48)
- $T_1^{(K_S^0)} \;\lesssim\; \left[\dfrac{\mathrm{Br}(\tau^- \to K_S^0\, e^-)}{2.6 \times 10^{-8}}\right] (8.4\ \mathrm{TeV})^{-4} \,, \qquad [53]$ (C49)
- $T_2^{(K_S^0)} \;\lesssim\; \left[\dfrac{\mathrm{Br}(\tau^- \to K_S^0\, \mu^-)}{2.3 \times 10^{-8}}\right] (8.6\ \mathrm{TeV})^{-4} \,. \qquad [53]$ (C50)

(5) Radiative charged lepton decay

The vector leptoquark contribution to the process $l_i \to l_j \gamma$ is induced at the loop level. Unlike for scalar leptoquarks, in the case of vector leptoquarks this effect cannot be computed in the general case, since the result is infinite and requires arbitrary subtractions that are well-defined only in a UV complete model. We parameterize our ignorance of this UV completion with the coefficients c_{LR} and c_{RL},

$$\Gamma(l_i^+ \to l_j^+ \gamma)_X = \frac{e^2 m_i^5}{4096\,\pi^5}\left[\left|\sum_{\alpha;\,k} \frac{f^{Ld}_{kj(\alpha)} f^{Ld*}_{ki(\alpha)}}{M_\alpha^2}\right|^2 + (L \leftrightarrow R)\right] + \frac{e^2 m_i^3 m_b^2}{4096\,\pi^5}\left[c_{LR}^2 \left|\sum_\alpha \frac{f^{Ld}_{3j(\alpha)} f^{Rd*}_{3i(\alpha)}}{M_\alpha^2}\right|^2 + (L \leftrightarrow R)\right] + \ldots \,, \quad \text{(C51)}$$

where $k = 1, 2, 3$ and we expect c_{LR} and c_{RL} to be $\mathcal{O}(1)$, with their values dependent on the UV details of the model. The ellipsis denotes interference and mass-suppressed terms.

If the matrices f_{ij} are proportional to unitary matrices, the terms in the first line of Eq. (C51) vanish. The experimental bounds, neglecting higher order terms, become,

- $c_{LR}^2 \left|\sum_\alpha \dfrac{f^{Ld}_{31(\alpha)} f^{Rd*}_{32(\alpha)}}{M_\alpha^2}\right|^2 + c_{RL}^2 \left|\sum_\alpha \dfrac{f^{Rd}_{31(\alpha)} f^{Ld*}_{32(\alpha)}}{M_\alpha^2}\right|^2 \lesssim \left[\dfrac{\mathrm{Br}(\mu \to e\,\gamma)}{4.2 \times 10^{-13}}\right] (332\ \mathrm{TeV})^{-4} \,, \qquad [68]$ (C52)
- $c_{LR}^2 \left|\sum_\alpha \dfrac{f^{Ld}_{31(\alpha)} f^{Rd*}_{33(\alpha)}}{M_\alpha^2}\right|^2 + c_{RL}^2 \left|\sum_\alpha \dfrac{f^{Rd}_{31(\alpha)} f^{Ld*}_{33(\alpha)}}{M_\alpha^2}\right|^2 \lesssim \left[\dfrac{\mathrm{Br}(\tau \to e\,\gamma)}{3.3 \times 10^{-8}}\right] (3.1\ \mathrm{TeV})^{-4} \,, \qquad [69]$ (C53)
- $c_{LR}^2 \left|\sum_\alpha \dfrac{f^{Ld}_{32(\alpha)} f^{Rd*}_{33(\alpha)}}{M_\alpha^2}\right|^2 + c_{RL}^2 \left|\sum_\alpha \dfrac{f^{Rd}_{32(\alpha)} f^{Ld*}_{33(\alpha)}}{M_\alpha^2}\right|^2 \lesssim \left[\dfrac{\mathrm{Br}(\tau \to \mu\,\gamma)}{4.4 \times 10^{-8}}\right] (2.9\ \mathrm{TeV})^{-4} \,. \qquad [69]$ (C54)

In our model the leading order terms contributing to $l_i^+ \to l_j^+ \gamma$ are $\mathcal{O}(m_b^2/M_{X_L}^2)$ and the resulting constraints are negligible compared to tree-level bounds.

(6) $l_i^+ \to l_j^+$ conversion

The effective Hamiltonian for the $l_i^+ \to l_j^+$ conversion consists of the dipole transition part corresponding to $l_i^+ \to l_j^+ \gamma$ and terms arising from integrating out the heavy vector leptoquarks, i.e.

$$\begin{aligned}\mathcal{H}^{\mathrm{eff}}_{l_i^+ \to l_j^+} = {} & \frac{e\, m_b}{16\,\pi^2} \sum_\alpha \frac{c_{LR}}{M_\alpha^2}\, f^{Ld}_{3j(\alpha)} f^{Rd*}_{3i(\alpha)}\, \bar{l}_{jR}\, \sigma_{\mu\nu}\, l_{iL} F^{\mu\nu} \\ & + \frac{e\, m_i}{16\,\pi^2} \sum_{\alpha,\,k} \frac{1}{M_\alpha^2}\, f^{Ld}_{kj(\alpha)} f^{Ld*}_{ki(\alpha)}\, \bar{l}_{jR}\, \sigma_{\mu\nu}\, l_{iL} F^{\mu\nu} \\ & + \sum_{\alpha,\,m} \frac{1}{M_\alpha^2}\Big[f^{Ld}_{mj(\alpha)} f^{Ld*}_{mi(\alpha)} \big(\bar{l}_{jL} \gamma^\mu l_{iL}\big)\big(\bar{d}_L^k \gamma_\mu d_L^k\big) \\ & \qquad - 2\, Q\, f^{Ld}_{mj(\alpha)} f^{Rd*}_{mi(\alpha)} \big(\bar{l}_{jL} l_{iR}\big)\big(\bar{d}_R^k\, d_L^k\big)\Big] \\ & + (L \leftrightarrow R) + \ldots \,, \end{aligned} \quad \text{(C55)}$$

where $m = 1, 2$. The steps required to match the effective Hamiltonian (C55) to the Hamiltonian at the nucleon level and compute the conversion rate are provided in [70, 71].

The tightest experimental constraint from $l_i^+ - l_j^+$ conversion arises from $\mu - e$ conversion on gold [54]. Since the resulting bound on the dipole transition contribution is less restrictive than the constraint from $\mu \to e\,\gamma$ in Eq. (C52), we concentrate only on the second part of the Hamiltonian (C55). Following [70], the $\mu - e$ conversion rate is then given by

$$\Gamma(\mu \to e) = m_\mu^5 \big|\tilde{g}^{(p)}_{LV} V_p + \tilde{g}^{(n)}_{LV} V_n + \tilde{g}^{(p)}_{LS} S_p + \tilde{g}^{(n)}_{LS} S_n\big|^2 + (L \leftrightarrow R) \,, \quad \text{(C56)}$$

where

$$\tilde{g}_{LV}^{(p)} = \tfrac{1}{2}\,\tilde{g}_{LV}^{(n)} = \sum_\alpha \frac{1}{M_\alpha^2} f_{12(\alpha)}^{Ld} f_{11(\alpha)}^{Ld*} \,, \tag{C57}$$

$$\tilde{g}_{LS}^{(p)} = -2\,Q \sum_\alpha \frac{1}{M_\alpha^2}\left[4.3\, f_{12(\alpha)}^{Ld} f_{11(\alpha)}^{Rd*} + 2.5\, f_{22(\alpha)}^{Ld} f_{21(\alpha)}^{Rd*}\right],$$

$$\tilde{g}_{LS}^{(n)} = -2\,Q \sum_\alpha \frac{1}{M_\alpha^2}\left[5.1\, f_{12(\alpha)}^{Ld} f_{11(\alpha)}^{Rd*} + 2.5\, f_{22(\alpha)}^{Ld} f_{21(\alpha)}^{Rd*}\right],$$

with similar relations obtained upon switching $(L \leftrightarrow R)$. The numerical coefficients were adopted from [72]. For the ${}^{197}_{79}\mathrm{Au}$ nucleus, which provides the most stringent bound, the parameters in Eq. (C56) are,

$$V_p = 0.0974\,,\;\; V_n = 0.146\,,\;\; S_p = 0.0614\,,\;\; S_n = 0.0918\,,$$

and are the result of the calculation using "method 1" in Sec. III A of [70]. The best bound on $\mu - e$ conversion is [54],

$$\frac{\Gamma(\mu \to e \text{ in Au})}{\Gamma(\mu \text{ capture in Au})} < 7 \times 10^{-13}\,. \tag{C58}$$

The constraints on general $(3,1)_{2/3}$ leptoquark models are derived by inserting Eq. (C56) into (C58) and using the total μ^- capture rate in ${}^{197}_{79}\mathrm{Au}$, $\Gamma(\mu \text{ capture in Au}) = 8.6 \times 10^{-18}$ GeV [73]. In the case of our model, with just LH leptoquark couplings, the constraint simplifies to

$$\left|\sum_\alpha \frac{f_{12(\alpha)}^{Ld} f_{11(\alpha)}^{Ld*}}{M_\alpha^2}\right|^{-1/2} \gtrsim\; 762 \text{ TeV}\,. \tag{C59}$$

Finally, let us note that the bounds on generic leptoquark models were considered in [5–10]. Our formulae reproduce those results up to the difference in the adopted values of quark masses, meson decay constants and form factors used.

Appendix D: Flavor constraints: $\mathrm{SU}(4)_L \times \mathrm{SU}(4)_R$ model

In our model $X^{(1)} \equiv X_1$ and $X^{(2)} \equiv X_2$ given by Eq. (15), therefore the coefficients in Eq. (C1) are

$$\begin{aligned}
f_{ij(1)}^{Lu} &\equiv \frac{g_L \cos\theta_4}{\sqrt{2}} L_{ij}^u\,, & f_{ij(1)}^{Ld} &\equiv \frac{g_L \cos\theta_4}{\sqrt{2}} L_{ij}^d\,,\\
f_{ij(1)}^{Ru} &\equiv \frac{g_R \sin\theta_4}{\sqrt{2}} R_{ij}^u\,, & f_{ij(1)}^{Rd} &\equiv \frac{g_R \sin\theta_4}{\sqrt{2}} R_{ij}^d\,,\\
f_{ij(2)}^{Ru} &\equiv \frac{g_R \cos\theta_4}{\sqrt{2}} R_{ij}^u\,, & f_{ij(2)}^{Rd} &\equiv \frac{g_R \cos\theta_4}{\sqrt{2}} R_{ij}^d\,,\\
f_{ij(2)}^{Lu} &\equiv -\frac{g_L \sin\theta_4}{\sqrt{2}} L_{ij}^u\,, & f_{ij(2)}^{Ld} &\equiv -\frac{g_L \sin\theta_4}{\sqrt{2}} L_{ij}^d\,.
\end{aligned} \tag{D1}$$

Constraints on the model parameters are obtained by substituting the expressions in Eq. (D1) into the bounds derived in App. C. In the limit $v_R \gg v_L$ and $v_R \gg v_\Sigma$, for which $\sin\theta_4 \simeq 0$, $X_1 = X_L$ and $X_2 = X_R$, one arrives at the constraints listed below. The numbering scheme indicates which equation in App. C a given constraint originated from.

K_L^0 decays

$$\frac{m_{X_L}}{g_L\sqrt{|\mathrm{Re}(L_{11}^d L_{21}^{d*})|}} \gtrsim\; 21.2 \text{ TeV}\,, \tag{D12}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{11}^d L_{22}^{d*} + L_{21}^d L_{12}^{d*}|}} \gtrsim\; 225 \text{ TeV}\,, \tag{D13}$$

$$\frac{m_{X_L}}{g_L\sqrt{|\mathrm{Re}(L_{12}^d L_{22}^{d*})|}} \gtrsim\; 51.0 \text{ TeV}\,. \tag{D14}$$

B^0 decays

$$\frac{m_{X_L}}{g_L\sqrt{|L_{11}^d L_{31}^{d*}|}} \gtrsim\; 0.24 \text{ TeV}\,, \tag{D15}$$

$$\frac{m_{X_L}}{g_L\sqrt[4]{|L_{11}^d L_{32}^{d*}|^2 + |L_{12}^d L_{31}^{d*}|^2}} \gtrsim\; 8.9 \text{ TeV}\,, \tag{D16}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{12}^d L_{32}^{d*}|}} \gtrsim\; 10.7 \text{ TeV}\,, \tag{D17}$$

$$\frac{m_{X_L}}{g_L\sqrt[4]{|L_{11}^d L_{33}^{d*}|^2 + |L_{13}^d L_{31}^{d*}|^2}} \gtrsim\; 2.6 \text{ TeV}\,, \tag{D18}$$

$$\frac{m_{X_L}}{g_L\sqrt[4]{|L_{12}^d L_{33}^{d*}|^2 + |L_{13}^d L_{32}^{d*}|^2}} \gtrsim\; 2.8 \text{ TeV}\,, \tag{D19}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{13}^d L_{33}^{d*}|}} \gtrsim\; 1.0 \text{ TeV}\,. \tag{D20}$$

B_s^0 decays

$$\frac{m_{X_L}}{g_L\sqrt{|L_{21}^d L_{31}^{d*}|}} \gtrsim\; 0.2 \text{ TeV}\,, \tag{D21}$$

$$\frac{m_{X_L}}{g_L\sqrt[4]{|L_{21}^d L_{32}^{d*}|^2 + |L_{22}^d L_{31}^{d*}|^2}} \gtrsim\; 6.4 \text{ TeV}\,, \tag{D22}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{22}^d L_{32}^{d*}|}} \gtrsim\; 6.0 \text{ TeV}\,, \tag{D23}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{23}^d L_{33}^{d*}|}} \gtrsim\; 0.7 \text{ TeV}\,. \tag{D24}$$

π^+ decays

$$\frac{m_{X_L}}{g_L\sqrt{\left|\mathrm{Re}\big[L_{11}^d (VL^d)_{11}^* - 4.3\, L_{12}^d (VL^d)_{12}^*\big]\right|}} \gtrsim\; 2.8 \text{ TeV}\,. \tag{D29}$$

K^+ decays

$$\frac{m_{X_L}}{g_L\sqrt{\left|\mathrm{Re}\big[L_{11}^d (VL^d)_{21}^* - 4.3\, L_{12}^d (VL^d)_{22}^*\big]\right|}} \gtrsim\; 2.2 \text{ TeV}\,, \tag{D30}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{11}^d L_{22}^{d*}|}} \gtrsim\; 27.0 \text{ TeV}\,, \tag{D37}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L_{12}^d L_{21}^{d*}|}} \gtrsim\; 67.8 \text{ TeV}\,. \tag{D38}$$

B^+ decays

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{11}L^{d*}_{32}|}} \gtrsim 11.4 \text{ TeV} , \tag{D39}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{12}L^{d*}_{31}|}} \gtrsim 11.4 \text{ TeV} , \tag{D40}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{21}L^{d*}_{32}|}} \gtrsim 13.6 \text{ TeV} , \tag{D41}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{22}L^{d*}_{31}|}} \gtrsim 12.5 \text{ TeV} , \tag{D42}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{22}L^{d*}_{33}|}} \gtrsim 2.3 \text{ TeV} , \tag{D43}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{23}L^{d*}_{32}|}} \gtrsim 2.3 \text{ TeV} . \tag{D44}$$

τ decays

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{11}L^{d*}_{13}|}} \gtrsim 3.6 \text{ TeV} , \tag{D47}$$

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{12}L^{d*}_{13}|}} \gtrsim 3.3 \text{ TeV} , \tag{D48}$$

$$\frac{m_{X_L}}{g_L\sqrt{\left|L^d_{21}L^{d*}_{13} - L^d_{11}L^{d*}_{23}\right|}} \gtrsim 5.0 \text{ TeV} , \tag{D49}$$

$$\frac{m_{X_L}}{g_L\sqrt{\left|L^d_{22}L^{d*}_{13} - L^d_{12}L^{d*}_{23}\right|}} \gtrsim 5.1 \text{ TeV} . \tag{D50}$$

$\mu - e$ conversion

$$\frac{m_{X_L}}{g_L\sqrt{|L^d_{12}L^{d*}_{11}|}} \gtrsim 539 \text{ TeV} . \tag{D59}$$

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