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REMARKS ON THE BETA STABILITY IN NEUTRON STARS*

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Abstract:

We show that in a relativistic theory of nuclear matter the usually surpressed beta decay of neutrons $n \rightarrow p + e^- + \bar{\nu}_e$ is allowed. It can serve as a cooling mechanism. The theory also predicts the existence of free pions inside dense neutron star. The conclusions rest primarily on the correct phenomenological description of symmetry energy in symmetric (N = Z) nuclear matter.

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The cooling of neutron stars, especially since the launch of Einstein X-Ray Observatory, has elicited a number of suggestions explaining the high neutrino luminosity required to understand the rapid cooling of neutron stars. A natural explanation of this, such as the beta decay of neutrons i.e., $n \rightarrow p + e^- + \bar{\nu}_e$ was ruled out, because of the difficulty in conserving momentum near the corresponding Fermi surfaces.[1] The Fermi-Dirac factor would significantly surpress such a cooling mechanism. Alternative processes such as pion condensation[2] and recently proposed beta decay of the down quark[3] $d \rightarrow u + e^- + \bar{\nu}_e$ have been suggested as a way to rapidly cool the neutron star. The pion condensate model implies that there exists a new collective mode inside neutron stars, while the quark model suggests that the interiors of neutron stars are composed of quarks. In view of these interesting suggestions to account for the rapid neutron star cooling, another look at the beta decay of ordinary neutrons in a dense neutron star at non zero temperature is quite appropriate. Perhaps a conventional explanation of neutron star cooling is still possible.

It is assumed that the ground state of zero temperature ($T = 0$) neutron star is beta stable. That is, the reaction $n \rightarrow p + e^- + \bar{\nu}_e$ has come to equilibrium. At non-zero temperature beta stability will be disturbed. The reaction $n \rightarrow p + e^- + \bar{\nu}_e$ will then proceed around the neutron and proton Fermi surfaces if the temperature T is not large. The rate of beta decay will be determined by the neutron and proton Fermi surfaces.

If the top of the neutron Fermi surface is given by the Fermi momentum $k_F(n) = 400$ MeV, Bahcall and Wolf estimate, based on a non-interacting, non-relativistic model, the proton concentration in beta stability with neutrons to be 1.5 percent that of the neutrons. This places the top of the proton Fermi surface at $k_F(p) = 100$ MeV. On account of charge neutrality, the electron Fermi momentum must also be $k_F(e) = k_F(p) = 100$ MeV. In this case $n \rightarrow p + e^- + \bar{\nu}_e$ as a cooling mechanism will be highly surpressed. There have been other estimates of proton concentration in neutron matter. Sprung and Nemeth[4)] estimate, in a Bruckner-Bethe-Goldstone theory, that for $k_F(n) =$ 315 MeV the relative proton concentration could be as high as 8 percent. A naive, linear extrapolation of their results indicates that for $k_F(n) = 400$ MeV, the proton concentration would be 12 percent. That is a factor of eight greater than the non-interacting model estimate. Furthermore, the calculations of Maxwell, Brown et al., in a conventional sigma model (see their footnote number 17) estimate that the proton concentration could be 6 percent at a neutron density 1.8 that of normal nuclear matter (this corresponds to $k_F(n) = 400$ MeV). This is a factor of 4 greater than Bahcall and Wolf estimate. What is the required proton concentration to approximately balance the momenta in the neutron beta decay $n \rightarrow p + e^- + \bar{\nu}_e$ without having the Fermi-Dirac factor significantly surpressing the reaction? Evidently we need $k_F(p) \approx 200$ MeV. This corresponds to a relative proton concentration of 12.5 percent and quite close to the extrapolated value predicted by Sprung and Wolf.

It is known that one has to pay the price of symmetry energy when separating neutrons from protons. It, together with Coulomb energy, plays an essential role in determining the line of beta stability in nuclei.[5] It stands to reason that it should be considered in studying the proton concentration in beta stable neutron stars. For this purpose we study nuclear matter in a relativistic quantum field theory proposed by Walecka.[6] As we shall show, the actual details of this model are unimportant to support our conclusions, as long as the symmetry energy of ordinary, symmetric (N = Z), nuclear matter is correctly parametrized. In fact, the sigma model of Maxwell, Brown et al., leads to the same conclusions as long as a neutral rho field R^0_μ is introduced to parametrize the symmetry energy. We prefer to work with the Walecka model for the simple reason that it is an excellent phenomenological model of nuclear matter and finite nuclei.[7]

The Lagrangian describing the interaction of nucleons ψ with a scalar field σ an isoscalar vector field ω_μ and an isovector vector field $\hat{R}_\mu$ is assumed to be

$$= -\bar{\psi}\left(\gamma_\mu \frac{\partial}{\partial X_\mu} + m_n\right)\psi - \frac{1}{4} F_{\mu\nu}F_{\mu\nu} - \frac{1}{4} \hat{G}_{\mu\nu}\hat{G}_{\mu\nu} - \frac{1}{2}((\partial_\mu\sigma)^2 + m_s^2\sigma^2)$$

$$- g_s\sigma\bar{\psi}\psi - \frac{1}{2} m_V^2\omega_\mu\omega_\mu - \frac{1}{2} m_V^2\hat{R}_\mu\cdot\hat{R}_\mu + ig_V\bar{\psi}\gamma_\mu\psi\omega_\mu + ig_r\bar{\psi}\gamma_\mu\hat{\tau}\psi\hat{R}_\mu$$

where

$$F_{\mu\nu} = \frac{\partial}{\partial X_\mu} \omega_\nu - \frac{\partial}{\partial X_\nu} \omega_\mu$$

$$\hat{G}_{\mu\nu} = \frac{\partial}{\partial X_\mu} \hat{R}_\nu - \frac{\partial}{\partial X_\nu} \hat{R}_\mu \tag{2}$$

The mean field approximation replaces the quantum field operators by their expectation values $\sigma \Rightarrow \langle\sigma\rangle = \sigma_0$, $\omega_\mu \Rightarrow \langle\omega_\mu\rangle = \delta_{\mu 0}\omega_0$ $R_\mu^{(k)} \Rightarrow \langle R_\mu^{(k)}\rangle = \delta_{k0}\delta_{\mu 0}R_0^{(o)}$. The equations of motion for infinite, translationally and rotationally invariant nuclear matter are

$$m_\sigma^2 \sigma_0 = -g_s(\rho_s(n) + \rho_s(p)) \tag{3a}$$

$$m_v^2 \omega_0 = g_v(\rho_v(n) + \rho_v(p)) \tag{3b}$$

$$m_v^2 R_0^{(o)} = g_r(\rho_v(n) - \rho_v(p)) \tag{3c}$$

with the Fermi energies for neutrons and protons given by

$$E_F(n) = g_v\omega_0 + g_r R_0^{(o)} + \sqrt{k_F(n)^2 + m^{*2}} \tag{4a}$$

$$E_F(p) = g_v\omega_0 - g_r R_0^{(o)} + \sqrt{k_F(p)^2 + m^{*2}} \tag{4b}$$

and the effective nucleon mass m^* is

$$m^* = m_n + g_s\sigma_0$$

The neutron and proton scalar densities $(\rho_s(n), \rho_s(p))$ are given in the Hartree approximation by

$$\rho_s = \frac{2}{(2\pi)^3} \int d^3k \frac{m^*}{\sqrt{k^2 + m^{*2}}} \qquad (5a)$$

$$\rho_v = \frac{2}{(2\pi)^3} \int d^3k = \frac{1}{3\pi^2} k_F^3 \qquad (5b)$$

where the appropriate neutron, proton indices should be attached in Eq. (5a) and Eq. (5b). The coupling constants $C_s = g_s(m_n/m_s)$, $C_v = g_v(m_n/m_v)$, $C_r = g_r(m_n/m_v)$ are determined from known properties of symmetric nuclear matter, such as saturation density, binding energy per particle and symmetry energy. For a saturation Fermi momentum of 1.34 fm^{-1} and a binding energy of –15.75/particle the values of C_s and C_v are $C_s = 17.96$, $C_v = 15.60$. The coupling constant C_r will be varied to yield a range of symmetry energy values. For the Walecka model the coupling will range from 0.0 to 5.0 corresponding to a symmetry energy of 20 MeV and 40 MeV. The high value of the symmetry energy even without the rho meson field (corresponding to $C_r = 0{,}0$) in the Walecka model is due to the fact that at normal nuclear densities the effectivemass m* is just half its free nucleon value, $m^* = 0.5\ m_n$. For such a low effective mass, the Fermi motion of the nucleons accounts for a substantial portion of the symmetry energy. Furthermore, it implies that the nucleons are relativistic. One can explore the behavior of the effective mass m* by changing the interaction in the Lagrangian of Eq. (1). A simple way to do this is to introduce non-linear self-interactions of the sigma field, that is, let the potential functional be $U(\sigma) = a\sigma^2 + b\sigma^3 + c\sigma^4$. The saturation and binding energy of symmetric nuclear matter can now be parametrized by a continuum set of parameters. For each choice of C_s

and C_v appropriate values of b and c can be found to achieve this. The Boguta-Bodmer[8)] model is of special interest, since it accounts for some properties of finite nuclei. In this model $C_s = 8.0$ and $C_v = 2.0$. Then the effective mass $m^* \sim 0.9\ m_n$ and the symmetry energy without the rho meson ($C_r = 0.0$) is then only 13 MeV. In this case the relative proton concentration is only 1 percent at $k_F(n) = 400$ MeV. The Boguta-Bodmer model corresponds most closely to a non-interacting model. We find that if C_r is adjusted to yield a net symmetry energy of 30 to 35 MeV for symmetric nuclear matter at normal densities, the proton concentration in beta stable neutron matter, as a function of neutron Fermi momentum, is about the same irrespective of the model considered.

To compute the proton concentration (or equivalently, the proton Fermi momentum) we require beta stability at zero temperature. For this, the chemical potentials must satisfy

$$\mu(n) - \mu(p) = \mu(e) \tag{6}$$

and charge neutrality requires that

$$k_F(p) = k_F(e) \tag{7}$$

In this calculation we assume that the chemical potential is just the Fermi energy (this is true only at zero temperature). Furthermore, we assume that the electrons are highly relativistic ($k_F(e) \gg m_e$). Then Eq. (6) becomes

$$2\left(\frac{g_r}{m_v}\right)^2 [\rho_v(n) - \rho_v(p)] + \sqrt{k_F(n)^2 + m^{*2}} - \sqrt{k_F(p)^2 + m^{*2}} = k_F(p) \qquad (8)$$

where we used Eq. (3c) and Eq. (7) to eliminate the rho field in favor of the densities and the electron Fermi momentum in favor of the proton Fermi momentum. For a given neutron Fermi momentum $k_F(n)$, those solutions of the field equations Eqs. (3a) and (3c) are beta stable that also satisfy Eq. (8). We solve the system of equations numerically.

In Fig. 1 we show the Walecka model prediction for the relative proton concentration in beta stable neutron star for $C_r = 0.0$, 3.5 and 5.0, corresponding to a symmetry energy of 20 MeV, 30 MeV and 40 MeV. As expected, the increasing symmetry energy increases the relative proton concentration. In Fig. 2 we show the Boguta-Bodmer ($C_s = 0.8$, $C_v = 2.0$) model predictions for $C_r = 0.0$, 5.5 corresponding to a symmetry energy of 13 MeV and 35 MeV respectively. Although both models have essentially different physical content at normal nuclear densities, it is clear that in both cases the symmetry energy is the deciding factor in determining the proton concentration as a function of the neutron Fermi momentum. In Fig. 3 we show the proton Fermi momentum in the Walecka model. For $k_F(n) = 400$ MeV, the top of the proton Fermi surface is at $k_F(p) \approx 200$ MeV and not at

100 MeV as predicted by Bahcall and Wolf. It is interesting to note that the neutron and proton chemical potential difference ($\mu_n - \mu_p \approx k_F(p)$) exceeds the mass of a free pion at about $k_F(n) \sim$ 350 MeV. Since the effective pion mass in nuclear matter is about 200 MeV or above, we expect a large number of free pions to exist when $k_F(n) > 400$ MeV.

A correct phenomenological parametrization of nuclear matter symmetry energy leads to two interesting conclusions. First, the concentration of protons inside a beta stable neutron star is much greater than previously estimated. Our results are not in contradiction with the Sprung and Nemeth calculations at low neutron Fermi momentum. A naive extrapolation of their results to higher neutron densities is consistent with our results. In our calculation the proton concentration is large enough to allow the ordinary neutron beta decay $n \rightarrow p + e^- + \bar{\nu}_e$ to be an important cooling mechanism in neutron stars. Perhaps what is being observed is just a confirmation of conventional nuclear physics extrapolated to higher densities. Second, since the rho field is an isospin one field and distinguishes between neutrons and protons, it must be very important in pion condensation in neutron matter. In the present calculation it is so important, that it allows free pions to exist at relatively low densities.

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Figure Captions

Fig. 1. Percentage proton concentration as a function of neutron Fermi momentum for various rho meson coupling strengths in Walecka model.

Fig. 2. Percentage proton concentration as a function of neutron Fermi momentum for various rho meson coupling strengths in Boguta-Bodmer model.

Fig. 3. Proton Fermi momentum as a function of neutron Fermi momentum in Walecka model, for symmetry energies of symmetric nuclear matter of 30 MeV and 40 MeV respectively.

The figures were printed from originals provided by the author.

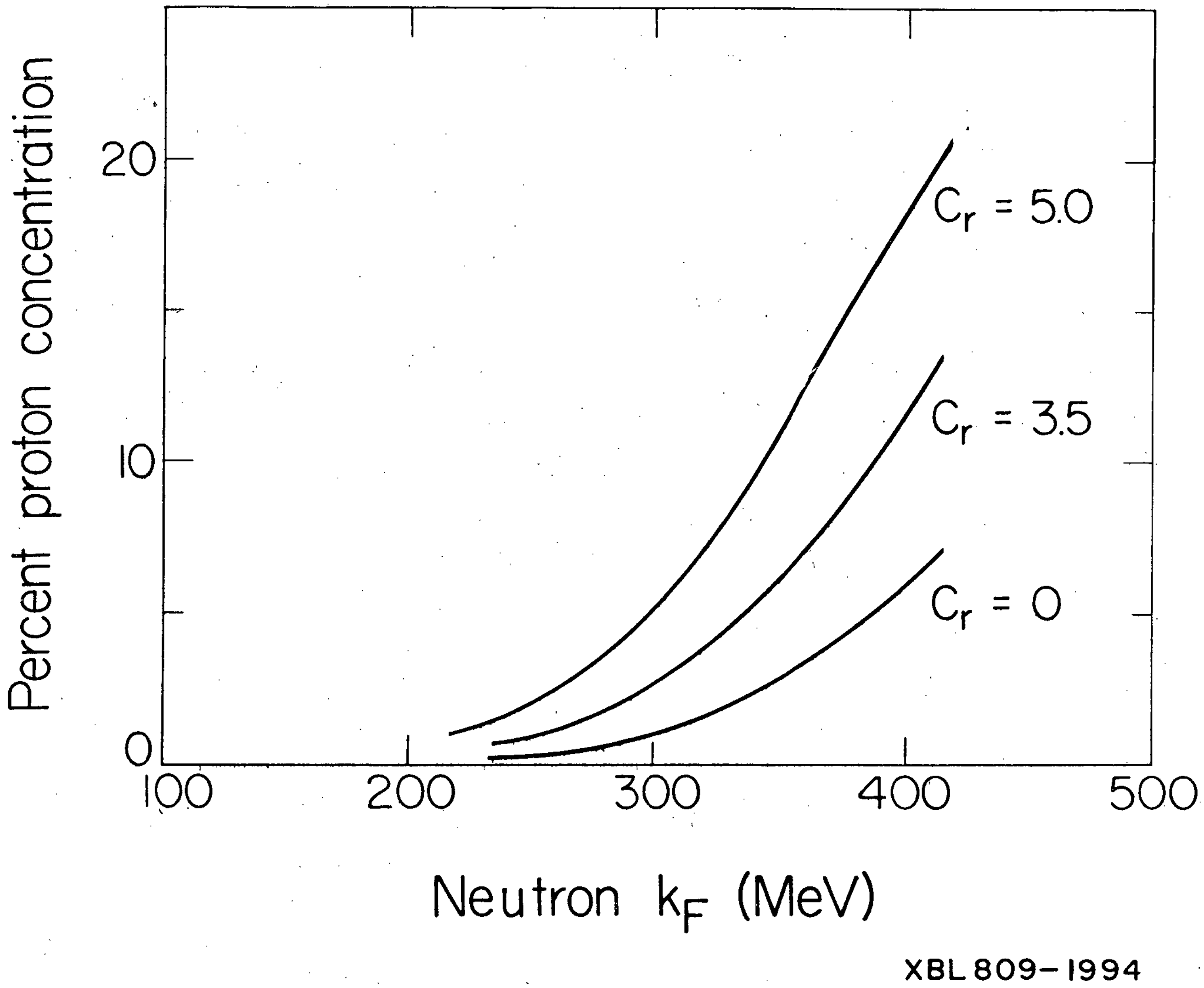
Percent proton concentration
0
10
20
100
200
300
400
500
Neutron k_F (MeV)
C_r = 5.0
C_r = 3.5
C_r = 0
XBL 809-1994

Fig. 1

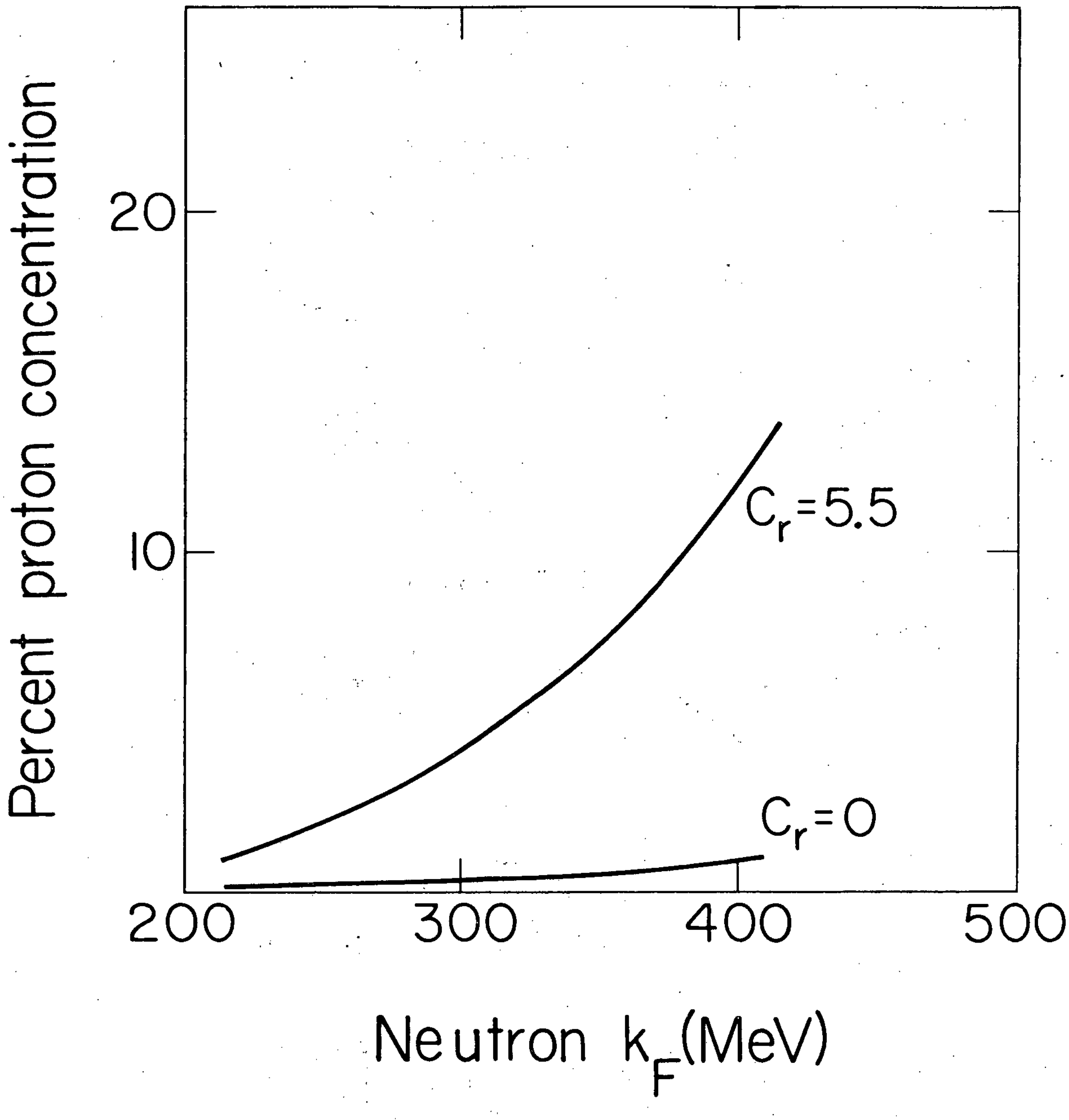

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Fig. 2

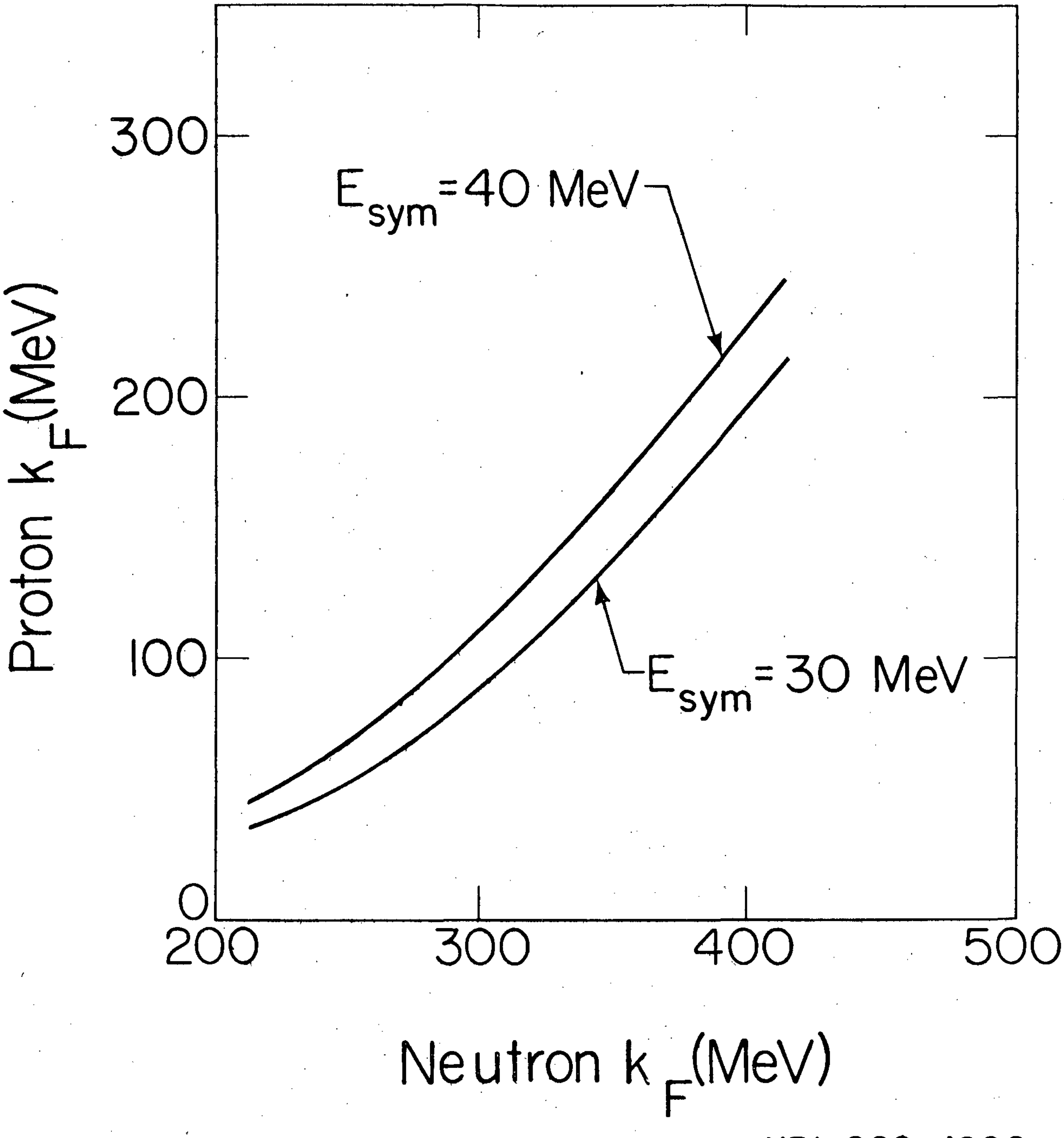

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Fig. 3

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