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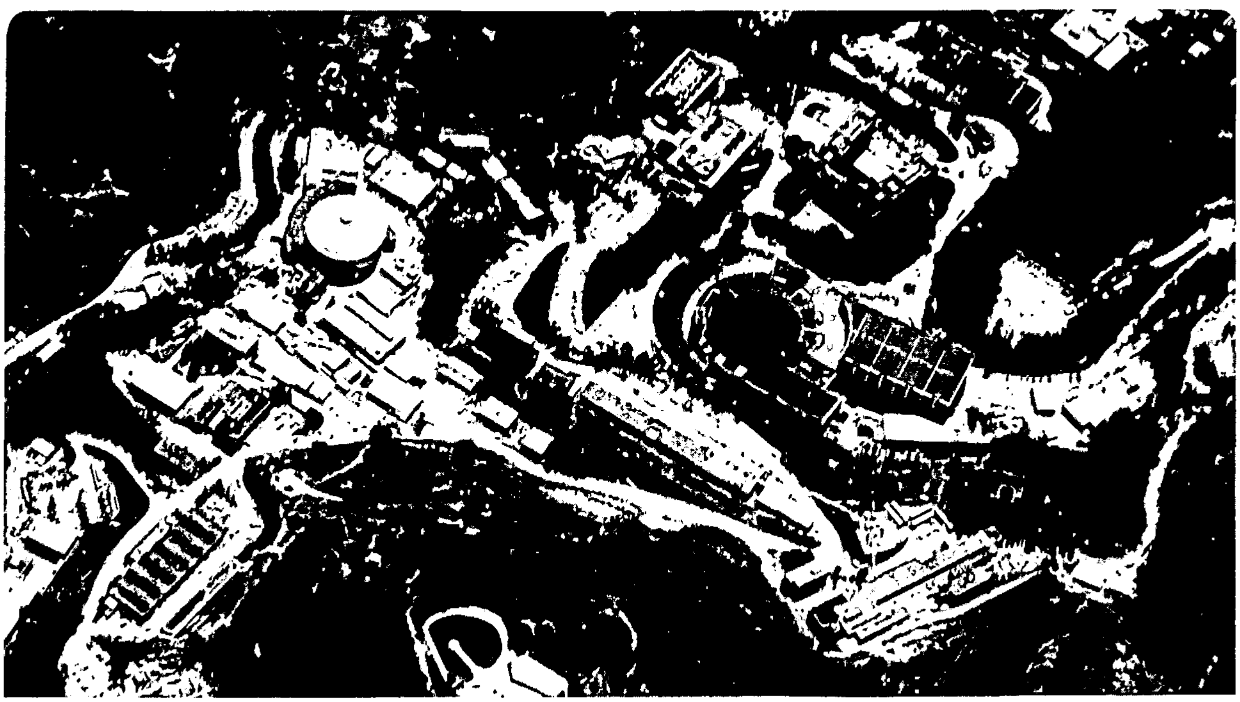

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The Electromagnetic Penguin Contribution to ϵ'/ϵ for Large Top Quark Mass*

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We investigate the electromagnetic penguin contribution to ϵ'/ϵ when the top quark mass is large. We find it depends sensitively on the top quark mass.†

1 Introduction

It is encouraging that the parameter ϵ'/ϵ is now being studied with a precision of 10^{-3} [1]. But it is clear that this measurement can only yield a detailed understanding of the standard model in conjunction with careful theoretical study. This investigation can be separated into two parts, the long and short distance calculations. The evaluation of low energy matrix elements is a nonperturbative problem which awaits lattice calculations (see other talks in this workshop for status and prospects). But the short distance calculation is straightforward in that it involves calculating the coefficients of quark operators in a regime where QCD perturbation theory should be reliable. To correctly relate the calculated matrix elements to the measured value of ϵ'/ϵ requires determining the dependence of these coefficients on the standard model parameters.

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Because current indications are that the top quark mass is not small compared to the W mass, this calculation is performed to all orders in m_t/m_W. This entails calculating the W box and Z penguin diagrams contributing to the $\Delta S = 1$ processes, in addition to the usual photon penguin diagram. We will see that both the magnitude and sign of this contribution is very sensitive to the value of the top quark mass. Furthermore, the operator transforming as $(27_L, 1_R)$, which also develops a CP violating amplitude when electromagnetic contributions are included, may also be significant because of a large top quark mass.

We first recall some definitions and the reason that the electromagnetic penguin contribution, suppressed by α_{em}/α_s, is nevertheless expected to be sizable. The quantity ϵ' is defined by

$$\epsilon' = -\frac{e^{i(\delta_2-\delta_0+\pi/2)}}{\sqrt{2}} \frac{\mathrm{Im}\, A_0}{\mathrm{Re}\, A_0} \omega \left(1 - \omega^{-1} \frac{\mathrm{Im}\, A_2}{\mathrm{Im}\, A_0}\right), \qquad (1)$$

where δ_2 and δ_0 are the isospin 2 and 0 final state strong interaction phase shifts, $A_{2[0]}$ is the isospin two[zero] $K \to \pi\pi$ amplitude, and ω, the ratio of their real parts, is 0.045. (We will be working in the quark basis.)

Notice that because of the factor of ω^{-1}, contributions to Im A_2 (such as is generated by the electromagnetic penguin) are 20 times more important than comparable contributions to Im A_0. The electromagnetic penguin operator, which is generated by a diagram in which the gluon of the usual penguin diagram is replaced by a photon (or a Z if we work to all orders in $(m_t/m_W)^2$), contributes to this isospin 2 amplitude. Furthermore, the electromagnetic penguin operator, which transforms nontrivially under both the groups $SU(3)_L$ and $SU(3)_R$, is expected to be chirally enhanced. The other operator generated by the photon and Z penguin diagrams (and also by a box diagram in which two W's are exchanged), which transforms as $(27_L, 1_R)$, also contributes to ϵ'/ϵ but is not chirally enhanced. However, we will see that this operator cannot necessarily be neglected.

Calculation and Results

The calculation is similar to that done to evaluate the short distance contribution to $K_L \to \pi^0\, e^+\, e^-$ (see the report by Jonathan Flynn at this workshop). We employ the effective field theory method to integrate out heavy degrees of freedom consecutively, using the renormalization group to scale between mass thresholds. The objective is to find the coefficients of operators involving only the light quarks at low energy. Our operator basis at low energies consists of the standard operators $O_1 \ldots O_6, O_c^+, O_c^-$, plus the two additional "electromagnetic penguin operators", O_7 and O_8. They are defined as:

$$
\begin{aligned}
O_7 &= 4\,(\overline{s}_{L\alpha}\gamma^\mu d_{L\alpha})[(\overline{u}_{R\beta}\gamma_\mu u_{R\beta}) - \tfrac{1}{2}(\overline{d}_{R\beta}\gamma_\mu d_{R\beta}) \\
&\quad - \tfrac{1}{2}(\overline{s}_{R\beta}\gamma_\mu s_{R\beta})] \qquad (2) \\
O_8 &= 4\,(\overline{s}_{L\alpha}\gamma^\mu d_{L\beta})[(\overline{u}_{R\beta}\gamma_\mu u_{R\alpha}) - \tfrac{1}{2}(\overline{d}_{R\beta}\gamma_\mu d_{R\alpha}) \\
&\quad - \tfrac{1}{2}(\overline{s}_{R\beta}\gamma_\mu s_{R\alpha})], \qquad (3)
\end{aligned}
$$

where α and β are color indices. The coefficients in the second factor are proportional to the quark charges. At high energies, one defines similar operators, but includes all the quarks lighter than the relevant scale. Therefore, our initial matching condition produces an analog of O_7 (with coefficient $\tilde{c}_7'$) which contains two additional terms involving the b and c quarks.

We work to all orders in m_t/m_W, but neglect QCD corrections between the top mass and W mass, which should be small. We simultaneously integrate out the W, Z, and top quark. We evaluate the amplitudes for the electromagnetic penguin diagrams to obtain

$$
\begin{aligned}
\tilde{c}_7'(m_W) &= \frac{\alpha_{em}(m_W)}{6\pi}\,(4C(x) + D(x)) \qquad (4) \\
\tilde{c}_8'(m_W) &= 0, \qquad (5)
\end{aligned}
$$

where the coefficient with a tilde over it is the imaginary part of the coefficient with the KM angles factored out.

$$
\begin{aligned}
C(x) &= \frac{x}{4}\left[\frac{x/2-3}{(x-1)} + \frac{3x/2+1}{(x-1)^2}\ln x\right] \qquad (6) \\
D(x) &= \left[\frac{-19x^3/36 + 25x^2/36}{(x-1)^3}\right. \\
&\quad \left. + \frac{-x^4/6 + 5x^3/3 - 3x^2 + 16x/9 - 4/9}{(x-1)^4}\ln x\right], \qquad (7)
\end{aligned}
$$

where $x = (m_t/m_W)^2$. $C(x)$ comes from the the Z penguin and $D(x)$ from the photon penguin.

Apart from the modified initial conditions for the operators and the absence of a W to t scaling region, the analysis is similar to those performed by other authors ([3,4,5,6]). Below, we plot the results for the coefficients $\tilde{c}_6, \tilde{c}_7/\alpha_{em}$, and $\tilde{c}_8/\alpha_{em}$, where the coefficient $\tilde{c}_6$ is necessary for determining the strength of the dominant strong penguin contribution to ϵ'/ϵ. The coefficients are shown for $\Lambda_{QCD} = 100$ MeV and 300 MeV.

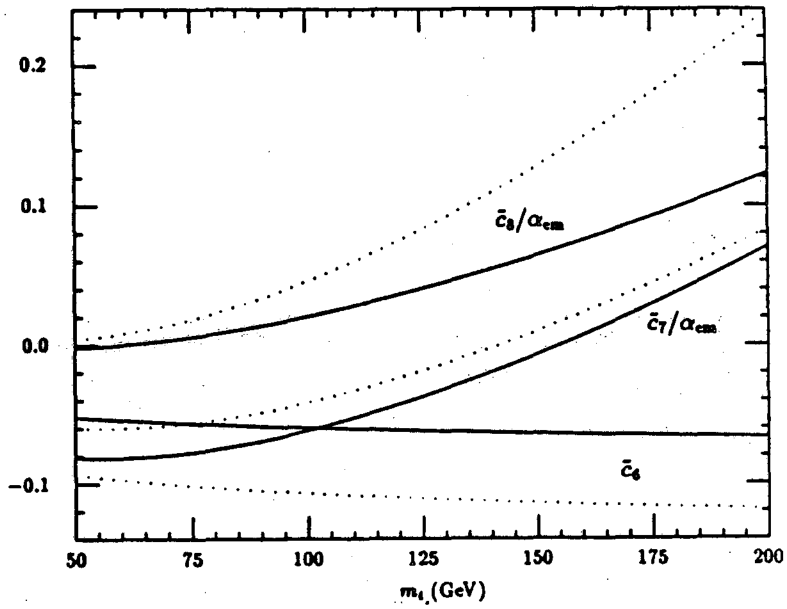

Figure 1 Coefficients $\tilde{c}_6$, $\tilde{c}_7/\alpha_{em}$ and $\tilde{c}_8/\alpha_{em}$ as functions of m_t. Solid curves are for $\Lambda = 100\,\text{MeV}$, dotted curves are for $\Lambda = 300\,\text{MeV}$.

From this figure, we see that $\tilde{c}_7/\alpha_{em}$ rises with increasing top mass, primarily as a result of the large increase in the Z-penguin contribution, and changes sign as the top mass is increased. Moreover, $\tilde{c}_8/\alpha_{em}$ is much bigger at large top quark mass than at small mass, and is almost always positive. The importance of these changes is reflected in the top mass dependence of Ω_{EMP}, defined as the ratio between the electromagnetic and strong penguin contributions to ϵ'/ϵ. It is approximately equal to

$$\Omega_{\rm EMP} \approx .20 \left[\frac{\tilde{c}_7\, B_7 + 3\tilde{c}_8\, B_8}{3\tilde{c}_6\, B_6 \alpha_{\rm em}}\right] . \tag{8}$$

Here the B_i parameterize the difference between the true and vacuum insertion matrix elements. In the next plot, we show for two values of Λ_{QCD} (100 and 300 MeV) the value of $\Omega_{\rm EMP}$ in the vacuum insertion approximation (where the B_i's are taken to be unity).

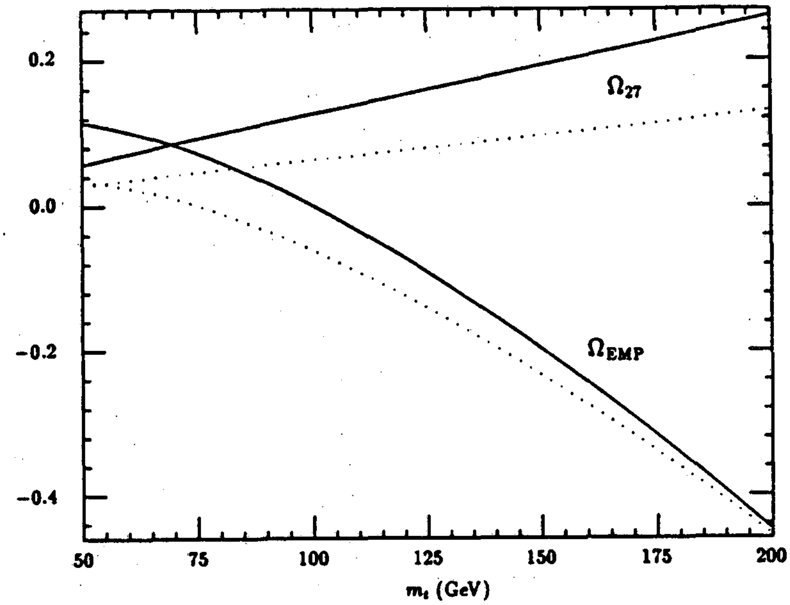

Figure 2 Variation of $\Omega_{\rm EMP}$ and Ω_{27} with m_t, for $\mu = 1\,\mathrm{GeV}$, with B parameters set to 1. Solid curves are for $\Lambda = 100\,\mathrm{MeV}$, dotted curves are for $\Lambda = 300\,\mathrm{MeV}$.

We see that $\Omega_{\rm EMP}$ varies between 0.12 and -0.45 for $\Lambda_{QCD} = 100$ MeV as m_t goes from 50 to 200 GeV (.035 and -.46 for $\Lambda_{QCD} = 300$ MeV), changing sign at a value of m_t of about 100 GeV. Therefore, the electromagnetic penguin contribution could *decrease* the value of ϵ'/ϵ from the strong penguin value. Clearly the contribution of the electromagnetic penguin is very sensitive to the value of m_t.

So far, we have considered only the electromagnetic penguin operator transforming as $(8_L, 8_R)$. We now consider the CP violating coefficient for the operator transforming as $(27_L, 1_R)$, which is:

$$\begin{aligned} O_{27} &= 4\Big[(\overline{s}_{L\alpha}\gamma^\mu d_{L\alpha})\left((\overline{u}_{L\beta}\gamma_\mu u_{L\beta}) - (\overline{d}_{L\beta}\gamma_\mu d_{L\beta})\right) \\ &\quad + (\overline{s}_{L\alpha}\gamma^\mu d_{L\beta})(\overline{u}_{L\beta}\gamma_\mu u_{L\alpha})\Big] \end{aligned} \tag{9}$$

and has coefficient $\tilde{c}_{27} = (\tilde{c}_2 + \tilde{c}_1)/3$.

We include this additional operator because although it is not chirally enhanced, the W coupling is only nonvanishing for the left-handed quarks, and is enhanced by $1/\sin^2\theta_W$. We approximate the coefficient by including only the matching condition at the W scale, which has this enhanced coupling, and neglect scaling contributions. The matching conditions for $\tilde{c}_1$ and $\tilde{c}_2$ are

$$\begin{aligned} \tilde{c}_1'(m_W) &= \frac{\alpha_{\rm em}}{2\pi}\Big(\frac{10}{\sin^2\theta_W}B(x) + \frac{4\sin^2\theta_W - 4}{\sin^2\theta_W}C(x) \\ &\quad + D(x)\Big) \qquad (10) \\ \tilde{c}_2'(m_W) &= 1 \end{aligned}$$

where $B(x)$, the box diagram contribution is defined by:

$$B(x) = \frac{1}{4}\left[\frac{-x}{(x-1)} + \frac{x}{(x-1)^2}\ln x\right] \qquad (12)$$

Defining Ω_{27} as the contribution of O_{27} relative to the strong penguin operator, we obtain

$$\Omega_{27} = 0.034\left[\frac{\tilde{c}_1 + \tilde{c}_2}{3\tilde{c}_6\,\alpha_{\rm em}}\right]\frac{B_{27}}{B_6} \qquad (13)$$

We see from this formula that unless the coefficient $\tilde{c}_{27}$ is bigger than $\tilde{c}_8$ and $\tilde{c}_7$ that we do not expect Ω_{27} to be important. However, as was anticipated from the discussion above, it is larger, and Ω_{27} can be a substantial fraction of $\Omega_{\rm EMP}$ if the top mass is large. This is illustrated in Figure 2, where both $\Omega_{\rm EMP}$ and Ω_{27} are plotted. We see that Ω_{27} lies in the range 0.015–0.19, and increases with increasing top mass. For small top mass it is negligible, but for large top quark mass, although smaller than $\Omega_{\rm EMP}$ in magnitude, it can partially cancel its contribution.

Conclusion:

We have seen that the relative importance of the electromagnetic penguin, and whether it increases ϵ'/ϵ depend sensitively on the mass of the top quark. Also, the operator transforming as $(27_L, 1_R)$ could be important if the top quark is heavy. We conclude that the precise value for ϵ'/ϵ will depend on the size of the mass of the top quark, as well as the value of the matrix elements of the low energy operators.

Acknowledgments

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